Dynamic modeling, stability and energy consumption analysis of a realistic six-legged walking robot

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Abstract

This paper deals with a detailed analysis on kinematics, dynamics, stability and energy consumption of a realistic six-legged robot. The aim of this study is to extend a previous work of Roy et al. [1], in order to estimate optimal feet forces and joint torques of the six-legged robot generating wave-gaits with four different duty factors and deal with its stability issues. Two different approaches are developed to determine optimal feet forces. In the first approach, minimization of the norm of feet forces is carried out using a least square method, whereas minimization of the norm of joint torques is performed in the second approach. The second approach is found to be more energy efficient compared to the first one. The maximum values of feet forces and joint torques are seen to decrease with the increase of duty factor. The effects of walking parameters, namely velocity, stroke and duty factors have been studied on energy consumption and stability of the robot. The variations of average power consumption and specific energy consumption with the velocity and stroke are compared for four different duty factors. Wave gait with a low duty factor is found to be more energy-efficient compared to that with a high duty factor at the highest possible velocity.

Keywords: Six-legged robot Wave gait Dynamics Energy consumption Stability

1. Introduction

A multi-legged robot possesses a tremendous potential for maneuverability over rough terrain, particularly in comparison with conventional wheeled or tracked mobile robot. It introduces more flexibility and terrain adaptability at the cost of low speed and increased control complexity. Moreover, multi-legged locomotion has the disadvantage of achieving poor energy efficiency [2]. Therefore, the minimization of energy consumption plays a key role in the locomotion of an autonomous multi-legged robot used for service applications. The autonomous walking robot cannot function satisfactorily at low energy efficiency due to the fact that it has to carry all driving and control units in addition to the payload and trunk body. It is to be remembered that long duration missions like exploration of planets, underground mining, locating and deactivating of bombs are subject to power supply constraints. Therefore, a reduction in energy consumption by such robots will not only make them travel more, but also help to reduce the size of actuators leading to reduction in their weight and cost. In this connection, authors' previous studies [1,3] are worth mentioning.

In the previous study of Roy et al. [1], only one type of gait pattern, i.e., alternating tripod gait was considered to determine energy efficient feet force and torque distributions. However, the effects of walking parameters on energy consumption and stability were not studied. This paper may be considered as an extension of Roy et al. [1], where a more detailed parametric and systematic study related to energy consumption and stability of a realistic six-legged robot has been conducted.

The paper is organized as follows: a brief review of the previous studies is included in Section 2. Section 3 describes the dynamic model of the robot. Energy consumption models are derived in Section 4. Section 5 presents mathematical formulation of normalized energy stability margin. Simulation results are stated and discussed in Section 6. Some conclusions are drawn in Section 7.

2. Review of previous work

In the last two decades, different approaches had been used by various investigators to achieve energy efficiency in statically balanced multi-legged robots. These approaches include: (a) design of energy efficient mechanical leg structure [4,5]; (b) employment of energy storage devices to recover energy [6,7]; (c) optimal selection of gait parameters [8–10]; (d) optimal solution to foot force distribution [11,12]. Several
algorithms for finding optimal contact forces had been proposed by various researchers, such as linear programming (LP) method [13,14], compact dual LP method [15], quadratic programming (QP) method [16,17] and analytical methods [18–23]. Klein and Kittivatcharapong [24] formulated the objective function as the sum of the squares of the tip point force components of the robot legs and concluded that the LP method leads to discontinuity in the solution and might be difficult to be applied for complex systems in real-time. Compact-Dual Linear Programming method had been used by Cheng and Orin [15] for solving the foot force distribution problem but it could not overcome the discontinuity problem. Nahon and Angeles [16] formulated the force distribution problem in mechanical fingers as a quadratic programming problem. They concluded that the quadratic programming could yield continuous and faster solutions compared to the LP method did. However, in a general multi-legged robot, the number of constraints is larger than that in a finger system. Therefore, it might be difficult to implement this method to multilagged robots in real-time. Chen et al. [25] combined the merits of compact formulation and Quadratic Programming (QP) to develop the compact QP method. The compact QP method was applied to resolve the force distribution problem using various objective functions, such as minimum force, minimum load balance and maximum safety margin on friction constraints.

The pseudo-inverse method had been utilized to obtain the foot force distribution of multi-legged walking robot [26–31]. This method could provide a possible solution to the indeterminate equilibrium equations, i.e., force and moment balance equation. In those studies, either direct pseudo-inverse of the matrix related to the equality constraints [27,32] or pseudo-inverse of the enlarged form of that matrix with linearized active constraints [28] was used to initialize the optimization of the forces, in the space determined by the inequality constraints [33]. In fact, the pseudo-inverse solution of the force equality constraints is indeed a practical and effective solution for foot force distribution in multi-legged walking robots, whether it is used directly or as a starting guess. The reason is that, the resulting forces are minimal; therefore, the effect of inequality constraints related to friction are either eliminated or reduced to a minimum. This is the underlying reason, why the pseudo-inverse solution for force distribution works so effectively in practice. However, as it was demonstrated in [17,30,34] that behind this computational efficiency, there was energy inefficiency due to formulating the problem as minimization of the tip point force components. It did not consider any other practical locomotion performance objectives, such as minimization of joint torques or minimization of energy consumption etc.

Kar et al. [34] used sequential quadratic programming method to determine foot force distribution for minimum energy consumption of a walking robot. Although the above attempt could find the optimal values of feet forces of the multi-legged robot, they might not be suitable for real-time implementations because the used optimization techniques were iterative in nature. An exhaustive search method had also been used to obtain optimal feet forces of six-legged robot [35]. Results of this method were compared with that of the pseudo-inverse method to infer that pseudo-inverse method generates suboptimal solutions. Zhou et al. [36] proposed a new force distribution method called Friction Constraint Method (FriCoM) to evaluate reaction forces at each of the ground legs of a quadruped robot during walking. Results of the FriCoM were compared with those obtained by the pseudo-inverse method and an incremental method [37]. The FriCoM was found to be more practical compared to the pseudo-inverse method and even computationally faster than the incremental method. Thus, this method was seen to be more suitable for real-time control of quadruped than the incremental method. However, the authors did not consider any locomotion performance objectives, as mentioned above.

Marhefka and Orin [17] utilized quadratic programming to solve foot force distribution in hexapod walking robots that minimizes the power consumption in DC motors. In that work, the power consumption was calculated based on a DC motor model in terms of the joint-torques. It was concluded that “minimization of sum of squares of joint-torques” produces good results considering real power dissipation; however “minimization of internal forces” may often produce poor power performance. Moreover, the distribution of forces and moments was formulated as a torque distribution problem with energy efficiency oriented objective function by Erden and Leblebicioglu [38]. The resultant quadratic programming problem was solved by eliminating the equality constraints, converting the problem into a linear complementary problem, and using Lemke’s Complementary Pivoting Algorithm. The simulation results demonstrated that by formulating the objective function as “minimizing the sum of the squares of the joint-torques”, rather than the more common approach of “minimizing the sum of the squares of the tip point force component”, a much more energy efficient distribution was achieved. The results of these papers are important for the justification of formulating the objective as “minimizing the sum of the squares of the joint-torques” for minimum energy consumption. Although the above attempt could find the optimal values of feet forces of the multi-legged robot, they might not be suitable for real-time implementations because the used optimization techniques were iterative in nature.

In connection with energy consumption analysis of multi-legged robot, Lapshin [39] proposed an energy efficient model of a walking machine and some results were obtained from the standpoint of gait-parameters only. Lin and Song [32] introduced a model for dynamic formulation and force distribution, and made an analysis of stability and energy efficiency with respect to the duty factor and velocity for a quadruped walking machine. Kar et al. [34] performed an analysis of energy efficiency with respect to structural parameters, interaction forces, coefficient of friction and duty factor of wave gait, based on a simplified model of six-legged robot. Their structural parameter analysis concluded that there is an optimum lateral offset which makes the ground reaction force vectors pass close to the hip or knee points of the legs throughout the walk. It was stated that power consumption decreases as the number of supporting legs increase. Moreover, power consumption decreases at the instants that supporting legs make a minimum extension. Kar et al. [34] did not make an analysis of the effect of stroke; therefore, it was not possible to observe the interfering effects of duty factor and stroke in their work. Kar et al. [34] and Lin and Song [32] considered instantaneous power to be the product of instantaneous joint torques and joint velocities. Such models ignored the fact that a considerable amount of energy may be dissipated through the motors connected to joints of the legs. In order to eliminate such drawbacks, the integral of the sum of squares of the joint torques could be considered as a criterion of dissipated energy in the actuators. Marhefka and Orin [8] performed an analysis of average power consumption with respect to the parameters of wave gaits, based on a simulation model of a hexapod robot. In their work, gains from power regeneration by the DC motors were not permitted in the optimization problem. Erden and Leblebicioglu [40] performed an energy efficiency analysis of a six-legged robot. Based on the analysis, some strategies were derived to determine the parameters of the wave gaits for minimum energy consumption. Moreover, the authors proposed a phase modified version of wave gaits for more energy efficient walking of the six-legged robot. Nishii [41] used the integral of weighted sum of the product of instantaneous joint torques and joint velocities and the sum of
squares of the joint torques as the energetic cost, which was analyzed with respect to duty factor and velocity in the walking of a two joint six-legged robot. Arikawa and Hirose [42] addressed the relationship between power consumption and foot positions for a quadruped robot. Silva et al. [43–45], Zhoga [46] and Zelinska [47] analyzed energy expenditure and efficiency of multi-legged locomotion systems by taking leg dynamics and torque into account, but they did not consider the type of joint actuator, although its contribution to energy consumption is significant. Recently, Gonzalez de Santos et al. [48] studied the energy required for a six-legged robot using alternating tripod gait and derived a method to minimize the energy consumption of the said robot. More recently, the locomotion of symmetric hexapods, that is, hexagonal six-legged robots had been studied by Wang et al. [49–50]. The proposed gaits for the said robots were compared with those of rectangular robots from the stability, terrain adaptability and walking ability points of view.

The most of the studies on walking dynamics were conducted with a simplified model of legs and body. However, in order to have a better understanding of walking and other important issues of walking, such as dynamic stability, energy efficiency and on-line control, kinematic and dynamic models based on a realistic walking robot design are necessary. In the present study, an attempt has been made to carry out dynamics, stability and energy consumption analysis of a realistic six-legged robot. The effects of walking parameters, namely stroke, velocity and duty factor on energy consumption and stability margin of the said robot will also be studied.

3. Kinematic and dynamic modeling of the robot

To carry out a detailed analysis of six-legged robot, a few important terms [2] related to gait generation of multi-legged robot and their mathematical relationships are to be defined first. These terms are explained below.

- **Gait:** it is defined as the time and location of the placing and lifting of each foot, coordinated with the motion of the body, in order to move the body from one place to another.
- **Transfer or swing phase:** it is that phase of a leg during which the foot is not on the ground.
- **Support phase:** it is the period during which the foot is placed on the ground.
- **Cycle time (T):** it is the time for a complete cycle of leg locomotion of a periodic gait.
- **Duty factor (β):** it is the time fraction of a cycle time (T), during which a particular leg is in support phase.
- **Stride (λ):** it is the distance through which the center of gravity translates during one complete locomotion cycle.
- **Leg stroke (Rs):** it is the distance through which the foot-tip of a leg is translated relative to the trunk body during the support phase. Leg stroke must be within the leg workspace. For a periodic and regular gait, $Rs=βλ$.
- **Wave gait:** it is a periodic, regular and symmetric gait, in which the sequence of placing events of the legs on each side runs from the rear leg and proceeds forward to the front leg. Therefore, the gait creates a wave of stepping from the rear to front.
- **Lateral offset (Yi):** it is the shortest distance between vertical projection of the hip on the ground and the corresponding track.

The relation between the leg transfer time ($t_0$) and support time ($t_s$) is determined by the duty factor: $β = t_0/T$, where $T = t_0 + t_s$. The velocity is given by the relation: $v = Rs/β$. Based on these equations, the stroke is given by $Rs = vt_0/β/(1 − β)$. For a constant stroke length and constant velocity, the leg transfer time decreases with increasing duty factor.

3.1. Structure of the robot

A CAD model of a realistic six-legged robot considered in the present work is shown in Fig. 1. It consists of a rectangular trunk body and six legs, which are similar and symmetrically placed around the trunk body on two sides. Each leg has three degrees of freedom and is composed of three links connected by three active rotary joints.

3.2. Kinematics and foot trajectory planning

A complete kinematics and dynamic model of a realistic six-legged robot is required to analyze complex relationships among gait parameters and determine its energy consumption and stability margin. In mathematical modeling of the six-legged robot, it is assumed that kinematic parameters (like linear and angular positions, velocities and accelerations) of the trunk body of the robot are given. The aim is to derive kinematics equations of all the links on the legs and to determine the required joint velocities and accelerations in order to achieve the given body
motion; and to derive dynamic equations related to all the links and determine the joint torques.

The full kinematics model of a realistic six-legged robot considered in this study is available in Ref. [1]. This section deals with a brief review of the kinematics, gait and trajectory planning of the said robot for completeness. The present analysis is based on the same set of assumptions considered in Ref. [1], as stated below.

(a) The trunk body is maintained at a constant height with respect to terrain surface.
(b) The trunk body moves at a constant velocity \( v_y \).
(c) In order to avoid the impact of foot tip with ground, zero velocities have been considered at the instants of landing and takeoff.

Body reference frame \( \{B\} \) (Fig. 2) is represented with respect to global reference frame \( \{G\} \) and hip reference frame of \( i \)th leg \( \{0\} \) is denoted in body reference frame \( \{B\} \) using transformation matrices as given below.

\[
\begin{bmatrix}
1 & 0 & 0 & v_x(t) \\
0 & 1 & 0 & v_y(t) \\
0 & 0 & 1 & h_B \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & \frac{L_1}{2} & 0 \\
0 & 1 & \frac{h_{10}}{2} & 0 \\
0 & 0 & 1 & h_B \\
0 & 0 & 0 & 1
\end{bmatrix},
\begin{bmatrix}
1 & 0 & 0 & \frac{L_2}{2} \\
0 & 1 & \frac{h_{20}}{2} & 0 \\
0 & 0 & 1 & h_B \\
0 & 0 & 0 & 1
\end{bmatrix},
\begin{bmatrix}
1 & 0 & 0 & \frac{L_3}{2} \\
0 & 1 & \frac{h_{30}}{2} & 0 \\
0 & 0 & 1 & h_B \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Forward and inverse kinematics analysis for each leg has been carried out to develop the overall kinematics model of the six-legged robot. The main structure for each leg is a serial chain with three links, as shown in Fig. 3.

Coordinate systems i.e., frames have been assigned to the joints of a leg using the Denavit–Hartenberg (D–H) notations [51]. The D–H parameters, namely link length \( a_i \), link twist \( z_i \), joint distance \( d_i \), and joint angle \( \theta_i \), required to completely describe the three joint legs are given in Table 1. The foot tip reference frame \( \{3\} \) can be expressed in the leg or hip reference frame \( \{0\} \) as given below.

\[
\begin{bmatrix}
\begin{bmatrix}
C_{01}C_{02} & C_{03} & S_{02}S_{03} \frac{L_1}{2} & S_{02}S_{03}C_{03} \frac{h_{10}}{2} \\
C_{03}S_{01} & -C_{02} & C_{01}C_{03} \frac{L_1}{2} & C_{01}C_{03}S_{03} \frac{h_{10}}{2} \\
-S_{02} & 0 & C_{02}S_{01} \frac{L_1}{2} & C_{02}S_{01}S_{03} \frac{h_{10}}{2} \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
C_{01}C_{02} & C_{03} & S_{02}S_{03} \frac{L_2}{2} & S_{02}S_{03}C_{03} \frac{h_{20}}{2} \\
C_{03}S_{01} & -C_{02} & C_{01}C_{03} \frac{L_2}{2} & C_{01}C_{03}S_{03} \frac{h_{20}}{2} \\
-S_{02} & 0 & C_{02}S_{01} \frac{L_2}{2} & C_{02}S_{01}S_{03} \frac{h_{20}}{2} \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
C_{01}C_{02} & C_{03} & S_{02}S_{03} \frac{L_3}{2} & S_{02}S_{03}C_{03} \frac{h_{30}}{2} \\
C_{03}S_{01} & -C_{02} & C_{01}C_{03} \frac{L_3}{2} & C_{01}C_{03}S_{03} \frac{h_{30}}{2} \\
-S_{02} & 0 & C_{02}S_{01} \frac{L_3}{2} & C_{02}S_{01}S_{03} \frac{h_{30}}{2} \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 0 & 1
\end{bmatrix}
\]

The position of foot tip \((p_x, p_y, p_z)\) can be determined with respect to hip reference frame \( \{0\} \) for the known values of joint angles through forward kinematics calculations [1].

A specific type of periodic gait called wave gait has been considered in the present study. Wave gaits are common in locomotion, the legs operate in two distinct phases: swing or support phase. In swing phase, a leg is lifted off the ground and moved to a new foothold. The foot is on ground and the leg assists in producing the desired body motion in the support phase. To ensure a smooth functioning, each joint trajectory (say \( j \)th of the swing legs is assumed to follow a polynomial of fifth degree in time \( t \)) [1].

The legs in support phase are assumed to be supporting the body without any slippage on their tip points. The foot trajectory in the support phase is simple to compute because the tip points of the support legs are stationary with respect to the global reference frame. This means that the tip point velocity of a support leg with respect to the robot body is equal to the negative of the velocity of the robot body with respect to the global reference frame. The trajectory of the foot relative to the body during leg stroke is a straight line because the motion along straight line only is being implemented here. The velocity and acceleration equations for each leg during the support phase can be expressed as follows:

\[
\dot{\theta}_i = J_i^{-1} \ddot{p}_i
\]

\[
\ddot{\theta}_i = J_i^{-1} \dot{p}_i - J_i \ddot{\theta}_i
\]

where Cartesian velocity vector \( \dot{p}_i = [-v_y, -v_y, 0]^T \), joint velocity vector \( \dot{\theta}_i = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]^T \) and \( J_i \) is the Jacobian matrix of \( i \)th leg. The longitudinal velocity \( v_x \) and lateral velocity \( v_y \) for the straight-forward walking is given as follows: \( v_x = v; v_y = 0 \). It is now necessary to build a dynamic model of the robot to evaluate the foot forces, joint torques and energy consumption, which is discussed below.

### Table 1

D–H parameters for three joint legs.

<table>
<thead>
<tr>
<th>Link no.</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( d_1 )</th>
<th>( \theta_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( L_1 )</td>
<td>90°</td>
<td>0</td>
<td>( \theta_1 )</td>
</tr>
<tr>
<td>2</td>
<td>( L_2 )</td>
<td>0</td>
<td>0</td>
<td>( \theta_2 )</td>
</tr>
<tr>
<td>3</td>
<td>( L_3 )</td>
<td>0</td>
<td>0</td>
<td>( \theta_3 )</td>
</tr>
</tbody>
</table>

![Fig. 3. Frames assigned at different joints of a leg.](image-url)
3.3. Optimum feet forces and torque distribution

A six-legged robot is a complex linkage system, whose legs are connected to one another through the trunk body and also through the ground, and thus, forms closed kinematic chains. The basic problem of controlling these kinematic chains is their coordination. In addition to the local coordination problem, which involves control of the individual joints of a leg to achieve the desired tip control, there is a global coordination problem involving coordination among several chains of the multiple legs. The forces and moments propagate through the kinematic chains from one leg to another, and therefore, dynamic coupling exists. The equations of motion for such a complex mechanism with six legs, each of which has three degrees of freedom, are derived by applying Lagrangian dynamics formulation together with Denavit–Hartenberg’s link coordinate representation, and the derived relationships are given in the vector-matrix form as follows:

\[ \mathbf{\tau}_i = [\mathbf{M}(\mathbf{\theta})][\dot{\mathbf{\theta}}] + [\mathbf{H}(\mathbf{\theta})] + [\mathbf{G}(\mathbf{\theta})] \times [\mathbf{J}_i] \mathbf{F}_i, \]

(4)

where \( \mathbf{M}(\mathbf{\theta}) \) is the mass matrix of the leg, \( \mathbf{H}(\mathbf{\theta}) \) is a vector of centrifugal and Coriolis terms, \( \mathbf{G}(\mathbf{\theta}) \) is a vector of gravity terms, \( \mathbf{J}_i \) is the Jacobian matrix and \( \mathbf{F}_i \) is the vector of ground reaction forces of \( i \)th foot. One of the major problems of inter-chain coordination is the distribution of feet forces among the supporting legs. Similar to the previous study [1], several assumptions are made to simplify the problem, as stated below.

(i) There is no slipping between tip of supporting leg and ground.
(ii) The contact between feet and ground is assumed to be point contact with friction, which indicates that the interaction between the tip of the feet and ground is limited to three components of force: one normal and two tangential to the surface.

With these assumptions, the problem of feet forces’ distributions has been solved using two approaches, namely minimization of norm of feet forces (approach I) and minimization of norm of joint torques (approach II), as in Ref. [1].

**Approach I: minimization of norm of feet forces**

The total load acting on the robot’s body is represented by the wrench \( \mathbf{W} = \begin{bmatrix} \mathbf{W}_F \mathbf{W}_M \end{bmatrix} \). It is to be noted that this wrench contains the forces \( \mathbf{W}_F = [F_x F_y F_z]^T \) and moments \( \mathbf{W}_M = [M_x M_y M_z]^T \) acting at the robot’s center of gravity \((x_c, y_c, z_c)\) and represents weight of the robot including payload, any applied loads and inertial effects of the robot’s body (refer to Fig. 1). However, the inertial effects of the legs on the trunk body have been neglected to simplify the study. At any instant, considering static equilibrium of the robot, the force and moment balance equations can be expressed in vector form as follows:

\[ \Sigma_i F_i + \mathbf{W}_F = 0 \]

(5)

\[ \Sigma_i \mathbf{r}_i \times F_i + \mathbf{W}_M = 0 \]

(6)

where, \( \mathbf{F}_i = [F_x F_y F_z]^T \) is the ground-reaction force vector of foot \( i \), \( \mathbf{r}_i \) is the position vector of ith foot-ground contact point with respect to the body reference frame \( \mathbf{B} \), located at the body’s geometric center. Here \( i = 1, 2, \ldots n; n \) is the number of the ground and support legs at a particular instant.

The Eqs. (5) and (6) can be written in a matrix form as follows:

\[ [\mathbf{A}] [\mathbf{F}] = -[\mathbf{B}] [\mathbf{W}] \]

(7)

where

\[ [\mathbf{A}] = \begin{bmatrix} 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \end{bmatrix}_{6 \times 9} \quad \text{and} \quad [\mathbf{F}] = \begin{bmatrix} F_x \quad F_y \quad F_z \end{bmatrix}^T \text{ for } \beta = 1/2 \]

\[ [\mathbf{A}] = \begin{bmatrix} 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \end{bmatrix}_{6 \times 15} \quad \text{and} \quad [\mathbf{F}] = \begin{bmatrix} F_x \quad F_y \quad F_z \quad F_r \quad F_t \end{bmatrix}^T \text{ for } \beta = 2/3 \]

\[ [\mathbf{A}] = \begin{bmatrix} 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \end{bmatrix}_{6 \times 15} \quad \text{and} \quad [\mathbf{F}] = \begin{bmatrix} F_x \quad F_y \quad F_z \quad F_r \quad F_t \quad F_s \end{bmatrix}^T \text{ for } \beta = 5/6. \]
For $\beta = 3/4$, during the periods: (0–3 T/12) and (6 T/12 to 9 T/12), [A] will have the dimensions of 6 x 15 (same as that with $\beta = 5/6$), whereas during the remaining period, [A] will have the dimensions of 6 x 12 (same as that with $\beta = 2/3$).

$\mathbf{[B]} = \begin{bmatrix} b_1 & b_0 & b_0 & b_0 & b_0 & b_0 & 1_{6 \times 6} \end{bmatrix}$ and $\mathbf{F}_i = [f_{ix} f_{iy} f_{iz}]^T$

$L_i$ is the $(3 \times 3)$ identity matrix, $\mathbf{0}_3$ is the $(3 \times 3)$ null matrix and $\mathbf{C}_i$ is the $(3 \times 3)$ skew-symmetric matrix of vector $[x_i, y_i, z_i]^T$.

This matrix defines the position of tip of a ground foot $i$ ($i = p, q, r$ for $\beta = 1/2$; $i = p, q, r, s$ for $\beta = 2/3$ and $\beta = 3/4$; $i = p, q, r, s, t$ for $\beta = 3/4$ and $\beta = 5/6$) or that of center of gravity ($i = c$) with respect to body reference frame.

The values of $F_x, F_y, M_x, M_y$, and $M_z$ are to be found for various terrains as given below:

$F_s = -m_a x_s, F_p = -m_a x_p, F_r = -m |g| z_s$; and $M_s = M_p = M_r = M_0$;

where $m$ is the total mass of the robot including payload, $a_s$ and $a_p$ are the accelerations of the trunk body along $x$ and $y$ directions, respectively.

With the known feet positions, the feet forces during a whole locomotion cycle can be computed using Eq. (7), which is indeterminate, because it consists of six equations but there are more than six unknowns. The solution of Eq. (7) has been obtained using the least square method [52], which gives the minimum norm solution of the indeterminate equilibrium equations.

**Approach II: minimization of norm of joint torques**

In this approach, the Eq. (7) can be re-formulated using the following relations.

$\mathbf{[F]} = \mathbf{[D]} \mathbf{[t]}$  \hspace{1cm} (8)

where

$\mathbf{[D]} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$ for $\beta = 1/2$

$\mathbf{[D]} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}$ for $\beta = 2/3$

$\mathbf{[D]} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}$ for $\beta = 5/6$.

For $\beta = 3/4$, during the periods: (0–3 T/12) and (6 T/12 to 9 T/12), [D] will have the dimensions of 15 x 15 (same as that with $\beta = 5/6$), whereas during the remaining period, [D] will have the dimensions of 12 x 12 (same as that with $\beta = 2/3$).

$\mathbf{[t]} = \begin{bmatrix} \mathbf{t} \end{bmatrix}$; $\mathbf{t}$ is the $(3 \times 3)$ Jacobian matrix of leg i. Here, $[\mathbf{t}] = [x_t \ y_t \ z_t]$, $[\mathbf{t}] = [x_t \ y_t \ z_t]$ for $\beta = 1/2$; $[\mathbf{t}] = [x_t \ y_t \ z_t]$ for $\beta = 2/3$ and $\beta = 3/4$; $[\mathbf{t}] = [x_t \ y_t \ z_t]$ for $\beta = 3/4$ and $\beta = 5/6$; and $\mathbf{t}_z = [t_{i1} \ t_{i2} \ t_{i3}]$ is the torque vector containing three joint torques at leg $i$.

The Eq. (7) can be re-written as follows:

$\mathbf{[A]} \mathbf{[D]} \mathbf{[t]} = -\mathbf{[B]} \mathbf{[W]}$  \hspace{1cm} (9)

$\mathbf{[A]} \mathbf{[t]} = -\mathbf{[B]} \mathbf{[W]}$  \hspace{1cm} (10)

The minimum norm solution of the above indeterminate equations has been obtained using a least square method [52].

The solution is written in matrix form as follows:

$[\mathbf{t}] = \mathbf{[A]}^{-1}[[\mathbf{A}][\mathbf{B}]^{-1}]^{-1}(-[\mathbf{B}][\mathbf{W}])$  \hspace{1cm} (11)

A systematic derivation of the Lagrange equations yields the torque expressions for $\mathbf{t}$ as follows:

$\mathbf{[t]} = \begin{bmatrix} \mathbf{M}_1 & -\mathbf{H}_1 & \mathbf{C}_1 \end{bmatrix} \begin{bmatrix} \mathbf{J}_1 \mathbf{f}_x \mathbf{J}_1 \mathbf{f}_y \end{bmatrix}$

where the terms: $\mathbf{M}_k, \mathbf{H}_k, \mathbf{C}_k$ can be obtained as follows:

$\mathbf{M}_k = S_j = \max_{i, m} T_{ij} \mathbf{U}_{ij} \mathbf{J}_j^m, \quad i, k, m = 1, 2, 3.$

$\mathbf{H}_k = S_j = \max_{i, m} T_{ij} \mathbf{U}_{ij} \mathbf{J}_j^m, \quad i, k, m = 1, 2, 3.$

$\mathbf{C}_k = S_j = \max_{i, m} T_{ij} \mathbf{U}_{ij} \mathbf{J}_j^m, \quad i, k, m = 1, 2, 3.$

Here, $\mathbf{g} = [g_x \ g_y \ g_z]$ is the acceleration due to gravity with respect to the body reference frame, $\mathbf{J}_j$ represents the inertia tensor. Now, $\mathbf{U}_{ij}$ and $\mathbf{U}_{ik}$ can be obtained as follows:

$\mathbf{U}_{ij} = \mathbf{A} \mathbf{U}_{ij}$

$\mathbf{U}_{ik} = \mathbf{A} \mathbf{U}_{ik}$

A more detailed expression of each term is given in Ref. [1].

**4. Energy consumption model**

The energy consumption in a legged robot is mainly due to that consumed by an actuator attached at each joint of the legs. As a joint is driven by DC geared-motor, its consumed energy during a time ($T$) is given by

$E = \int_0^T u_i dtdt,$  \hspace{1cm} (13)

where $u_i$ is the applied voltage and $i_k$ represents the armature current.

The behavior of DC motors can be explained by its torque and voltage equations [53], as mentioned below.

$\tau_m = K \omega_i$  \hspace{1cm} (14)

$\omega_e = K_i \theta_m$  \hspace{1cm} (15)

$u_i = u_e + R_i \omega_e + L_i \frac{d\omega_e}{dt},$  \hspace{1cm} (16)

where $\tau_m$ is the torque (N m) at the rotor, $K_i$ is the torque constant, $K_i$ is the voltage constant, $\theta_m$ is angular velocity of the rotor, $R_i$ is the armature resistance, $L_i$ is armature inductance, $u_i$ is the induced voltage in the armature windings opposing the applied voltage.

The output angular velocity ($\theta$) and torque ($\tau$) of the geared-motor are computed as follows:

$\omega_e = \frac{\theta}{K_i},$  \hspace{1cm} (17)
\[ \tau_m = 

Margin (NESM) as

$$S_N = \frac{S_{ESM}}{mg} = \min \left\{ h_i \right\}$$  \hspace{1cm} (27)

A stability analysis based on NESM has been performed in this study.

6. Results and discussion

In this section, a case study related to straight-forward motion of a six-legged robot over flat terrain has been presented to test the feasibility of the above mathematical model for solving the problem. In computer simulations, physical parameters of the six-legged robot have been kept similar to those used in Ref. [1]. The values of mass, moment of inertia and positions of center of gravity of the said realistic robot have been computed using CATIA CAD/CAE software [62]. Fig. 5 illustrates a flowchart describing overall procedure adopted in computer simulations for the said robot.

The information related to walking parameters, like height of the body, velocity, leg stroke and duty factor, are fed as inputs for testing the performance of developed approaches through computer simulations. The outputs of interest are the feet forces’ distributions, joint torques, average power consumption, specific energy consumption and normalized energy stability margin.

In this simulation, body height, total mass of the robot, velocity of trunk body, and leg stroke are assumed to be equal to 0.13 m, 3.5 kg, 0.03 m/s, and 0.10 m, respectively. It is important to mention that the above values of input parameters have been decided through a careful study on the leg’s workspace and dynamic limits of the joint actuators. The coefficient of friction between the concrete terrain and the rubberized leg tips is taken to be equal to 0.8. Figs. 6–9 display the distributions of feet reaction forces obtained by approaches I and II over two locomotion cycles for four different duty factors ($\beta$).

It is seen from Fig. 6 that the front and rear legs complement each other in terms of support force and the middle legs are subjected to the maximum amount of force. However, the sum of vertical components of feet forces of all the ground legs at any
given instant of time obtained by any approach is equal to the weight of the robot. In this case, the front and rear legs of one side and middle leg of the other side of the longitudinally symmetric six-legged robot are always in support. Moment balance about the longitudinal axis, as required to prevent the robot from rolling sideways, reveals that the middle leg must exert more force compared to the front and rear legs do. As the duty factor is increased, the support force contribution of each leg is reduced (refer to Figs. 7–9). Moreover, the maximum values of feet forces are seen to decrease, as duty factor increases. It happens due to the fact that the force required to support the body is distributed over more number of legs, as the number of support legs increases with duty factor.

It is observed from the results of numerical simulations of approach 1 that the feet forces with either zero or almost zero horizontal components occur during the support phase of the leg and the robot does not make a good use of the friction. However, in approach 2, the horizontal components of the feet forces are found to be significant. These results are quite similar to that reported by Erden and Leblebicioglu [38].

Fig. 8. Distributions of feet forces over two locomotion cycles obtained by Approaches I and II for duty factor $\beta = 3/4$.

Fig. 9. Distributions of feet forces over two locomotion cycles obtained by Approaches I and II for duty factor $\beta = 5/6$.

Fig. 10 represents the variations of torques in each joint of the legs during two locomotion cycles as obtained by approaches I and II, for duty factor $\beta = 1/2$. It is interesting to note that for a particular ground leg, the maximum torque generated at joint 2 has turned out to be more compared to that at other two joints. Moreover, joint torques of the legs during the support phase have become much more than those during the swing phase, as expected. It is interesting to note that the maximum torque required at joint 2 of middle leg is found to be much higher than that at other joints (namely joints 1 and 3). In approach II, the variations of torque requirement at different joints of the middle and other legs are seen to be relatively less than that of approach I.

Similarly, Figs. 11–13 show the variations of joint torques at each joint of the legs obtained by both the approaches, over two locomotion cycles for the duty factors of $2/3$, $3/4$ and $5/6$. It can be observed that the variations of torque at different joints of a leg, obtained by approach II are relatively less than that of approach I.

It is also observed from the above figures that the maximum values of torque at different joints decrease, when the duty factor increases. This has happened due to the fact that the feet forces
are distributed evenly, as the duty factor increases, as explained earlier.

Table 2 shows the average values of the squares of joint torques of the robot for straight-forward motion with wave gait for four different duty factors, as obtained by approaches I and II. The average value of the squares of joint torques of the robot as obtained by approach I is seen to be higher than that yielded by approach II for all the duty factors. For example, the average of the squares of joint torques of the robot as obtained by approach I is seen to be approximately 2.77 times of that yielded by approach II in case of duty factor \( \beta = \frac{1}{2} \). Since the average of the squares of joint torques is considered to be proportional to average dissipated energy (average heat loss) of the joint motor, the decrease in the average of the squares of joint torques corresponds to proportional amount of decrease in the dissipated energy of the motors. Therefore, it can be concluded that approach II is more energy efficient than approach I for all duty factors. It is also observed that the average value of the squares of joint torques during one complete locomotion cycle decreases with the increase of duty factor for both the approaches. This is so, because more number of legs will be in support phase when the duty factor is increased and as a result, the forces required to support the body are distributed more evenly among the legs, when the duty factor increases and thereby, the contribution (in terms of torque and energy) of each support leg is reduced. Since the average of the squares of joint torques is considered to be proportional to average dissipated energy of the joint motor, it may be concluded that energy dissipation becomes less in wave gait with the higher duty factor in comparison with that involving the lower duty factors.

6.1. Effect of velocity and duty factor on energy consumption

Wave gaits are widely used in six-legged robots due to their high stability. The choice of a particular wave gait for a legged robot depends on the desired characteristics of the robot. For example, the choice of a particular wave gait for a legged robot depends on the desired characteristics of the robot. For example, Table 2 shows the average values of the squares of joint torques of the robot for straight-forward motion with wave gait for four different duty factors, as obtained by approaches I and II. The average value of the squares of joint torques of the robot as obtained by approach I is seen to be higher than that yielded by approach II for all the duty factors. For example, the average of the squares of joint torques of the robot as obtained by approach I is seen to be approximately 2.77 times of that yielded by approach II in case of duty factor \( \beta = \frac{1}{2} \). Since the average of the squares of joint torques is considered to be proportional to average dissipated energy (average heat loss) of the joint motor, the decrease in the average of the squares of joint torques corresponds to proportional amount of decrease in the dissipated energy of the motors. Therefore, it can be concluded that approach II is more energy efficient than approach I for all duty factors.
robot corresponds to determining the gait parameters (i.e., duty factor, stroke, gait velocity), in accordance with the mechanical and electrical design limitations (i.e., maximum stroke length, maximum acceleration or minimum swing time etc.). In this study, minimization of energy consumption is taken to be the criterion to determine optimal walking parameters.

The effects of velocity on average power consumption and specific energy consumption for different duty factor are studied here. In Table 3, the values of average power consumption in one locomotion cycle are given with respect to the duty factor and velocity. For a particular value of duty factor, average power consumption is found to increase with the increase in velocity, as expected. It is observed that approach II is able to provide more energy efficient gait compared to approach I for all duty factors. It is to be noticed that the velocity should be as low as possible to minimize power consumption for a particular duty factor. However, traveling with a low velocity takes more time to cover a fixed distance, and consequently, total energy consumption may be increased. The energy required to travel a fixed distance can be quantified using a parameter called specific energy consumption, that is, the energy consumed per weight per traveled distance. Table 4 displays the effects of variation of velocity on specific energy consumption for four different duty factors. Approach II has provided an energy saving of 2%–44% in comparison with approach I. The data shown in Tables 3 and 4 related to approaches I and II have been plotted in Figs. 14 and 15, respectively. Specific energy consumption is found to decrease with the increase of velocity for a particular value of duty factor. However, average power consumption is seen to increase with the increase in velocity. The results obtained here are seen to be similar to that reported by Marhefka and Orin [8]. It is also important to note that for a high value of duty factor, the robot may not be able to move with high velocity due to dynamic constraints of the joint motors because of the requirement of the faster leg transfers. The blank (-)
6.2. Effect of stroke and duty factor on energy consumption

Results related to the effects of stroke on average power consumption during straight-forward motion of the robot with wave gaits of different duty factors are presented in Table 5. It is to be noted that the range of stroke values has been made fixed after a careful study of leg’s workspace and body height. Approach II has provided an energy saving of 14%–40% in comparison with approach I. The variations of average power consumption and specific energy consumption with stroke as obtained by approaches I and II for four duty factors have been plotted in Figs. 16 and 17, respectively. For a given velocity, both average power consumption and specific energy consumption are found to increase with stroke for all duty factors. This happens due to the fact that for a larger stroke the feet of ground legs are to be placed at farther points with respect to the body compared to those corresponding to the smaller stroke. In order to place the feet appropriately, on an average the joints of the robot’s leg will have larger displacement, angular velocity and accelerations, which results into the higher numerical values of joint torques and thus, the energy consumption increases.
Moreover, for a particular stroke, average power consumption and specific energy consumption are seen to decrease with the increase of duty factor for both the approaches I and II. It is interesting to observe that approach II has provided more energy efficient solutions compared to approach I for all the strokes. It is also to be noted that a few boxes of Table 5 have been kept blank (-), as no feasible solutions are obtained for some combinations of the stroke and duty factor due to dynamic constraints of the robot’s joint motors.

6.3. Effect of stroke, duty factor and angle of slope on NESM

Table 6 shows the variations of minimum values of NESM (that is, \( S_N \)) with stroke and angle of slope in one locomotion cycle obtained by the robot during straight motion with four different values of duty factors.

For a particular stroke, \( S_N \) is seen to increase with the increase in duty factor. It happens due to the fact that the number of ground leg increases, as duty factor increases and thereby,
Table 3
Variations of average power consumption with velocity for four duty factors.

<table>
<thead>
<tr>
<th>Velocity (m/s)</th>
<th>$\beta = 1/2$</th>
<th>$\beta = 2/3$</th>
<th>$\beta = 3/4$</th>
<th>$\beta = 5/6$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Approach I</td>
<td>Approach II</td>
<td>Approach I</td>
<td>Approach II</td>
</tr>
<tr>
<td>0.020</td>
<td>0.2694</td>
<td>0.1417</td>
<td>0.1894</td>
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<tr>
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<tr>
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<td>0.1646</td>
<td>0.1920</td>
<td>0.1500</td>
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<td>0.2763</td>
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<tr>
<td>0.060</td>
<td>0.2775</td>
<td>0.2345</td>
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<td>–</td>
</tr>
</tbody>
</table>

Stroke $= 0.1$ m, height of trunk body $= 0.13$ m, lateral offset $Y_i = 0.19$ m

Table 4
Variations of specific energy consumption with velocity for four duty factors.

<table>
<thead>
<tr>
<th>Velocity (m/s)</th>
<th>Specific energy consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta = 1/2$</td>
</tr>
<tr>
<td></td>
<td>Approach I</td>
</tr>
<tr>
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</table>

Stroke $= 0.1$ m, height of trunk body $= 0.13$ m, lateral offset $Y_i = 0.19$ m

Fig. 14. Variations of (a) average power consumption and (b) specific energy consumption with velocity for four duty factors using Approach I.

Fig. 15. Variations of (a) average power consumption and (b) specific energy consumption with velocity for four duty factors using Approach II.
stability margin improves. It is also to be observed that \( SN \) decreases with stroke particularly for low values of duty factor (that is, \( \beta = 1/2 \) and \( \beta = 2/3 \)). For higher values of duty factor (say \( 2/3 \) and \( 3/4 \)), \( SN \) does not vary much with the stroke, due to a large number of ground legs. It is important to note that the value of \( SN \) decreases with the increase of angle of slope of the terrain, as expected.

### 7. Conclusions

Besides kinematic analysis, two approaches, such as minimization of norm of feet forces (approach I) and minimization of norm of joint torques (approach II) have been developed to estimate optimal distributions of feet forces during straight-forward motion of a six-legged robot with four different duty factors. An energy consumption model has been derived for the statically stable robot after considering both mechanical energy and energy loss due to heat emission by DC motors. The effects of walking parameters have been studied on energy consumption and stability margin of the said robot. It has been observed that approach II provides more energy efficient distributions of feet forces compared to approach I. Moreover, approach II has yielded less variation in joint torques of the six-legged robot in comparison with that obtained by approach I. It is also important to notice that the joint torques of the supporting legs have turned out to be much more compared to that of the swing legs. The maximum values of feet forces and joint torques are found to

<table>
<thead>
<tr>
<th>Stroke (m)</th>
<th>Average power consumption (W)</th>
<th>( \beta = 1/2 )</th>
<th>( \beta = 2/3 )</th>
<th>( \beta = 3/4 )</th>
<th>( \beta = 5/6 )</th>
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<tr>
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<td>0.1777</td>
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<tr>
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<td>0.1954</td>
<td>0.2011</td>
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<td>0.1812</td>
</tr>
</tbody>
</table>

Velocity = 0.03 m/s, height of trunk body = 0.13 m, lateral offset \( Y_i = 0.19 \) m

Fig. 16. Variations of (a) average power consumption and (b) specific energy consumption with stroke for four duty factors using Approach I.

Fig. 17. Variations of (a) average power consumption and (b) specific energy consumption with stroke for four duty factors using Approach II.
decrease with the increase of duty factor. The variations of average power consumption and specific energy consumption with velocity and stroke have been studied for four different duty factors. It has been observed that at a particular velocity, average power consumption and specific energy consumption increase with the stroke for all duty factors. It is also interesting to note that the velocity should be as high as possible for a particular duty factor in order to minimize total energy consumption. However, as velocity increases, the maximum reachable duty factor becomes restricted due to the dynamic constraints of joint actuators. It is important to notice that NEMS increases with the increase in duty factor. It happens due to the fact that the number of ground leg increases with the increase in duty factor and thus, stability margin improves. The developed kinematics and dynamic models have been examined for straight gait generation.

Table 6

<table>
<thead>
<tr>
<th>Angle of slope</th>
<th>Stroke (m)</th>
<th>Minimum value of NEMS ($S_m$) (m)</th>
</tr>
</thead>
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<tr>
<td>0°</td>
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Acknowledgments

The first author, Shibendu Shekhar Roy, acknowledges all helps from Mr. Ajay Kr. Singh, Department of Mechanical Engineering, NIT, Durgapur, India.

References
