VIRTUAL METROLOGY SOFTWARE TOOLS AND TECHNIQUES
FOR FEATURE EXTRACTION OF SPIRAL BEVEL GEAR ERRORS

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ABSTRACT

The main objectives of multi-axis machining are to achieve perfect surface quality, precision, and speed without any need for re-machining. Within cutting processes, a strong integration of technologies such as virtual machining and inspection algorithms is important [1]. Closed Loop Machining (CLM) is an important method to maximise the efficiency of cutting processes, and automatic data collection and inspection of the part to be machined are essential elements for continuous improvements [2]. This is especially true for free form shaped parts, which are often machined under extremely tight tolerances, as free form functional surfaces often have a great influence on the performance of a product [3]. As online inspection of parts is often not possible to the required accuracy, offline CMM inspection is used to collect the measured data of the free form part.

However, in order to close the loop between inspection and re-machining, understanding the error sources of the part to be re-machined is essential. The main focus of this work is to develop a virtual metrology software tool, and associated techniques, to allow the correct identification of 5 axis machining error sources of spiral bevel gears. The virtual metrology platform has been developed based on the open source ‘ParaView’ software, and involves creating an extensive error library by virtually modifying gear parameters. These error sources, once understood, can then form part of an adaptive machining process to allow for the reduction of machining uncertainty, and an increase in part quality.

KEYWORDS: Closed Loop Machining, Decision Support, Virtual Metrology

1. INTRODUCTION

The mathematical description of bevel gears can be extremely complex, and among the various types of gears, spiral bevel gears (SBG’s) are the most complex [4]. Traditionally, manual gear hobbing machines or specialized CNC gear cutting machines are the fastest and most efficient methods for producing high-precision SBG sets, especially in high volumes. However, for low to mid-volumes, another emerging option is the use of five axes machining centres. For prototype production, five axes machining of SBG’s has the distinct advantage of significant cost savings compared to serial production.

This is in part because of the extremely complex machining setups and considerably large initial investments involved in serial production. A broad range of gears can be manufactured with 5 axes machining, including highly specialised gear types. Normally, this cannot be done by dedicated gear machine tools, except in very limited cases [4]. As a reflection of the complex design and manufacturing processes involved in their serial production, SBG flanks are defined as free form surfaces [5]. Precision is the key factor in determining the
performance of a spiral bevel gear [6], and the determination of their geometric features therefore is a quantitative measure of their quality.

With this in mind, a lot of work has been realised in the gear inspection domain. The functional tolerances of gear teeth concern the geometric specifications on the teeth features. This means that geometric deviations from a given nominal design can be translated into qualitative variations of the functionality of the gear set. For gears, in addition to the general geometric specifications, particular specialised specifications are necessary. These specifications include cumulative pitch error, pitch variation, and tooth profile error [7]. Pitch measurements in particular are an excellent way of indicating possible sources of error in the manufacturing process of an SBG, particularly static errors, for example coordinate system errors due to influencing factors such as clamp design or poor machine set-up. However, determining the error sources in a SBG machining process is quite often experience led, and therefore very prone to human error, and quite often regarded as more of an art than a science.

Following from this, the aim of this work was to develop a stand-alone software application that would enable the knowledge capture of specific error sources relating to five axes machining of SBG pinions. The application was designed in order to replace the purely experience led machining adaption process which typically characterises this production method, in order to reduce the gap between manufacturing, measurement, and subsequent process adaption. Using a statistical approach, actual cumulative pitch measurement results from manufactured pinions are input into the program, after which this actual data is compared to an error library database consisting of thousands of specific errors sources. The database of error characteristics will act as a reference for actual errors found during machining, and the creation of the error library involved simulating static gear alignment and position errors and building trend models. The error library database was constructed using the open-source 3D visualisation software ParaView. The application was built using Microsoft Visual Basic for Application (VBA) and Microsoft Excel. This paper outlines the steps taken in order to produce the decision support application, as well as providing real world results showing the usage of the tool in industry.

2. BACKGROUND

The determination of pitch deviations on the tooth flanks belongs to the most important single tests on gears. The reference circle pitch, measured in the transverse section, is defined as the arc on the circumference of the reference circle of diameter $d$, containing two consecutive right or left tooth flanks [8]. The DIN standard 3965-1 [9] references the tolerances for spiral bevel gears, and is the standard used in this research.
As is represented in Figure 2.1(A) and 2.1(B), ISO based standards define the pitch using the midpoints of two adjacent flank faces which lie on the intersection between the gears pitch reference cone and a “sampling cone”. This intersection between the two cones forms a circle whose diameter is defined in the gears ideal parameters. The DIN standard 3965-1 defines the cumulative pitch error, $F_P$, as: the difference between the sum of the actual pitches between any two teeth on the pitch circle and the correct value. Pitch data will be extracted from a virtual model as opposed to a real part because no real part can represent the ideal geometry to a high enough standard. All manufactured parts will contain sources of error, thereby introducing a bias into any attempt to create a ‘pure’ error source.

3. DECISION SUPPORT TOOL DEVELOPMENT

3.1. Data Extraction and 3D Visualisation
The virtual metrology concept of this work is divided in to three main steps: A) correct data extraction using a virtual gear model within ParaView, B) correct calculation of the pitch results using the extracted X and Y point coordinate data, and C), the comparison of actual pitch data to the virtual measurement results which make up the error library. VDI/VDE 2613 [10] is the German national standard which governs the CMM measurement of SBG sets. This acted as the main source of standardisation in order to develop guidelines in the creation and usage of the decision support tool, with the aim of ensuring compliance between virtual data and real data received from CMM measurement results.

The standard specifies a number of parameters about pitch measurement which consist as the core basis for the methodology of this work. The determination of the cumulative pitch error cannot be based on pitch measurements taken in the direction of the normal of the gear tooth, as the measured points would not lie in the same transverse plane, as specified in DIN 3960 [11]. The CMM measurement method therefore consists of measuring the pitch points in a transverse section on the inspection circle. These measured points are used to determine the transverse pitch, $P_t$, on the reference circle.

Corresponding to the fact that the transverse axis must remain constant during measurement, the 3D gear itself is rotated in ParaView, instead of the reference circle. This will recreate the situation of eccentric clamping during
machining or measurement. Using this method, it is not necessary to calculate the Z coordinates, as the X and Y coordinates are those which are changing. This corresponds to calculations 1 and 2 required to process the transverse pitch, $P_t$, described in [12] as:

$$P_t = \frac{d}{2} (\theta_i - \theta_{i-1}) - \frac{\pi d}{z}$$  \hspace{1cm} (1)$$

Where $d$ is the diameter of the reference circle, $z$ is the number of gear teeth, and $\theta_i$ is the phase angle between two consecutive gear flanks calculated as:

$$\theta_i = \arctan\left(\frac{pX_i}{pY_i}\right)$$  \hspace{1cm} (2)$$

In order to create the 3D dataset representing the pinion within ParaView, the nominal CAD data of a single pinion tooth, represented as X, Y, and Z coordinates, was first converted into the legacy VTK file format. The location of the X, Y, and Z coordinates can be seen in Figure 3.1. The dataset was represented as an unstructured grid, as a matrix of 15*19 coordinate data points representing each flank, shown in Figure 3.1 (A). This enabled the coordinate data to be rendered as a surface within ParaView, seen in Figure 3.1 (B). Once a single tooth had been created, it was duplicated 13 times in order to create a full pinion, as displayed in Figure 3.1 (C-D). The degree of rotation between each tooth is 27.69°, which is the ideal pitch angle between consecutive flanks. This data represents a 3D model of the ideal pinion flank geometries.

![Figure 3.1: Generation of the ideal flank geometry within ParaView](image)

Several feature extraction algorithms are applied to the data in order to recreate the standardised measurement of a pinion via CMM. When a CMM probe touches a gear flank which intersects with the inspection circle, the X and Y coordinates of that point are recorded. To replicate this procedure, a reference circle is created in ParaView by first creating a sphere whose diameter is equal to
that of the inspection circle used by the CMM. This sphere is then cut by a plane parallel to the Z axis. The X and Y coordinates of the sphere and plane are set at 0, 0, while their height from the Z axis is the same height as the centre point of all gear flanks. The result of this combination is a circle identical to the inspection circle used by the CMM’s to measure the machined pinions. The X, Y, and Z coordinates are recorded where this circle intersects the surfaces of the 3D mesh flanks, as shown in Figure 3.2.

As this represents an ideal gear with deviation free flanks, taking pitch measurements in this state results in a zero error condition. When displayed, this is characterised by a line parallel to the X axis passing through 0, 0. This is the ideal state, meaning any pitch deviation always represents a source of error. In order to recreate machining and measurement defects, the virtual ideal gear model is transformed about its axes according to user inputs, while the inspection circle remains fixed. The axes which are transformed are the X and Y axes in positive and negative translational directions, as well as angular tilt errors introduced along the same axes, again in positive and negative directions. This represents the summation of all individual errors which can be detected using pitch results. Deviations in the Z axis and rotational errors are undetectable by pitch, and are commonly determined using tooth thickness evaluation and flank form error respectively. A combination of all errors is also possible, and after the coordinates are found the data is extracted as a .CSV file for further evaluation and generation of pitch results. As all error combinations can be calculated from the individual error pitch results, ParaView is no longer needed after the individual errors have been defined.

![Figure 3.2: Creation of coordinates for determining virtual pitch measurements.](image)

The coordinates are highlighted as spheres for ease of visualisation

### 3.2. Pitch Error Generation

Single pitch error, \( f_p \), is the difference between the desired and the actual transverse pitch. Successive summing of the single pitch errors results in the characteristic curve of the cumulative pitch error. Location errors of the gearing axis with respect to the reference axes used during measurement and machining
influence the pitch measurement results. This is because if a SBG is off-centre during machining or measurement due to eccentric clamping for example, the centre of the resulting inspection circle will not correspond to the intended axis of the SBG. Such errors are represented by distinctive pitch curves. To create the single pitch error results, the X and Y coordinates are calculated using equations 1 and 2. After this, the X and Y coordinates are converted to degrees transposed about 360°, as opposed to two 180° segments. The difference in degrees between subsequent corresponding flanks is then converted into arc lengths, which is directly compared to the ideal arc length of the pinion, in this case, 18.493mm. Any difference is recorded as the single pitch error. These values are then summed to generate the cumulative error, given in mm.

<table>
<thead>
<tr>
<th>Raw Data Input</th>
<th>Data Analysis</th>
<th>Data Results</th>
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<tbody>
<tr>
<td>Flank</td>
<td>Single Error</td>
<td>Cumulative Error</td>
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<tr>
<td>5</td>
<td>-0.0007</td>
<td>0.0000</td>
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<tr>
<td>6</td>
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<tr>
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**Figure 3.3:** Part of the developed application used to calculate single and cumulative pitch error using raw data input coordinates taken from ParaView.

In order to determine whether the virtual pitch measurements were similar to what a CMM would measure given a defective part, a pinion was machined using standard 5 axes techniques developed during this research. All pinion flanks were then measured using a CMM, and the subsequent measured coordinates were converted into legacy VTK format in order to be analysed in ParaView.

When measuring the pinion, it was important that the CMM had the same protocols as were implied in the virtual measurement, i.e., pitch measured using a transverse section counter-clockwise direction of movement etc. The CMM pitch results were then compared to the virtual results which were obtained in ParaView in the same fashion as the method described in section 3.1, as shown in Figure 3.4. When the two results are plotted against one another, it can clearly be seen visually that there is a strong correlation between both flanks of the CMM generated pitch and the virtual results. The square of the correlation coefficients between the concave and convex flanks is 0.999. The maximum difference in values between the concave flanks is 9µm, while the max difference in values between the convex flanks is 11µm.

In reality, the methodology is quite probably more accurate than this, as the transformed data from the CMM measurements has filters applied to it to smooth the data within ParaView, leading to inaccuracies. Given this level of accuracy, it was decided that the error library would be created in 10µm steps, from -60µm to 60µm. This range covers the entire range of accuracies possible within the DIN
tolerance grade band governing the pinions used in this research, DIN 3965 Grades 1 – 7.

![CMM Pitch Vs. Virtual Pitch](image)

**Figure 3.4:** Similarity between virtual pitch results and CMM pitch results

### 3.3. Error Library

The next stage of development was the creation of the error library to be used as a comparative database against which actual data would be evaluated. The first step was to recreate all single error sources. This involved transforming the coordinate system of the 3D data in ParaView in controlled steps. The X axis was first transformed in the positive direction by 10µm, represented as X+0.01mm. This was then stepped to X+0.02mm, and followed this pattern of 10µm steps up to X+0.06mm. The X axis was then transformed in the negative X direction, to create the errors in similar steps from X-0.01mm to X-0.06mm. The X axis was then tilted in both the positive and negative directions, in steps of 0.01°, to create Tilt_X+0.01° to Tilt_X+0.06°, and Tilt_X-0.01° to Tilt_X-0.06°.

It was noted that there appeared to be a linear mathematical relationship between the error sources as they were increased. It was discovered that all errors were governed by equation 3 below;

\[
\Delta_i = \frac{i}{\omega} \times \Delta_\omega
\]  

(3)

Where \( \Delta \) is the deviation, in millimetres or degrees, of any defined error. Defined errors are +/- X, +/- Tilt_X, +/- Y, and +/- Tilt_Y errors. \( i \) and \( \omega \) are the magnitudes of those errors, in this case, 0.01 to 0.06. After further testing it became apparent that errors in the negative direction were the inverse values of their positive counterparts. After the X axis values were determined, the singular Y axis values were also recreated, which are +/- Y and +/- Tilt_Y.

The next stage of development was to determine the combined errors, and also any possible mathematical relationship between the combined errors and their single error constituents. After creating several combined error examples in ParaView, a mathematical relationship was defined. It was revealed that all combined errors are the summation of the deviations of the individual errors, as seen in equation 4;

\[
\Delta_{i,\omega} = \Delta_i + \Delta_\omega
\]  

(4)
This holds true for all error combinations, regardless of sign, and regardless of how many single error constituents are summed. A possible reason for the linear behaviour of the errors is that over the very small area on the tooth flank that the errors are derived from, the part surface is relatively flat. For this reason, it is also quite probable that these error source relationships would hold true for most other gears types, except for those with extreme free form surface flanks.

Defining the mathematical relationships between both single errors and single errors and combined errors allowed for the much more efficient mathematical construction of the comparative error library. Using these methodologies, an error library of approximately 30,000 entries was created, which represents the totality of all possible single and combined errors, both positive and negative, that the pinion can inherit within the tolerance range specified.

3.4. Construction of Decision Support Tool

The Decision Support Tool (DST) was constructed using Visual Basic for Applications in Microsoft Excel 2007. The reason for this is due both to the ease of usability of the two applications, and the high level of product integration into many work environments. The structure of the application is split across several functions. The two main functions are those that calculate the pitch errors based on coordinate data, as described in section 3.2, and the function that determines which of the error library entries matches closest to the CMM results.

The data in the application is split into two columns, concave and convex flank deviations. The concave and convex flanks are treated as separate entities, rather than the whole gear considered as one. This is because of the complicated machine movements involved during pinion machining. As not all errors are derived from clamping issues for example, it cannot be assumed that the concave and convex flanks will always show the same source of error. If for example a machine spindle lengthens during the course of machining, this may result in the Z and B axes being affected in such a way as to influence two flanks differently.

The user of the application is required to simply input pitch results taken directly from a measured part, and select the ‘Analyse’ button on the app user interface. The app is then coded so that the pitch results that were input get compared one by one to the errors in the error library, using Pearson’s Correlation method. The application then performs two main tasks. Firstly, for every flank, it returns the name of the three errors which have the highest correlation coefficients. This represents the three errors with the closest match to the input data. Secondly, the app returns the one error which has the highest combined correlation coefficient to the input data, derived from the correlation coefficients of the concave and convex flanks. This displays to the user the ‘best match’ scenario, meaning that error with the highest combination of both concave and convex flank correlation coefficients. The CMM data is then plotted separately against the top three results for every flank and the ‘best match’ error or error combination. This enables the user to also visually determine the error most probable to be influencing the part.

Importantly, for future development, the application is structured so as to allow any comparison method or algorithm to be used, if it is deemed more accurate or statistically robust. A picture of the DST user interface is shown below in Figure 3.5.
3.5. Initial Results

The initial results of the DST look very promising. The tool has been used to assess the errors present in several pinions machined during the course of this work. An example of one of the ‘Best Match’ errors is displayed in Figure 3.6, and the relationship between the CMM pitch data and the Virtual Metrology (VM) error library data is visually evident. The concave and convex flanks have squared coefficients of correlation of 0.99 and 1.00 respectively.

The DST has also identified several repeatable errors which were present in a series of pinions recently produced. Three gears out of the four produced had negative tilts in the X axes of both flanks, averaging -0.046°. The same three gears also had very similar Y axis failures present displayed by both flanks, with an average of -0.0275mm per gear. In figure 3.6, the most probable source of error is displayed as the title of the graph, in this case, a positive tilt in the X and Y axes, given in degrees, and a negative translational X axis failure, displayed in mm.
4. CONCLUSION

The aim of the work was to develop techniques and a software tool to allow the correct identification of 5 axis machining error sources of spiral bevel gears, which, once understood, could form part of a broader adaptive machining process. The early results have been encouraging, and as the modularity of the app will allow further developments if necessary. Even though corrective adaption methods may not be readily available to a machine operator through simple machine interactions, the DST provides a supportive statistical aid to help determine error sources. It is hoped this will help to reduce the gap between machining, measurement, and machine adaption, to facilitate better part quality.

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6. REFERENCES