Optimizing transportation and storage of final products in the sugar and ethanol industry: a case study

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Abstract

COPERSUCAR Ltda (the acronym for the Sugarcane and Ethanol Producers’ Cooperative in São Paulo state) is a Brazilian cooperative of sugarcane producers and the largest sugar and ethanol manufacturer in Brazil, producing 4.4 million metric tons of sugar and 2.7 billion liters of ethanol. The cooperative is composed of 34 sugar mills with centralized sales and marketing. This organization establishes the amount of each product that will be manufactured in each mill to reduce total transportation and storage costs and, consequently, increase overall gain. Critical aspects of this problem are seasonal production and, therefore, the need to store final products to meet demand during the off-season period. This study focuses on the application of a multi-period linear programming model that provides optimal assignment of production, transportation, and storage of final products subject to manufacturing and flow capacity constraints. The expected annual benefits of implementing the proposed solution are 3.3 million dollars. In addition, a sensitivity analysis was carried out to investigate the possibility of increasing the capacity of the installed mills.

Keywords: logistics system; multi-period linear programming; case study

1. Introduction

In recent years, there has been a growing interest in the application of optimization techniques in Brazilian production systems (see, for example, Taube-Netto 1996; Yoshizaki et al. 1996; Colin et al. 1999; Caixeta-Filho 1999; Kiyuzato et al. 2002). One of the possible reasons for this is that managers are concerned with operational efficacy, i.e., they need to be interested due to the effects of strong, professional competition. Additionally, due to the still moderate use of these techniques in Brazil, there is ample opportunity to improve operations systems using classical methods (Colin et al. 1999). This work is another example of this trend.

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Founded in July 1959, COPERSUCAR is composed of 34 sugar mills, 31 of which are concentrated in the state of São Paulo, as shown in Fig. 1. With centralized management of sales and marketing, this group serves clients throughout Brazil. This cooperative is the largest sugarcane manufacturer in Brazil, with a production of 4.4 million metric tons of sugar and 2.7 billion liters of ethanol (data for the 1999/2000 crop). It is also the market leader, accounting for 26% of the sugar market and 23% of the ethanol market in the central and southern regions of Brazil.

This study focuses on the application of a linear programming (LP) model that provides the optimal assignment of production, transportation, and warehousing for each period within the logistics system managed by COPERSUCAR. Critical aspects of this problem are seasonal production and the resulting need to store final products to meet demand during the off-season. Thus, production planning involves decisions such as distribution of the production mix between several plants according to their individual capacity, the need for totally external storage, and management of inventory levels for each plant.

Numerous mathematical programming models applied to production and distribution problems can be found in the literature. Gehring et al. (1991) used LP in the integrated planning of production and distribution of cement. Gutierrez (1996) developed a multi-period, multi-product LP model to optimize production, inventory placement (raw materials and final products), and distribution of a company that deals with agricultural input with high seasonal demand. Schuster and Allen (1998) approached the problem of aggregate planning for a manufacturer of fruit-derived products, minimizing transportation, manufacturing, and storage costs. Hindi et al. (1998) presented an application of a multi-product transshipment model for the distribution of commodities, where the supplying plant needed to be tracked. Jayaraman (1998) simultaneously approached the problem of inventory replenishment, plant location, and transportation mode selection by means of a mixed integer LP model. Rabinowitz and Mehrez (2001) presented a non-linear model for the problem of redesigning the logistics system of a
potassium extraction company. In the sugarcane industry, Yoshizaki et al. (1996) applied a transshipment model to the distribution of ethanol fuel, and Colin et al. (1999) approached sugar distribution of a single sugar mill using a multi-period model.

This paper presents a multi-period, multi-product LP model whose objective is to facilitate the production planning decision process at the cooperative. This model takes into account freight and external storage costs, respecting technical constraints specific to the logistics system, and complete fulfillment of the forecast demand.

In the next section, the production and distribution system is described and the problem is defined. In Section 3, the mathematical model is presented and the fundamental hypotheses are discussed. In Section 4, the main results are analyzed. In the last section, a critical analysis of the model is performed, improvement possibilities are identified, and options for further development of this study are suggested.

2. Problem description

The sugar–ethanol industry in Brazil uses sugarcane as the major raw material. Its syrup is extracted and used to obtain two main final products: sugar and ethanol. Figure 2 shows the main products obtained from a sugarcane mill.

The manufacturing process of these products is seasonal, as raw material is available only during some months (generally from May to November). Such seasonality requires an efficient planning system for production, storage, and distribution in order to meet demand all year round.

Currently, the cooperative uses an optimization model solely for determining the production and distribution of sugar. The quotas for different kinds of sugar are shared among the mills to minimize transportation costs. For ethanol, each mill determines production on its own and the central administration is responsible for marketing and distribution. This is possible because it is a commodity market, and sales can be boosted by cutting prices, allowing for complete production outflow. Although external storage costs for both products and transportation costs for ethanol are significant, 29% of the total cost, they are not considered in this model. It should be observed that, as storage issues are secondary, the system presents periods of surplus of storage space in some mills, while other mills have to rent external storage capacity.

Fig. 2. Final products.
Analyzing the actual methodology, weak points were identified and we proposed a new methodology that could improve the logistics system. The proposed scheme is based on a new optimization model that minimizes transportation and external storage costs for both product families (ethanol and sugar). In this new system, production quotas for all products are determined in an integrated manner. Therefore, each mill has to provide the central administration with the total amount of sugarcane available and it, in turn, is in charge of evaluating demand and imposing product quotas (hydrated ethanol, anhydrous ethanol, sugar type 1, sugar type 2, sugar type 3, sugar type 4, VHP sugar) based on demand location, availability of raw materials, and the production, storage, and distribution capacities of each mill.

3. The model

The model is based on the classic transshipment problem, with constraints of logistics and production capacity, and considering multiple products and periods. The model intends to minimize transportation and external storage (tanks and warehouse) costs by means of production and distribution planning for a 12-month period.

For each period, one can quantify the following: the amount of each product to be produced at each origin; the available inventory of each product at each origin or transshipment point; the amounts to be stored in external facilities for each product family (sugar or ethanol); the volumes of each product to be moved from each origin (or transshipment point) in a specific type of packaging to each destination; and all costs related to each transportation and external storage operation.

The following notation has been used:

\[
\begin{align*}
f &= 1, \ldots, m & \text{index set for product families} \\
v &= 1, \ldots, n & \text{index set for products} \\
h &= 1, \ldots, e & \text{index set for packaging types} \\
i &= 1, \ldots, o & \text{index set for production plants (origins)} \\
j &= 1, \ldots, d & \text{index set for demand zones (destinations)} \\
k &= 1, \ldots, c & \text{index set for external storage warehouses (transshipment)} \\
r &= 1, \ldots, g & \text{index set for production regions} \\
l &= 1, \ldots, p & \text{index set for time periods} \\
t &= 1, \ldots, u & \text{index set for transportation modes}
\end{align*}
\]

Decision variables are as follows:

\[
\begin{align*}
P_{vil} & \quad \text{amount of finished product } v \text{ at origin } i \text{ produced in period } l \\
P_{fjl} & \quad \text{amount of family } f \text{ at origin } i \text{ produced in period } l \\
X_{avhijlt} & \quad \text{amount of product } v \text{ in package } h \text{ sent from origin } i \text{ to demand zone } j \text{ in period } l \text{ using mode } t \\
X_{bhiklt} & \quad \text{amount of product } v \text{ using package } h \text{ sent from origin } i \text{ to transshipment point } k \text{ in period } l \text{ using mode } t \\
X_{cvhjkl} & \quad \text{amount of product } v \text{ using package } h \text{ sent from transshipment point } k \text{ to demand zone } j \text{ in period } l \text{ using mode } t
\end{align*}
\]
The model parameters are described as follows:

- $S_{a_{fil}}$: amount of product family $f$ sent from origin $i$ to demand zone $j$ in period $l$ using mode $t$
- $S_{b_{fiklt}}$: amount of product family $f$ sent from origin $i$ to transshipment point $k$ in period $l$ using mode $t$
- $S_{c_{fkjl}}$: amount of product family $f$ sent from transshipment point $k$ to demand zone $j$ in period $l$ using mode $t$
- $S_{d_{frjl}}$: amount of product family $f$ sent from region $r$ to demand zone $j$ in period $l$ using mode $t$
- $S_{e_{frklt}}$: amount of product family $f$ sent from region $r$ to transshipment point $k$ in period $l$ using mode $t$

- $E_{p_{vil}}$: amount of finished product $v$ stored at origin $i$ at the end of period $l$
- $E_{q_{vkl}}$: amount of finished product $v$ stored at transshipment point $k$ at the end of period $l$
- $E_{f_{ilo}}$: amount of product family $f$ stored at origin $i$ at the end of period $l$
- $E_{g_{ijkl}}$: amount of product family $f$ stored at transshipment point $k$ at the end of period $l$
- $W_{fil}$: amount of product family $f$ at origin $i$ to be stored externally during period $l$

- $F_{tv}$: production ratio of raw material to finished product $v$
- $M_{p_{il}}$: total amount of raw material to be processed at origin $i$ in period $l$
- $D_{e_{vijl}}$: demand for finished product $v$ in package $h$ at demand zone $j$ in period $l$
- $A_{mf_i}$: storage capacity of product family $f$ at origin $i$
- $C_{af}$: monthly storage cost per unit of product family $f$
- $C_{fa_{vijlt}}$: per unit transportation cost of finished product $v$ in package $h$ from origin $i$ to demand zone $j$ in period $l$ using mode $t$
- $C_{fb_{vihkl}}$: per unit transportation cost of finished product $v$ in package $h$ from origin $i$ to transshipment point $k$ in period $l$ using mode $t$
- $C_{fc_{vihkl}}$: per unit transportation cost of finished product $v$ in package $h$ from transshipment point $k$ to demand zone $j$ in period $l$ using mode $t$
- $C_{m_{fil}}$: daily maximum production capacity for product family $f$ at origin $i$ in period $l$
- $C_{p_{vil}}$: daily maximum production capacity for finished product $v$ at origin $i$ in period $l$
- $C_{t_{vi}}$: maximum production capacity for finished product $v$ relative to the total amount produced at origin $i$ (%)
- $D_{p_{il}}$: total number of production days at origin $i$ in period $l$
- $D_{x_{il}}$: total number of days for dispatching products in period $l$
- $D_{r_{il}}$: total number of days for receiving materials in period $l$
- $D_{t_{il}}$: total number of days for transportation in period $l$
- $E_{x_{a_{fil}}}$: daily capacity for dispatching products at origin $i$ for product family $f$ in period $l$ using mode $t$
- $E_{x_{b_{fkl}}}$: daily capacity for dispatching products at transshipment point $k$ for product family $f$ in period $l$ using mode $t$
- $R_{ca_{fil}}$: daily capacity for receiving materials for product family $f$ at demand zone $j$ in period $l$ using mode $t$
- $R_{cb_{fkl}}$: daily capacity for receiving materials for product family $f$ at transshipment point $k$ in period $l$ using mode $t$
\( \text{Tra}_{fijlt} \) daily transportation capacity for product family \( f \) from region \( r \) to demand zone \( j \) in period \( l \) using mode \( t \)

\( \text{Trb}_{frkl} \) daily transportation capacity for product family \( f \) from region \( r \) to transshipment point \( k \) in period \( l \) using mode \( t \)

\( \text{Tr}_{ckjl} \) daily transportation capacity for product family \( f \) from transshipment point \( k \) to demand zone \( j \) in period \( l \) using mode \( t \)

The model does not consider intermediate stages, i.e., we assume that all system inputs and outputs take place at the end of periods. We also assume that all initial inventories are 0, although it is possible to input any known amount. The final version of the model is shown below.

\[
\min Z = CF_1 + CF_2 + CF_3 + CA_1 + CA_2,
\]

subject to:

\[
CF_1 = \sum_{v=1}^{n} \sum_{h=1}^{e} \sum_{i=1}^{o} \sum_{j=1}^{d} \sum_{l=1}^{p} \sum_{t=1}^{u} X_{vhijlt} C_{f vhijlt},
\]

\[
CF_2 = \sum_{v=1}^{n} \sum_{h=1}^{e} \sum_{i=1}^{o} \sum_{k=1}^{c} \sum_{l=1}^{p} \sum_{t=1}^{u} X_{hvikhlt} C_{fbvikhlt},
\]

\[
CF_3 = \sum_{v=1}^{n} \sum_{h=1}^{e} \sum_{k=1}^{c} \sum_{j=1}^{d} \sum_{l=1}^{p} \sum_{t=1}^{u} X_{vkhjl} C_{fcvkhjl},
\]

\[
CA_1 = \sum_{f=1}^{m} \sum_{i=1}^{o} \sum_{l=1}^{p} W_{fil} C_{af},
\]

\[
CA_2 = \sum_{f=1}^{m} \sum_{k=1}^{c} \sum_{l=1}^{p} E_{gijkl} C_{af},
\]

\[
\sum_{i=1}^{o} \sum_{t=1}^{u} X_{vhijlt} + \sum_{k=1}^{c} \sum_{l=1}^{p} X_{vkhjl} \geq D_{vhijlt} \quad \text{for all } v, h, j \text{ and } l,
\]

\[
Ep_{vi} = 0 \quad \text{for all } v \text{ and } i,
\]

\[
Eq_{vk} = 0 \quad \text{for } v \text{ and } k,
\]
\[ E_{p_{vl}} = E_{p_{vl(l-1)}} + P_{vl} - \sum_{h=1}^{e} \sum_{j=1}^{d} \sum_{t=1}^{u} X_{a_{vhjt}} - \sum_{h=1}^{e} \sum_{k=1}^{e} \sum_{t=1}^{u} X_{b_{vhlkt}} \quad \text{for all } v, i \text{ and } l, \] (10)

\[ E_{q_{vkl}} = E_{q_{vkl(l-1)}} + \sum_{h=1}^{e} \sum_{i=1}^{o} \sum_{t=1}^{u} X_{b_{vhlkt}} - \sum_{h=1}^{e} \sum_{j=1}^{d} \sum_{t=1}^{u} X_{c_{vhlkt}} \quad \text{for } v, k \text{ and } l, \] (11)

\[ E_{g_{1kl}} = \sum_{v=1}^{2} \sum_{y_{vl(l-1)}} + \sum_{h=1}^{e} \sum_{k=1}^{o} \sum_{t=1}^{u} X_{b_{hklkt}} - \sum_{h=1}^{e} \sum_{j=1}^{d} \sum_{t=1}^{u} X_{c_{hklkt}} \quad \text{for } u, k \text{ and } l, \] (12)

\[ E_{g_{2kl}} = \sum_{v=3}^{7} \sum_{y_{vkl(l-1)}} + \sum_{h=1}^{e} \sum_{k=1}^{o} \sum_{t=1}^{u} X_{b_{hklkt}} - \sum_{h=1}^{e} \sum_{j=1}^{d} \sum_{t=1}^{u} X_{c_{hklkt}} \quad \text{for } u, k \text{ and } l, \] (13)

\[ E_{f_{vvl}} = \sum_{v=1}^{2} \sum_{y_{vvl(l-1)}} + \sum_{h=1}^{e} \sum_{j=1}^{d} \sum_{t=1}^{u} X_{h_{vlkt}} - \sum_{h=1}^{e} \sum_{k=1}^{o} \sum_{t=1}^{u} X_{b_{hklkt}} \quad \text{for all } v, i \text{ and } l, \] (14)

\[ E_{f_{2kl}} = \sum_{v=3}^{7} \sum_{y_{vvl(l-1)}} + \sum_{h=1}^{e} \sum_{j=1}^{d} \sum_{t=1}^{u} X_{h_{vlkt}} - \sum_{h=1}^{e} \sum_{k=1}^{o} \sum_{t=1}^{u} X_{b_{hklkt}} \quad \text{for all } v, i \text{ and } l, \] (15)

\[ W_{f_{il}} \geq E_{f_{il}} - Am_{fi} \quad \text{for all } f, i \text{ and } l, \] (16)

\[ P_{r_{1il}} = \sum_{v=1}^{2} \sum_{y_{vvl(l-1)}} + \sum_{h=1}^{e} \sum_{j=1}^{d} \sum_{t=1}^{u} X_{h_{vlkt}} - \sum_{h=1}^{e} \sum_{k=1}^{o} \sum_{t=1}^{u} X_{b_{hklkt}} \quad \text{for all } v, i \text{ and } l, \] (17)

\[ P_{r_{2il}} = \sum_{v=3}^{7} \sum_{y_{vvl(l-1)}} + \sum_{h=1}^{e} \sum_{j=1}^{d} \sum_{t=1}^{u} X_{h_{vlkt}} - \sum_{h=1}^{e} \sum_{k=1}^{o} \sum_{t=1}^{u} X_{b_{hklkt}} \quad \text{for all } v, i \text{ and } l, \] (18)

\[ P_{f_{il}} \leq C_{m_{cil}} D_{p_{cil}} \quad \text{for all } f, i \text{ and } l, \] (19)

\[ P_{v_{il}} \leq C_{p_{vil}} D_{p_{cil}} \quad \text{for all } i, l \text{ and } v = 1, 2, \] (20)

\[ \sum_{s=3}^{i} P_{s_{il}} \leq \sum_{v=3}^{n} P_{v_{il}} \sum_{s=3}^{v} \sum_{s=3}^{n} C_{s_{il}} \quad \text{for all } i, v = 3, \ldots, n \text{ and } s = 3, \ldots, n, \] (21)
\[
\sum_{v=1}^{n} P_{vil} \times F_{tv} = M_{pil} \quad \text{for all } i \text{ and } l, \tag{22}
\]

\[
S_{afijlt} = \sum_{v=1}^{n} \sum_{h=1}^{e} X_{a_{vijlt}} \quad \text{for all } f, i, j, l \text{ and } t, \tag{23}
\]

\[
S_{bfiklt} = \sum_{v=1}^{n} \sum_{h=1}^{e} X_{b_{vihklt}} \quad \text{for all } f, i, k, l \text{ and } t, \tag{24}
\]

\[
S_{cfkjlt} = \sum_{v=1}^{n} \sum_{h=1}^{e} X_{c_{vkhjlt}} \quad \text{for all } f, k, j, l \text{ and } t, \tag{25}
\]

\[
\sum_{j=1}^{d} S_{afijlt} + \sum_{k=1}^{e} S_{bfiklt} \leq E_{x_{afijlt}} \quad \text{for all } f, i, l \text{ and } t, \tag{26}
\]

\[
\sum_{j=1}^{d} X_{c_{fkjlt}} \leq E_{x_{bfklt}} \quad \text{for all } f, k, l \text{ and } t, \tag{27}
\]

\[
\sum_{i=1}^{o} S_{afijlt} + \sum_{k=1}^{e} S_{cfkjlt} \leq R_{a_{fijlt}} \quad \text{for all } f, j, l \text{ and } t, \tag{28}
\]

\[
\sum_{i=1}^{o} S_{bfiklt} \leq R_{b_{fiklt}} \quad \text{for all } f, k, l \text{ and } t, \tag{29}
\]

\[
\sum_{i \in \text{region } r} S_{afijlt} = S_{dfjlt} \quad \text{for all } f, r, j, l \text{ and } t, \tag{30}
\]

\[
\sum_{i \in \text{region } r} S_{bfiklt} = S_{cfklr} \quad \text{for all } f, r, k, l \text{ and } t, \tag{31}
\]

\[
S_{dfjlt} \leq T_{a_{frjlt}} \quad \text{for all } f, r, j, l \text{ and } t, \tag{32}
\]

\[
S_{cfklr} \leq T_{b_{frklr}} \quad \text{for all } f, r, k, l \text{ and } t, \tag{33}
\]
\[
S_{fkljt} \leq Tr_{fkljt} Dt \quad \text{for all } f, k, j, l \text{ and } t, \tag{34}
\]

\[
Xa_{vhijlt} \geq 0 \quad \text{for all } v, h, i, j, l \text{ and } t, \tag{35}
\]

\[
Xb_{vhiklt} \geq 0 \quad \text{for all } v, h, i, k, l \text{ and } t, \tag{36}
\]

\[
Xc_{vhjlt} \geq 0 \quad \text{for all } v, h, j, l \text{ and } t, \tag{37}
\]

\[
W_{fil} \geq 0 \quad \text{for all } v, i \text{ and } l, \tag{38}
\]

\[
Ep_{vil} \geq 0 \quad \text{for all } v, j \text{ and } l, \tag{39}
\]

\[
Eq_{vkl} \geq 0 \quad \text{for all } v, k \text{ and } l, \tag{40}
\]

\[
P_{vil} \geq 0 \quad \text{for all } v, i \text{ and } l. \tag{41}
\]

Equation (1) represents the objective function to be minimized, with its component costs being defined in equations (2–6). Equation (2) represents the total transportation costs for finished products from origin \(i\) to final destination (demand) \(j\), which is obtained by multiplying the total shipped amount by its corresponding unit costs. Equation set (3) gives the total transportation costs for finished products from origin \(i\) to transshipment point \(k\). Equation (4) represents the total transportation costs for finished products from transshipment point \(k\) to final destination (demand zone) \(j\). Equation (5) defines the calculation method for external storage costs at each origin \(i\). The external storage cost is calculated by multiplying the amount of product stored externally by its corresponding unit cost.

The amount of finished products stored at transshipment points is also considered external storage. Equation set (6) calculates external storage costs by multiplying the inventory at transshipment points by the corresponding unit cost. We assume that there are no costs for external storage in the aggregate periods (the first and the last). This is a valid assumption as inventories are always below internal storage capacity in the first period (the beginning of production season – May–August), and product output is preferably allocated to externally stored inventories in the fifth period (off-season – December–April).

The constraints of this model are the operational limitations of the system and other characteristics of the logistics system. The set of equation (7) ensures that the demand is fully met, as the amount sent to a demand zone \(j\) is equivalent to the local demand. Equation (8) represents the initial inventory of finished products at origin \(i\), while the set of equation (9) represents the initial inventory of finished products at transshipment points \(k\). The set of expressions (10) defines the calculation of the final inventory of finished product \(v\) at origin \(i\) in each period \(l\) as the sum of the initial inventory plus the corresponding amount produced minus the amounts sent to demand
zones $j$ and to transshipment points $k$. Similarly, the set of expressions (11) represents the same calculations for inventories at transshipment points $k$.

Equations (12) and (13) define the inventory amount for each product family $f$ at transshipment points $k$ in each period $l$. Inventories at transshipment points are classified as external storage and considered as such in cost calculations. Equations (14) and (15) define the total amount of inventory for each product family $f$ at origin $i$ in each period $l$. The set of equation (16) determines the amount of inventory to be stored externally for each product family $f$ at origin $i$ in period $l$. The final inventory for each product family $f$ is then compared with the corresponding storage capacity. As this is an external storage minimization problem, variable $W_{fli}$ assumes value $E_{fli} - A_{fli}$, if $E_{fli} > A_{fli}$, or 0 (zero), if not.

The set of constraints (17) calculates the total amount produced for the “ethanol” product family using the sum of hydrated and anhydrous ethanol at origin $i$ in period $l$. The set of constraints (18) calculates the total production for the “sugar” product family using the sum of different types of sugar at origin $i$ in period $l$. The set of constraints (19) considers the production capacity limitations for product families $f$ based on the installed capacity of the plant. In this case, the total amount produced must be smaller than the installed capacity at the plant in origin $i$ in time period $l$. Similarly, the set of constraints (20) limits the production amount of hydrated and anhydrous ethanol (respectively, $v = 1$ and $v = 2$) to the installed capacity of the plant.

Constraints (21) limit sugar production for different types of product. The quality of sugar is affected by factors such as an excess of rain, sugarcane ripeness, and characteristics of the process, among other factors that can occur during crop season. Therefore, there are limitations regarding the quality of the sugar obtained, which results in a limitation of different types of sugar production (1–4 and VHP). The solution used to assess this constraint is to use a historical database to gather production capacity data in terms of the ratio of each variety to the total obtained. For instance, one origin $i$ can produce 60% of its total output as sugar type 3.

The set of constraints (22) ensures complete utilization of the available raw material through its conversion into product families $f$. In this case, we used conversion ratios for raw material into a given product family based on historical data.

Constraints (23) indicate the total amount of each product family $f$ sent from origin $i$ to demand zone $j$, given by the sum of products of the same family, regardless of the type of packaging used. Similarly, the set of constraints (24) indicates the total amount of each product family $f$ sent from origin $i$ to transshipment point $k$. The set of constraints (25) represents the total amount of product family $f$ sent from transshipment point $k$ to demand destination $j$, regardless of the packaging method.

Constraints (26) show limitations of handling capacity for each product family $f$ at origin $i$ using transportation mode $t$ in period $l$. Similarly, the set of constraints (27) demonstrates the handling capacity for transshipment points $k$. The set of constraints (28) shows the receiving capacity at destination for each product family $f$ by transportation mode $t$ to each destination $j$ in the period $l$. Similarly, the constraint set (29) limits the receiving capacity at each transshipment point $k$.

Transportation capacity is addressed according to regions of production, i.e., the availability of vehicles is evaluated for the geographical region of each origin. Mills that are located close to each other geographically form production regions. Constraints (30) and (31) represent how origins are clustered into production regions. The first set establishes the flux between origins $i$ from one
given region $r$ and the demand destinations $j$, while the second set establishes the flux between origins $i$ and transshipment points $k$.

The set of constraints (32) defines the transportation limitations for product family $f$ from each production region $r$ to destination $j$ via transportation mode $t$ in period $l$. The same can be applied to transportation directed toward transshipment points $k$, represented in the set of constraints (33). The set of constraints (34) represents the transportation limitations between transshipment point $k$ and destination demand zones $j$ via transportation mode $t$ in period $l$. Constraints (35) to (41) ensure that decision variables are non-negative.

4. Computational results

The model has been implemented using the “What’s Best!” software package, industrial version 5.0, on an 800 MHz Pentium III microcomputer with 512 Mb of RAM. It included: two product families (sugar and ethanol); seven products (hydrated ethanol, anhydrous ethanol, sugar type 1, sugar type 2, sugar type 3, sugar type 4, VHP sugar); four types of packaging (bulk, container, single sack, double sack); 34 origins; 70 destinations; four transshipment points; nine production regions; five periods (May–August, September, October, November, December–April); and two types of transportation modes (road and rail). The final instance of the model with 16,104 variables and 3950 constraints was within the software’s limitations (32,000 variables and 16,000 constraints). The processing time was 1 min and 26 s.

Data input and information output for this class of optimization software is performed using electronic spreadsheets (in this case, Microsoft Excel 2000), making the model interface very friendly. Schuster and Allen (1998) discussed the pros and cons of this kind of modeling software. Its advantages include the fact that it is appropriate for LP and has a user-friendly interface. Its major disadvantage is that large-scale models are difficult to manipulate due to the large amount of work necessary to update model instances or to modify the model structure.

The model solution predicted a cost reduction compared with current procedures. Table 1 shows the final model solution compared with the estimated cost for the current system.

It can be inferred from Table 1 that there is both an overall (US$3.3 million) as well as a relative (7.3%) cost reduction. All costs decline, especially those for external storage of sugar. In addition,
the solution can be immediately implemented with no need for additional investment, as the model considers the current infrastructure limitations.

The results also showed that the reduction in transportation costs is related mainly to the definition of the mix of sugar and ethanol for each mill, allowed by flexible production assignment. The reduction of external storage costs is influenced by a more efficient use of warehouses, provided by the optimization of product dispatch. The former dispatch criterion (proportional to current inventory) is replaced by an as-needed criterion, i.e., dispatch decisions are made based on the amount of product needed in the period in order to minimize external warehousing. Conversely, as dispatch criteria are changed, there is an imbalance in loading and dispatching operations, with periods of full utilization of dispatch capacity and very long idle periods. A similar result occurs with the production of several types of product, with periods of maximum production of a certain type interspersed with production peaks of other products. This change is due to the need to meet demand completely, together with the minimization of storage costs.

With the intention of providing useful information to the cooperative, without dramatically changing the current configuration, a sensitivity analysis was carried out to investigate a possible production capacity expansion. This analysis was carried out modifying constraints (19) and (20). The set of constraints (19) considers the production capacity limitations for product families according to the installed capacity of the plant, while constraints (20) limit the production amount of hydrated and anhydrous ethanol to the installed capacity of the plant.

With the objective of determining whether or not the increase in capacity could be advantageous, these two sets of constraints were relaxed as follows:

\[
Pr_{fil} \leq (1 + Inc)Cm_{fil}Dp_{fil} \quad \text{for all } f, i \text{ and } l,
\]

\[
Pv_{il} \leq (1 + Inc)Cp_{vil}Dp_{il} \quad \text{for all } i, l \text{ and } v = 1, 2,
\]

where \( Inc \geq 0 \) represents the increase in mill capacity.

The relaxed model (substituting constraints (19) and (20) by (19') and (20')) was solved for several values (0.1, 0.2, ..., 2) of \( Inc \). For each allowed increase, it is possible to identify which mills should be expanded and quantify the expected gain.

Figure 3 illustrates the behavior of the system for each relaxed scenario. If we observe the incremental savings obtained by consecutive values of capacity increase, it can be seen that the largest absolute difference is obtained with a capacity increase of 10%. This relative increment diminishes as the capacity increases and tends to be null when the increase is larger than 120%. By analyzing Fig. 3, the managers can gain a clear understanding of the possible benefits that can be obtained in the event of investments in the general production capacity of the system.

Figure 4 presents a bar chart that represents the mean real increase of capacity of each product for each relaxed scenario. It can be observed that for all cases the mean increase is below 55%.

This approach allows the cooperative to be more flexible. With the knowledge of the estimated savings for each relaxed scenario and the costs involved in expanding capacities, it can decide whether this enlargement is feasible or not and how it should be made. For example, different measures, such as buying new machines or buying old machines or just increasing the capacity of bottleneck equipment, can be analyzed. With a clear idea of expected benefits, the managers can select and implement the best alternatives.
5. Conclusions

This work presented a multi-period, multi-product LP model, developed to aid the production planning process of a cooperative of agricultural companies named COPERSUCAR. This model considered transportation and external storage costs, respecting the system’s technical limitations and completely satisfying the anticipated demand. Based on computational results, we can state

![Figure 3](image1.png)

**Fig. 3.** Savings obtained with an increase in the capacity of the mills.

![Figure 4](image2.png)

**Fig. 4.** Real mean increase of capacity for each maximum allowed increase.

5. Conclusions

This work presented a multi-period, multi-product LP model, developed to aid the production planning process of a cooperative of agricultural companies named COPERSUCAR. This model considered transportation and external storage costs, respecting the system’s technical limitations and completely satisfying the anticipated demand. Based on computational results, we can state
that the model was able to supply feasible solutions that meet the needs of the organization with respect to the problem described above.

The formulation of the problem as an LP model provided some advantages, such as reduced processing time (compared with non-linear or mixed integer programming), and the guarantee that the solutions provided are global optimal solutions.

As for any iterative process, model development requires continuous improvement; thus, it is important to identify points for further exploration. One of these issues is the choice of software. Although “What’s Best!” can be used for this kind of application, spreadsheets are not recommended for such a large-scale model. Other optimization languages are more suitable for manipulating large models, such as GAMS or AMPL.

During model development, several assumptions and simplifications were made. Although they did not impair the solution, they underestimate some aspects of the system, and should be improved. Some of the improvements are: to use a larger number of periods (limited by the maximum model size supported by the software available), to detail storage constraints according to the product type, and to consider other transportation modes. Furthermore, possible improvements could expand the model to consider profit maximization, including product prices, as well as to optimize crop season and production periods according to the ripeness curves of raw material.

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