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Comparison of DC Offset Effects on LMS Algorithm and its Derivatives

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Abstract— It is well known that DC offset degrades the performance of analog adaptive filters. The effects of DC offset on LMS derivatives such as sign-data LMS, sign-error LMS and sign-sign LMS have been studied to much extent but that on MLMS, VSSLMS and NLMS algorithms have remained relatively ignored. The present paper reports the effects of dc offset on LMS algorithm and its four variations Sign LMS, Momentum LMS, variable stepsize LMS and normalized LMS algorithm which are analyzed with the help of computer simulation. It was found that the performance of the LMS, MLMS and VSSLMS algorithm is adversely affected by presence of DC offset where as the SLMS and NLMS algorithms perform better in the presence of dc offset.

***Index Terms*— Adaptive equalization; DC offset, LMS algorithm, LMS derivatives, convergence, simulation**

I. INTRODUCTION

Adaptive equalization has a number of applications in system identification, channel equalization, signal enhancement and signal prediction. Adaptive algorithms are used to adjust the weights of adaptive equalizers towards an optimum configuration. In recent years a great deal of research has been directed towards the development of an algorithm which will converge more shortly. Recursive Least Square (RLS), Kalman, Least Square Lattice (LSL), Feast A Error Sequential technique (FAEST) and Least Mean Square (LMS) algorithms are examples of algorithms used for adaptive equalization. The simplest and most extensively employed adaptive algorithm is Least Mean Square (LMS) stochastic gradient algorithm [1-3]. It utilizes a gradient search technique to determine the filter coefficients that minimizes the mean square gradient error [4]. The LMS algorithm requires 2N operations per data (N for complex data)[5] and no explicit determination of the coefficients of input data[6]. This algorithm offers significant advantages over Weiner filters. Computational reduction can be realized by the use of the frequency domain adaptive algorithms [7-11]. To improve the performance of the LMS algorithm various modifications were made to this standard algorithm such as sign LMS (SLMS)[12-14], Dual sign LMS (DLMS)[15], momentum LMS(MLMS)[16-17], variable stepsize LMS (VSSLMS) [18-19], Normalized LMS(NLMS) [20,21], order static algorithm[22-23] etc. The performance of these algorithms degrades due to the presence of DC offset. From the literature survey it is seen that the DC offset effect is studied in a few derivatives of LMS algorithm namely, sign-data LMS, sign-error LMS and sign-sign LMS. It also shows that sign-error LMS and sign-sign LMS algorithms achieve better MSE performance in the presence of DC offset[19]. Then it is important to study the DC offset effects on the performance of the other algorithms. During the present research work attempts were made to study the effect of DC offset on the performance of the LMS algorithm and its variations such as SLMS, MLMS, VSSLMS and NLMS algorithm.

Before discussing the simulation results, mathematical derivations used for the simulation are presented. This is followed by results of simulation study. The DC offset effect on these algorithms is studied with the help of computer simulation for change of input signal level and channel characteristics keeping the other parameters constant. The results obtained from simulation study are used to discus and compare the performance of these algorithms in presence of DC offset.

II MATHEMATICAL REPRSENTATIONS OF ALGORITMMS

A. LMS algorithm

The LMS algorithm which is used to update the filter coefficients is given by

$$W(n+1) = W(n) + \mu\, u(n)\, e(n) \qquad (1)$$

where $W(n) = [W_o(n)\ W_1(n)\ \ldots\ W_{N-1}(n)]^T$ is the vector of N coefficient values at time, n, u(n) is input factor to the filter, μ is the time invariant convergence parameter and $e(n) = d(n) - W^T(n)\, u(n)$ where e(n) is error, d(n) is the scalar desired response. Then the LMS algorithm which is used to update filter coefficients with DC offset is given by

$$W(n+1) = W(n) + \mu\, [(u(n)+ m_x)\, (e(n)+m_e) + m] \qquad (2)$$

where $m_x = [m_{x1}, m_{x2}, \ldots.. m_{xN}]^T$ is the vector representing the unwanted DC offset on each of the gradient signals, m_e represents the unwanted DC offset on the error signal and m is the vector representing unwanted equivalent DC offsets at the input of the accumulator and at the multiplier.

B. SLMS algorithm:

To reduce the complexity of the LMS algorithm, sign algorithm is proposed in which only the polarity information of the error signal or data signal or both data and error is used for the filter coefficients update. During this study, sign-error LMS algorithm is considered and in this the polarity information of the error is used for the filter coefficients update.

The adaptation equation of the sign error LMS algorithm is given by

$$W(n+1) = W(n) + \mu\, u(n)\, [\mathrm{sgn}\, e(n)] \quad (3)$$

where

$$\mathrm{sgn}\, e(n) = \begin{cases} +1 & \text{if } d(n) - y(n) > 0 \\ 0 & \text{if } d(n) - y(n) = 0 \\ -1 & \text{if } d(n) - y(n) < 0 \end{cases}$$

Then the adaptation equation for the SLMS algorithm with DC offset is given by

$$W(n+1) = W(n) + \mu\, [(u(n)+m_x)(\mathrm{sgn}\, e(n) + m_e) + m] \quad (4)$$

C. MLMS algorithm:

The MLMS algorithm corresponds to a second order adaptive algorithm in which two previous weight vectors are combined at each iteration to updated weight. The second order weight update expression for the MLMS algorithm is

$$W(n+1) = W(n) + 2\,\mu e(n) u(n) + \alpha\, [W(n)-W(n-1)] \quad (5)$$

where α scales the momentum term added to the gradient descent. The second order weight update expression for the momentum LMS algorithm with DC offset is represented by

$$W(n+1)=W(n) + 2\mu[(u(n)+m_x)(e(n)+m_e) + m] + \alpha[W(n)-W(n-1)] \quad (6)$$

D. VSSLMS algorithm:

In LMS, SLMS and MLMS the stepsize considered is fixed or constant. The choice of the stepsize reflects the tradeoff between the misdjustment but gives the longer convergence time constant. In VSSLMS the stepsize is kept varying, therefore its value is determined by the number of sign changes to an error surface gradient estimate. The adaptation equation for the VSSLMS algorithm is given by

$$W(n+1) = W(n) + \alpha\, \mu(n) + \nu e(n)^2 u(n)\, e(n) \quad (7)$$

Then the equation for the VSSLMS algorithm with DC offset becomes

$$W(n+1) = W(n) + (\alpha\mu(n) + \nu\, e(n)^2)\, ([u(n) + m_x] + [e(n) + m_e] + m) \quad (8)$$

E. NLMS algorithm

In NLMS algorithm, the decreasing stepsize proves almost sure convergence of the homogenous algorithm assuming fixed condition and is not much affected by the change in input signal level and channel characteristics. The adaptation equation for NLMS algorithm is given by

$$W(n+1) = W(n) + \alpha\, u(n)\, e(n)\, [u^T(n)\, u(n) + \beta]^{-1} \quad (9)$$

where α is the new normalized adaptation constant which is generally less than or equal to one, β is a small positive term included in the denominator by which the update term does not becomes excessively large when $u^T(n)u(n)$ temporarily becomes small. The inclusion of $u^T(n)\, u(n)$ in the denominator increases another N multiplications and additions which can be avoided if N has extra large storage locations available. The adaptation equation for NLMS algorithm with DC offset is given by

$$W(n+1) = W(n) + \alpha([e(n) + m_e] + [u(n)+m_x] + m) + [\beta + u^T(n)\, u(n)]^{-1} \quad (10)$$

III. SIMULATION SOFTWARE DEVELOPMENT

During this simulation study software was developed in higher language C++. The inputs are all real values which are in polar form and the channel characteristics are assumed arbitrarily. Channel characteristics 1 is considered for moderate and phase distorted channel, channel characteristics 2 is considered for heavy amplitude and no phase distorted channel where as channel characteristics 3 is considered for heavy amplitude and phase distorted channel.

IV. SIMULATION RESULTS

A. LMS algorithm:

It is seen that as the input signal level increases, the mean square error decreases and rate of convergence of equalizer to converge to its steady state increases, as shown in Fig. 1.a.

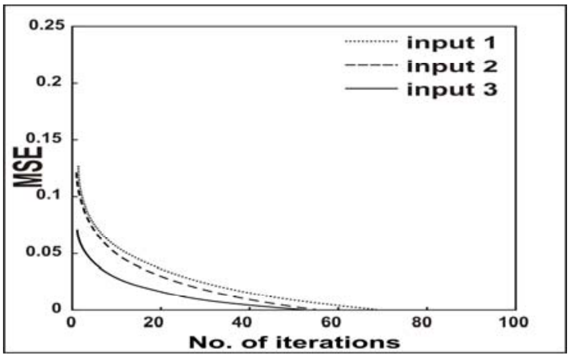

Fig.1.a. Effect of input signal level on LMS algorithm

For change in channel characteristics, it is seen that for heavy amplitude and no phase distorted channel and heavy amplitude and phase distorted channel, the convergence decreases and steady-state value of the mean squared error(and hence the misadjustment) increases with increased eignevalue spread as shown in Fig. 1.b.

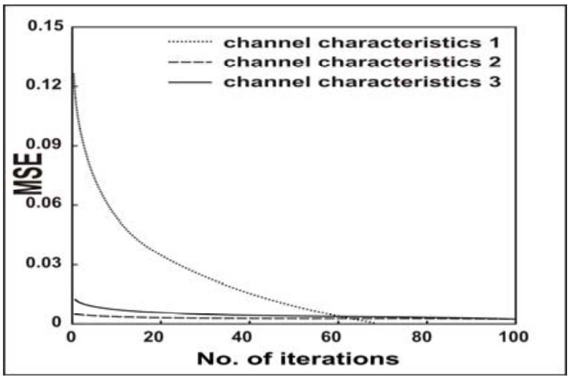

Fig. 1.b. Effect of channel characteristics on LMS algorithm

The DC offset effect on LMS algorithm is studied for change in input signal level and channel characteristics. From the simulation study it is seen that for change in input signal level the rate of convergence of LMS algorithm decreases. Lower input signal powers for fixed offset level produces higher excess MSE as shown in Fig. 1.c. The results obtained are in well agreement with results reported by others [24].

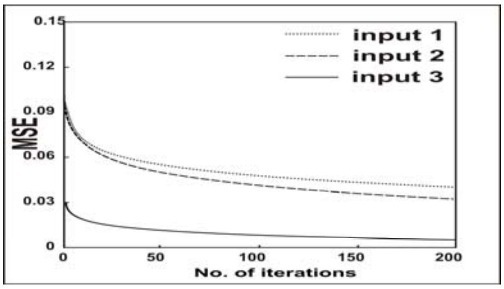

Fig. 1.c. Effect of input signal level with DC offset

For channels having heavy amplitude and no phase distortion and heavy amplitude and phase distortion, the equalizer diverges showing increase in its steady-state value of the error. This is due to the presence of the DC offset (Fig. 1.d.)

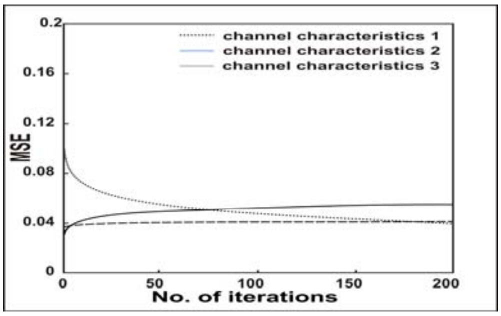

Fig. 1.d. Effect of channel characteristics with DC offset

SLMS Algorithm

For change in input signal level it is seen that as input signal increases the rate of mean squared error decreases and rate of convergence of the equalizer to converge to its steady-state increases.(Fig. 2. a).

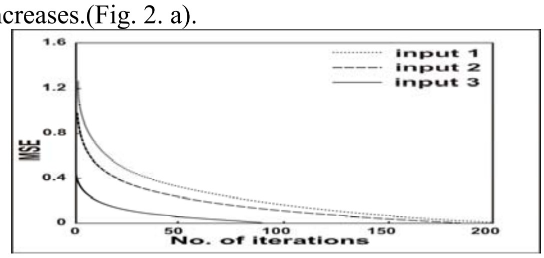

Fig. 2.a. Effect of input signal level on SLMS algorithm

For the change in channel characteristics, it is seen that when the eignevalues of the correlation matrix are widely spread, the rate of convergence slows down, because the correlation matrix is ill conditioned for heavy amplitude and phase distorted channel. The equalizer requires large number of iterations to converge as shown in Fig. 2.b.

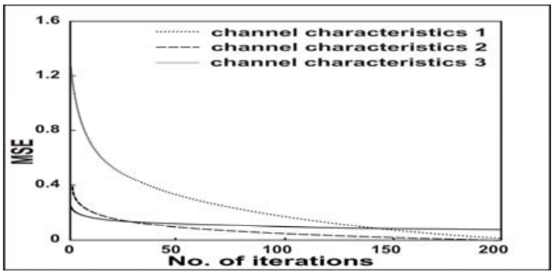

Fig. 2.b. Effect of channel characteristics on SLMS algorithm

In case of change in input signal level in the presence of DC offset, it is seen that sign-error LMS algorithm achieves better performance as compared to LMS algorithm and the results are in well agreement with results reported earlier [24]. (Fig. 2. c.)

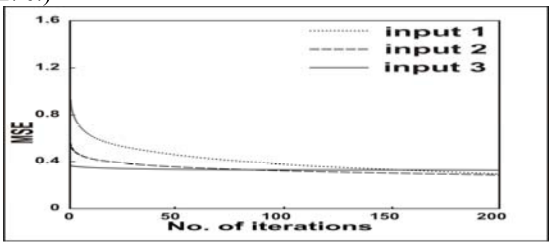

Fig. 2.c. Effect of input signal level with DC offset

For change in channel characteristics with DC offset it is seen that for heavy amplitude and no phase distorted channel and heavy amplitude and phase distorted channel, the equalizer converges with decrease in MSE as well as with decreased steady state value of error as shown in Fig. 2.d. This is due to the dc offset [24].

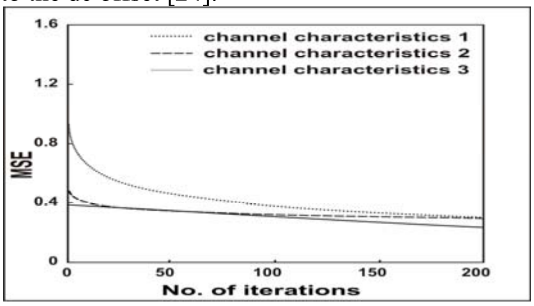

Fig. 2.d. Effect of channel characteristics with DC offset

MLMS algorithm

For change in input signal level there is not much effect on the convergence properties of the equalizer which may be due to the momentum term added in the adaptation equation which serves as an estimate of the previous gradient. The increase in misadjustment is proportional to the fractional amount of momentum term added to the standard LMS algorithm as shown in Fig. 3.a.

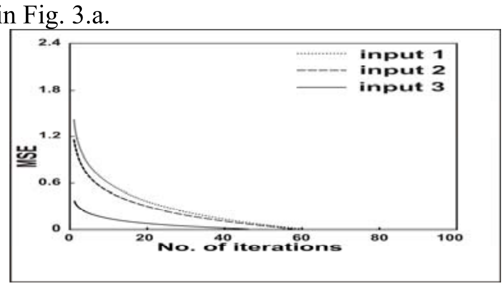

Fig. 3.a. Effect of input signal level on MLMS algorithm

For change in channel characteristics, it is seen that for heavy amplitude and phase distorted channel, the speed of convergence slows down. The increase in MSE is large as compared to LMS and SLMS algorithm.

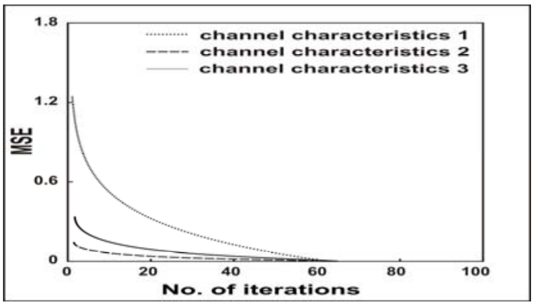

Fig. 3. b. Effect of channel characteristics on MLMS

In the presence of DC offset, for change in input signal level, it is seen that the performance of MLMS algorithm degrades. The increase in the steady-state value of the error is due to momentum term and the presence of DC offset [24]. (Fig. 3. c.)

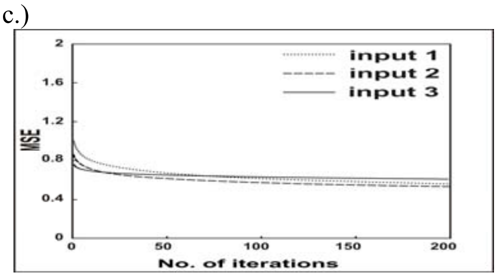

Fig. 3.c. Effect of input signal level with DC offset

With DC offset, for heavy amplitude and no phase distorted and heavy amplitude and phase distorted channel, the equalizer diverges with decrease in MSE and increase in steady-state value of error. For heavy amplitude and phase distorted channel, the equalizer diverges showing increase in steady-state value of error [24] as shown in Fig. 3.d.

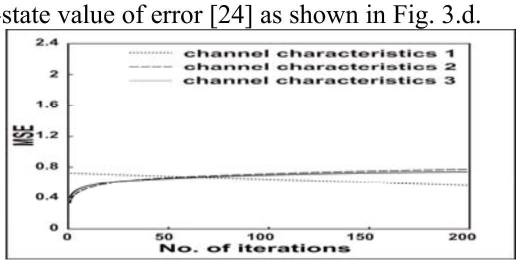

Fig. 3.d. Effect of channel characteristics with DC offset

VSSLMS algorithm

It is seen that as input signal level increases the rate of mean squared error decreases and rate of convergence of the equalizer to converge to its steady-state increases due to variable stepsize as shown in Fig. 4. a.

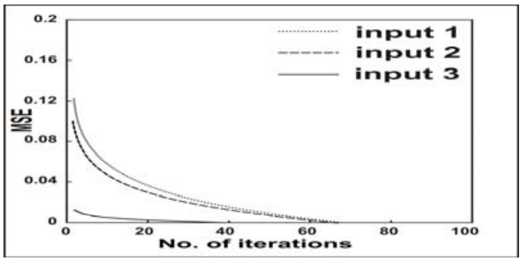

Fig. 4. a. Effect of input signal on VSSLMS algorithm

For change in channel characteristics it is seen that when the eignevalues of the correlation matrix are widely spread, the rate of convergence slows down, because the correlation matrix is ill conditioned for heavy amplitude and phase distorted channel. The equalizer requires large number of iterations to converge (Fig. 4.b.).

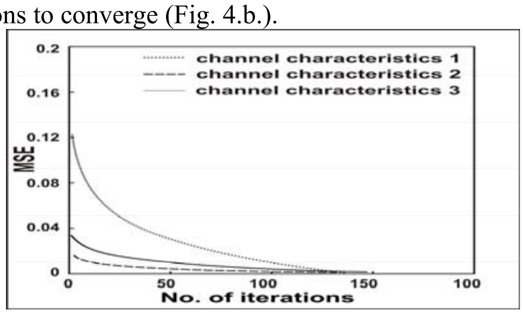

Fig. 4.b. Effect of channel characteristics on VSSLMS

With presence of DC offset, for change in input signal level the equalizer converges with decreased speed and increased MSE. The number of iterations required is many times greater than the number of iterations required for convergence without DC offset. The results obtained are in well agreement with the results reported by others [24] (Fig. 4.c.)

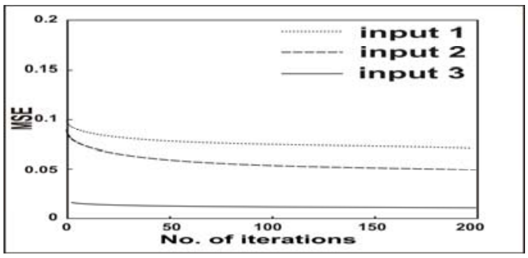

Fig. 4. c. Effect of input signal level with DC offset

For change in Channel characteristics with DC offset, the equalizer converges with decreasing steady-state value of error for heavy amplitude and phase distorted channel. For other channel characteristics, the equalizer diverges (Fig. 4. d).

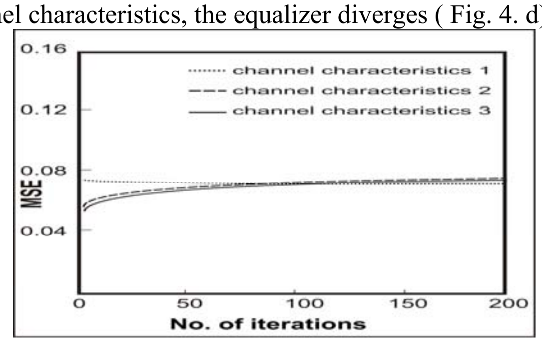

Fig. 4.d. Effect of channel characteristics with DC offset

NLMS Algorithm

For change in input signal level, it is seen that there is no effect of input signal level on the convergence. The independence of input signal level on the convergence performance of the equalizer can be explained by the adaptation equation of the algorithm which makes the rate of change of convergence independent of signal power as shown in Fig. 5.a.

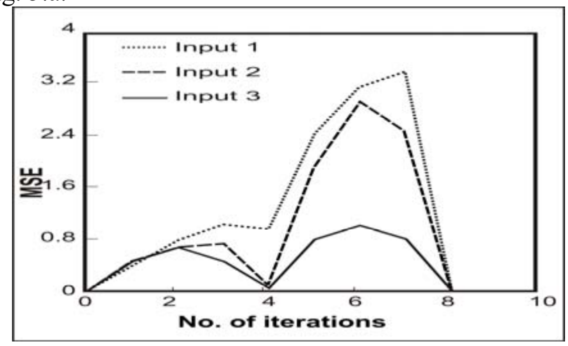

Fig. 5. a. Effect of input signal level on NLMS algorithm

For change in channel characteristics on NLMS algorithm, it is seen that for all channel characteristics, the algorithm converges with equal number of iterations but for heavy amplitude and phase distorted channel, the MSE is increased because the correlation matrix is ill conditioned. (Fig. 5.b.)

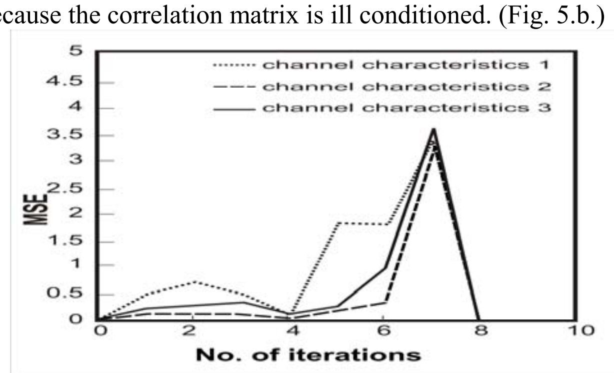

Fig. 5. b. Effect of channel characteristics on NLMS algorithm

In presence of DC offset as input signal level increases, the MSE increases but there is no effect on speed of the convergence of the equalizer as shown in Fig. 5.c. This is may be due to the normalized stepsize used in NLMS algorithm.

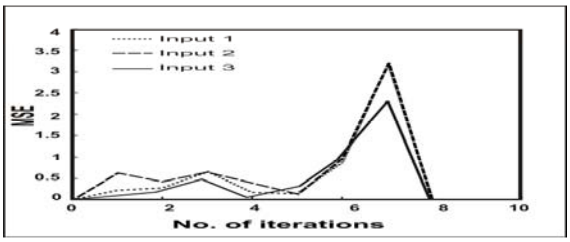

Fig. 5.c. Effect of Input signal level with DC offset

There is no change in the rate of convergence of the equalizer with change in channel characteristics with dc offset on NLMS algorithm, but the MSE is increased as shown in Fig. 5.d. This may be due to normalized stepsize used in NLMS algorithm.

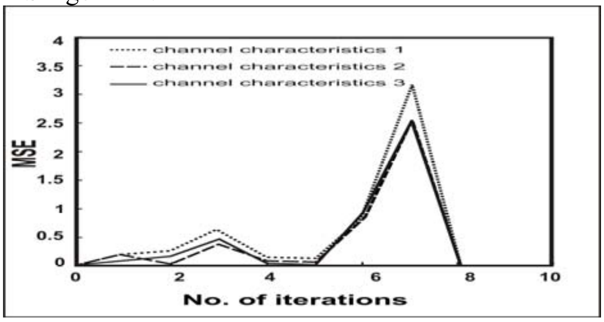

Fig. 5.d. Effect of channel characteristics with DC offset

V. CONCLUSIONS

The results obtained after the simulation of LMS algorithm and its derivatives such as SLMS, MLMS, VSSLMS and NLMS algorithms with and without DC offset are presented above for change in input signal level and channel characteristics. The results obtained are in well agreement with the results obtained for SLMS algorithm. From above results it is seen that the performance of LMS algorithm, MLMS algorithm and VSSLMS algorithm are affected by the presence of DC offset. The convergence performance of Sign-Error LMS algorithm and NLMS algorithm is better in the presence of DC offset. The performance of these algorithms is not affected by the presence of DC offset.

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