

Design and Implementation of Proportional Integral Observer based Linear Model Predictive Controller

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Abstract—This paper presents an interior-point method (IPM) based quadratic programming (QP) solver for the solution of optimal control problem in linear model predictive control (MPC). LU factorization is used to solve the system of linear equations efficiently at each iteration of IPM, which renders faster execution of QP solver. The controller requires internal states of the system. To address this issue, a Proportional Integral Observer (PIO) is designed, which estimates the state vector, as well as the uncertainties in an integrated manner. MPC uses the states estimated by PIO, and the effect of uncertainty is compensated by augmenting MPC with PIO-estimated uncertainties and external disturbances. The approach is demonstrated practically by applying MPC to QET DC servomotor for position control application. The proposed method is compared with classical control strategy-PID control.

Index Terms—Model Predictive Control, Interior-Point Method, LU factorization, DC Servomotor, Proportional Integral Observer.

I. INTRODUCTION

This paper considers Interior-point algorithm used on-line with Linear Model Predictive Control (MPC). Linear MPC assumes a linear system model, linear inequality constraints and a convex quadratic cost function [1]. MPC can be formulated as a quadratic programming (QP) problem and solved at each sampling interval. It thus has the natural ability to handle physical constraints arising in industrial applications [2].

At each sampling instance MPC solves an online QP optimization problem, computes the sequence of optimal current and future control inputs by minimizing the difference between set-points and future outputs predicted from a given plant model over a finite horizon in forward time. Then, only the current optimal input is applied to the plant. The updated plant information is used to formulate and solve a new optimal control problem at the next sampling instance. This procedure is repeated at the each sampling instance and the concept is called as receding horizon control MPC strategy. Since this quadratic program can be large depending upon control problem, MPC requires long computation times at each sampling instants, therefore it is usually restricted to systems with slow dynamics and large sampling intervals, such as chemical processes [3]. The ability to solve the QP problem online become critical when applying MPC to systems with

fast response time and/or embedded applications where computational resource may be limited [4].

Recently, many reports in the literature address applying MPC to control applications with short sampling intervals, by adapting fast optimization algorithms. These algorithms solve the resulting QP by exploiting the special structure of the control problem the MPC sub problem. IPM approaches the solution of Karush-Kuhn-Tucker (KKT) equations by successive decent steps. Each decent step is Newton's like step and the solution is obtained by solving system of linear equations using appropriate numerical methods in order to determine search direction. Matrix factorization approach provides a means to simplify the computation involved in linear solver. Different matrix factorization methods are used such as Gauss elimination, QR, LU, and Cholesky [3], [5], [6].

Model predictive control has received considerable attention driven largely by its ability to handle hard constraints. An inherent problem is that model predictive control normally requires full knowledge of the internal states. In the physical implementation of control strategy based on the system states, if the states are not available for measurement then state feedback will not be the implementable.

The problem of robust control system design with an observer based on the knowledge of input and output of the plant has attracted many researchers now-a-days [7], [8], [9], [10], [11], [12], [20]. The observer proves to be useful in not only system estimation and regulation but also identifying failures in dynamic systems. This requirement of obtaining the estimates of uncertainty, as well as state vector in an integrated manner is fulfilled by PIO. PIO approach basically works when the system uncertainty varies slowly with respect to time. It is an observer in which an additional term, which is proportional to the integral of the output estimation error, is added in order to estimate the uncertainties and improve the system performance [7], [8], [9].

In this paper, we present an interior-point algorithm to solve MPC problem, which utilizes Mehrotra's predictor-corrector algorithm [13], and the linear system at each IPM iteration is solved by LU factorization based Linear Solver. PIO is designed for state and uncertainties estimation. The remainder of the paper is organized as follows: section II describes MPC problem formulation. Section III presents the interior-point method. In section IV, the PIO theory is briefly

reviewed and applied for state and uncertainty estimation. Mathematical modeling of DC servomotor is derived in Section V. IPM QP solver results, the effectiveness of observer based MPC scheme with simulation results is presented in section VI. MPC implementation on Quanser QET DC servomotor is presented in section VII.

II. MPC PROBLEM FORMULATION

A discrete time linear time-invariant model of a system in a state space form is given as,

$$\left. \begin{aligned} x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \right\} \quad (1)$$

where, $y(t) \in \mathbb{R}^m$ are output vector, $u(t) \in \mathbb{R}^l$ are input vector and $x(t) \in \mathbb{R}^n$ are internal states vector.

A and B are system parameter matrices. Given a predicted input sequence, the corresponding sequence of state predictions is generated by simulating the model forward over the prediction horizon (N_p) intervals.

$$\left. \begin{aligned} x(t+2) &= Ax(t+1) + Bu(t+1) \\ x(t+2) &= A^2x(t) + ABu(t) + Bu(t+1) \end{aligned} \right\} \quad (2)$$

$$x(t+N_p) = A^{N_p}x(t) + \sum_{j=0}^{N_p-1} (A^{N_p-1-j}B)u(j). \quad (3)$$

The prediction model is given by,

$$\hat{y}(t+N_p) = Cx(t+N_p). \quad (4)$$

Putting the value of $x(t+N_p)$ in (4), we get,

$$\hat{y}(t+N_p) = C[A^{N_p}x(0) + \sum_{j=0}^{N_p-1} (A^{N_p-1-j}B)u(j)]. \quad (5)$$

In matrix form (5) is written as,

$$Y = \Phi x(0) + \Gamma U. \quad (6)$$

with,

$$Y = [\hat{y}(t+1) \quad \hat{y}(t+2) \quad \dots \quad \hat{y}(t+N_p)]^T,$$

$$U = [u(t) \quad u(t+1) \quad \dots \quad u(t+N_u-1)]^T,$$

$$\Phi = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_p} \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} CB & 0 & \dots & 0 \\ CAB & CB & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ CA^{N_p-1}B & CA^{N_p-2}B & \dots & CA^{N_p-N_u}B \end{bmatrix}$$

where, N_u is control horizon. The goal of the controller is to make the difference between the output, $\hat{y}(t+N_p)$ and the reference, $y_{ref}(t+N_p)$ as small as possible. This can be defined by using a standard least squares problem. The objective function is defined as,

$$J = \frac{1}{2} \sum_{i=1}^{N_p} \|\hat{y}(t+i) - y_{ref}(t+i)\|_Q^2 + \frac{1}{2} \sum_{i=1}^{N_u-1} \|\Delta u(t+i)\|_R^2. \quad (7)$$

where, $\Delta u(t+i) = u(t+i) - u(t+i-1)$

subjected to linear inequality constraints on system inputs,

$$u_{\min} \leq u(t) \leq u_{\max}$$

where, y_{ref} is the set-point and while Q and R are the positive definite weight matrices [14]. To get optimal solution, the MPC problem can be formulated as a standard quadratic programming (QP) problem as,

$$J(U) = \left\{ \frac{1}{2} U^T H U + f^T U \right\} \text{subjected to } AU - b \leq 0. \quad (8)$$

where, H is $(lN_u \times lN_u)$ Hessian matrix, f is $(lN_u \times 1)$ column vector [14], [16].

III. INTERIOR-POINT METHOD

Consider the following standard QP problem,

$$\min_x \frac{1}{2} x^T Q x + C^T x \text{ subjected to } Ax - b = 0, Lx - k \leq 0. \quad (9)$$

Except equality constraints, (9) resembles (8). The KKT Conditions for optimality are given by,

$$\left. \begin{aligned} Qx + A^T y + L^T z &= -C \\ Ax - b &= 0 \\ Lx + s - k &= 0 \\ z^T s &= 0 \\ z, s &> 0 \end{aligned} \right\} \quad (10)$$

where, y and z represents the Lagrange multipliers for equality and inequality constraints respectively. The slack variable s is introduced for converting inequality into equality. By applying Newton's method, search direction $(\Delta x, \Delta y, \Delta z, \Delta s)$ is obtained as shown in (11), where $J(x, y, z, s)$ denotes Jacobian of $f(x, y, z, s)$,

$$J(x, y, z, s) \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta s \end{bmatrix} = - \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix}. \quad (11)$$

Considering only inequality constraints, augmented form of original system is given as,

$$\begin{bmatrix} Q & L^T \\ L & -Z^{-1}S \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta z \end{bmatrix} = - \begin{bmatrix} r_1 \\ r_3 - Z^{-1}r_4 \end{bmatrix}, \quad (12)$$

$$\Delta s = Z^{-1}(r_4 - S\Delta z). \quad (13)$$

We use Mehrotra's Predictor-corrector algorithm [13] to solve (12) and (13) for $(\Delta x, \Delta z, \Delta s)$. Predictor or affine and corrector-centering steps uses (r_1, r_3, r_4) values from (14) and (15) respectively to form the right-hand side of (12) and (13) which in turn finds $(\Delta x^{aff}, \Delta z^{aff}, \Delta s^{aff})$ and $(\Delta x^{cc}, \Delta z^{cc}, \Delta s^{cc})$ [1], [15], [17], [18], [19].

$$\begin{bmatrix} r_1 \\ r_3 \\ r_4 \end{bmatrix} = - \begin{bmatrix} Qx + C + L^T z \\ Lx + s - k \\ ZSe \end{bmatrix}. \quad (14)$$

$$\begin{bmatrix} r_1 \\ r_3 \\ r_4 \end{bmatrix} = - \begin{bmatrix} 0 \\ 0 \\ \text{diag}(\Delta z_{aff}) \text{diag}(\Delta s_{aff}) e - \sigma \mu e \end{bmatrix}. \quad (15)$$

where, σ is centering parameter and μ is complementarity measure or duality gap. Thus, solution of linear system in (13) is to be calculated twice at each iteration of IPM, which puts major computational load on overall performance of IPM, and eventually on MPC. Linear system in (13) uses two different right-hand side components but the same coefficient matrix, so just one factorization of this matrix is required per iteration. Such system can be solved using Gauss elimination, QR, LU, Cholesky factorization methods.

The coefficient matrix in a linear system (13) involved in IPM is symmetric and indefinite so conventional and more numerically stable Cholesky factorization cannot be used to solve the linear system. Compared to computational cost of QR decomposition ($2/3n^3$) and the problem of associated pivoting with ill condition of Gauss elimination, LU factorization proves to be more effective and accurate to solve system of linear equations [5], [6]. We propose LU factorization based Linear Solver for the same. The pseudo code of LU factorization Linear Solver for (13) is given as follows:

Pseudo Code-LU factorization

$$A = \begin{bmatrix} A_{11} & A_{12}; & A_{21} & A_{22} \end{bmatrix};$$

$$\text{where, } A_{11} = Q; A_{12} = L^T; A_{22} = L; A_{21} = -Z^{-1}S$$

$$B = \begin{bmatrix} B_{11}; & B_{22} \end{bmatrix};$$

$$\text{where, } B_{11} = r_1, B_{22} = (r_3 - Z^{-1}r_4);$$

$$A = LU;$$

Solve $LY = B$; //forward substitution

$$UX = Y; //backward substitution$$

$$X = \begin{bmatrix} X_1; & X_2 \end{bmatrix};$$

Interior-point method(IPM) algorithm

Initialization and input

$$X_0 = \begin{bmatrix} 0 & 0 & \dots & 0 \end{bmatrix}^T;$$

$$Z_0 = \begin{bmatrix} 1 & 1 & \dots & 1 \end{bmatrix}^T; S_0 = \begin{bmatrix} 1 & 1 & \dots & 1 \end{bmatrix}^T; Z_0, S_0 \succ 0;$$

$$e = \begin{bmatrix} 1 & 1 & \dots & 1 \end{bmatrix}^T; \alpha = 0.25$$

Input Q, C, L, k matrices from given QP problem

$$\mu = \frac{Z_0^T S_0}{m}; \text{ m- no. of constraints}$$

Start the loop and terminate if stopping criterion are satisfied

Predictor or affine step

$$S = \text{diag}(S_0); Z = \text{diag}(Z_0);$$

$$r_{1aff} = QX_0 + C + L^T Z_0; \quad r_{3aff} = LX_0 + S_0 - k;$$

$$r_{4aff} = ZSe;$$

Solve (13), (12) by linear solver and compute

$$\Delta x^{aff}, \Delta z^{aff}, \Delta s^{aff}$$

Compute centring parameter σ

$$\mu_{aff} = \frac{(Z_0 + \alpha \Delta z_{aff})^T (S_0 + \alpha \Delta s_{aff})}{m};$$

$$\sigma = \left(\frac{\mu_{aff}}{\mu} \right)^3;$$

Corrector and centring step to obtain search direction

$$Z_{aff} = \text{diag}(\Delta z_{aff}); S_{aff} = \text{diag}(\Delta s_{aff});$$

$$r_{1cc} = 0; \quad r_{3cc} = 0; \quad r_{4aff} = Z_{aff} S_{aff} e - \sigma \mu e;$$

Solve (13), (12) by linear solver and compute

$$\Delta x^{cc}, \Delta z^{cc}, \Delta s^{cc}$$

Update $(\Delta x, \Delta z, \Delta s)$ as

$$(\Delta x, \Delta z, \Delta s) = (\Delta x^{aff}, \Delta z^{aff}, \Delta s^{aff}) + (\Delta x^{cc}, \Delta z^{cc}, \Delta s^{cc})$$

Update X_0, Z_0, S_0 as

$$X_0 = X_0 + \alpha \Delta x; \quad Z_0 = Z_0 + \alpha \Delta z; \quad S_0 = S_0 + \alpha \Delta s;$$

End of loop

IV. OBSERVER DESIGN

The implementation of MPC requires the measurements of all the states. As only an encoder is used for measurement of the motor shaft position in practice. PIO is designed in this section for present application.

Consider a linear time-invariant system

$$\left. \begin{aligned} \tilde{x}(t+1) &= A\tilde{x}(t) + Bu(t) + L(y(t) - \hat{y}(t)) + B\tilde{e}(t) \\ \hat{y}(t) &= C\hat{x}(t) \\ \hat{e}(t+1) &= \hat{e}(t) + M(y(t) - \hat{y}(t)) \end{aligned} \right\} \quad (16)$$

where $\tilde{x}(t)$ is the estimated state vector and $\tilde{e}(t)$ is the estimation of uncertainties of p-dimensional vector. L proportional and M integral observer gain matrices of appropriate dimension.

The system described by Eq. (16) is said to be a full-order PI observer for system (1) if and only if

$$\left. \begin{aligned} \lim_{t \rightarrow \infty} \tilde{x}(t) &= 0 \\ \lim_{t \rightarrow \infty} \tilde{e}(t) &= 0 \end{aligned} \right\} \quad (17)$$

where, $\tilde{x}(t) = x(t) - \hat{x}(t)$ represents the observer's state estimation error and $\tilde{e}(t) = e(t) - \hat{e}(t)$ represents the error in uncertainty estimation.

We are assuming the following for derivation.

1. Matrices A, B, C are known. All uncertainties associated with the matrices A and B are lumped into $e(t)$.
2. The pair (A, B) is controllable.
3. The pair (A, C) is observable.

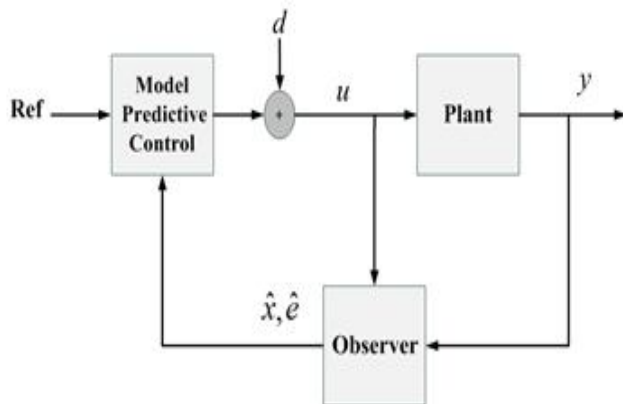


Figure 1. Generalized block diagram of observer based MPC system From (1) and (16), the dynamic of the observer error becomes

$$\tilde{x}(t+1) = (A - LC)\tilde{x}(t) + B\tilde{e}(t). \quad (18)$$

which together with (16) results in;

$$\begin{bmatrix} \tilde{x}(t+1) \\ \tilde{e}(t+1) \end{bmatrix} = \begin{bmatrix} A - LC & B \\ -MC & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \tilde{e}(t) \end{bmatrix}. \quad (19)$$

Simplifying (13)

$$\begin{bmatrix} \tilde{x}(t+1) \\ \tilde{e}(t+1) \end{bmatrix} = \left(\begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} L \\ M \end{bmatrix} \begin{bmatrix} C & 0 \end{bmatrix} \right) \begin{bmatrix} \tilde{x}(t) \\ \tilde{e}(t) \end{bmatrix}. \quad (20)$$

It follows that $\hat{x}(t)$ is an estimate of $x(t)$ if and only if,

$\Re \lambda_q [R] < 0$ for $q = 1, \dots, (n + p)$. The generalized block diagram of the observer based MPC system is shown in Fig. 1.

V. MATHEMATICAL MODELLING OF DC SERVOMOTOR

For mathematical model derivation of DC servomotor, the electrical & torque characteristic equations are formulated.

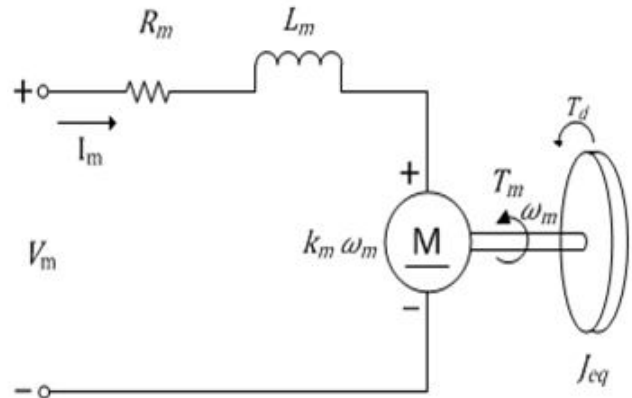


Figure 2. Schematic diagram of DC Servomotor

Fig. 2, shows the schematic diagram of armature controlled DC servomotor. Using Kirchoff's voltage law (KVL),

$$V_m = R_m I_m + L_m \left(\frac{\partial}{\partial t} \right) I_m + k_m \Omega_m. \quad (21)$$

where, k_m is the back-emf constant, R_m is the motor winding resistance and L_m is the motor winding inductance, I_m is the current through the motor winding.

TABLE I. NOMINAL DC SERVOMOTOR PARAMETERS [21]

Description	Symbol	Value	Unit
Motor electric time constant	τ_e	$7.74 * 10^{-5}$	s
Moment of inertia	J_{eq}	$2.21 * 10^{-5}$	$Kg.m^2$
Motor maximum velocity	u_{max}	298.8	rad/s
Motor maximum current	I_{max}	1.42	A
Motor maximum torque	T_{max}	0.068	N.m
Motor torque constant	k_m	0.0502	N.m/A
Motor armature resistance	R_m	10.6	Ohm
Motor armature inductance	L_m	0.82	mH
Open loop time constant	T	0.0929	s
Open loop steady state gain	K	19.9	rad/(V.s)

Representing (21) in Laplace domain as follows:

$$R_m I_m + L_m I_m s = V_m - k_m \Omega_m. \quad (22)$$

The armature electrical time constant of system is represented as,

$$\tau_e = \frac{L_m}{R_m}. \quad (23)$$

Expressing the torque equation into the Laplace domain,

$$J_{eq} \Omega_m s = k_m I_m + T_d. \quad (24)$$

Combining (21) and (23),

$$J_{eq} \Omega_m (s) s + \frac{k_m^2 \Omega_m (s)}{R_m} = \frac{k_m^2 V_m (s)}{R_m} + T_d (s). \quad (25)$$

Transfer function from motor shaft position to the motor input voltage can be given as,

$$G_m (s) = \frac{\theta (s)}{V_m (s)}. \quad (26)$$

The plant system parameters can be expressed as,

$$G_m (s) = \frac{k_m}{R_m \left(J_{eq} s + \frac{k_m^2}{R_m} \right)}. \quad (27)$$

Simplifying the above equation further,

$$G_m (s) = \frac{b}{s(s+a)} = \frac{K}{s(s+\tau)}. \quad (28)$$

where, $a = \frac{k_m^2}{J_{eq} R_m}$ and $b = \frac{k_m}{J_{eq} R_m}$

k_m is the steady state gain and τ is the time constant of the system.

Converting the transfer function into the state space form,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -1/\tau \end{bmatrix}, B = \begin{bmatrix} 0 \\ K/\tau \end{bmatrix}, C = [1 \quad 0], D = [0] \quad (29)$$

VI. MATLAB SIMULATION

A. QP Solver

The proposed algorithm is coded in MATLAB. Fig. 3, shows per iteration computation time comparison of proposed method with QR factorization and MATLAB's QP solver *quadprog*.

B. MPC Implimentation

The designed MPC with PIO is tested through experimentation for position control application of a DC

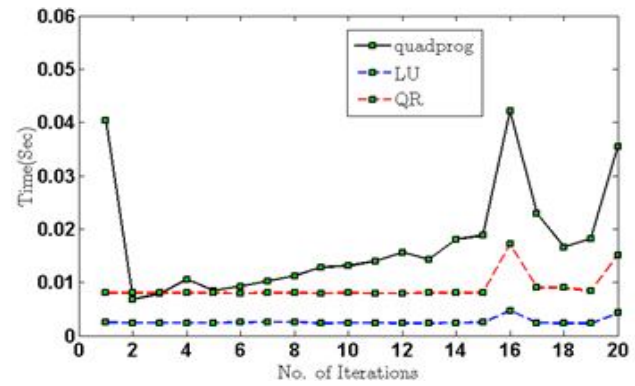


Figure 3. Per iteration computation time comparison of QP solving methods

servomotor.

Substituting the parameter values from Table I in (29), the state space representation of DC servomotor model can be obtained as,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 210 \end{bmatrix}, C = [1 \quad 0], D = [0] \quad (30)$$

QET DC servomotor model as given in (30) was considered for the MATLAB simulation of MPC. The simulation results of the proposed Observer based MPC and high pass filter (HPF) based MPC are shown in Fig. 4, Fig. 5, and Fig. 6.

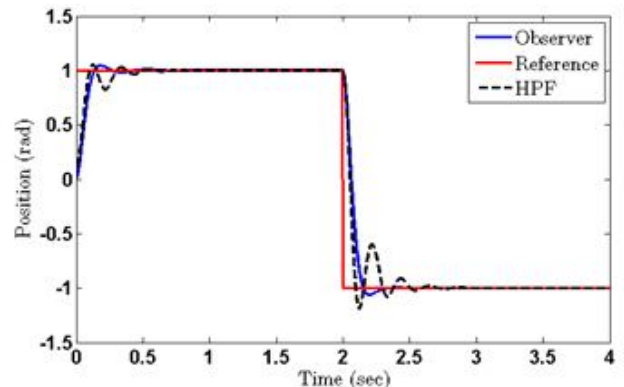


Figure 4. Comparative plot of position response

From the results it is observed that observer does better state estimation compared to HPF in transient time. Due to better state estimation of actual states, control input is conservative and states have less oscillation in the transient response.

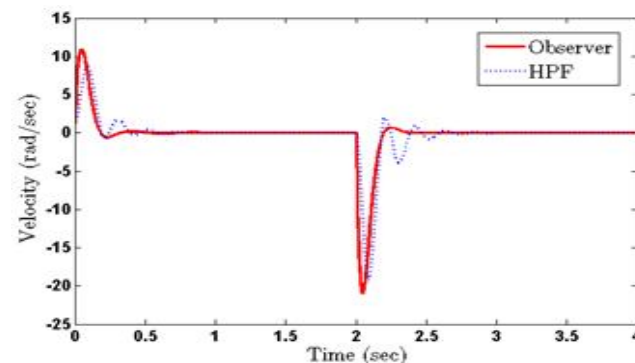


Figure 5. Comparative plot of second state

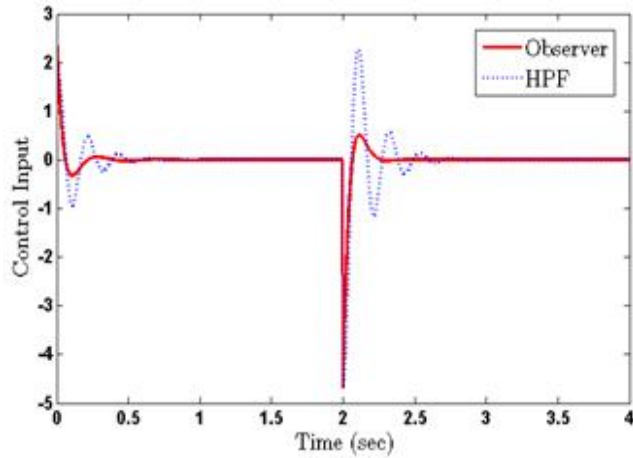


Figure 6. Comparative plot of control input

VII. HARDWARE IMPLEMENTATION

Efficacy of the proposed approach is demonstrated practically on Quanser QET DC servomotor plant with 0.01s sample time.

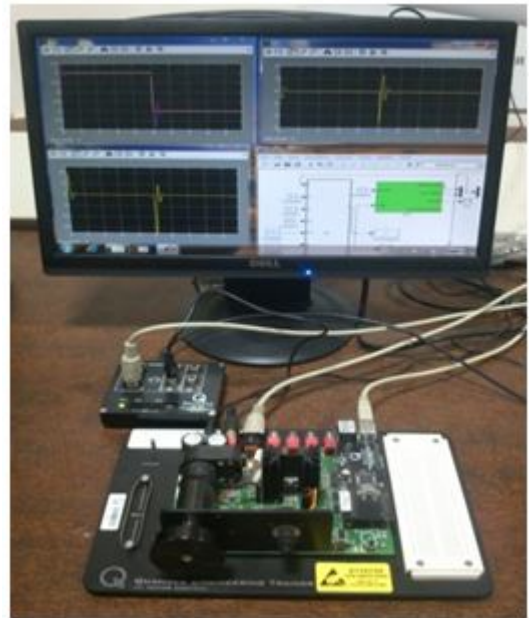


Figure 8. Quanser QET DC servomotor setup

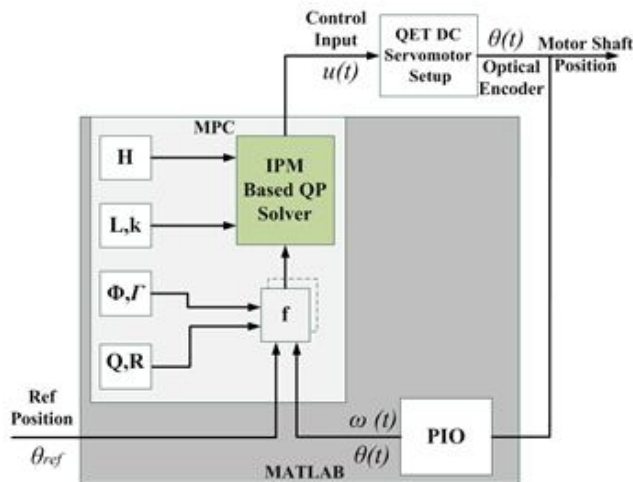


Figure 7. Real-time MPC Implementation strategy for position control of QET DC servomotor

The experimental setup has DC servomotor with Q2 USB based data acquisition card and a PC equipped with proprietary QuaRc 2.1 software. QuaRc provides hardware-in-loop simulation environment [21]. Motor shaft position $\theta(t)$ is measured by an encoder. To provide state estimation of plant, PIO is developed. The proposed real-time MPC implementation strategy and the experimental setup are shown in Fig. 7 and Fig. 8 respectively.

The performance of designed observer based MPC scheme is compared with classical controller i.e. PID controller. Experimentation has been carried out with constant load to DC servomotor as an external disturbance. Fig. 9, shows the plot of position response using PID controller and MPC. It shows that proposed MPC scheme has better transient response compare to classical controller.

Plot of control input of MPC and PID controller is shown in Fig. 10. Control input of MPC is conservative as compare to PID, which can be clearly observed from the plot.

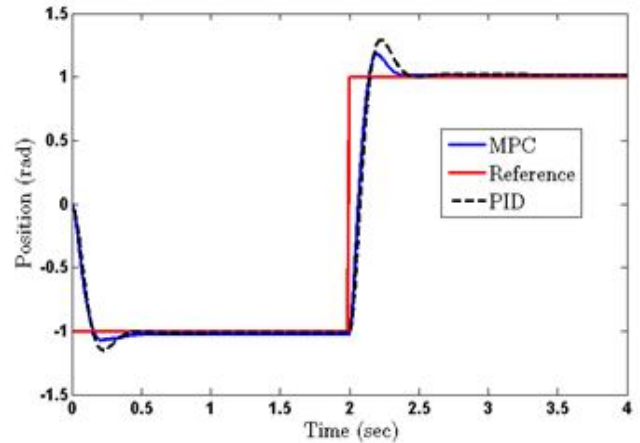


Figure 9. Comparative plot of position response using PID controller and MPC

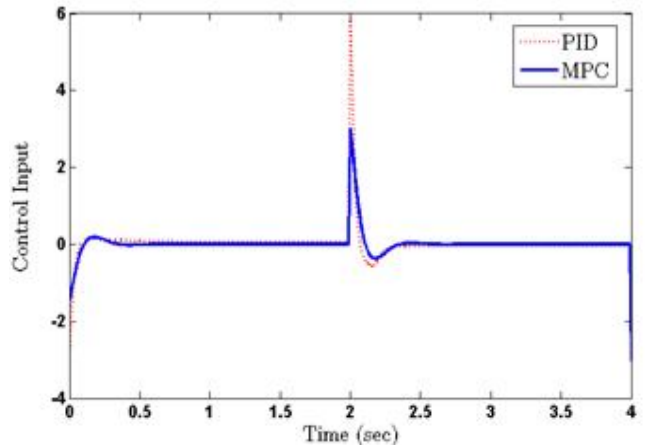


Figure 10. Comparative plot of control input of PID controller and MPC

Fig. 11, shows the position responses of the plant using two different prediction horizon with the control horizon kept constant at $N_u = 3$. From Fig. 12, it is observed that

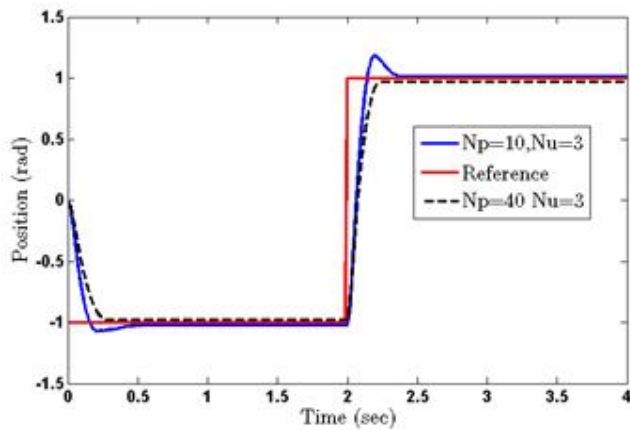


Figure 11. Plant response to reference position for different prediction horizon

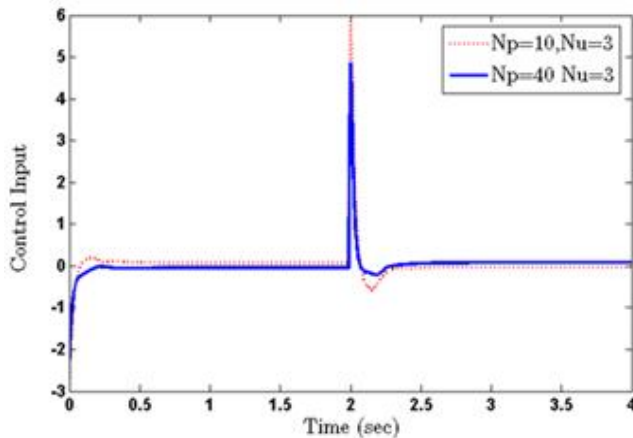


Figure 12. Control input for different prediction horizon

$N_p = 10$ results in a shorter rise time and a shorter settling time in the position response than $N_p = 40$. Thus the responses are faster while decreasing the Prediction Horizon N_p . Fig. 12, shows that this improvement is achieved at the cost of larger control signal.

CONCLUSIONS

In this paper, LU factorization based linear solver for solution of IPM is presented, which is used for real-time implementation of constrained linear model predictive controller for position control of DC servomotor. The issue of unavailability of internal state vector for implementation of MPC is addressed by designing a PIO. Simulation results are presented to demonstrate the effectiveness of PIO. Simulation result shows the superior performance of observer to estimate the state vector as well uncertainty compared to HPF. From experimentation results it is observed that, the proposed scheme solves QP problem efficiently within the specified sample period and performs well for tracking a reference position of DC servomotor which is relatively a fast dynamic system. Performance of the designed MPC has been compared with PID control via experimental results. It is also observed that, the controller is robust to handle parameter uncertainties and load disturbances.

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