An EPQ model for deteriorating items with variable demand rate and allowable shortages

Deepa Khurana*

Department of Mathematics,
Banasthali University,
Banasthali, Rajasthan, India
Email: Khurana.deepa12@gmail.com

Shilpy Tayal* and S.R. Singh

Department of Mathematics,
C.C.S. University,
Meerut, India
Email: agarwal_shilpy83@yahoo.com
Email: shivrajpundir@gmail.com
*Corresponding authors

Abstract: The fundamental assumption of an economic order quantity (EOQ) model is that 100% of items in an ordered lot are perfect. This assumption is not always pertinent for production processes because of process deterioration or other factors. This is an economic production quantity model for deteriorating items. To fulfil the market demands and expectations, the production rate is taken as a function of demand rate. The demand and deterioration of the products are time dependent function. Shortages are allowed and partially backlogged. The backlogging phenomenon in the literature is often modelled using backordering and lost sale costs. The backlogging option gets used only when it is economic to do so. The inventory policy proposed here considers the optimal production run time, production quantity and shortage period such that the total average cost can be minimised. Numerical examples are provided to illustrate and sensitivity analyses of optimal solutions are given for the proposed inventory model.

Keywords: deterioration; shortages; variable production rate; economical production quantity; EPQ; partial backlogging.

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Biographical notes: Deepa Khurana is a Research Scholar of Banasthali Vidhyapeeth Banasthali, Rajasthan. She has published a few papers in international journals. She is working in the field of operation research and inventory modelling.

Shilpy Tayal is working in the field of inventory modelling. She published so many research articles in various national and international journals and dynamically working in the field of operational research, inventory modelling and supply chain management.

S.R. Singh is working as an Associate Professor in theDepartment of Mathematics, C.C.S. University, Meerut, UP, India. He is dynamically working in the field of operations research, inventory control, two-warehouse inventory models and supply chain management.

1 Introduction

An economical production quantity (EPQ) model is that inventory control model which determines the optimal production amount of a product, to meet a deterministic demand over an infinite planning horizon at the lowest possible price. Industry changes over the past several decades have been driven by several factors including production, demand, deterioration, and shortages. One of the early works related to JELP was due to Goyal (1977). He assumed the infinite production rate and suggested a solution to the problem for the vendor and lot-for-lot policy for the shipments from the vendor to the buyer. According to it, complete production lot is shipped into a single shipment. After that Goyal (1988) assumed that the production lot is shipped in a number of equal size shipments, but only after the entire lot is produced. Hill (1999) originates the optimal shipment and production policy for the solitary vendor only buyer integrated production inventory problem. Rahman and Sarker (2007) investigated the material procurement and delivery policy in a production system where raw materials enter the assembly line from two different flow channels. Sarkar and Moon (2011) developed a production-inventory model considering stochastic demand with the effect of inflation. Singh et al. (2012) presented an economic production inventory model with rework and flexibility. Tayal et al. (2014c) considered a production inventory problem with space restriction and allowable shortages. In this the extra ordered quantity is returned to the supplier with a penalty cost. Tayal et al. (2016) presented an integrated production inventory model for perishable products. The rate of deterioration is taken as a linear function of time. This paper is developed with permissible delay in payment.

Deterioration and shortages are that concept, which play a very important role in the development of an inventory model. The rate of deterioration cannot be neglected and it is necessary to consider the product's rate of deterioration in the determination of lot size of the products. Wee (1995) developed a joint pricing and replenishment policy for inventory with a constant deterioration rate. Hsu et al. (2006) developed a deteriorating inventory model for season pattern demand with expiration date. Singh and Singh (2011) presented an integrated supply chain model for deteriorating items under imprecise environment. Tayal et al. (2014b) presented a multi item inventory model for deteriorating and expiry products with partial backlogging. In this model the occurring shortages are partially backlogged and the rate of backlogging depends on the waiting time up to the arrival of next lot. Tayal et al. (2015) developed an inventory model for non-instantaneous deteriorating items. In this model partially deteriorated items are allowed to sale with a discount rate from original one, and the units which are completely deteriorated, are superfluous. Tayal et al. (2014a) introduced an inventory model for deteriorating seasonal products with the option of an alternate market and allowable shortages. In this the occurring shortages are partially backlogged and backlogging rate depends on the waiting time. Tayal et al. (2014d) introduced a two echelon supply chain model for deteriorating items with effective investment in preservation technology. In this a preservation technology cost is incurred to reduce the product's rate of deterioration. Shortages are allowed and partially backlogged in this model.

In 2014, Zhang et al. presented artificial intelligence (AI). AI has revolutionised information technology. The new economy of information technology has shaped the way we are living. Recently, AI algorithms have attracted close attention of researchers and have also been applied successfully to solve problems in engineering. There is a potential requirement to develop efficient algorithm to find solutions under the limited resources, time, and money in real world applications. This special issue aims to report the latest advances in every aspect of AI technology, including machine learning, data mining, computer vision, multi agent systems, evolutionary computation, and fuzzy logic. It is further extended in 2016. Singh et al. (2016) presented an economic order quantity (EOQ) model for deteriorating products having stock dependent demand with trade credit period and preservation technology. Khurana and Chaudhary (2016) presented an optimal pricing and ordering policy for deteriorating items with price and stock dependent demand and partial backlogging.

As a result, market prospect around the world have increased significantly, as customers expect higher customisation. So to complete the customer's demand it is necessary to consider the production rate as a function of occurring demand. Through the present paper, we have discussed a production inventory model for deteriorating products over a finite planning horizon in which production rate is demand rate dependent. The shortages are allowed and partially backlogged at a constant rate.

2 Assumptions

1. the demand for the products is time dependent and linear in nature i.e., $D = a + bt$
2. the production rate is taken as a function of demand rate and is defined as $P = \alpha (a + bt)$
3. the product deterioration rate is a function of time t and given by Kt
4. shortages are allowed and partially backlogged
5. the warehouse has unlimited capacity
6. the deteriorated items are completely discarded.

3 Notations

$I(t)$	inventory level at any time t
A	set up cost per production run
H	holding cost per unit
K	deterioration coefficient, $K > 0$
α	production coefficient, $\alpha > 1$
a	initial demand rate

b demand coefficient

M maximum inventory level

T Duration of inventory cycle

v time at which inventory level becomes zero

t_1 time up to production occurs

c_d deterioration cost per unit

m production cost per unit

s shortage cost per unit

l lost sale cost per unit

θ rate of backlogging.

4 Mathematical modelling

In the model show in the Figure 1 the production starts at $t = 0$ and it is continued up to $t = t_1$ after satisfying the occurring demand and deterioration. During the time period $[t_1, v]$ the inventory level depletes due to the effect of demand and deterioration only and after that shortages occur. A differential equation always shows the change in one parameter with respect to another parameter. Here in this model we presented the change in inventory level with respect to time. So the differential equations leading the transition as per the system are given below:

$$\frac{dI_1(t)}{dt} = P - D - KtI_1(t), \quad P = a + bt, \quad D = a(a + bt)$$

$$\frac{dI_1(t)}{dt} + KtI(t) = (\alpha - 1)(a + bt) \quad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} + KtI(t) = -(a + bt) \quad t_1 \le t \le v \tag{2}$$

The boundary conditions associated with these equations are:

$$I_1(0) = 0, \quad I_2(t_1) = M \tag{3}$$

Solutions of these equations are given by:

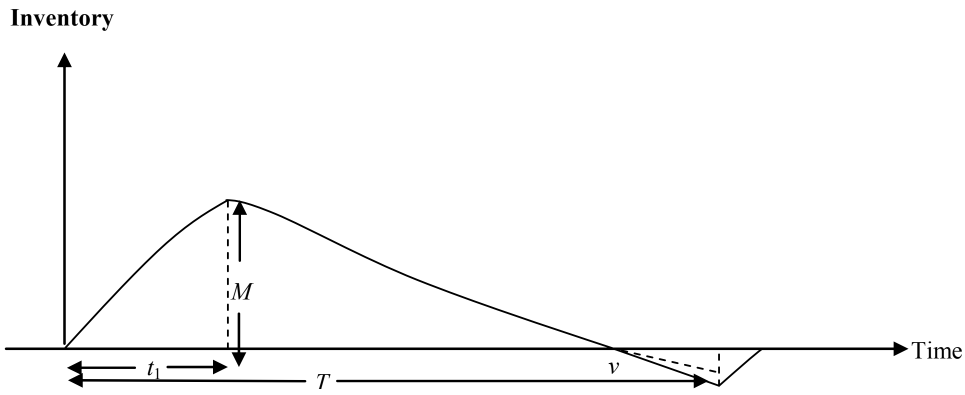
$$I_1(t) = (\alpha - 1)\left(at + \frac{bt^2}{2} + \frac{Kat^3}{6} + \frac{Kbt^4}{8}\right)e^{-\frac{Kt^2}{2}} \quad 0 \le t \le t_1 \tag{4}$$

$$I_2(t) = \left\{\begin{array}{l} Me^{\frac{Kt_1^2}{2}} + a(t_1 - t) + \frac{b}{2}\left(t_1^2 - t^2\right) \\ + \frac{aK}{6}\left(t_1^3 - t^3\right) + \frac{bK}{8}\left(t_1^4 - t^4\right) \end{array}\right\} e^{-\frac{Kt^2}{2}} \quad t_1 \le t \le v \tag{5}$$

From equation (5) we know that $I_2(v) = 0$. Then:

$$M = \left\{ a(v - t_1) + \frac{b}{2}(v^2 - t_1^2) + \frac{aK}{6}(v^3 - t_1^3) + \frac{bK}{8}(v^4 - t_1^4) \right\} e^{-\frac{Kt_1^2}{2}} \tag{6}$$

Figure 1 Representation of inventory level



5 Cost analysis

- *Set up cost:*

  The set up cost associated with the model is given by:

  $$\text{Setup cost} = A \tag{7}$$

- *Carrying cost:*

  The associated holding cost is given by:

  $$\begin{aligned} H.C. = h\left\{ \int_0^{t_1} I_1(t)dt + \int_{t_1}^{v} I_2(t)dt \right\} &= h\Bigg[(\alpha - 1)\left(\frac{at_1^2}{2} + \frac{bt_1^3}{6} - \frac{Ka}{12}t_1^4 - \frac{Kb}{40}t_1^5 \right) \\ &+ \Bigg\{ Mve^{\frac{Kt_1^2}{2}} + a\left(t_1 v - \frac{v^2}{2} \right) + \frac{b}{2}\left(t_1^2 v - \frac{v^3}{3} \right) + \frac{aK}{6}\left(t_1^3 v - \frac{v^4}{4} \right) \\ &+ \frac{bK}{8}\left(t_1^4 v - \frac{v^5}{5} \right) - \frac{MKv^3}{6} e^{\frac{Kt_1^2}{2}} - \frac{Ka}{2}\left(t_1 \frac{v^3}{3} - \frac{v^4}{4} \right) - \frac{Kb}{4}\left(\frac{t_1^2 v^3}{3} - \frac{v^5}{5} \right) \\ &- Mt_1 e^{\frac{Kt_1^2}{2}} - \frac{at_1^2}{2} - \frac{bt_1^3}{3} - \frac{aKt_1^4}{12} - \frac{bKt_1^5}{15} + \frac{MKt_1^3}{6} e^{\frac{Kt_1^2}{2}} \Bigg\} \Bigg] \end{aligned} \tag{8}$$

- *Deterioration cost:*

  The deterioration cost is given by:

  Deteriorated units = Total production − total demand

$$\text{Det. cost} = c_d \left\{ \int_0^{t_1} \alpha(a+bt)dt - \int_0^{v} (a+bt)dt \right\}$$

$$\text{D.C.} = c_d \left\{ \alpha\left(at_1 + \frac{bt_1^2}{2} \right) - \left(av + \frac{bv^2}{2} \right) \right\} \tag{9}$$

- *Production cost:*

$$\text{Total produced units} = \int_0^{t_1} \alpha(a+bt)dt$$

$$\text{P.C.} = \alpha\left(at_1 + \frac{bt_1^2}{2} \right) m \tag{10}$$

- *Shortage cost:*

$$\text{Total shortages} = \theta \int_v^{T} (a+bt)dt$$

$$\text{S.C.} = s\theta\left(aT + \frac{bT^2}{2} - av - \frac{bv^2}{2} \right) \tag{11}$$

- *Lost sale cost:*

$$\text{L.S.C.} = \int_v^{T} (1-\theta)(a+bt)dt$$

$$\text{L.S.C.} = l(1-\theta)\left(aT + \frac{bT^2}{2} - av - \frac{bv^2}{2} \right) \tag{12}$$

- *Total average cost:*

  Total cost per unit time is given by:

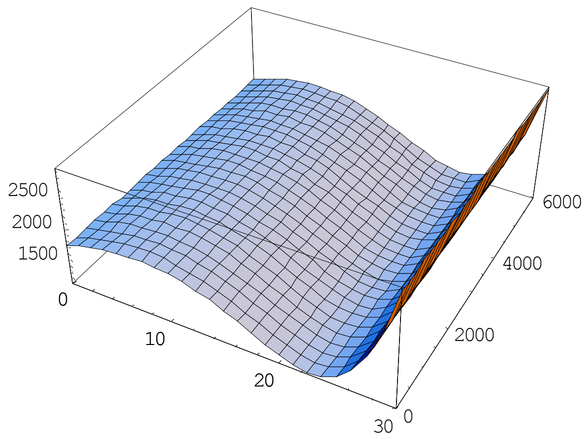
$$\text{T.A.C.} = \frac{1}{T}[\text{Production cost} + \text{deterioration cost} + \text{inventory holding cost} + \text{set up cost} + \text{shortage cost} + \text{lost sale cost}] \tag{13}$$

6 Numerical example

$a = 100$ units, $b = 2$ unit, $K = 0.01$, $\mathrm{l} = 4$ Rs/unit, $h = 0.2$ Rs/unit, $c_d = 12$ Rs/unit, $s = 3$ Rs/unit, $m = 10$ Rs/unit, $T = 30$, $a = 1.2$, $A = 250$ Rs/production run

Corresponding to these values the optimal values of production period (t_1) and critical point (v) comes out to be 19.3188 days and 22.7054 days respectively. The optimal value of T.A.C. is given by 1,152.97 Rs.

Figure 2 Behaviour of the T.A.C. with respect to t_1 and v (see online version for colours)



7 Sensitivity analysis

Sensitivity analysis is carried out with respect to different system parameters. Table 1 to Table 6 show the change in T.A.C. with the discrepancy in a, b, θ, K, h and α taking one at a time and other variables unchanged.

Table 1 Sensitivity analysis for the variation in demand parameter (a)

a	v	t_1	*T.A.C.*
80	22.7091	19.3219	963.242
85	22.7081	19.321	1,010.67
90	22.7071	19.3202	1,058.11
95	22.7062	19.3195	1,105.54
100	22.7054	19.3188	1,152.97
105	22.7046	19.3182	1,200.41
110	22.7039	19.3176	1,247.84
115	22.7032	19.3171	1,295.28
120	22.7026	19.3166	1,342.72

Figure 3 Variation in T.A.C. with variation in demand parameter a

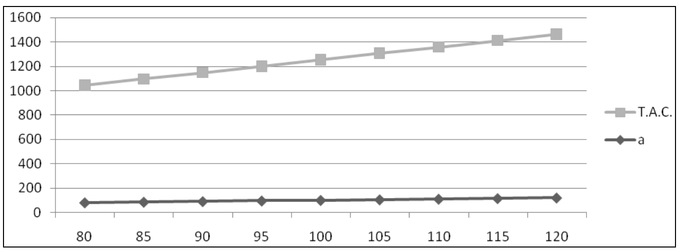


Table 2 Sensitivity analysis for the variation in demand parameter (b)

b	v	t_1	$T.A.C.$
1.6	22.702	19.3161	1,113.79
1.7	22.7029	19.3168	1,123.58
1.8	22.7037	19.3175	1,133.38
1.9	22.7046	19.3182	1,143.18
2	22.7054	19.3188	1,152.97
2.1	22.7062	19.3195	1,162.78
2.2	22.7069	19.3201	1,172.57
2.3	22.7077	19.3207	1,182.37
2.4	22.7084	19.3213	1,192.17

Figure 4 Variation in T.A.C. with variation in demand parameter b

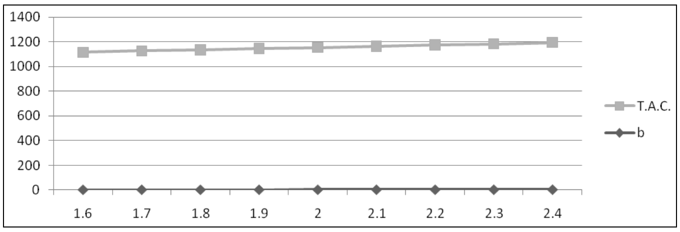


Table 3 Sensitivity analysis for the variation in backlogging rate (θ)

θ	v	t_1	$T.A.C.$
0.64	22.7257	19.3352	1,160.62
0.68	22.7206	19.3311	1,158.71
0.72	22.7156	19.3271	1,156.81
0.76	22.7105	19.323	1,154.9
0.8	22.7054	19.3188	1,152.97
0.84	22.7003	19.3147	1,151.06
0.88	22.6952	19.3106	1,149.14
0.92	22.69	19.3065	1,147.22
0.96	22.6849	19.3023	1,145.29

Figure 5 Variation in T.A.C. with variation in backlogging rate θ

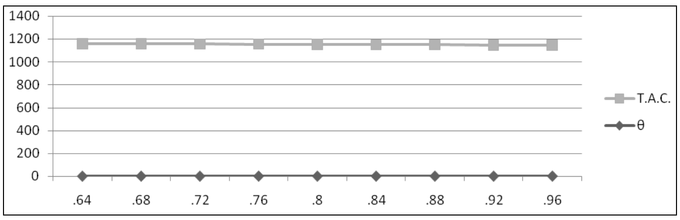


Table 4 Sensitivity analysis for the variation in deterioration rate (*K*)

K	*v*	t_1	*T.A.C.*
0.008	20.8799	17.846	1,093.98
0.0085	21.292	18.1786	1,107.21
0.009	21.7315	18.5331	1,121.37
0.0095	22.2013	18.9122	1,136.58
0.01	22.7054	19.3188	1,152.97
0.0105	23.248	19.7566	1,170.72
0.011	23.8343	20.2296	1,189.99
0.0115	24.4706	20.7431	1,211.04
0.012	25.1646	21.303	1,234.14

Figure 6 Variation in T.A.C. with variation in deterioration parameter *K*

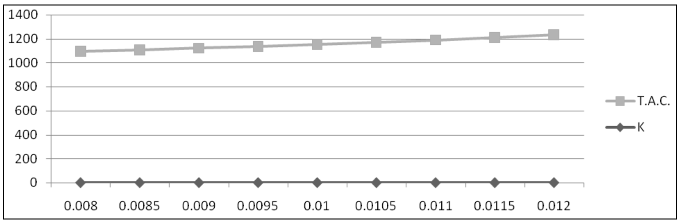


Table 5 Sensitivity analysis for the variation in holding cost (*h*)

h	*v*	t_1	*T.A.C.*
0.16	23.1475	19.6756	1,165.19
0.17	23.0255	19.5771	1,161.76
0.18	22.9117	19.4853	1,158.6
0.19	22.8053	19.3994	1,155.68
0.2	22.7054	19.3188	1,152.97
0.21	22.6113	19.243	1,150.47
0.22	22.5225	19.1713	1,148.13
0.23	22.4384	19.1035	1,145.94
0.24	22.3587	19.0391	1,143.89

Figure 7 Variation in T.A.C. with variation in holding cost *h*

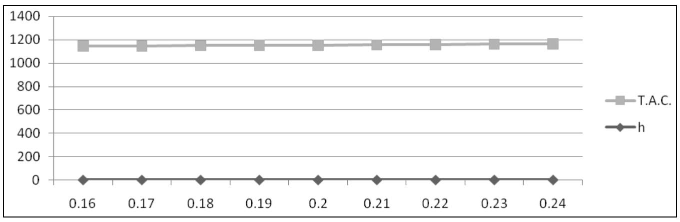
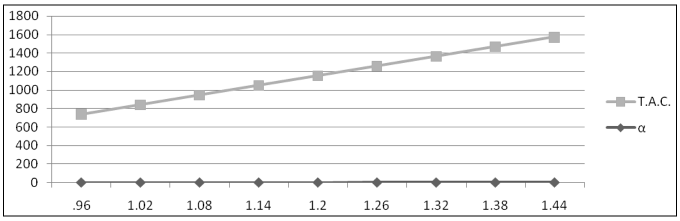


Table 6 Sensitivity analysis for the variation in demand coefficient (α)

α	v	t_1	*T.A.C.*
0.96	22.7054	19.3188	734.299
1.02	22.7054	19.3188	838.968
1.08	22.7054	19.3188	943.637
1.14	22.7054	19.3188	1,048.31
1.2	22.7054	19.3188	1,152.97
1.26	22.7054	19.3188	1,257.64
1.32	22.7054	19.3188	1,362.31
1.38	22.7054	19.3188	1,466.98
1.44	22.7054	19.3188	1,571.65

Figure 8 Variation in T.A.C. with variation in production coefficient α



8 Observations

While dealing with time-varying demand patterns, the researchers usually take the demand rate to be a linear function of time. This type of demand rate functions of the form $D(t) = a + bt$, $a \geq 0$, $b \neq 0$, implies steady increase ($b > 0$) or decrease ($b < 0$) in the demand rate.

1 With the increment in demand parameter rate (a), the purchasing rate is increasing and inventory level decrease rapidly. With the help of sensitivity analysis we observe that in this case the value of t_1 and v decreases and the value of T.A.C. increase.

2 'b' is demand parameter dependent on time. As the value of demand parameter (b) increases with time, purchasing rate is also increasing as well as the value of t_1, v and T.A.C. increases.

3 When the value of backlogging rate (θ) increases, the shortage cost and the other variables remain unchanged and the value of t_1, v and T.A.C. decreases.

4 The change in deterioration rate (K) leads to a positive change on the present value of total cost (T.A.C) i.e., T.A.C. increases with the increase of (K).

5 As the value of holding cost (h) increases, the value of T.A.C. also increases.

6 With the increment in production coefficient (α), the value of t_1 and v remains constant and the value of T.A.C. increases.

9 Conclusions

This model incorporates some realistic features that are likely to be associated with some kinds of inventory. Basically this model based upon stock dependent i.e., when we go to market for purchasing articles in garment shop. If retailer is having quantity of articles/stock it would attract the customer and if retailer is having shortage of goods then no purchaser would like to purchase goods on account of shortage and limited stock. As such the quantity of stock makes the reputation of retailer in the market. Therefore we can say all this things states about the salient feature of our model. Deterioration over time is a natural feature for goods and occurrence of shortages in inventory is a natural phenomenon in real situations. In this paper we have studied a production inventory model, in which deterioration rate is time dependent and production rate is a function of demand rate. The main contribution of this paper has been the development of a dynamic heuristic to determine the optimal production run time and optimal total cost. In totality, the setup that has been chosen boasts of uniqueness in terms of the conditions under which the model has been developed.

A numerical assessment of the theoretical model has been done to illustrate the theory. The solution obtained has also been checked for sensitivity. With the results the model is found to be quite suitable. All these facts together make this study very unique and straight forward. The model developed here may further be extended for more conditions of shortages and deterioration.

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