A probability method for prediction on High Cycle Fatigue of blades caused by aerodynamic loads

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**Abstract**

A probability method for prediction on High Cycle Fatigue (HCF) of blades caused by aerodynamic loads (PHBA) is erected and two approaches are adopted: improving the numerical method to obtain the dynamical responses of blades caused by aerodynamic loads and introducing probability theory into predicting HCF. The basic governing equations of fluid domain, solid domain and fluid–solid interface are given for flow-blade interaction system. Furthermore a numerical method named bidirectional sequential method is illuminated, and the corresponding physical process of vibration from unsteady state to steady state is described. The dynamical response can be obtained by this time domain method conveniently by the use of commercial softwares. Then PHBA is put forward based on vibration stress according to the probabilistic accumulative damage model. Finally an engineering sample is discussed to illuminate PHBA flow clearly. PHBA is performed on two types of second-stage stators to evaluate the operation security, thus the probabilistic accumulative damages altering with operation time are calculated, and the operational reliabilities are also obtained. The results can describe the possibility of HCF intuitively and quantitatively, which shows PHBA is very helpful in the HCF prediction and failure analysis.

**1. Introduction**

High Cycle Fatigue (HCF) of blades caused by forced vibration is an outstanding problem for turbo engines. Campbell method and Goodman method are widely used to evaluate the security of blades presently [1,2]. But Campbell method appears to be unsuited because of the denseness of modes and unpredictability of aerodynamic loads. Campbell method does not deal with response amplitude, however the high cycle life of blade lies on the vibration stress ultimately, so improving the numerical method to obtain the dynamical responses of blades caused by aerodynamic loads is necessary in predicting HCF of blades.

On the other hand, the traditional methods are deterministic approaches, but there are many uncertain factors during design and operation, such as computation, material, manufacture, operating environment and aerodynamic loads. Traditional methods employ empirical coefficients to evaluate the effects of above uncertain factors. It is not accurate and has shortages obviously [3]. Many researches show probabilistic approaches are effective on forecasting HCF of blades. Cesare et al. [4] holds that probabilistic approach is an improvement over the traditional deterministic method that defines an avoidance zone to ensure a level of protection against HCF. He suggests probabilistic approaches to be implemented in engine design practice, however he does not show a complete solution to the problem of HCF in blades design. Herman Shen [5] develops a probability based procedure for predicting and designing reliable blades against various potential HCF problems for all relevant vibration. A reliability prediction is performed on gas turbine blades at high frequency modes using a probabilistic vibratory stress distribution in conjunction with the modified Goodman diagrams.

In comparison with other deterministic or probabilistic prediction developments done in the past, a successful prediction on HCF of blades caused by aerodynamic loads should emphasize on two approaches: improving the numerical method to obtain the dynamical responses of blades caused by aerodynamic loads and introducing probability theory into predicting HCF of blades. In this paper, a numerical method named bidirectional sequential method is employed to obtain the dynamical response, and then operational reliabilities can be obtained according to the probabilistic accumulative damage model. The purpose of this paper is to extend an integrated probability method for predicting HCF of blades caused by aerodynamic loads (PHBA).

**2. Analysis on the dynamical responses of blades caused by aerodynamic loads**

To predict HCF of blades caused by aerodynamic loads, the primary topic is to improve the numerical method on solving...
flow-blade coupled system. It is a typical fluid–solid interaction (FSI) problem, which is caused by the interaction of elastic forces and aerodynamic loads. Arbitrary Lagrange–Euler formulation [6,7] and sequential method [8–10] are widely used to solve FSI problems. Numerical methods to solve problems for fluid dynamics or elastic dynamics have been developed intensively in the last decades and have reached a stage where commercial software is a powerful tool for engineering, so sequential method shows more advantages in engineering problems for its convenience on using commercial softwares. Therefore, the numerical method named bidirectional sequential method is employed and improved here to obtain the dynamical responses by the use of ANSYS and CFX.

2.1. Governing equations

Chiefly, we should present the basic governing equations for sequential method. The flow-blade system is as Fig. 1. The computational domain is divided into three parts, which are solid domain, fluid domain and interaction face. The motion equations and boundary conditions are given for each domain. Solid domain is considered as linear elastic, and its motion equations are expressed by displacement \( u_i \). Fluid domain is considered as viscous, compressible and small disturbance, and its equations are expressed by pressure \( p \) and velocity \( v_i \), \( u_i \) and \( p \) are used as the transmitting variables on interaction face.

2.1.1. In solid domain (\( V_s \))

The dynamical equations can be expressed by tensors as follows.

Equilibrium equations of force:

\[
\sigma_{ij} + F_i = \rho \ddot{u}_i + \mu \dot{u}_i \quad (\text{in } V_s) \tag{1}
\]

Strain–displacement relations:

\[
\varepsilon_{ij} = \frac{1}{2} (\dot{u}_i + \dot{u}_j)
\]
\[ e_{ij} = \frac{1}{2} (u_j + u_i) \quad (\text{in } V_s) \]

Constitutive relations:
\[ \sigma_{ij} = D_{ijkl} e_{kl} \quad (\text{in } V_s) \]

Boundary conditions:
\[ u_i = \bar{u}_i \quad (\text{on } \Gamma_{in}) \]
\[ \sigma_{ij} n_j = \bar{T}_i \quad (\text{on } \Gamma_{so}) \]

Initial conditions:
\[ u_i(x,y,z,0) = u_i(x,y,z) \quad \bar{u}_i(x,y,z,0) = \bar{u}_i(x,y,z) \]

2.1.2. In fluid domain \((V_f)\)

Ignoring the effect of body force in flow of the turbo machine, the governing equations are as follows.

Motion equations:
\[ \ddot{v}_i + v_j v_{ij} = -\frac{1}{\rho} p_j + \tau_{ij} \quad (\text{in } V_f) \]

Continuity equations:
\[ \dot{\rho} + \rho_0 v_i = 0 \quad (\text{in } V_f) \]

State equations:
\[ p = \Theta \rho = \frac{k}{\rho_0} \rho \quad (\text{in } V_f) \]

Constitutive equation:
\[ \tau_{ij} = v_i (v_{ij} + v_{ji}) - \frac{P}{\rho^2} \delta_{ij} \quad (\text{in } V_f) \]

Boundary conditions are as follows.

Kinetic wall boundary:
\[ \bar{V}_f = \bar{V}_w \quad (\text{on } \Gamma_{iw}) \]

Fixed wall boundary:
\[ \bar{V}_f = 0 \quad (\text{on } \Gamma_{in}) \]

Inlet and outlet boundary:
\[ \frac{\partial p}{\partial n_f} = \bar{V}_w \quad (\text{on } \Gamma_{in}) \]
\[ \frac{\partial p}{\partial n_f} = \bar{V}_w \quad (\text{on } \Gamma_{out}) \]

2.1.3. On interaction face \((\Gamma_{sf})\)

The follows should be satisfied on interaction face.

Kinematic conditions: the normal velocity should be equal on \(\Gamma_{sf}\) thus
\[ \nu_n = \bar{v}_i \cdot \bar{n}_k = \bar{v}_i \cdot \bar{n}_k = -\bar{v}_i \cdot \bar{n}_k = v_{n} \quad (\text{on } \Gamma_{sf}) \]

Dynamical conditions: the normal pressure should be equal on \(\Gamma_{sf}\) thus
\[ \sigma_{ij} n_j = \tau_{ij} n_j = -\tau_{ij} n_j \quad (\text{on } \Gamma_{sf}) \]
2.2. Bidirectional sequential method on FSI system

The solving process through bidirectional sequential method is shown in Fig. 2 as follows.

(1) Initial conditions of flow should be given as the normal unsteady aerodynamic analysis in turbo machines. And we can obtain the initial conditions by steady aerodynamic analysis.

(2) The solid is regarded as fixed wall during unsteady aerodynamic analysis in the first external loop, and the equations of fluid domain are solved through finite volume method in CFX. While the internal loop in CFX is finished, the values of $p(t)$ and $v(t)$ at a certain time are obtained.

(3) The transient value of $p(t)$ is transmitted to solid domain by APDL (ANSYS Parametric Design Language) based on function (16).

(4) The transient result of $p(t)$ is regarded as the initial boundary conditions of solid, thus the dynamical responses are solved by finite element method in ANSYS.

(5) The transient value of $u(t)$ is obtained and transferred to fluid domain based on function (15).

(6) Further boundary conditions of fluid are updated according to $u(t)$. Therefore, steps from (2) to (5) are iterated and the external loop is formed. We can regard the computation as convergent while the flow velocity, flow pressure, solid displacement and solid stress are all altering periodically.

Here ANSYS is used to solve solid domain, while CFX is used to solve fluid domain. Moreover, APDL is adopted to modify the transmitting loads in the external loops.

2.3. The corresponding physical process of bidirectional sequential method

The corresponding physical process of bidirectional sequential method is similar to perform transient analysis on solid or on fluid solely as shown in Fig. 3. Here $T$ is the time period of structural vibration and $T_p$ is the pitch time of flow. The vibration of blade is caused by the aerodynamic force, so the response period $T$ should be equal to the period of exciting force $T_p$ when the computation is convergent. As normal transient analysis requiring, the time step $\Delta t$ and $\Delta t_1$ must be small enough to satisfy the computation requirements in each domain. Besides $\Delta t$ should be equal to $\Delta t_1$ in order that the loads can be transmitted in each time step.

While the computation is convergent, we can regard the fluid domain and solid domain as coming to a new balance, so the parameters of each domain will be altering periodically. We can estimate whether the computation is convergent according to the convergence criterion: the relative residual error of each parameter in neighboring cycles is less than 1.0e-3. The accuracy of this method will not be introduced detailedly in this paper, and it is validated by experiments that can be consulted in the author’s other work [11].
3. Probability prediction on HCF of blades caused by aerodynamic loads

3.1. Basic theory of probability prediction on HCF

As most HCF analysis [4,12–14], Miner rule is chosen here and the value of accumulative damage is expressed as function (17).

\[ D = \sum_{i=1}^{n} \frac{n_{ai}}{N_{ai}} \]  

(17)

The residual life of blade is as function (18).

\[ M_{\text{HCF}} = D_c - D \]  

(18)

Two sorts of reasons lead to the uncertainties of \( M_{\text{HCF}} \) caused by aerodynamic loads. They are the material resistance uncertainty and vibration response uncertainty. The material resistance uncertainty is not altering with operation time or operation cycles, such as the ultimate strength and fatigue limit. It determines the distribution of \( D_c \). The vibration response uncertainty is altering with operation time, such as the structure randomicity and load randomicity. It determines the distribution of \( D \).

Considering the uncertainties, while the residual life \( M_{\text{HCF}} \geq 0 \), the corresponding operation reliability coefficient can be expected as [15]

\[ C_{\text{rel}} = P[D_c - D > 0] \]  

(19)

Therefore, we can get the probabilistic accumulative damage model as Fig. 4a. Here x-axis is the operation cycle. The red curves represent the probability density of accumulative damage \( D \), which is altering with time. The green curves represent the probability density of material resistance uncertainty \( D_c \), which is not altering with time. So the intersection area of \( D \) and \( D_c \) represent the unreliability coefficient \( 1/C_{\text{rel}} \), which is increasing with time.

Because the characteristic function of the distribution for \( D_c \) is affected by the nonuniformity of microstructure, it is not studied in the paper, so \( D_c \) is fixed as 1 here. Thus, the probabilistic accumu-
The relative damage model is as Fig. 4b. The residual life is 
\[ M_{HCF} = \frac{1}{C_0 D} \],
the corresponding operation reliability can be expected as
\[ C_{rel} = P(1 - D > 0) \]

3.2. Numerical method on probability vibration response analysis

In the actual operating of blades, the vibration response uncertainty is caused by structure randomicity (SR) and load randomicity (LR). SR includes the randomicity of the structure dimension, elastic modulus, density, damping, etc. LR includes the randomicity of the load frequency, load value, load distribution, etc. They affect the distribution of vibration response synchronously, furthermore they determine the distribution of \( D \). Because of the complexity of multi random parameters, it is hard to use theoretical method to obtain the characteristic function of distribution for vibration response. Monte Carlo method and response surface method are integrated here to get the characteristic function of distribution for vibration response. The schematic diagram is as Fig. 5.

(1) Based on the given characteristic functions of distribution for input random variables, we can get the samples of input random variables through Monte Carlo method.

(2) The samples are solved through numerical method iteratively and the samples of output random variables such as vibration stress and vibration displacement are obtained.

(3) The output random variables are fitted by a quadratic polynomial as shown by function (21), thus the response surface function is obtained.

\[ \bar{Y} = C_0 + \sum_{i=1}^{NRV} C_i X_i + \sum_{i=1}^{NRV} \sum_{j=1}^{NRV} C_{ij} X_i X_j \] (21)

Here \( C_0, C_i, C_{ij} \) are unattached coefficients. \( NRV \) is the number of input random variable.

(4) Because the computing time of numerical method mentioned in Section 2.2 is too long to perform thousands of iterations. The response surface is used to replace the numerical method to obtain more output samples. Therefore, the solving speed is improved greatly.

(5) Lastly, the statistical parameters on output variables are counted based on response surface results and the characteristic functions of distribution for output variables are obtained.

3.3. Analysis flow and program

In this section, the analysis flow of PHBA will be illuminated. Assumptions are applied here in PHBA as follows.

(1) The uncertainty of material resistance is ignored, only factors that may affect the vibration response are considered as randomicity.

(2) Each stage is regarded as tuned.

(3) The characteristic functions of distribution for all input random variables are regarded as known.

For PHBA is based on Miner rule, the limitations are obvious. We can only solve typical HCF problems though PHBA, i.e. we cannot take some factors into account such as the effect of load sequence and the plastic strain effect. During traditional predictions on HCF, vibration stresses including mean stress (\( S_m \)) and alternated stress (\( S_a \)) are obtained by experiments or numerical methods. They are determinate values and the values of high cycle life can be obtained by nominal stress method directly. While during probability prediction, considering the randomicity, \( S_m \) and \( S_a \) are expressed by characteristic functions of distribution through numerical method in Section 3.2. So probability Goodman curve and Monte Carlo method are employed here, and then FORTRAN program is written to obtain the probabilistic accumulative damages based on the characteristic functions of distribution for \( S_m \) and \( S_a \).

During PHBA, the load uncertainty is primarily caused by the randomicity of flow, thus the uncertainty of the rotation speed, flow rate, inlet pressure and so on should be considered. Analysis flow should be as Fig. 6a. Firstly, the random parameters of flow are sampled by Monte Carlo method. Secondly, bidirectional method on FSI system is carried out to get vibration response. Thirdly, the characteristic functions of distribution for \( S_m \) and \( S_a \) are obtained. Lastly, the probability accumulative damages and reliabilities are gained by FORTRAN program. Although Monte Carlo
method and response surface method are integrated to get the characteristic function of distribution for vibration response as discussed in Section 2.2, we still have to perform hundreds of bidirectional numerical analysis to obtain the response surface. So it is impractical to be performed in actual engineering problems.

The computation time of bidirectional sequential method is too long so that the ideal method is hard to carry out. Therefore, an approximate method is put forward as Fig. 6b. As discussed above, the coupled system behaves as the steady vibration of blade and periodic movement of flow while the computation is convergent. Hence, firstly we can take bidirectional sequential analysis to obtain the period pressure distribution on blade, and then we perform transient analysis on solid domain to substitute bidirectional analysis in Monte Carlo samples. Thus, the bidirectional numerical analysis will be needed only once, and the computation time is reduced greatly. Of course there are still numerous CFD calculations and FEA calculations in one bidirectional numerical analysis as described in Section 2.2.

Rotation speed primarily affects the load frequency, while flow rate and inlet total pressure primarily affect the pressure amplitude and pressure distribution, so frequency, pressure amplitude and pressure distribution are sampled to get the characteristic functions of distribution for $S_m$ and $S_n$ as shown in Fig. 6b.

4. Engineering case

A certain gas engine is taken as an example here to illuminate PHBA. Failures occurred on the second-stage stator twice at rated operating speed, and they were caused by the crack initiation near the trailing edge through metallurgical examinations. There were two types of second-stage stators named D07 and D11 used in the engines, and the failures only occurred on the engines assembling D11. There was a tiny difference between D07 and D11 for the profiles. But no correlation between profiles and failures could be described through traditional dynamical studies including natural characteristic analysis and Campbell analysis, and no reasonable explanations of the failures could be given. Further PHBA is carried out to find the roots of failures. The results of PHBA describe the possibility of HCF failures intuitively and quantitatively, which shows PHBA is very helpful in the HCF prediction and failure analysis.

4.1. Introduction of models

The profiles of the two types are quite similar, therefore only the mesh of D11 are shown here in Fig. 7. Material parameters
and general operation parameters are same for the two types as shown in Table 1.

In solid domain, only the second-stage stator is modeled with hexahedral element mesh. The nodes on root are fixed according to the assembling status.

In fluid domain, the second-stage rotor, second-stage stator and third-stage rotor are taken into account during modeling. The numbers of each stage are 27, 47 and 31 orderly, so the degrees of fluid domains are 13.33, 7.66 and 11.61 respectively. H–O–H mesh is adopted in flow as shown in Fig. 7. A rotational periodicity is employed as boundary condition on each domain respectively. The transient rotor stator model is employed interstage in CFX to consider transient interaction effects at a sliding (frame change) interface. The transient rotor stator model is a kind of frame change/mixing model used in CFX, which can predict the true transient interaction of the flow between a stator and rotor passage considering the phase lag. The interface position is updated each time step, as the relative position of the grids on each side of the interface changes. The solution with transient rotor stator model will contain most of the overall flow features in simulating a periodic-in-time quasi-steady state. So it is suitable in flow-blade coupled system, for we care the pressure acted on blade especially.

The walls of second stator are considered as interaction face, and the walls of other blades are considered as fixed wall boundary.

Fig. 9. Pressure responses in time domain and in frequency domain.
Fig. 10. Displacement on blade in time domain and in frequency domain.

(a) Contour map of displacement on blade altering with time

(b) Displacement curves of blade tip in time domain and in frequency domain
4.2. Natural characteristic analysis on stators

Natural characteristic analysis and Campbell analysis are carried to seek the possible vibration modes during operation. The natural frequencies and corresponding modes of D07 and D11 are as shown in Fig. 8.

From Fig. 8, we can see there may be resonances for either D07 or D11 at operation speed. But failures only occurred in engines assembling D11, which is hard to explain by traditional dynamical design methods such as Campbell diagrams. Besides, what is the value of vibration stress and when will the failure occurs? They are all significant questions and we can obtain the answers.
through PHBA. Because of the sameness of analysis flow, only PHBA on D11 is illuminated detailedly in the following, but the results of both D07 and D11 are mentioned.

4.3. Vibration response analysis

Firstly, the bidirectional numerical analysis should be performed as the first step shown in Fig. 6b. The time step $\Delta t$ is given as $3.4 \times 10^{-6}$ during computing, which satisfies the need of solid domain and fluid domain. The periodic flow of fluid and steady vibration response of solid are obtained from computation results while the calculation converges.

4.3.1. Results of fluid domain

While the computation is convergent, the pressure distribution of middle section in the cascade passage is as Fig. 9a for a certain time, here the key point A is defined at leading edge while the key point B is defined at trailing edge. Then we get the pressure characteristics of A and B in the time domain and in frequency domain as shown in Fig. 9b. From Fig. 9b, we can see the main load frequencies at leading edge and at trailing edge are different, so a multifrequency load is exciting the second stator. In the following, we will see the response of the stator excited by a multifrequency load is quite different from excited by a harmonic load.

4.3.2. Results of solid domain

Correspondingly, we obtain the computation results for the solid domain which we are more concerning about. The displacement contour maps of the second stator altering with time are as Fig. 10a, we can see the second stator is vibrating as the combination of the 3rd natural mode and the 4th natural mode. The displacement history of key point B and its corresponding frequency

Table 2
Input random parameters.

<table>
<thead>
<tr>
<th>Variety</th>
<th>Distribution type</th>
<th>Mathematical expectation</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR $E$ (MPa)</td>
<td>Lognormal</td>
<td>0.63e5</td>
<td>0.63e3</td>
</tr>
<tr>
<td>$\rho$ (tn/mm$^3$)</td>
<td>Lognormal</td>
<td>2.72e−9</td>
<td>2.72e−11</td>
</tr>
<tr>
<td>LR $f_s$ (Hz)</td>
<td>Normal</td>
<td>12,600</td>
<td>126</td>
</tr>
<tr>
<td>$c_s$</td>
<td>Normal</td>
<td>14,400</td>
<td>144</td>
</tr>
<tr>
<td>$c_{sa}$</td>
<td>Normal</td>
<td>1</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 3
Statistical characteristics of vibration response.

<table>
<thead>
<tr>
<th>Varieties</th>
<th>Mathematical expectation</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean stress $S_{m1}$ (MPa)</td>
<td>141.18</td>
<td>9.09</td>
</tr>
<tr>
<td>$S_{m16}$ (MPa)</td>
<td>56.72</td>
<td>8.01</td>
</tr>
<tr>
<td>Alternated stress $S_{a1}$ (MPa)</td>
<td>86.55</td>
<td>3.37</td>
</tr>
<tr>
<td>$S_{a16}$ (MPa)</td>
<td>42.52</td>
<td>7.48</td>
</tr>
</tbody>
</table>

Fig. 12. Histogram of samples for Sm3 and Sa3.
response are drawn as Fig. 10b. In time domain, the displacement history presents beat vibration characteristic, whose vibration amplitude is not constant but periodic. The vibration possesses a maximum amplitude and a minimum amplitude in a cycle time $T = 5.542e^{-4}$ s. The beat vibration phenomenon is caused by two close frequencies that root in the multifrequency aerodynamic load. We can see the peaks in frequency domain as Fig. 10b are consistent with the pressure characteristic of flow as Fig. 9b, the reason has been discussed in Section 2.3.

The contour maps of von-Mises stress altering with time are as Fig. 11a, we can see the maximum stress location is changing in a cycle. The maximum stress location appears at Max_1 and Max_2 in sequence during a cycle, Max_1 is near the trailing edge at stator tip while $t = T_p$, Max_2 is near the trailing edge at stator root while $t = T_p + 6\Delta t$. That is because the stator is vibrating as the combination of the 3rd natural mode and the 4th natural mode. The stresses of both locations should be noticed, and we can contrast them to find the most dangerous location for further analysis. The von-Mises stress history curves of Max_1 and Max_2 are obtained as Fig. 11b. We can see the stress history also presents beat vibration characteristic. If the maximum value occurs at the same time for Max_1 and Max_2 in Fig. 11b, which is to give a horizontal
translation for one curve, then the von-Mises stress of Max_1 is bigger than that of Max_2 at any moment, so the most dangerous location is sought as Max_1. Because of the beat vibration characteristic of stress history, there are still multistage HCF damages even in one cycle.

4.4. PHBA on the second-stage stator

Based on the above results, the pressure distribution of the stator surface altering with time need to be abstracted as the second step shown in Fig. 6b. The pressure acted on every node of the solid domain shows multifrequency characteristic, so it is expressed like Fourier series. From Fig. 9b, we can see the primary frequency of pressure near the leading edge is 12,600 (Hz), while the primary frequency of pressure near the trailing edge is 14,400 (Hz).

4.4.1. Discretization of random parameters

The random parameters should be discretized and sampled as the third step shown in Fig. 6b. For this study, SR is considered including the elastic modulus $E$ and density $\rho$, while LR is...
4.4.2. Probability numerical analysis on vibration response

APDL is applied here to carry out the loop including performing Monte Carlo sample and transient analysis on solid domain as the fourth step shown in Fig. 6b. The random input varieties are given as Table 2 and the transient analysis is carried during each sample. The theory and flow of this step are also described detailed in Section 4.2 and Fig. 5.

As the results of the solid domain discussed in Section 4.3.2, the most dangerous location is sought as Max_1. Considering beat vibration characteristic of von-Mises stress, there are 16 different amplitudes of stress in one cycle for Max_1 as Fig. 11b. So we need to get the characteristic functions of distribution for mean stress $S_{m1}, S_{m2}, \ldots, S_{m16}$ and alternated stress $S_{a1}, S_{a2}, \ldots, S_{a16}$ though enough samples. The average values and standard deviation values of $S_{m1}, S_{m2}, \ldots, S_{m16}$ and $S_{a1}, S_{a2}, \ldots, S_{a16}$ are steady-going when we sample 80 times through Monte Carlo method, then we can consider that the sample time is enough. Response surface is fitted based on the above samples. Further the fitted response surface is used to sample 10,000 times instead of transient analyses. Then histograms of samples for $S_{m1}, S_{m2}, \ldots, S_{m16}$ and $S_{a1}, S_{a2}, \ldots, S_{a16}$ are obtained. Here taking $S_{m3}$ and $S_{a3}$ for an example, the histograms are shown as Fig. 12.

Based on the results of Fig. 12, the characteristic functions of distribution for $S_{m3}$ and $S_{a3}$ can be equivalent to normal distribution, and the statistical parameters are obtained including mathematical expectation and standard deviation. For the other stress stages, we can also get corresponding statistical parameters. Thus the statistical characteristics of $S_{m1}, S_{m2}, \ldots, S_{m16}$ and $S_{a1}, S_{a2}, \ldots, S_{a16}$ are gained as Table 3. Here we have reached the fifth step shown in Fig. 6b.

4.4.3. Probability prediction on HCF of second-stage stators caused by aerodynamic loads

With the statistical characteristics of vibration response, we can use the FORTRAN program mentioned in Section 3.3 to acquire the probability accumulative damages and operation reliabilities of D11, which is the sixth step shown in Fig. 6b. Then we can get results as Fig. 13.

In Fig. 13a, x-axis is the operation time, the curves represent the probability density of accumulative damage, which is altering with time. The vertical plan is the critical damage plan. If we just calculate the reliability coefficient at a certain operation time, we can extract the corresponding curve from Fig. 13a and can get Fig. 13b. In Fig. 13b, the area on the left of the critical plan represents the reliability coefficient, while the area on the right of the critical plan represents the unreliability coefficient. It is intuitive that the reliability is decreasing from 5 min to 60 min in Fig. 13.

In order to find the differences between D07 and D11, PHBA is also performed on D07, the result is as Fig. 14. Comparing Figs. 13 and 14, the accumulative damage curves are far away from the critical plan in Fig. 14 even having operated 60 min, but the intersection area is obvious in Fig. 13 while having operated 60 min. So D07 will be more reliable during working.

From the results above, we can also describe operation reliabilities at each operation time quantitatively, which are displayed as Table 4. The second stator is used in a short life engine, and it desires 30 min at operation speed. We can see from Table 4, as operation time is increasing, the reliability of D07 retains 1.0000, so D07 satisfies the operation requirement. However, the reliability of D11 decreases to 0.7723 rapidly in 30 min, so D11 is not suggested to be installed.

Based on the above results, all the engines assembling D11 are replaced by D07, and the failures are eliminated.

5. Conclusions

Based on the bidirectional sequential method and probability accumulative damage model, a probability method for prediction on HCF of blades caused by aerodynamic loads is put forward in the paper, and following conclusions are obtained.

1. The vibration stress of blade can be obtained by bidirectional sequential method by the use of ANSYS and CFX. The computation can be regarded as convergent while the key parameters are all altering periodically. The results can be employed in the further probability HCF study conveniently, for it is a time-domain numerical method and the vibration stress of blade is expressed in time domain.

2. Because of the multifrequency characteristic of aerodynamic load in actual engines, the vibration response of blade shows beat vibration characteristic, so the HCF damages of a blade are caused by multistage stresses even though it is operating at a constant rotation speed.

3. An engineering sample is discussed to illuminate PHBA flow clearly. PHBA is performed on two types of second-stage stators to evaluate the operation security. The results of PHBA describe the possibility of HCF failures intuitively and quantitatively, which shows PHBA is very helpful in the HCF prediction and failure analysis.

This paper emphasizes the integral process of PHBA, some details in basic theory and numerical analysis are not presented here. Some correlative work can be referred in [11]. Further, some experimental study is to be carried out to validate the accuracy of PHBA.

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References


Table 4 Operational reliabilities altering with operation time.

<table>
<thead>
<tr>
<th>Operation time (min)</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D07</td>
</tr>
<tr>
<td>1</td>
<td>1.0000</td>
</tr>
<tr>
<td>3</td>
<td>1.0000</td>
</tr>
<tr>
<td>5</td>
<td>1.0000</td>
</tr>
<tr>
<td>10</td>
<td>1.0000</td>
</tr>
<tr>
<td>30</td>
<td>1.0000</td>
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Zhang Dayi, male, born on 1st July 1979, a native of PR China, graduated from Beijing University of Aeronautics and Astronautics (BUAA) in 2008, with doctor’s degree in Engineering, and now is dedicating himself to do research as post-doctor in BUAA. In recent years, he is engaged in vibration control and HCF prediction on aero-forced blades.

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