Spread Spectrum Modulation by Using Asymmetric-Carrier Random PWM

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Abstract—This paper presents a new fixed-carrier-frequency random pulsewidth modulation method, where a new type of carrier wave is proposed for modulation. Based on simulations and experimental measurements, it is shown that the spread effect of the discrete components from the motor current spectra and acoustic spectra is very effective and is independent from the modulation index. The flat motor current spectrum generates an acoustical noise close to the white noise, which improves the acoustical performance of the drive. The new carrier wave is easy to implement digitally, without employing any external circuits. The modulation method can be used in both open- and closed-loop motor control applications.

Index Terms—AC motor drives, acoustic noise, modulation, pulsewidth modulation (PWM) power converters.

I. INTRODUCTION

To reach controllability, high efficiency, and dynamic performance in electrical drives, power electronic converters based on on/off control of power switches are employed (Fig. 1). To control the power switches from the converter, several modulation methods were proposed from which the PWM method is the most used technique. A drawback of this method is that it gives rise to discrete frequency components in the current spectrum which lead to EMI [1]–[4] and acoustic noise in the drive [5]–[7]. A cost-effective strategy to distribute the discrete components from the current spectrum of the motor is RPWM. Several concepts of RPWM strategies can be found in the literature [8]. The RPWM strategies from a switching frequency point of view can be classified into two main categories:

1) RCF-PWM;
2) FCF-RPWM.

The spread of the discrete components from the motor current spectrum using RCF-PWM is more effective compared to the FCF-RPWM method [9]–[11]. In most of the practical applications, the control algorithm is synchronized with the switching; therefore, the variable switching frequency affects the performance of closed-loop applications [12], [13]. Usually, FCF-RPWM methods are based on the fact that the redistribution of zero vectors does not have an effect on the fundamental component but changes the high frequency content of the motor current spectra [14]–[16]. In [17], several FCF-RPWM methods were analyzed and compared. The authors concluded that the methods where positions of pulses were randomized (RPP-PWM) have good performance only at low fundamental amplitude. An RCF-PWM method where the slope of the carrier wave varies randomly can be found in [18] and [19]. In [20], the authors introduce a limitation on variable-slope modulation method to realize a new FCF-RPWM method. In this method, the increasing and decreasing slopes of a triangular carrier wave were chosen such that the modulation period constant is maintained.

A drawback of the variable-slope modulation technique is that external circuits are needed to generate PWM pulses. This paper proposes a new FCF-RPWM method called AC-RPWM. The advantage of the new AC-RPWM method compared to...
other FCF-RPWM methods is that it has good performance for both low and high m.i. values (m.i.-normalized output voltage amplitude [15]). A digital implementation requiring no external circuits is also proposed in this paper. Finally, the error introduced by the motor current sampling in the top and bottom of the carrier wave is analyzed.

II. FCF-RPWM

In a conventional construction of an inverter (Fig. 1), six active vectors and two zero sequence vectors can be generated. The six active voltage vectors represented in the \( \alpha - \beta \) plane [Fig. 2(a)] form a hexagon, where each active vector points to the corner of the hexagon. When one of the six active vectors is generated, the load takes energy from the dc link, forming a circuit from the load impedances as shown in Fig. 2(a) and (b). The generation of the zero sequence vector is done by connecting the three legs of the load to the plus \((V_{111})\) or minus \((V_{000})\) of the dc bus. The position of a voltage vector \(U_s\) in the \(\alpha - \beta\) plane is defined by the ratio between the applied time lengths for the two adjoined active vectors. The zero sequence vectors are responsible for reducing the amplitude of the resultant voltage vector \(U_s\). During a modulation period where a triangular carrier wave is used, two resultant voltage vectors, with arbitrary amplitude and position in the \(\alpha - \beta\) plane, can be generated. The first voltage vector is generated during the first part of the modulation period (rising slope, noted \(T_{\text{rising}}\)), and the second vector is generated during the second part of the modulation period (falling slope, noted \(T_{\text{falling}}\)).

The fundamental requirement of FCF-RPWM is to have constant update frequency and to generate the same voltage vector in the \(\alpha - \beta\) plane as it was before randomization. Due to the fact that the distribution of zero vectors \((V_{111}, V_{000})\) does not influence the fundamental frequency component [15], the time-length distribution of the zero vectors can vary randomly. The random variation of the zero sequence vectors gives a random position for the active region [the shadowed area in Fig. 2(b)] during the first and the second part of the modulation period. Using symmetrical modulation (the reference voltage vector is updated once per modulation period), the same resultant voltage vector is generated in the first and the second part of the modulation period. Fig. 3 shows the effect of repositioning of the pulse for one phase of the inverter. The pulses using symmetrical modulation for legs named \(q_a, q_b,\) and \(q_c\) are represented by continuous lines, while the repositioning of the pulse for leg \(q_c\) is represented by the dashed line. The average phase voltage during the modulation period in leg \(q_c\) is the same, independently of the pulse position. The generated voltage vectors \(U_s, U_{s1},\) and \(U_{s2}\) differ in amplitude and position in the \(\alpha - \beta\) plane, which means that the fundamental component is
distorted by the individual randomized pulse position in one leg. The distorted fundamental component affects the stability of the drive, introducing current and torque ripple. Usually, the three phases of the motor are in delta or star connection; therefore, the current through one phase depends on the current in the other two phases. As a consequence, the PWM pulses have to be synchronized to have controllability of the motor current in each individual phase of the motor. This proves that the position of a pulse cannot be changed in an arbitrary manner for each leg. In the case of RPP-PWM, the pulse position of each leg is modified. However, the time spent for generating the active vectors is maintained the same.

III. DESCRIPTION OF THE PROPOSED MODULATION METHOD

The PWM unit (used for motor control) of a commercial microcontroller usually consists of an up–down counter, three CRs, and a PR. An up-down counter is used for generating the carrier wave for the PWM unit. In traditional modulation methods like space vector modulation (SVM), the time required for up-counting mode \( T_{\text{rising}} \) is equal to the time required for down-counting mode \( T_{\text{falling}} \), as it is shown in the first modulation period in Fig. 4. By changing the ratio between \( T_{\text{rising}} \) and \( T_{\text{falling}} \), the resultant voltage vectors generated in the first and the second part of the modulation period will be similar in position and amplitude, with the difference being that the resultant voltage vector is created with different modulation frequency. Generating the same voltage vectors after and before randomization fulfills the second requirement for FCF-RPWM described in the previous paragraph. To fulfill the first requirement for FCF-RPWM, the distribution of the time length between the first and the second part of the modulation \( (T_{\text{rising}} \text{ and } T_{\text{falling}}) \) is done such that the modulation period \( T_{\text{mod}} \) constant is maintained. The following equation has to be fulfilled to maintain constant modulation period:

\[
T_{\text{mod}} = T_{\text{rising}} + T_{\text{falling}} = \text{constant} = \frac{1}{f_{\text{sw}}}
\]

where \( T_{\text{mod}} \) is the modulation period, \( T_{\text{rising}} \) is the time length where the counter is counting up (the first part of the modulation period), \( T_{\text{falling}} \) is the time length where the counter counts down (the second part of the modulation period), and \( f_{\text{sw}} \) is the switching frequency.

The second modulation period in Fig. 4 represents the proposed asymmetrical carrier waveform, where the modulation period \( T_{\text{mod}} \) is maintained constant, but the distributions of the time length between \( T_{\text{rising}} \) and \( T_{\text{falling}} \) are not equal. Having a constant time base \( T_{\text{clk}} \) for the PWM module counter, the slope of the carrier wave cannot be changed. As it is shown in Fig. 4, for digital implementation of the asymmetrical carrier waveform, two different values are used for the PR, creating an asymmetrical carrier wave for modulation. From a mathematical point of view, after the duty cycles are calculated conformably [21], the compare values can be calculated for the first and second parts of the modulation period by using

\[
\begin{align*}
\text{PR}_{\text{rising}} &= \frac{T_{\text{rising}}}{T_{\text{clk}}} \\
\text{PR}_{\text{falling}} &= \frac{T_{\text{falling}}}{T_{\text{clk}}} \\
CV_1 &= d_{\text{av0}} \cdot \text{PR}_{\text{rising}} \\
CV_2 &= CV_1 + d_{\text{av1}} \cdot \text{PR}_{\text{rising}} \\
CV_3 &= CV_2 + d_{\text{av2}} \cdot \text{PR}_{\text{rising}}
\end{align*}
\]

where \( \text{PR}_{\text{rising}} \) and \( \text{PR}_{\text{falling}} \) are the PR values for the PWM timer in the first and second parts of the modulation period, \( d_x \) is the duty cycle for the active and zero sequence voltage vectors, and \( CV_x \) represents the compare values in the CRs of the PWM module of the microcontroller.

By choosing a random time length for \( T_{\text{rising}} \) or \( T_{\text{falling}} \) in every modulation period, the time length for the active vector regions (highlighted in Fig. 4) will vary randomly. In other words, the voltage vector generated in the \( T_{\text{rising}} \) period is generated at different switching frequency than the voltage vector generated in the \( T_{\text{falling}} \) period. From this point of view, the AC-RPWM method can be interpreted as an RCF-PWM, where the update of the new voltage vector is done with constant frequency. The spread effect of discrete components from the motor current spectra using this method is effective even at high m.i., where the time spent for generation of the active vectors is longer than the time spent for generation of zero vectors. For low m.i., a very good spread effect of the discrete components can be reached by redistributing the time length of the zero sequence vectors (using the RPP-PWM strategy). As it was presented in the introduction, the redistribution of the time length between the zero voltage vectors affects the current ripple, without having any influence on the fundamental component [14]. Redistribution of the zero vectors modifies the position of the active vector regions (highlighted in Fig. 5) in the rising and falling modulation periods. In the first modulation period in Fig. 5, redistribution of the zero vectors modifies the position of the active vector; both active regions are centered \( (T_{\text{av0}} = T_{\text{av1}}) \). In the second modulation period,
the active vectors are repositioned \( T_{zv0} \neq T_{zv1} \). From the point of view of the current ripple, the optimal position for active vectors is in the middle like in the SVM. In Table I, the acoustic performances of four different modulation methods are presented, at an average switching frequency of 4 kHz. The SPL was measured with a Bruel and Kjaer sound level meter type 2230 using A-weighting. The increased current ripple causes louder acoustic noise in the motor (Table I) but transforms the whistling noise into a white noise.

### IV. DIGITAL IMPLEMENTATION OF AC-RPWM

In Fig. 6, a general block scheme of a typical motor control structure in open or closed loop is shown. The outcome of the control block is always a calculated reference voltage vector in the \( \alpha-\beta \) plane. This voltage vector \( U_s \) from Fig. 2(a) is decomposed into two adjacent active voltage vectors, and the compare values for the PWM module are usually calculated using the SVM block. The AC-RPWM block is an additional block which makes the randomization of the active and zero sequence vectors’ time length within a modulation period. In this block, \( PR_{\text{rising}} \) and \( PR_{\text{falling}} \) are also calculated, and the duty ratios are converted into compare values for the PWM module, based on (2). It can be seen from the block diagram in Fig. 6 that the main advantage of AC-RPWM is that it is very straightforward to include into an existing closed- or open-loop control algorithm, without the need for changing the control structure or adding hardware components. As a disadvantage, it can be mentioned that the PWM module has to be updated twice during the modulation period (double update is needed).

### V. SIMULATION RESULTS

To compare the spectra of the motor current obtained by using different modulation techniques, a number of simulations were carried out in the power electronic simulation software PLECS 3.1 [22]. In order to make the comparison between different modulation methods with maximum randomization freedom, the nonlinearities like dead time and minimum pulse filter have been neglected in the simulation.

Fig. 7 shows the current spectra produced by four modulation methods: SVM, RPP-PWM, AC-RPWM, and the combination of AC-RPWM with RPP-PWM. The simulations were made for three different values of the m.i.: at low speed with m.i. \( = 0.1 \), at medium speed with m.i. \( = 0.5 \), and at high speed with m.i. \( = 0.9 \).

At low speed, the RPP-PWM method has good performance; the discrete frequency components disappear from the spectrum around the switching frequency. However, the discrete components are present at double the switching frequency. In the case of AC-RPWM, the discrete components are located around the switching frequency. The combination of the two methods nearly entirely eliminates the discrete components from the current spectrum.

At medium speed, the time spent for generation of zero sequence vectors is less; the possibility for repositioning the active vectors is reduced. This results in the appearance of the discrete components in the case of RPP-PWM. At double the switching frequency, the amplitude of the discrete components is considerably reduced when AC-RPWM is used. The combination of the two methods (RPP–AC-RPWM) gives again the best performance.

At high speed, the time spent for generation of zero sequence vectors is minimal; there is not a big difference between the spectra obtained by using SVM or RPP-PWM. For high speed, the performance of AC-RPWM is better than that of RPP-PWM.

In conclusion, the combination of the two modulation methods, RPP-PWM and AC-RPWM, has the best performance of the considered techniques in all speed ranges.

In Table II, the total harmonic distortion (THD) of the simulated motor current is presented for the three different m.i. values. It should be noticed that, for high m.i., the THD of the RPP-PWM method is lower but the discrete components are still present in the spectrum (the randomization effect is small); the THDs of RCF-PWM and AC-RPWM are almost the same in this case.
VI. EXPERIMENTAL RESULTS

To show the acoustic performance and the effect of the hardware limitations (dead time and minimum pulsewidth) of the proposed modulation method, an experimental setup was built, as shown in Fig. 8. The setup consists of a 2.2-kW asynchronous motor, a 2.2-kW Danfoss VLT AutomationDrive FC 302, and a motor control running on a Texas Instruments 320F28335 floating-point microcontroller. The motor current and vibrations on the motor shell were measured with a Bruel and Kjaer pulse multianalyzer type 3560. In this work, vibration measurements have been used to analyze the performance of the proposed modulation method. It is well known that the vibration spectrum is similar to that of the radiated acoustic noise [23]. For randomization purposes, the built-in pseudorandom number generator function from “C” programming language was used. Figs. 9 and 10 show the measured spectra of the phase current and the vibrations on the shell of the asynchronous motor for m.i. values of 0.1, 0.5, and 0.9. The measured current spectrum shows similar results as the simulation from the previous section despite the fact that, for the experimental tests, limitations to ensure the minimum pulsewidth were introduced.

![Fig. 7. Simulation results of the motor current spectrum. SVM with m.i. values of (a) 0.1, (e) 0.5, and (i) 0.9. RPP-PWM with m.i. values of (b) 0.1, (f) 0.5, and (j) 0.9. AC-RPWM with m.i. values of (c) 0.1, (g) 0.5, and (k) 0.9. RPP–AC-RPWM with m.i. values of (d) 0.1, (h) 0.5, and (l) 0.9.]

![Fig. 8. Block diagram of the experimental setup.]

### TABLE II

THD OF THE MOTOR CURRENT

<table>
<thead>
<tr>
<th>m.i.</th>
<th>SVM (4kHz) [%]</th>
<th>RCF PWM (3-5kHz) [%]</th>
<th>RPP PWM 4kHz [%]</th>
<th>AC-RPWM 4kHz [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.0</td>
<td>1.1</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>0.5</td>
<td>4.0</td>
<td>4.6</td>
<td>4.8</td>
<td>5.4</td>
</tr>
<tr>
<td>0.9</td>
<td>4.9</td>
<td>6.2</td>
<td>5.1</td>
<td>6.5</td>
</tr>
</tbody>
</table>

VII. CURRENT SAMPLING

The current sampling error can cause torque oscillation which deteriorates the performance of the control in the case of closed-loop applications [24]–[27]. Usually, the acquisition of the motor currents is done on the top and/or the bottom of the triangular carrier wave [25], [28]. In the case of SVM, this means that the sampling of the motor currents is done in the middle of the time length spent for zero sequence vector generation. To quantify the current sampling error, the ideal case was simulated without nonlinearities (like dead time, minimum pulse filter, and saturation). The motor from the schematic shown in Fig. 1 was replaced with an $R-L$ load.
The impedance of the simplified $R–L$ circuit was set to be similar with the impedance of the motor. Using a symmetrical regular sampled SVM method, a balanced three-phase sinusoidal voltage can be generated. The $R–L$ circuit will act as a first-order low-pass filter, creating a current with the same fundamental frequency as the motor phase voltage. To extract the fundamental current component (purely sinusoidal without ripple) from the measured current through the inductance, a resonant filter was used. The advantages of the resonant filter are that its phase shift is zero at its resonant frequency and that it has high attenuation outside of the resonant frequency. Subtracting the fundamental current value from the measured current value gives the current measurement error when the sampling is made on the top and on the bottom of the carrier wave. Fig. 11 shows the simulation results of the macroscopic and microscopic scales of the filtered current and the measurement error.

As it can be concluded from Fig. 11, the sampling error is high when the reference voltage vector is in the middle of a sector, and it is low when the reference voltage vector has the same position as an active vector. This means that, when the reference voltage vector is close to an active voltage vector, the sampling in the top and bottom of the triangular carrier has minimal error. At zero crossing, the slope of the current is maximum, which results in maximum error. Fig. 11(c) shows that, when zero vectors are generated, the reference signal is always between the values of the current sampled in the first and second parts of the modulation period. This error is mainly caused by the regular sampling. The maximum error for this case is approximately $\pm 0.05$ A. For this example, where 10 A
is the peak nominal current, 15 A can be considered as a maximum measurable current. Considering the relatively high current measurement error (1%), even a resolution as low as 8 b is enough for analog-to-digital (AD) conversion.

By using the RPP-PWM method and by sampling the motor current on the top and bottom of the triangular carrier, the current will not be sampled in the middle of the time spent for generation of the zero sequence vectors, which further increases the sampling error.

This error can be reduced by using the same position for the active vector region in the first and the second half of the modulation period (single update for the voltage vector). However, by introducing limitations on the possible position for the active region, the spread effect of the discrete components from the motor current spectra is reduced.

Fig. 12 shows the inductor current in macroscopic time scale when the RPP-PWM and AC-RPWM techniques are used. The inductor current is sampled in both cases in the middle and in the top of the triangular carrier; the sampled value is subtracted from the filtered current, which gives the sampling error. As it was expected, the sampling error increases in the case of random modulation.

In the case of AC-RPWM, the error was slightly smaller than that in the case of RPP-PWM because the active vector region is placed in the middle of the half modulation period. Taking into consideration the error made by sampling on the top and bottom of the carrier, for the random modulation, a resolution of 6 b is enough for AD conversion. In the case of applications where this sampling error can cause problems, limitations like maximizing the time length $T_{\text{rising}}$ or $T_{\text{falling}}$ to, for example, 80% of the time length of the modulation period can be introduced. However, this kind of limitations reduces the effectiveness of the randomization (the discrete components are going to be more dominant). The usage of a low-pass filter and a high sampling frequency for current measurement could be a solution [26]. However, low-pass filters affect the phase of the filtered signal which can introduce even higher error. Using a notch filter (like the previously presented resonant filter) which
not affect the phase of the current can cause problems by filtering out the low-frequency components in the measured current. In this case, the current regulators from the closed-loop control will not be able to compensate for these unwanted low-frequency components.

VIII. CONCLUSION

A new FCF-RPWM method called AC-RPWM has been presented and analyzed in this paper. The modulation method has fixed update frequency, which has the advantage of easy implementation and integration into an existing open- or closed-loop motor control algorithm, without using any additional hardware. The simulations and the experimental measurements show that the AC-RPWM method effectively spreads the discrete components of the current and vibration spectra independent of the m.i. This flat motor current spectrum is the main advantage compared to PWM methods like SVM and discontinuous PWM (DPWM). For those applications where high demands for shaft-torque dynamics are needed, the current sampling has to be improved. However, the AC-RPWM method is well suited for applications like heating ventilation and air conditioning (HVAC), where the demand on shaft-torque dynamics is moderate and the acoustic noise issue is more important. The proposed method can be used for those HVAC applications where the less efficient asynchronous motor is replaced by a permanent-magnet synchronous motor, which requires closed-loop control.

REFERENCES

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