

3D Surface Reconstruction Using Structured Circular Light Patterns

Deokwoo Lee and Hamid Krim

Department of Electrical and Computer Engineering
North Carolina State University
Raleigh NC 27606, USA
{dlee4, ahk}@ncsu.edu
<http://www.vissta.ncsu.edu/>

Abstract. Reconstructing a 3D surface in \mathbb{R}^3 from a 2D image in \mathbb{R}^2 has been a widely studied issue as well as one of the most important problems in image processing. In this paper, we propose a novel approach to reconstructing 3D coordinates of a surface from a 2D image taken by a camera using projected circular light patterns. Known information (i.e. intrinsic and extrinsic parameters of the camera, the structure of the circular patterns, a fixed optical center of the camera and the location of the reference plane of the surface) provides a mathematical model for surface reconstruction. The reconstruction is based on a geometrical relationship between a given pattern projected onto a 3D surface and a pattern captured in a 2D image plane from a viewpoint. This paper chiefly deals with a mathematical proof of concept for the reconstruction problem.

Keywords: 3D reconstruction, Structured light system, Circular light pattern, Geometrical relationship.

1 Introduction

3D image reconstruction from a 2D projected image on the basis of structured light patterns has been of much interest and an important topic in image processing. Applications are in the areas of object recognition, medical technologies, robotic visions and inspections of properties of target images, etc ([1], [2]). 3D surface reconstruction from a 2D projected image can be solved using a geometric relationship between the target image(3D) and the projected image(2D). There are two main approaches to the reconstruction problem : active and passive. The general principle involved in the passive method is triangulation using 2 or more cameras(usually 2 cameras) [3]. The relative positions of two cameras and an object in each image plane provides the necessary information to reconstruct 3D coordinate information ([3], [4]). This is also called a stereo correspondence problem, for which many techniques have been proposed ([3], [7]). In the passive

method, prior knowledge (i.e. projection of object points in the image plane) necessitates high computational complexity for the solution. The 3D measurement also depends on the distance between two cameras and the result is sometimes distorted [10]. The alternative approach to the reconstruction problem is an active stereo vision, or an active method, using a structured light system which is used widely [5]. Generally one camera is replaced by a light source such as an LED or a laser beam that projects a known pattern. Only one camera is used to capture the projected pattern on a 3D object to be measured. Structured light patterns with high resolution can achieve a large number of sampling points over the surface and result in high accuracy [6]. In this paper, we assume that the position of a camera and a light source are known, a camera is modeled as an ideal camera (often called a pinhole model) and the light projection is parallel [7](Fig. 6). In practice, however, a camera is not usually calibrated to a pinhole model and structured patterns do not exactly preserve their shape by a set of lenses (lens distortion), but it is very difficult to model the system [4]. The observed deformation of the projected pattern on a 3D object provides information of its real world 3D coordinates (x_w, y_w, z_w) . In order to improve the accuracy of the reconstruction, stripe patterns ([1], [8], [9]), pyramidal laser rays resulting in a matrix of dots (genetic algorithm) [10], occluding contours [11] and coded structured light [4] techniques were developed. Known information about the patterns(i.e. structure of patterns or the location of the points belonging to each pattern) are required prior to the projection in the previous works. Circular patterns have very good characteristics such as a closed and continuous form, and a symmetric shape leading to a lower computational complexity. Moreover, only a single point(i.e. the center of a circle) and the radii of the circles are needed. This advantage can improve the algorithm efficiency. Our approach is based on analyzing a deformed circular pattern acquired by the camera providing the 3D coordinate information of an object. (Fig. 1). This paper gives an account of proof of concept for a reconstruction and some simple simulated results. The outline of this paper is as follows. The overview of our system is described and some initial assumptions are explained in Section 2. Section 3, containing the major contribution of this paper, presents notations for image representation(Section 3.1) and details the proposed mathematical model to achieve the reconstruction procedure (Section 3.2). Some preliminary experimental results are shown in Section 4 to

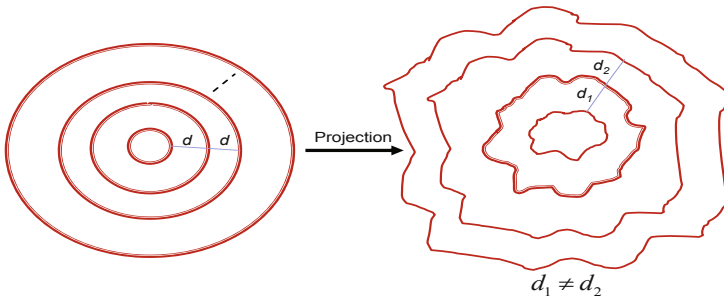


Fig. 1. Ideal and deformed circles

substantiate the mathematical model proved in Section 3.2. Finally, Section 5 presents the conclusion and future works.

2 System Architecture

2.1 Overall System Description

Basically, the structured light system (Fig. 3) ([12], [13]) is composed of a high-power and a noninvasive LED light source, a spatial light modulator(LC2002) generating a structured light pattern and a series of lenses to collimate a light beam. This is low-cost and effective system to generate a structured light patterns, to try using different kind of light sources and to establish a mathematical

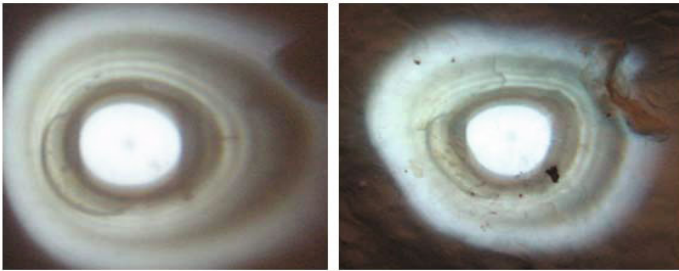


Fig. 2. Ideal and projected(observed) circular patterns

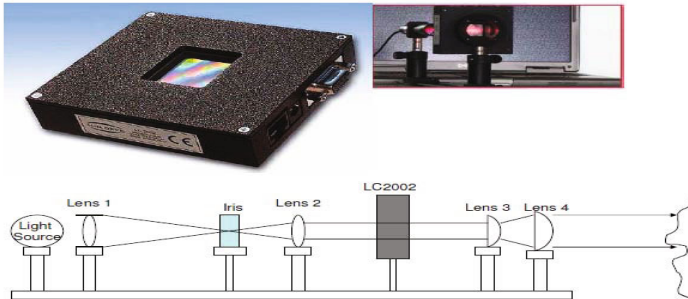


Fig. 3. Setup of structured light projection system using LC2002

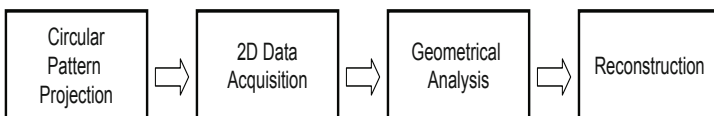


Fig. 4. Block diagram of an overall system

modeling for a 3D coordinates reconstruction. Overall system (Fig. 4) consists of a pattern projection, acquisition of 2D data points, geometrical analysis of 2D data and real world 3D coordinates, and a reconstruction. As discussed previously, our structured light pattern is circular and a reconstruction is based on the relationship between ideal circular patterns and deformed ones. In this paper, we assume that the patterns are projected in parallel, a camera is calibrated(i.e. known intrinsic and extrinsic parameters) to a pinhole model, and all the components(a camera, a 3D surface and a light source) are in fixed positions.

3 3D Surface Reconstruction

3.1 Notations and Geometrical Representation

Let $S \in \mathbb{R}^3$ be a domain of a 3D object of interest, then a point $P_w \in S$ is represented as

$$P_w = \{(x_w, y_w, z_w) \in \mathbb{R}^3\}, \tag{1}$$

where an index w is used to denote real world coordinates. Let $L \in \mathbb{R}^3$ be a domain of a circular structured light source and the origin defined as a center of a pattern(or a curve), then a point $P_L \in L$ is represented as

$$P_L = \{(x_{Lij}, y_{Lij}, z_{Lij}) \in \mathbb{R}^3 \mid x_{Lij}^2 + y_{Lij}^2 = R_j^2, z_{Lij} = 0\}, \\ i = 1, 2, \dots, M, j = 1, 2, \dots, N. \tag{2}$$

Let $S_3 \in \mathbb{R}^3$ be a domain of projected circular patterns on a 3D object, then $P_3 \in S_3$ is represented as

$$P_3 = \{(x_{wij}, y_{wij}, z_{wij}) \in \mathbb{R}^3\}, \quad i = 1, 2, \dots, M, j = 1, 2, \dots, N. \tag{3}$$

After the patterns projected, P_3 and P_w defined in the intersection of S and S_3 are identical,

$$P_3 = \{P_w \mid P_w \in S \cap S_3\} \text{ or } P_w = \{P_3 \mid P_3 \in S \cap S_3\}. \tag{4}$$

Let $S_2 \in \mathbb{R}^2$ be a domain of 2D image plane of a camera, then $P_2 \in S_2$ is represented as

$$P_2 = \{(u_{ij}, v_{ij}) \in \mathbb{R}^2\}, \quad i = 1, 2, \dots, M, j = 1, 2, \dots, N, \tag{5}$$

where M is a number of patterns and N is a number of sampled points in each pattern. The 3D reconstruction problem is analyzing a relationship between $P_3 \in S_3$, $P_L \in L$ and $P_2 \in S_2$ (Fig. 5). Let $f : L \rightarrow S_3$ be a map of a light projection and $g : S_3 \rightarrow S_2$ be a map of reflection respectively, then a reconstruction problem can be described as functions

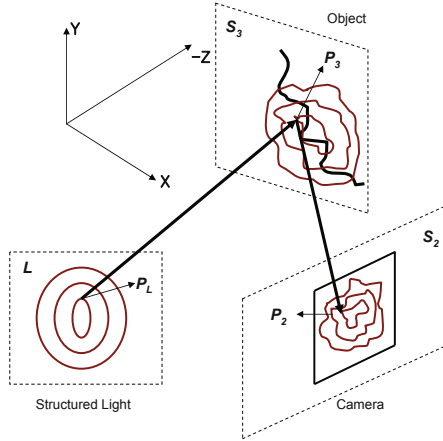


Fig. 5. Geometrical representation of the experimental setup

$$f(P_L) = P_3, \tag{6}$$

$$g(P_3) = P_2. \tag{7}$$

Recall that we assume parallel light projection which preserves (x_{Lij}, y_{Lij}) after projection onto a 3D object such as

$$I : (x_{Lij}, y_{Lij}) \rightarrow (x_{wij}, y_{wij}), \forall i, j, \tag{8}$$

$$i = 1, 2, \dots, M, j = 1, 2, \dots, N,$$

where I is an identity function. As discussed previously, under the assumption of parallel projection, (x_{Lij}, y_{Lij}) and (x_{wij}, y_{wij}) obey same constraints as follows:

$$x_{Lij}^2 + y_{Lij}^2 = R_i^2, \tag{9}$$

$$x_{wij}^2 + y_{wij}^2 = R_i^2, \tag{10}$$

where i denotes the i th positioned pattern. While preserving (x_{Lij}, y_{Lij}) coordinates, parallel projection makes the depth (z_{wij}) varies after projection onto an object. We call these variation of depth, z_{wij} , a *deformation factor*. 3D reconstruction problem is composed of analyzing a deformed circular patterns and depth recovery.

3.2 Mathematical Model

This section details the reconstruction technique of real world 3D coordinates of an object from a planar image of its patterned lighted surface. The geometrical structure, describing the physical measurement setup, is defined in 3D space and the reference plane is chosen prior to the reconstruction (Fig. 6). We assume that a camera is calibrated to a pinhole model[7]. To solve a reconstruction problem,

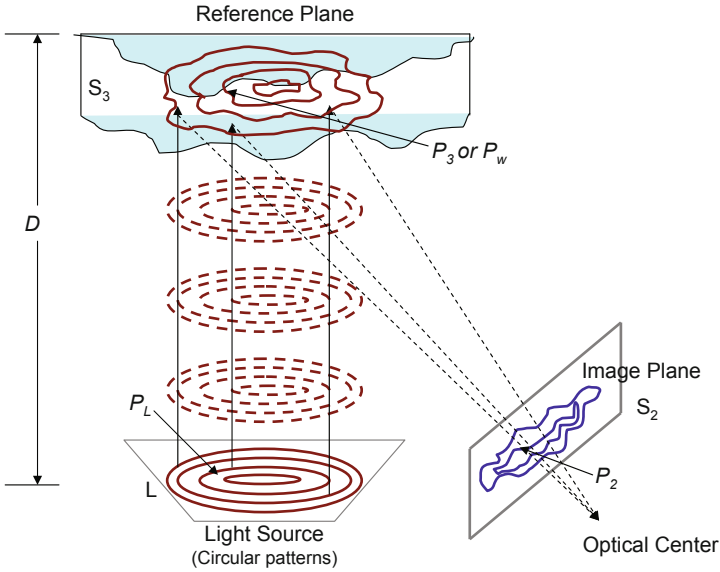


Fig. 6. Reconstruction experimental setup based on parallel projection

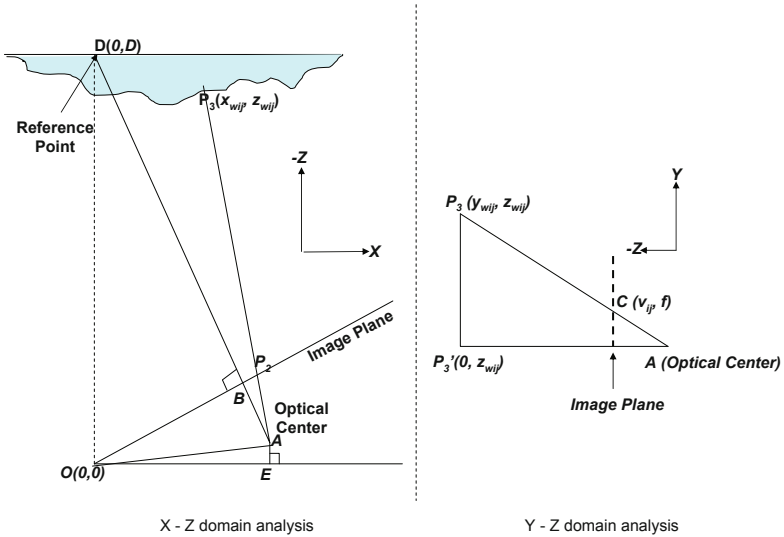


Fig. 7. (X - Z) and (Y - Z) domain analysis

we opt for two distinct coordinate systems. One lies on (X, Z) domain and the other lies in (Y, Z) domain (see Fig. 7). From Fig. 7 along with associated attributes, we can solve the 3D reconstruction problem. Assuming again that the structured light patterns remain parallel, camera is calibrated to a pinhole

model, we know the locations of a camera, reference plane of an object and light source, we can write as follows:

$$\begin{aligned} \overline{AO} &= d, \quad \overline{AB} = f, \\ \overline{BO} &= \sqrt{d^2 - f^2} = d_1, \\ d \cos(\angle AOB) &= d \cos \theta_2 = \sqrt{d^2 - f^2}, \\ \overline{OP_2} &= |\overrightarrow{BO} + \overrightarrow{BP_2}|, \end{aligned} \tag{11}$$

where the point $P_2(u_{ij}, v_{ij})$ defined in the 2D image plane, is the result of reflection from the point P_3 , A is the optical center and B is the origin point in the 2D image plane. Since the coordinate system of a 3D object and that of a 2D image plane are different, denoting the $\angle(AOE)$ by θ_1 , we transform the domain S_2 to S_3 associated with $(X - Z)$ domain as follows:

$$\begin{aligned} \theta_1 + \theta_2 &= \theta, \\ A &: (-d \cos \theta_1, d \sin \theta_1), \\ B &: (-d_1 \cos \theta, d_1 \sin \theta), \\ P_2 &: (-d_2 \cos \theta, d_2 \sin \theta), \end{aligned} \tag{12}$$

where $\theta_1, \theta_2, \theta, d, d_1$ and d_2 are known information. Using a property that the lines $\overline{P_2P_3}$ and \overline{DB} meet at the point A (see Fig. 3), we can write the relationship between x_w and z_w as

$$\begin{aligned} d \sin \theta_1 &= \frac{-d \cos \theta_1}{x_{wij} + d_2 \cos \theta} \left(z_{wij} + \frac{d_2^2}{2} \sin 2\theta \right) + \frac{d_2 x_{wij} \sin \theta - z_{wij}}{x_{wij} + d_2 \cos \theta}, \\ \Rightarrow z_{wij} &= F(x_{wij}). \end{aligned} \tag{13}$$

To completely reconstruct 3D coordinates (x_w, y_w, z_w) , we can show the $(Y - Z)$ domain analysis(Fig. 3) as

$$\begin{aligned} \frac{v_{ij}}{f} &= \frac{y_{wij}}{z_{wij} - d \sin \theta_1}, \\ z_{wij} &= \frac{f}{v_{ij}} y_{wij} + d \sin \theta_1 \\ &= \frac{f}{v_{ij}} \sqrt{R^2 - x_{wij}^2} + d \sin \theta_1, \end{aligned} \tag{14}$$

$$\Rightarrow z_{wij} = H(x_{wij}). \tag{15}$$

Concerning all above steps, we can determine 3D coordinates of the deformed curves on a 3D object,

$$F(x_{wij}) = z_{wij}, \tag{16}$$

$$H(x_{wij}) = z_{wij}, \tag{17}$$

$$x_{wij}^2 + y_{wij}^2 = R_j^2. \tag{18}$$

4 Examples of Reconstruction

To substantiate these reconstruction steps, some examples are shown in Figs. 8, 9, 10 and 11. In experiments, 9 circular patterns are projected using a LC2002 light modulator which is a device for displaying images(e.g. structured light patterns) with a resolution up to SVGA format(800×600 pixels)[12]. A regular camera is used to take a picture of patterns projected on the target object. 2D data points (u_{ij}, v_{ij}) are manually selected, usually in pixel unit, (Fig. 8) and used to reconstruct $(x_{wij}, y_{wij}, z_{wij})$.

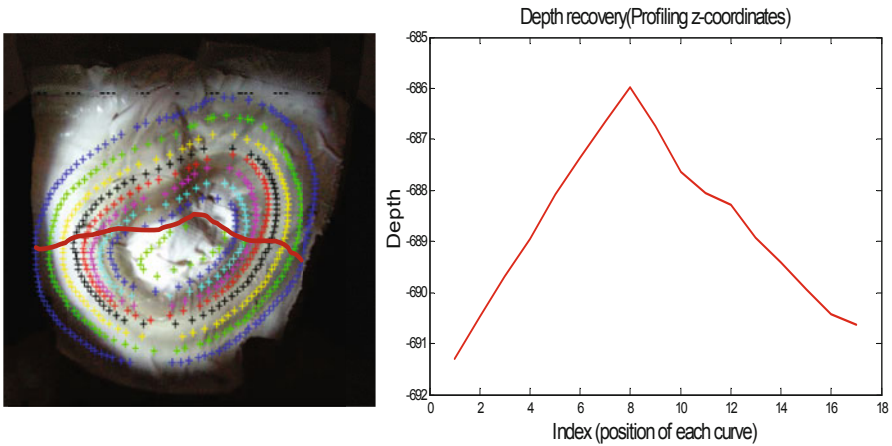


Fig. 8. Simulation of a profiling z_{wij} coordinates(relative depths) of a red line

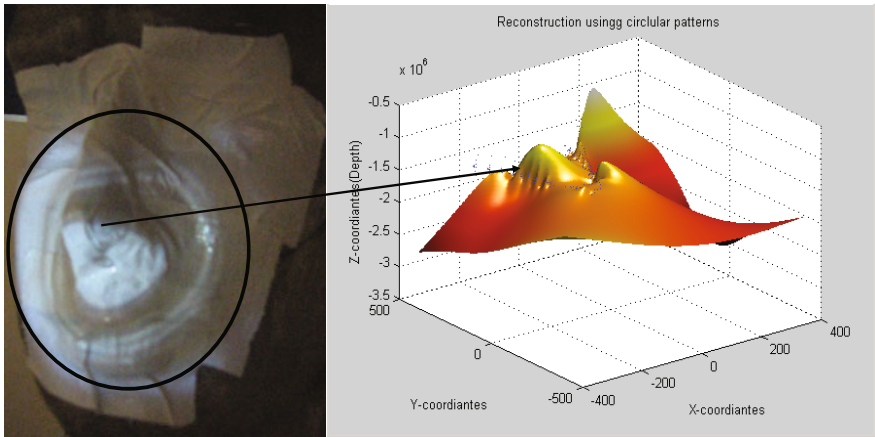


Fig. 9. Reconstructed 3D face from a measurement of 3D coordinates (x_w, y_w, z_w)

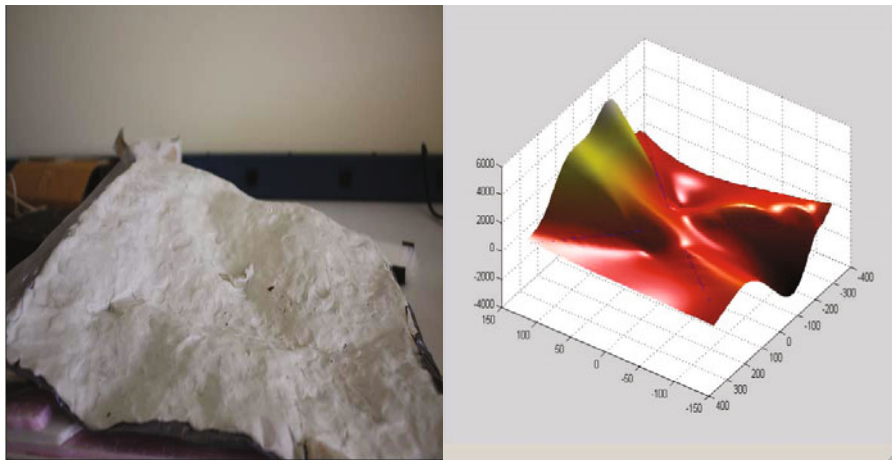


Fig. 10. Reconstruction example of a terrain model

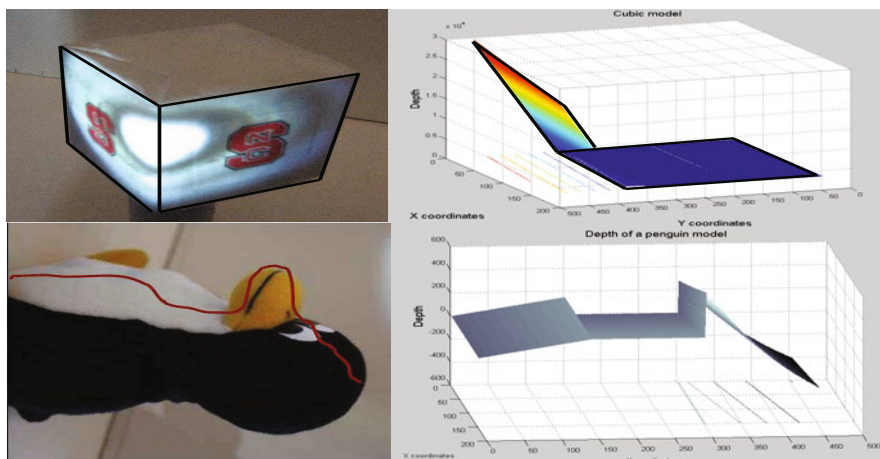


Fig. 11. Reconstruction examples of a cubic and a penguin model(depth recovery)

5 Conclusion and Further Works

In this paper we have presented an algorithm for 3D image reconstruction using circular structured light patterns. However our purpose in this paper is to emphasize the theoretical proof of the reconstruction procedure. 3D image reconstruction is of great importance to many fields of image processing : surveillance and inspection, and the proposed algorithm can be used for reconstructions of an unknown object. 3D coordinates (x_w, y_w, z_w) were computed using a relationship between ideal and deformed circular patterns. The example reconstructions are preliminary results and there are some technical issues to be resolved in

future work. The first issue is the projection : an LED light source and a spatial light modulator are used to project a light pattern onto the 3D surface, and the accuracy of the reconstruction results have not been examined and this is also an important aspect for continued work. In this paper, low resolution of a structured light pattern and manually extracted 2D data points result in low quality of reconstruction results. From a practical perspective, we need a better projection system to increase resolution leading to high quality of reconstruction results. The better projection system also plays a very important role in acquisition of 2D data points in imaged patterns. The second one is the calibration of the camera : in this paper, we assume that the camera is calibrated(i.e. constant intrinsic parameters) to make a possible mathematical model for a measurement setup and a geometrical reconstruction. Calibration is also one of the most important task in an active stereo vision and a lot of works have been presented ([14], [15], [16], [17]). Circular patterns provide important information and algorithm efficiency for 3D reconstruction problems, in particular when the objects are generic surfaces. Especially the characteristics of circular patterns(Section. 1) contributes an improvement in the mathematical model for the reconstruction process and computation efficiency. Quantifying the algorithm efficiency (i.e. computation times) compared to other method is also required in the future. Furthermore, with high sampling density (i.e. a great number of circular patterns), the reconstruction result is extremely accurate. Provided this accurate result, we can calculate the minimum sampling rate(minimum number of circular patterns) to successfully reconstruct the 3D surface. In the future, we are planning to determine the minimum sampling rate for a generic 3D signal, and the algorithm proposed in this paper is promising as an initial step toward the sampling rate. In the future, our goal is determining a sampling criterion specifying the necessary number of circular patterns required for 3D reconstruction and this work is in progress.

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