Transshipment Between Competitive Retailers

Xuan Zhao\textsuperscript{1}

Operations and Decision Sciences, School of Business and Economics, Wilfrid Laurier University,

Waterloo, N2L 3C5, Canada, xzhao@wlu.ca

Derek Atkins

Operations and Logistics Division, Sauder School of Business, The University of British Columbia, Vancouver, V6T 1Z2, Canada, derek.atkins@sauder.ubc.ca

December 2007 and accepted November 2008

\textsuperscript{1}The corresponding author.

Phone: 519-8840710 ext 2814
Fax: 519-8840201

1
Abstract

This paper is based on observations that competing retailers have the option of either agreeing in advance to transship excess inventory to each other or seeing unsatisfied customers switch to the competitor for a substitute. A transshipment game and a substitution game between competing retailers is studied. After establishing the existence and uniqueness of a pure-strategy Nash Equilibrium in retail prices and safety stocks for each game, we show that transshipment never leads to a lower retail price and a higher safety stock, so transshipment never leads to a situation that definitely benefits consumers. We also show that when the transshipment price is low and competition strong (perhaps because of low retailer differentiation), retailers should prefer consumers substituting. However, when the transshipment price is high and competition weak (with high retailer differentiation), then transshipment benefits them. Transshipment becomes less attractive as competition increases or retailer differentiation decreases. Competitive retailers serving the same market need to be cautious in agreeing to transship because they face larger opportunity costs than retailers serving independent markets. The results provide guidance for management as well as for public policy regarding transshipment in a competitive market.

Key words: Demand substitution; Transshipment; Price competition; Equilibrium comparison

[Supplementary materials are available for this article. Go to the publisher’s online edition of IIE Transactions for the following free supplemental resources: Proofs of statements in Transshipment Between Competitive Retailers]

1 Introduction

Stockouts are endemic in the retail industry. In the supermarket industry alone, stock-outs lead to lost sales of $7-12 billion annually (Ton 2002). “It’s no small secret that retailers are still out of stock on 20% to 30% of the SKUs consumers want almost all the time,” according to the Vice President and CIO of Liz Claiborne (Hill 1998).

When stocking out, a retailer is at risk of losing customers to competitors in search for sub-
stitutes. A survey by P&G finds about 50% of its customers switching to another retailer after a stockout (Tierney 2004). Verbeke, Farris and Thurik (1998) find 34% of Coca Cola customers switch after a store is out of stock. Anticipating this, an option sometimes available to retailers is to arrange in advance to reduce lost sales by the transshipment of inventory or capacity from a competitor. The purpose of this paper is to understand the following: What economic conditions favor a transshipment agreement ex-ante between competing retailers? How are the retailers pricing and inventory decisions affected by such a transshipment agreement? The following examples further motivate this research.

1. In the British Columbia forest industry, a small number of independent trucking companies compete to transport logs from scaling yards, where logging operators have weighed and sorted stems, over public roads to mills. For short periods the capacity of trucking companies can be considered fixed. The demand for transport is quite variable because of terrain, weather and a number of factors beyond their control. Under these circumstances, the cost of the capacity that a transport company deploys in any region and the price it charges determine whether it will survive. Faced with an unmet need to move logs because their usual trucking company is at capacity, a logging operator needs to contact another trucker. However, this creates a problem for the operator as their relationship with the transport provider is much richer than mere transportation: it also includes such services as payment systems and credit arrangements. A working arrangement usually arises among truckers, such that, given unmet demand by one and idle equipment at another, the idle trucks and drivers will be re-assigned on a short-term basis. This is not a formal arrangement in the sense of a legal contract, but the terms of trade are common knowledge to all truckers and the “deal” is transparent to loggers, who are charged the same price. They are not concerned if a different logo is on the side of the truck. So an unmet demand from a buyer (logger) is satisfied by sellers (truckers) transshipping their excess capacity (trucks) rather than by the buyer switching

---

2 According to the Executive Director for the Central Interior Logging Association of British Columbia, “there is often very little opportunity for the rate negotiations between the trucking contractor and the sub-contractor. The price paid to the trucking contractor and the sub-contractor will be usually the same. In a few cases, the regular trucker can obtain a 1%-2% markup to cover its administration costs.”
2. Most automobile sales are done through dealerships that are independent of (although associated with) manufacturers. It is primarily a decentralized industry. The cars available, possible deviations from the manufacturers’ recommended list price, credit terms and a number of other dealer specified services determine the competitive environment. A customer who desires a brand but simply “must have” a blue one, not the red one on the lot, is almost a daily occurrence for a salesman. Having an inventory of the entire range of models and colors is simply impractical for most dealers. Over time, the practice has arisen of sharing models among dealers. In some cases this is highly formalized with shared web-based inquiry tools; in others, a simple call by the salesman seals the deal. An alternative solution for dealers is to see the customer leave to search others’ lots that might stock blue ones.

3. Story three is common to all of us. We enter a store for a fashionable item, only to find it stocked out. We leave and find another store where a reasonable substitute satisfies us. If the store had multiple outlets in the same city, the salesperson might well have been able to find the product in the same chain and transshipped it, but this would be most unusual for independent and competing retailers.

Although the above three stories are from different industries, they have much in common. First, the business entities in all three industries have a choice: they could let the unsatisfied customer “walk away” to find a substitutable product elsewhere, or, they could set up relationships with competitors to transfer unused capacity/inventory to satisfy the extra demand. Of course, more options exist than simply to agree to transship or not. For example, a more laissez-faire approach, without formal prior agreements, could see a combination of customers switching and one-off transshipments between competitors. However, there are many situations where such choices should be made in advance. It might be helpful in this context to keep in mind situations in which up-front investments are needed; for example, some car dealers have built significant IT

---

3Discussions with Toyota, Honda, Ford and GM car dealerships indicate that transshipment (so-called “dealer trade”) is typically at the manufacturers’ wholesale price, which is a common knowledge to all dealers.
infrastructure to facilitate transshipment.

Second, all the entities are individually owned and sell substitutable products/services, and operate in the same market boundary. Such entities compete on both price and inventory/capacity. All cases have significant retailer-level differentiation from either value added services or simple parking or location conveniences. Also, in all examples, if transshipment is not available, unsatisfied consumers search for and switch to an alternative—so-called “inventory competition.”

Third, all examples can be described by a single-period model. We typically use the newsvendor model to approximate situations where perishable goods are involved. In these examples, truck capacity is highly perishable, this year’s model/color of car is reasonably so, and the retail fashion item mainly so. In all cases, demand is uncertain, as in the newsvendor paradigm, and capacity or inventory must be in place before demand is known.

Given these considerations, how does the choice of transshipment among competitors affect the underlying pricing and inventory decisions? What drives the strategy to choose transshipment?

Summary of Results

Our methodology is to compare two competitive games: a transshipment game where competitors agree to transship in the case that one retailer has surplus inventory and another has extra demand, and a substitution game where, instead, unsatisfied consumers switch to an alternative retailer for a substitute. The following is a summary of our results.

1. We extend the literature on the transshipment game (transshipment among decentralized business units) by considering competition between retailers. In the substitution game we explicitly link consumer switching behavior with imperfect competition between differentiated products.

2. Using a technique developed by Zhao and Atkins (2008), we prove the existence of a pure-strategy Nash equilibrium in retail prices and safety stocks, for both the transshipment game and the substitution game. We further provide conditions for the uniqueness of a pure-strategy Nash equilibrium for both games.

3. Considering a symmetric equilibrium, for the transshipment game, we find that a stronger degree of competition leads to a lower equilibrium retail price and safety stock, which results in
more unsatisfied demand and a greater need for transshipment. Further, a larger transshipment price results in a higher retail price and safety stock at equilibrium.

4. When comparing the symmetric equilibria of two games, we find that transshipment between competing retailers never leads to a lower retail price and a higher level of safety stock, so transshipment never leads to a situation that definitely benefits consumers.

6. With low transshipment prices and strong competition (a low level of differentiation), retailers are better off without transshipment. However, with high transshipment prices and weak competition (a high level of differentiation), transshipment benefits retailers. Transshipment is less attractive as the degree of competition increases.

7. We also consider scenarios where retailers transship at either the receiver’s or the sender’s retail price, in addition to an exogenous transshipment price considered above. With the receiver’s price, transshipment again becomes less attractive as the degree of competition increases; however, with the sender’s retail price, transshipment is preferred by retailers.

After reviewing the literature in section 2, we introduce the model in section 3. We provide equilibrium analyses of the two games in section 4, and compare them in section 5. Section 6 provides additional results where the transshipment price is either the receiver’s or the sender’s retail price, and section 7 discusses the key results, the contribution and the future research. All proofs appear in the Appendix.

2 Relationship and Contribution to the Literature

To the best of our knowledge, the previous literature either studies transshipment alone by assuming that transshipping parties belong to distinct and non-competitive markets, or studies inventory competition alone without considering the transshipment option. This paper considers a commonly-observed situation in which competitive businesses have the option of either transshipping leftover inventories or leaving consumers to substitute. These are termed the “transshipment game” and the “substitution game” respectively. For each game, players are allowed to make cross-functional
decisions—inventories and prices. So this paper connects three streams of literature: the transshipment game literature and the demand substitution/inventory competition literature in operations management, and price competition literature in marketing/economics.

The literature on transshipment studies retailers transshipping inventories when one has excess inventory and another excess demand and constitutes a major stream in the area of risk pooling. Rudi et al. (2001) divide research on transshipment into two categories: “intrafirm” vs. “interfirm”. The first category has transshipment among retail outlets within the same firm, and a single decision maker makes both inventory replenishment and transshipment decisions. Most transshipment literature is in this category, belonging to a centralized optimization problem. Recent years have witnessed more research in the second category: transshipment (more precisely, inventory trade) among individually-owned retailers, with each retailer making decisions in their own interests—the transshipment game. For example, Rudi et al. (2001) study transshipment between independent retailers at a fixed price. Gullu et al. (2005) consider a decentralized supply chain with one supplier and two retailers and allow retailers to arrange transshipment (at the wholesale price) after demand updating at the end of the supplier’s production/procurement lead time. Chod and Rudi (2006) consider capacity trading between independent firms and use a Nash bargaining equilibrium to allocate surplus profit from trade. There are many other papers along these lines. For example, Kouvelis and Gutierrez (1997) consider the possibility of capacity transfer in the secondary market, and Granot and Sosic (2003) model transshipment between retailers using a cooperative game methodology. The transshipment game in this paper differs from previous papers by including the pricing decision and price competition between retailers.

The second stream of literature, “inventory competition”, considers the possibility of consumer switching or demand substitution when demand is not fully satisfied. Previous literature assumes a fixed retail price. Parlar (1988), Netessine and Rudi (2003) and Lippman and McCardle (1997) assume that unsatisfied customers have only one attempt at substitution and leave the market if they are again unsuccessful. Mahajan and van Ryzin (2001) consider that customers have multiple attempts at switching and choose the next available product by maximizing their utility. Current
research extends the literature by considering pricing and price competition. Zhao and Atkins (2008), assuming one-attempt at substitution, study newsvendors selling substitutable products and competing not only in inventory, but also in prices. Using a novel approach, they establish the existence of a pure-strategy Nash equilibrium for the retailer game. They also examine the properties of the equilibrium and investigate the impact of competition on retailers’ behavior. Zhao (2008) considers a decentralized supply-chain system with one supplier selling to $N$ newsvendors competing in both price and inventory and explores how the supplier should design the contract to align the competing retailers in a way that achieves the best system-wide performance. Hopp and Xu (2007) approximate the dynamic multiple-attempt substitution process using a static fluid network model and study a retailer game with both service and price competition. In contrast, the substitution game in this paper considers newsvendors competing in both price and inventory. Furthermore, we explicitly recognize that the substitution rate is a function of retailer differentiation. Thus the “degree of competition” or “degree of differentiation” is not assumed to be independent of the “spill rate”. On the contrary, the two are explicitly intertwined. In a subsequent working paper, Atkins and Zhao (2007) have generalized this to the case with $N$ competitive sellers under linear stochastic demand systems.

The three examples above motivated us to integrate these two streams of literature. Both transshipment or demand substitution occur after the realization of random demand. The driving force for transshipment comes from retailers, who satisfy demand by outsourcing inventory/capacity. The driving force for demand substitution comes from consumers, who satisfy their demand by switching to another alternative. Transshipment has usually been considered as exogenous and occurring between non-competing retailers. This paper explicitly models consumer switching behavior between substitutable products and studies the basic trade-off made by competing retailers: Should they agree in advance and gain income from selling inventory/capacity to their competitor or wait to gain income from consumers switching to them from their competitor?
3 The Model

Consider two independent retailers, competing by selling imperfectly substitutable and perishable products in the same market. Before the selling season, each retailer needs to make three decisions in advance: (i) whether to enter a transshipment agreement with the competitor,\(^4\) (ii) how much to order, and (iii) how much to charge for a product. The detailed sequence of events are as follows:

1. Retailers decide whether to enter into a transshipment agreement or not. If yes, surplus inventories will be traded at either the market-listed prices or one of the retailers’ retail prices (more discussion in Section 3.1).

2. Each retailer simultaneously decides on how much to order and the selling price of their products.

3. The primary random demand for each retailer is realized when consumers choose their preferred retailer.

   4a. If there is a transshipment agreement, and if one retailer has surplus inventory and another extra demand, transshipment occurs.

   4b. If there is no agreement, then a fraction of the unsatisfied consumers switch to a substitute if available (more discussion in Section 3.2).

5. Retailers collect profits.

The two games differ in step 4. We first describe the common features.

When the selling season starts, retailers observe their demands. Following the literature that considers the price-sensitive newsvendor model (e.g., Petruzzi and Dada 1999; Agrawal and Seshadri 2000), we use \( D_i = q_i(\overrightarrow{p}) + \epsilon_i \) to model a retailer’s direct demand, where \( \overrightarrow{p} = (p_i, p_j) \). The first part, \( q_i(\overrightarrow{p}) \), represents the deterministic portion of the demand, having the usual properties associated with the economics of price competition: 

\[
q_i^{(i)}(\overrightarrow{p}) = \text{def} \frac{\partial q_i(\overrightarrow{p})}{\partial p_i} \leq 0, \quad q_i^{(j)}(\overrightarrow{p}) = \text{def} \frac{\partial q_i(\overrightarrow{p})}{\partial p_j} \geq 0
\]

The second part, \( \epsilon_i \), represents the price-independent uncertain demand, having a general PDF

\(^4\)Note that transshipment will occur only if both retailers agree to do so. For competing retailers, it is very unlikely that a retailer is interested in transshipping to his competitor when the competitor runs out of stock whereas he never gets the same deal when the situation is reversed. See more discussions on this in Section 7.
$f_i(\cdot)$ and CDF $F_i(\cdot)$ on a non-negative support (without loss of generality). We assume that the $\epsilon_i$’s are independent and have increasing failure rate (IFR) distributions. Retailers need to decide on retail prices $p_i$ and safety stocks $y_i$ before knowing the demand. So the total order of a retailer is $Y_i = q_i(\bar{p}) + y_i$, where $y_i$ is usually viewed as a “hedge” or protection against uncertain demand, determining the availability or service level. We further assume $q_i(\bar{p}) = a_i - b_i p_i + \theta(p_j - p_i)$, where $\theta$ measures the degree of competition (or degree of differentiation) between retailers and $b_i$ is the self-price sensitivity. This imperfect competition is a result of retailer-based differentiation; e.g.: services, parking, location, reputation, payment and delivery. Put differently, the physical products offered by the two retailers are essentially identical, but consumers see them as imperfect substitutes because of retailer-based differentiation. The above linear demand form is commonly used to study issues related to the operations/marketing interfaces (Tsay and Agrawal 2000, Boyaci and Ray 2004).

After random demands are realized, and if there is a transshipment agreement, unmet demand is satisfied as much as possible by a transshipment between retailers (4a). Otherwise, some customers will take their unfulfilled demand to another retailer (4b). Next, we model the retailer’s profit under 4a and 4b respectively. We use superscripts $T$ and $S$ respectively for the two modes of operations and refer to them as the transshipment game and the substitution game.

### 3.1 Retailers’ Profits under the Transshipment Game

Assume a transshipment agreement has been signed in advance. Then, after the random demand is realized, if retailer $i$ has surplus inventory and retailer $j$ has excess demand, then transshipment of $\min\{(Y_i - D_i)^+, (D_j - Y_j)^+\}$ at a price $\tau_{ij}$ will occur (Rudi et al. 2001). In our case, this can be rewritten as $\min\{(y_i - \epsilon_i)^+, (\epsilon_j - y_j)^+\}$. As the physical products are essentially homogeneous and the product differentiation occurs at retailer level, the transshipped product is essentially the same to the consumer. So we assume a 100% acceptance. However, the structural results from this paper will still hold if only a fraction of surplus inventory is transshipped or if only a fraction of consumers accept transshipment. This will, however, reduce the benefit of the transshipment.
A key parameter here is the transshipment price $\tau_{ij}$. We investigate two types of price setting. In the first type, $\tau_{ij}$ will be fixed and exogenous, a price that has been existing \textit{ex ante} as conventional wisdom or common knowledge in the industry. This is motivated by the car dealership example discussed in Section 1 and some other industries where transshipment is commonly observed. \footnote{5}

This assumption can also be found in Rudi et al. (2001) and Van Mieghem (1999). Although exogenous, we still investigate what happens as $\tau_{ij}$ is varied parametrically. It is reasonable to assume that $0 \leq \tau_{ij}(\tau_{ji}) \leq p_i(p_j)$. The lower bound zero represents an extreme situation reflecting that the perishable product is otherwise worthless to the sender. The upper bound represents another extreme situation where the buyer pays the full market price—an emergency replenishment situation.

In addition, motivated by the British Columbia forest industry (and the maritime industry mentioned in footnote 6), we consider a second type of setting where the transshipment price is given by a pre-determined \textit{formula}: We allow the price to be set at either the selling or the buying retailers’ retail price. This model can incorporate the situation that the transshipment price is a proportion of the retailer’s retail price. That is, $\tau_{ij} = p_i \ast \text{some proportion}$. As the proportion varies from zero to 1, we have again that $0 \leq \tau_{ij} \leq p_i$. See Section 6 for more discussion.

For simplicity, we assume a zero penalty cost and zero salvage value, so the overage cost is the cost of procurement and the underage cost is the loss of profit. Discussions with people in the forest industry and in car dealerships indicate that the actual costs associated with transacting the transshipment are low compared with other costs. This might not always be the case, but for simplicity, we assume that the costs of undertaking a transshipment have been internalized in the transshipment prices, and this assumption will not affect the main insights from the paper.

\footnote{5}{For example, in the maritime industry of the U.S., the pooling rate (transshipment price) is \textit{an established formula or scheme commonly known by all ocean carriers} (from Ocean Common Carrier and Marine Terminal Operator Agreements). In the railroad and motor industry of the U.S., the Department of Transportation states to \textit{eliminate authority to provide anti-trust immunity for railroad and motor carrier pooling and rate agreements} (from Administration’s Bill to Reauthorize the Surface Transportation Board).}
Given the above, the retailer’s problem is to maximize $\pi_i^T$:

$$
\pi_i^T = \pi_i^d - wy_i + p_i E[\min\{\epsilon_i, y_i\}] + \tau_{ij} E[\min\{(y_i - \epsilon_i)^+, (\epsilon_j - y_j)^+\}] \\
+ (p_i - \tau_{ji}) E[\min\{(y_j - \epsilon_j)^+, (\epsilon_i - y_i)^+\}].
$$

(1)

where $w$ is the unit cost of the product and $\pi_i^d = (p_i - w)q_i(\bar{p})$ is the deterministic profit.

If we consider the reasonable case (as in the dealership example) of $\tau_{ij} = \tau_{ji} = \tau$ (Rudi et al. 2001), then we can further simplify (1) by defining the effective stochastic demand for the transshipment game. First, rewrite (1) as:

$$
\pi_i^T = \pi_i^d - wy_i + (p_i - \tau) E[\min\{\epsilon_i, y_i\} + \min\{(y_j - \epsilon_j)^+, (\epsilon_i - y_i)^+\}] + \tau E[\min\{\epsilon_i, y_i\} + \min\{(y_i - \epsilon_i)^+, (\epsilon_j - y_j)^+\}],
$$

(2)

where $D_i^+ = \epsilon_i + (\epsilon_j - y_j)^+$ is the effective demand for a sender, and $D_i^- = \epsilon_i - (y_j - \epsilon_j)^+$ is the effective demand for a receiver. As IFR is preserved by convolution of independent variables, $D_i^+$ and $D_i^-$ also have IFR distributions, thus $r_{D_i^+} = \frac{-\partial \Pr(D_i^+ > y_i)}{\partial y_i}$ is increasing, similarly for $r_{D_i^-}$.

Finally, define $D_i^T$ as $\Pr(D_i^T > y_i) = \frac{\tau}{p_i} \Pr(D_i^+ > y_i) + \frac{\rho - \tau}{p_i} \Pr(D_i^- > y_i)$. Note that $D_i^T$ is not a real physical quantity but a useful surrogate that reflects the total effective demand from transshipment. Then (1) can be written as

$$
\pi_i^T = \pi_i^d - wy_i + p_i E[\min\{D_i^T, y_i\}] + (p_i - \tau) E[(y_j - \epsilon_j)^+].
$$

(3)

We use function (2) in the main body of the paper but sometimes we use (1) and (3) in the proofs in the appendix.

---

6 IFRs of $D_i^+$ and $D_i^-$ are guaranteed but not restricted by the independence and IFR of $(\epsilon_i, \epsilon_j)$. We provide a graphical explanation of this in the Appendix (2).
3.2 Retailers’ Profits under the Substitution Game

Without a transshipment agreement, if retailer $i$ has surplus inventory, the unsatisfied consumers of retailer $j$ can switch to $i$ and purchase a substitute. This constitutes the indirect demand of retailer $i$, \( \min\{(Y_i - D_i)^+, \Delta_j(D_j - Y_j)^+\} = \min\{(y_i - \epsilon_i)^+, \Delta_j(\epsilon_j - y_j)^+\} \), where \( \Delta_j \) represents the proportion of unsatisfied consumers at retailer $j$ that would switch to a different retailer and substitute.

The key parameter here is the spill rate—the proportion of the unsatisfied demand at retailer $j$ that switches to the competitor, as two retailers are imperfect substitutes to consumers. In the Appendix, we review the most common derivation of the linear demand model and derive the spill rate formula \( \Delta_j = \frac{\theta}{(b_j + \theta)} \gamma_{ji} \). The first part, \( \theta/(b_j + \theta) \), is the substitution rate as a result of product and price differentiation. Following the literature (e.g., Netessine and Rudi 2003 and the references there), we also use a fixed proportion, \( \gamma_{ji} \) where \( 0 \leq \gamma_{ji} \leq 1 \) to capture the reduction in spill-over due to other factors, for example, physical costs related to switching, such as travel and search costs. When \( \theta = 0 \), there is no competition or equivalently products are not substitutable, and then \( \Delta_j = 0 \). When \( \theta \to \infty \), retailers are under perfect competition: they are perfectly substitutable and have no differentiation, and then \( \Delta_j = \gamma_{ji} \). This simple formula effectively connects the “degree of competition” in economics with the “spill rate” in operations. Thus inventory competition also depends on consumers’ willingness to substitute: Not all disappointed consumers will be prepared to substitute—consistent with the implication of imperfect competition. Put simply, the degree of competition \( \theta \) and the spill rate \( \Delta \) are not independent.

The total demand experienced by retailer $i$ is \( D_i + \gamma_{ji} \theta/(b_j + \theta)(\epsilon_j - y_j)^+ = q_i(\overline{p}) + D^*_i \), where \( D^*_i = \epsilon_i + [\gamma_{ji}\theta/(b_j + \theta)](\epsilon_j - y_j)^+ \) is the effective stochastic demand experienced by retailer $i$ under substitution. Similarly, \( D^*_i \) also has IFR distribution, that is, \( r_{D^*_i} \) is increasing.

So in the substitution game, each retailer needs to set $p_i$ and $y_i$ to maximize \( \pi^S_i \) where

\[
\pi^S_i = \pi^d_i - w y_i + p_i E[\min\{\epsilon_i, y_i\}] + p_i E[\min\{\epsilon_i, \gamma_{ji}\theta/(b_j + \theta)(\epsilon_j - y_j)^+\}] \tag{4}
\]

\[
\pi^d_i - w y_i + p_i E[\min\{D^*_i, y_i\}].
\]
Both $\pi^T_i$ and $\pi^S_i$ are functions of $(p_i, y_i, p_j, y_j)$. We assume that the retailers’ strategy sets are compact: $\{(p_i, y_i) : w_i \leq p_i \leq p_i^{\text{max}}, y_i^{\text{min}} \leq y_i \leq y_i^{\text{max}}\}$. The upper bounds are large enough to not impact any of the choices. So the game with a compact set behaves just as the original game with an unbounded strategy space (Cachon and Netessine 2004). The following table summarizes the two games.

Table 1. Summary of Two Games

<table>
<thead>
<tr>
<th></th>
<th>Substitution game</th>
<th>Transshipment game</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost parameter</td>
<td>$w$</td>
<td>$w$</td>
</tr>
<tr>
<td>Demand function</td>
<td>$D_i = q_i(p_i^-) + \epsilon_i$</td>
<td>$D_i = q_i(p_i^-) + \epsilon_i$</td>
</tr>
<tr>
<td>Demand parameters</td>
<td>$a_i, b_i, \theta$, PDF $f_i(\cdot)$ and CDF $F_i(\cdot)$</td>
<td>$a_i, b_i, \theta$, PDF $f_i(\cdot)$ and CDF $F_i(\cdot)$</td>
</tr>
<tr>
<td>Profit from initial demand</td>
<td>$\pi^d_i - w y_i + p_i E[\min{\epsilon_i, y_i}]$</td>
<td>$\pi^d_i - w y_i + p_i E[\min{\epsilon_i, y_i}]$</td>
</tr>
<tr>
<td>Profit after demand realization</td>
<td>$p_i E[\min{(y_i^- - \epsilon_i)^+, \Delta_j(y_j^-)^+}]$</td>
<td>$(\tau_{ij} E[\min{\min{(y_i^- - \epsilon_i)^+, (\epsilon_j - y_j)^+}}] / (p_i - \tau_{ij}) E[\min{\min{(y_i^- - \epsilon_i)^+, (\epsilon_j - y_j)^+}}])$</td>
</tr>
</tbody>
</table>

4 Game Analyses

4.1 The Transshipment Game

This transshipment game differs from previous research by including price competition between independent retailers. We first study the existence and uniqueness of a pure-strategy Nash equilibrium in retail prices and safety stocks for the general game setup and characterize the equilibrium solutions. Then we examine equilibrium properties, which are based on a symmetric equilibrium. Recall that a game is symmetric in the sense that the players’ strategies and payoffs are identical (Cachon and Netessine 2004). Mahajan and van Ryzin (2001) discuss that for many business situations, it is appropriate to focus on a symmetric equilibrium. We find that both retail prices and safety stocks decrease with $\theta$, the competition factor, but increase with the transshipment price $\tau$.

Theorem 1 i. The retailer’s profit function (2) is jointly quasiconcave in $(p_i, y_i)$; a Nash equilibrium in pure strategy exists, characterized by

$$\frac{d\pi^d_i}{dp_i} + E[\min\{D_i^-, y_i\} + (y_j - \epsilon_j)^+] = 0,$$  \hspace{1cm} (5)
\[ -w + (p_i - \tau) \Pr(D_i^- > y_i) + \tau \Pr(D_i^+ > y_i) = 0. \] (6)

ii. Further assuming \( r_{D_i^-} \geq r_{D_i^+} \), there exists a unique symmetric equilibrium for the symmetric transshipment game.

A discussion on the conditions is in the appendix. For the rest of the paper, the uniqueness of the transshipment game is not strictly necessary.

**Proposition 1** At a symmetric equilibrium, the retail price and safety stock decrease with \( \theta \) but increase with \( \tau \).

Increasing the degree of competition reduces not only the retail price but also the safety stock, which leads to more unsatisfied demand and more need for transshipment. An agreement to transship creates an externality that encourages retailers to rely on each other; so they maintain lower levels of safety stock and also set lower prices. When transshipment is more expensive, retailers have incentives to keep higher levels of safety stock and rely less on outsourcing inventory, and retail prices also increase.

### 4.2 Substitution Game

The substitution game differs from Zhao and Atkins (2008), Zhao (2008) and the reference therein in that we also explicitly develop a substitution rate between independent retailers as a function of price and retailer-level differentiation. We first show that a pure-strategy Nash equilibrium in retail price and safety stock exists, and that there is a unique symmetric equilibrium for the game. We then explore how the degree of competition, \( \theta \), affects the equilibrium.

**Theorem 2** i. \( \pi_i^S \) is jointly quasi-concave in \( (p_i, y_i) \). So there is a pure-strategy Nash equilibrium for the game. The best response of retailer \( i \) solves

\[
d\pi_i^d/dp_i + E \min\{D_i^s, y_i\} = 0
\] (7)
\[ -w + p_i \Pr(D_i^* \geq y_i) = 0. \] (8)

\( \text{ii. There exists a unique symmetric equilibrium in the symmetric substitution game.} \)

Similar proofs have been given by Zhao and Atkins (2008) and Zhao (2008). We provide them in the appendix for the convenience of readers.

At this point, economic intuition might lead to the expectation that a larger \( \theta \) would decrease equilibrium retail prices and increase total inventories, thus leading to less unsatisfied demand. However, unsatisfied demand does not depend on total inventories but on the safety stocks. Note that \( \theta \) plays a double role here. A larger \( \theta \) drives the equilibrium price down because of the increased degree of competition, which also leads to lower levels of safety stock (from (8), it is easy to check that \( dy_i/dp_i > 0 \)). On the other hand, a larger \( \theta \) leads to more switching; thus retailers tend to increase safety stocks in order to capture more spill demand from the rival. By having the spill rate as a function of \( \theta \), the competition factor, the fact that consumer preference affects both competition in economics and spill in operations is explicitly modeled, which helps to provide a richer picture on how competition affects the equilibrium decisions. Proposition 2 shows that only one possibility can be excluded.

**Proposition 2** At a symmetric equilibrium, the only scenario that cannot occur is that an increase in \( \theta \) leads to a decrease in the safety stock and an increase in retail price.

A larger \( \theta \) decreases the profit margin \( p_i - w \) but increases the indirect sales \( Q_{ij} \). If \( p_i - w \leq dQ_{ij}/d\theta \), that is, the decreased profit margin is less than the rate at which indirect sales increase, then indeed retailers will set higher prices and higher levels of safety stock under stronger competition—\( dy/d\theta > 0 \) and \( dp/d\theta > 0 \). However, \( dQ_{ij}/d\theta \) is a probability function and less than one; so it is unlikely to be greater than \( p_i - w \). So the interesting cases occur when \( p_i - w \geq dQ_{ij}/d\theta \), that is, the negative impact of \( \theta \) dominates. Then retailers will set lower prices, but either a lower or a higher level of safety stock can occur. Numerical work shows that \( (dy/d\theta < 0, dp/d\theta < 0) \), that is, both safety stocks and retail prices decreasing is by
far the most common, but that \( (dy/d\theta > 0, dp/d\theta < 0) \) occurs regularly. So increasing competition commonly decreases safety stocks, and thus increases unsatisfied demand.

5 The Impact of a Transshipment Agreement between Competing Retailers

Traditional transshipment and inventory competition models seldom admit of closed form solutions, and our much more complex model is no exception to this. It is even more difficult to compare players’ strategies and payoffs for the two games. We attempt as much analytical insight as possible and supplement this with numerical results where such analysis is not possible.

5.1 How Does a Transshipment Agreement Affect the Decisions of Competing Retailers?

The first question we would like to address is: In a competitive situation, how does a transshipment agreement affect the pricing and safety stock decisions of competing retailers? Can we draw some conclusions on whether consumers benefit from transshipment between competing retailers? Proposition 3 compares symmetric equilibrium strategies (and omitting the subscripts) between the two games.

**Proposition 3** There exists a unique \( \tau \) and a unique \( \tau' \) such that when \( \tau > \tau' \), \( p^T > p^S \) and \( y^T > y^S \); when \( \tau < \tau' \leq \tau \), \( p^T > p^S \) and \( y^T \leq y^S \); and finally when \( \tau \leq \tau' \leq \tau \), \( p^T \leq p^S \) and \( y^T < y^S \).

So neither mode of operation consistently gives higher levels of safety stock and retail prices. The comparison depends on a key economic factor—the transshipment price. With a transshipment agreement and a high transshipment price (close to \( p \)), retailers provide high levels of safety stock and charge high prices for the selling season. At the other extreme, when surplus inventories are not valued (low \( \tau \)), retailers set lower levels of safety stock and charge less. Between these extremes values of \( \tau \), retailers set lower safety stocks but charge more than what they would do
with consumers switching, leading to a situation that a transshipment agreement would actually hurt consumers. Note that, transshipment never results in more safety stock and lower retail prices; thus transshipment never leads to a situation that definitely benefits the consumers.

5.2 Should Competing Retailers Sign a Transshipment Agreement?

Do competing retailers have an incentive to enter into a transshipment agreement? Without competition and substitution, the answer would be yes. Otherwise, as shown below, the answer changes as \( \tau \) and \( \theta \) vary. Proposition 4 provides some answers for limiting cases. Lemma 1 first establishes a stochastic order of the random demands.

**Lemma 1** \( D_i^- \leq_{st} D_i^s \leq_{st} D_i^+ \).

**Proposition 4** For a symmetric equilibrium (omitting the subscript),

i. if \( \tau = 0 \) and \( \theta \to +\infty \), then \( \pi^S > \pi^T \).

ii. if \( \tau = p \) and \( \theta = 0 \), then \( \pi^S < \pi^T \).

Intuitively, we can view \( \tau \) as reflecting the marginal revenue from transshipment and \( \theta \) as indicating the cost of switching consumers: the larger the \( \theta \), the less costly for consumers to accept a substitution. Then Proposition 4 says that when the revenue from transshipment is small, but switching is cheap, competing retailers are better off without transshipment. On the other hand, when the revenue from transshipment is substantial, and consumer switching is costly, then transshipment is beneficial. For general values of \( \tau \) and \( \theta \), we have:

*Observation 1.* A retailer’s profit initially increases as \( \tau \) increases but might eventually decrease as \( \tau \) gets larger.

*Observation 2.* When the competition factor \( \theta \) is large, transshipment becomes less attractive to retailers.

The appendix provides analytical support for these observations. The first observation indicates that when everything else is equal, retailers might prefer to transship inventories at intermediate levels (and Section 5.3 confirms that a transshipment price in the range of \( w < \tau < p \) usually leads
to higher profit ratios between transshipment and substitution games). The second observation implies that retailer profits without transshipment are less reduced as competition becomes fiercer. These two observations complement the extreme results of Proposition 4. We might expect that transshipment is not a good choice when \( \theta \) is large even when \( \tau \) is not at the extreme value of zero. The following numerical results confirm this.

### 5.3 Numerical Results

To gain further insights, we undertook some numerical experiments. Experiments were performed using uniform, normal and exponential distributions, and included both symmetric and asymmetric retailers over a comprehensive data set. However, the qualitative conclusions from these studies varied little, and as our intention is simply illustrative, we present just a representative example using identical and independent exponential distributions for \((\epsilon_1, \epsilon_2)\) with means of 100, and \(\gamma = 1\).

The deterministic part of the demand is set with \(a = 240\) and \(b = 12\). As \(p \geq w\), this means that the deterministic demand must be less than \(240 - 12w\). When the wholesale price is set low, i.e., \(w = 6\), this leads to a deterministic demand in the \((0, 168)\) range, and hence on average roughly commensurate with the average random demand. When the wholesale price is higher, i.e., \(w = 12\), the average deterministic demand is closer to half that of the random demand. So \(w = 6\) represents a situation where the two sources of demand are about equal on average, but \(w = 12\) makes the random component about twice the deterministic. The degree of competition \(\theta\) is chosen from \([b/2, b, 4b]\), and we refer to these three values as weak, medium and strong competition. As the degree of competition increases, the equilibrium retail prices decrease, making reasonable values of the exogenous transshipment price to be considered smaller. We display \(\tau\) from \([w/2, p]\), and low, medium, and high transshipment prices refer to when \(\tau\) is around \(w/2\), around \(w\), and greater than \(w\), respectively.
5.3.1 Profit Comparison

In figures 1 and 2, the vertical axis is the ratio of the profit in the transshipment game to the profit in the substitution game, both at their respective equilibria, as functions of $\theta$ and $\tau$. Thus a ratio above 1 would mean that transshipment has the higher profit. We see that neither game is an outright “winner,” but that the transshipment game performs better when competition is weaker ($\theta$ smaller), and the substitution game becomes more attractive when competition is fiercer ($\theta$ larger). Further, the range of transshipment prices that favour transshipment is larger for weaker price competition.

![Figure 1. Profit ratio for a low w](image1)

![Figure 2. Profit ratio for a high w](image2)

For competitive retailers, the benefit of signing a transshipment contract varies with both $\theta$ and $\tau$. If they could choose the transshipment price to get the best possible profits, then that price should also vary with the competition factor ($\theta$), as seen in the figures. This insight is not available if consumer switching between retailers is ignored.

In the car dealership example, we learned that transshipment at the wholesale price ($\tau = w$) is typical. In both graphs we observe that when $\tau \geq w$, transshipment will be outperformed only when competition is quite fierce. For the retail clothes-fashion industry, it is reasonable to assume that competition is the fiercest, with less differentiation between retail outlets and ease of consumer switching. Consistent with the graphs, transshipment between competing retail outlets is seldom observed in this industry. On the other hand, for the logging truck example, with quite well differentiated services, letting consumers switch is less preferred. The loggers actually benefit from
transshipment across a broad range of transshipment prices. Section 6 will discuss more about this logging example.

5.3.2 Strategy Comparison

Figures 3 and 4 are consistent with Proposition 3. If we focus on the range of transshipment prices probably of most interest, \( \tau \geq w \), we observe that transshipment can be associated with both higher prices and safety stocks, but that this effect is less pronounced for more competitive situations. We have noted that fiercer competition reduces retail prices and safety stocks for both games, but in the transshipment game the reduction is more significant, since the double role of \( \theta \) under the substitution game somewhat mitigates the intense price competition, leaving less distorted retail prices and safety stocks.

![Figure 3. Safety stock ratio](image1)

![Figure 4. Retail price ratio](image2)

5.3.3 Total Inventory Comparison

At a symmetric equilibrium, the total inventory is \( Y = a - bp + y \). We have seen that typically we can expect that \( \partial p / \partial \theta < 0 \) and \( \partial y / \partial \theta < 0 \), but this gives little information about the total inventory since it increases in \( y \) but decreases in \( p \). From Figures 5 and 6, no monotonic result is available for \( \partial Y / \partial \theta \). Both figures show that the ratio increases with \( \tau \). Retailers have more total inventory under transshipment with a high price. However, if the transshipment price is low, retailers order less in total. Varying the degree of competition \( \theta \) makes a small difference to this conclusion. If we assume that both retailers order from a supplier using a fixed wholesale price, then the supplier’s profit
is \( \sum_{i=1,2} w_i Y_i(w_i) \). The insight here is that the supplier usually agrees with the retailers’ decision about when to use transshipment, although the supplier does so over a relatively smaller range of \( \tau \).

6 Additional Results

In this section, we consider two scenarios where the transshipment price is set in advance to be either the selling or buying retailer’s retail price. The model can easily incorporate scenarios where the transshipment price is set at a fraction of the selling or buying retailer’s retail price (which makes transshipment less advantageous), as well as incorporating a fixed per-unit transshipment cost. This will not change the main insights. If the retailers agree in advance to transship at the receiver’s price—that is, retailer \( i \) charges retailer \( j \)’s retail price when \( i \) transships to \( j \) (as in the logging truck example), the transshipment game becomes

\[
\pi_i^T = \pi_i^d - wy_i + p_iE[\min\{\epsilon_i, y_i\} + p_jE[\min\{(y_i - \epsilon_i)^+, (\epsilon_j - y_j)^+\}]]. \tag{9}
\]

If retailers agree in advance to transship at the sender’s price—retailer \( i \) charges their own retail price when \( i \) transships to \( j \)—then the transshipment game becomes

\[
\pi_i^T = \pi_i^d - wy_i + p_iE[\min\{\epsilon_i, y_i\} + p_iE[\min\{(y_i - \epsilon_i)^+, (\epsilon_j - y_j)^+\}]
+ (p_i - p_j)E[\min\{(y_j - \epsilon_j)^+, (\epsilon_i - y_i)^+\}]. \tag{10}
\]
Similar methods to those used in Theorems 1 and 2 show the existence of a pure-strategy Nash equilibrium in each game. We first compare the choice between transshipment and substitution analytically, then compare profits numerically.

**Proposition 5**  
i. Let transshipment prices be set ex ante as the receiver’s retail price; the following three scenarios occur \( \{p^T < p^S \text{ and } y^T < y^S; p^T < p^S \text{ and } y^T > y^S; p^T > p^S \text{ and } y^T > y^S\} \).

ii. If transshipment prices are set ex ante as the sender’s retail price, then the only possible scenario is \( y^S \leq y^T \text{ and } p^S \leq p^T \).

For the case of transshipping at the receiver’s retail price, we find through numerical results that when the degree of competition is weak or medium, transshipment provides higher profits to retailers, (and this is consistent with the observation from the logging example); otherwise, retailers will prefer to let consumers switch, consistent with the results of Section 4.3. For the case of transshipping at the sender’s retail price, retailers are always better off under transshipment. The idea is that if a retailer can trade his surplus inventory at the same price as his regular retail price, then arranging transshipment is beneficial.

7 Conclusion

The main question explored in this paper is: How does transshipment between competing retailers affect their pricing, operational strategies and profits? This question is important from three aspects: First, it helps explain when transshipment benefits competing businesses. Second, it informs policy makers about the impact of such “cooperation” on consumers. Third, it links three areas of the literature: transshipment, inventory competition and price competition between differentiated products.

Two games are studied: a transshipment game and a substitution game. For each game, we first established the existence of a general pure-strategy Nash equilibrium in retail prices and safety stocks and characterize the equilibrium solutions, and we found conditions for a unique symmetric equilibrium in pure strategy. We then analyzed how key economic factors such as the transshipment
price \( (\tau) \) and the competition factor \( (\theta) \) affect equilibrium retail prices and safety stocks. Finally, we compared the two games in terms of competing retailers’ strategies and profits at equilibrium.

Whether transshipment is beneficial for retailers depends on both the transshipment price \( (\tau) \) and the competition factor \( (\theta) \). The former measures the revenue from trading inventory, and the latter reflects the competition between retailers and the cost of substitution for consumers. We found a low transshipment price but a high degree of competition favors the option of letting consumers substitute. With a high transshipment price and a low degree of competition, retailers should choose transshipment. Furthermore, a larger \( \theta \) makes transshipment less attractive.

We also considered cases where transshipment occurs at the retail prices. When retailers are able to sell the surplus inventory at their own retail prices, then transshipment benefits retailers and also encourages them to set higher retail prices and safety stocks. On the other hand, when the surplus inventory is sold at the competitor’s retail price, transshipment is beneficial except for the case with the fiercest competition.

Comparing our findings with observations from three examples, we observe that for the dealer trade in the automobile industry, the common-knowledge transshipment price equals the wholesale price \( (\tau = w) \), and transshipment performs better except under fierce competition. The logging truck companies usually charge the competitor’s price if surplus capacities are subcontracted, so choosing transshipment is the best option for them as consumer switching is costly. With less differentiation between retail outlets and ease of consumer switching, fashion retailers let consumers switch, consistent with our findings as well.

To obtain analytical results on properties of the equilibria and on the comparison of the two games, we assumed that the retailers are identical. This assumption is appropriate when no one player (e.g. retailer) has a significant dominant position in the market, as in this paper. However, there are many other circumstances where the effect of players being very different (heterogeneous) in size or power is exactly at the very core of the research questions. The question is, then, how robust our results are when the retailers are non-identical. We have looked at this numerically. We found that the patterns of the graphs for each retailer are similar to that of the symmetric
cases presented in Figures 1–6. For example, the graphs of profit ratios show that as the degree of competition increases, the benefit of transshipment diminishes, especially at low transshipment prices. Due to the asymmetric parameter values, there is a small range of transshipment prices where one retailer prefers transshipment but the other retailer prefers substitution. However, given an exogenous transshipment price in the market like the industries we have cited in the paper, transshipment occurs only if both retailers have an incentive to do so: transshipment occurs in situations where the market transshipment prices are such that both retailers have a greater profit under transshipment than that under substitution.

The following-up question, then, is, what if retailers can provide some incentives to each other such that one of the retailers is willing to send but not receive transshipment and vice versa for the other retailer? To answer this question, two extra asymmetric games need to be formulated, analyzed and compared with the two games studied in the paper; and some incentive schemes need to be proposed to make the asymmetric games possible. For example, if retailer $i$ receives but does not send transshipment, then the profit after demand realization (Table 1) will be $p_i E[\min\{(y_i - \epsilon_i)^+, \Delta_j(\epsilon_j - y_j)^+\}] + (p_i - \tau_{ji})E[\min\{(y_j - \epsilon_j)^+, (\epsilon_i - y_i)^+\}]$, and retailer $j$ who sends but does not receive transshipment has a profit after demand realization as $\tau_{ji}E[\min\{(y_j - \epsilon_j)^+, (\epsilon_i - y_i)^+\}]$. Similar formulation can be obtained when the positions of retailers $i$ and $j$ are changed. A detailed analysis of the asymmetric games and the incentive issues involved, however, is outside the scope of this paper and we leave it for future extensions.

There are also many other possible extensions and avenues for future research. This paper has considered industry examples where the transshipment price is either given as an exogenous market parameter or a function of retail price set ex ante. Extensions that include either a cooperative or a competitive setting of transshipment prices might be interesting to pursue. However, to do this, a simplification of the current model would be needed in order to remain tractable. The paper has assumed a game with complete information. Otherwise, a retailer might prefer to cheat by hiding the actual surplus inventory and hope to get some switching consumers in the case that the transshipment price is low and the substitution rate is high. This consideration, we believe, will make
transshipment even less attractive. However, a detailed investigation is needed to draw a thorough conclusion and some “truth-telling” mechanism might be needed. Extensions to multiplicative or general demand models would be interesting; however, the spill rate formula would need to be researched anew as ours is rooted in the linear demand function. We have broadly ignored the supplier, who has been assumed to be a wholesale-price taker. Examining the contracting problem between the supplier and retailers to understand how this affects the transshipment decision could be interesting.

Acknowledgement

This research is supported by the Natural Sciences and Engineering Research Council of Canada, No. 312572-05 and No. 4743.

References


