Disturbance estimation for sensorless PMSM drive with Unscented Kalman Filter

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Abstract—This paper describes a study and experimental verification of sensorless control of Permanent Magnet Synchronous Motor in mechatronics application. There are proposed novel estimation strategy based on the Unscented Kalman Filter, using only the measurement of the motor current for on-line estimation of speed, rotor position and disturbance – load torque. Information about the load is important for complex drive control systems like robot arm. It is seldom obtained by estimation way especially in sensorless systems. Used Kalman filter is an optimal state estimator and is usually applied to a dynamic system that involves a random noise environment. Control structure with unscented algorithm, in real time requires a very efficient signal processor. Experimental results have been carried out to verify the effectiveness and applicability of the novel proposed estimation technique.

I. INTRODUCTION

There is currently working on several methods to find the status of the propulsion mechatronic system based on measurement of readily available terminal variables [1], [2]. These methods are called observers at all [3]. One of them is the Kalman filter, it is based on mathematical dependencies formulated by Rudolf Kalman [4].

Julier and Uhlman proposed completely novel solution of estimation theory problem in [5], next developed by Van Der Merwe in [6], and authors in [7]. Their solution based on Unscented Transformation. These authors found, it is easier to approximate the Gaussian distribution associated with each state vector variable, rather than approximated nonlinear function transformation. Based on this assumption it is possible to find state but also mean and deviation of its disturbances.

In simple model of electromechanicals one axis drive system, the natural state variables are shaft position and speed with disturbance which is load torque [8], [9]. So it is possible to obtain information about position, speed and torque. Permanent Magnet Synchronous Motor are widely used in industrial high precision and high dynamic mechatronics drives. Their inherent high torque to inertia ratio, small sizes and enhanced dynamic performances are predominant features [2], [8]. Systems called sensorless have the potential to get rid of any mechanical sensors placed on the machine shaft, usually of position and velocity measurements [1], [2]. Some method have been developed in order to obtain mechanical quantities of PMSM and induction motors [1], [2], [10]–[13]. Interesting issue is drive parameter estimation estimation [14]. The main assumption is how to find actual value of load torque, there are some solutions of disturbance estimation [9], [15]–[20].

Observers and filters can be used for sensorless control. Observers method use only direct signals – stator voltages, as well as variables measuring terminal – phase stator currents. So we can say that from a pragmatic point of view, information about disturbances, when the motor load, and what they relate to this, the strength of interaction is quite important [21]. Most tension or load measurements are performed using strain gauge bridges, which can be mounted at various locations of the robot arms. Construction of an load torque observer which could estimate an interaction is quite an important step towards the development of methods to control the torque and arm forces in conjunction with the sensorless control.

Another important use of load values are control systems designs for load torque compensation [10], [22], [23]. Load torque signal is often added to the output of speed controller for fast acting load compensation what is very important in sensorless drives [9], [19]. The speed controller reaches immediately the speed reference value avoiding offsets which must be compensated by the integrator (part of PI).

Moreover, a better response to load torque variations which are detected and compensated leading to small speed variations is obtained [24], [25].

The proposed work playback system allows to estimate the state variables – all variables occurring in the classical mathematical model of PMSM [8] and additional load torque. This variable brought into the effects on the motor shaft, which is often treated as a disturbance.

II. SENSORLESS CONTROL

Sensorless control of PMSM is based on eliminating the necessity of the shaft angular position and speed transducers, also to eliminate any mechanical sensors placed on and near the motor shaft. Avoiding of mechanical sensors provides profits such as: increased reliability drive, ability to work in
adverse environmental conditions, a radically decline in the cost of the drive.

Vector control of Permanent Magnet Synchronous Motor is an suit control method of controlling electric drives, where field oriented theory is used to control space vectors of magnetic flux, currents, and voltages [26], [27]. It is possible to set up the coordinate system to decompose the vectors into how much electromagnetic field is generated and how much electromagnetic torque would be produced. The structure should be very simple and close into separately excited DC motor. This control technique achieves dynamic performance of PMSM [27], [28].

Simplified structure of proposed subordinated sensorless control of PMSM is shown in figure 1. Control diagram consists of PI current controllers subordinated to speed controller and adequate frame coordinate transformations. The main state of presented system is an estimator, realised by Kalman Filter theory operation. The PI speed controller feeds active current \( i_q^* \) in order to keep Field Oriented Control [26]. The demanded current is computing by using the difference between requested speed \( \omega_r \) and speed \( \dot{\omega}_r \), which is estimated by Kalman filter. Motor operating does not require the field weakening, as assumed, but there is no huge problem to introduce nonzero current \( i_d \). Therefore desired current \( i_d^* \) in \( d \) axis is maintained to zero. These signals are inputs of PI current controllers, which provides desired voltages in \( dq \) reference frame. Basing on estimated shaft position \( \gamma \), \( dq \) voltage is converted into the stationary two axis frame \( \alpha \beta \) and send to control Pulse Width Modulation three phase inverter.

III. MATHEMATICAL MODEL OF THE DRIVE WITH DISTURBANCES CONSIDERATION

Mathematical model of PMSM concerns in three main parts: electrical network, electromechanical torque production, and third is mechanical subsystem [8]. The stator of PMSM and Induction Motor are similar. The rotor consists permanent magnets, there are a modern rare-earth magnets with high strength.

During investigations some simplified assumptions are made: saturation is neglected, inducted electromagnetic force is sinusoidally, eddy currents and hysteresis losses are neglected, no dynamical dependencies in air-gap, rotor cage is not present [8]. With these assumptions, the rotor oriented \( dq \) electrical network equations of PMSM can be described as:

\[
\begin{align*}
u_d &= R_s i_d + L_d \frac{d i_d}{dt} + \rho \omega_r L_q i_q, \\
u_q &= R_s i_q + L_q \frac{d i_q}{dt} + \rho \omega_r L_d i_d + \rho \omega_r \Psi_m.
\end{align*}
\]

(1)

(2)

where \( u_d, u_q \) are \( dq \) axis voltages, \( i_d, i_q \) are \( dq \) axis currents, \( L_d, L_q \) are \( dq \) axis inductances, \( R_s \) is stator resistance, and \( \Psi_m \) is magnetic flux produced by permanent magnets placed on rotor.

The electromagnetic torque which is produced has value:

\[
T_e = \frac{3}{2} p [\Psi_m - (L_q - L_d) i_d] i_q,
\]

(3)

where \( p \) is number pole pairs, and fraction \( \frac{3}{2} \) stems from frame conversion: perpendicular stator \( \alpha \beta \) into rotor \( dq \).

Drive mechanical dynamics can be describes as a simple differential equation:

\[
T_e - T_{load} = J \frac{d \omega_r}{dt},
\]

(4)

with only two quantities: variable \( T_{load} \) — sum of load torques and constant \( J \) — summary moment of inertia of mechanical kinematic chain. Based on (3) and (4) movement equation is:

\[
\frac{d \omega_r}{dt} = \frac{3}{2} (\Psi_m - (L_q - L_d) i_d) \cdot i_q - \frac{T_e}{J}.
\]

(5)

Position \( \gamma \) can be described by derivative equation of rotational speed:

\[
\frac{d \gamma}{dt} = p \cdot \omega_r.
\]

(6)

Many speed observers described in literature do not recognise load torque and other disturbances [1], [2], [29]–[31]. In these observers, the velocity is assumed to be constant in short period time without any disturbances. For this work the disturbances of each variable is recognised.

State space model can be described as a set of stochastic discrete equations (based on Euler single order factorisation):

\[
\ddot{\xi}_k = F_k (\ddot{\xi}_{k-1}) + B_k (\ddot{\xi}_{k-1}) \omega_k + w_k,
\]

(7)

\[
\dot{\xi}_k = H_k (\dot{\xi}_k) + \epsilon_k,
\]

(8)

with actual step \( k \) and computation time equal \( T_s \), which \( \omega_k \) and \( \epsilon_k \) are disturbances of system and output respectively.

System matrix \( F_k \) is:

\[
F_k (\ddot{\xi}_k) =
\begin{bmatrix}
1 - T_s \cdot \frac{R_s}{L_d} & T_s \cdot \omega_r \frac{L_q}{L_d} & 0 & 0 \\
-T_s \cdot \omega_r \frac{L_q}{L_d} & 1 - T_s \cdot \frac{R_s}{L_q} & -T_s \cdot \frac{\Psi_m}{L_q} & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & T_s & 1
\end{bmatrix},
\]

(9)

where:

\[
T_1 = T_s \cdot \frac{3}{2} p \left( \Psi_f - (L_q - L_d) i_d \right)
\]

Output matrix \( H_k \) is:

\[
H_k (\ddot{\xi}_k) =
\begin{bmatrix}
\cos \gamma & -\sin \gamma & 0 & 0 \\
\sin \gamma & \cos \gamma & 0 & 0
\end{bmatrix},
\]

(10)

and input matrix \( B_k \) is as follow:

\[
B_k (\ddot{\xi}_k) = T_s \cdot \frac{1}{L_d}
\begin{bmatrix}
\cos \gamma & \sin \gamma \\
-\sin \gamma & \cos \gamma \\
0 & 0 \\
0 & 0
\end{bmatrix}.
\]

(11)

The first three elements of disturbance vector \( w_k \) appear in (7) are natural stochastic disturbances:

\[
w_k =
\begin{bmatrix}
w_{i_d} & w_{i_q} & w_{\gamma} & w_{\omega}
\end{bmatrix}.
\]

(12)
This values can not be determined analytically, so the fourth element can be described by 5:

\[ w_\omega = -T_s \cdot \frac{1}{J} \cdot T_{load}. \quad (13) \]

So it is possible to determine estimation of load torque:

\[ \hat{T}_{load} = -J \cdot T_s \cdot w_\omega \quad (14) \]

There is main assumption of the whole work. There is also possible to determine other properties of proposed system based on other values of estimated disturbances vectors \( w_k \) and \( v_k \), what is subject of further work.

IV. Unscented Kalman Filter

Primary issue nonlinear systems estimation is difficult to determine the nonlinear function of state and output and probability distribution [5], [7], [32], [33]. It appears, that the nonlinear transformation of what is to deviate and Jacobians counting for an Extended Kalman Filter does not determine real covariances. Extended Kalman Filter based on the classical Kalman filter assumptions about the linearity of the object and it is in this way are covariances calculated [4], [33]. For nonlinear objects covariances are associated with the process, but it can not be linked into linearised model of object. Based on the particular analysis of nonlinear systems where the covariance of the state should not be associated with linearised system, and can even be far from them.

Unscented Kalman filter is an improvement over the Extended Kalman Filter algorithm. Simon Julier and Jeffrey Uhlman proposed completely novel solution of estimation theory problem in [5]. Their solution based on Unscented Transformation. These authors found it easier, to approximate the Gaussian distribution associated with each state vector variable, rather than approximated nonlinear function transformation. Made it possible to simplify the algorithm by eliminating the need for calculating Jacobians at each step and working point. There is based on two assumptions. The first is the determination of non-linear transform of the function at work, and not in the whole range of probability density distribution function. The second point concerns the search for work in which this density corresponds to the actual decomposition of the nonlinear system.

This filter, like its classical form is based on two cycles performed procedures: prediction and correction.

A. Prediction

The UKF prediction can be used independently from the UKF update, in combination with a linear update. As assumed in the classical Kalman filter approach, and as is the case for the extended filter prediction proceeds in a similar way for
each solution. In this case, however, extend estimation state vector of the value of disturbances. Such a procedure makes it possible to estimate the state vector and its environment. Strictly, this environment will transform non-linear distortion [5].

Defined new \( \dot{x}_{k-1|k-1} \) vector is as:

\[
x_{k-1|k-1}^a = [\dot{x}_T^T]_{k-1} - E(u_T^T) \ E(u_T^T)^T.
\] (15)

So it is natural to define its covariances, which is formed by the making of the covariance of the state vector \( P_{k-1|k-1} \), the known process noise covariance \( Q_k \) and distortion measurement \( R_k \). Therefore assumes the form:

\[
P_{k-1|k-1}^a = \begin{bmatrix}
P_{k-1|k-1} & 0 & 0 \\
0 & Q_k & 0 \\
0 & 0 & R_k
\end{bmatrix}.
\] (16)

A set of \( 2L + 1 \) sigma points – \( \chi_{k-1|k-1} \) is derived from the augmented state and covariance where \( L \) is the dimension of the augmented state:

\[
\chi_{k-1|k-1}^0 = x_{k-1|k-1}^0 - 2L, \quad \chi_{k-1|k-1}^i = x_{k-1|k-1}^0 + \sqrt{(L+\lambda)P_{k-1|k-1}^0}, \quad \text{for } i = 1, \ldots, L - 1, \\
\chi_{k-1|k-1}^{i+1} = x_{k-1|k-1}^0 - \sqrt{(L+\lambda)P_{k-1|k-1}^0}, \quad \text{for } i = L + 1, \ldots, 2L.
\] (17)\( \vdots \) (19)

The matrix square root \( \sqrt{(L+\lambda)P_k} \) should be calculated using numerically efficient and stable methods such as the Cholesky decomposition, where \( (L+\lambda) \) coefficient was choose based on [6]. The sigma points \( \chi_{k-1|k-1} \) are propagated through the state space transition function:

\[
\chi_{k|k-1}^i = F_k \chi_{k-1|k-1} + B_k u_{vk}, \quad i = 0, \ldots, 2L.
\] (20)

The weighted sigma points \( \chi_{k|k-1}^i \) are recombined to produce the predicted state \( \dot{x}_{k|k-1} \) and covariance \( P_{k|k-1} \), like:

\[
\dot{x}_{k|k-1} = \sum_{i=0}^{2L} W_s^i \chi_{k|k-1}^i, \\
P_{k|k-1} = \sum_{i=0}^{2L} W_c^i [\chi_{k|k-1}^i - \dot{x}_{k|k-1}] [\chi_{k|k-1}^i - \dot{x}_{k|k-1}]^T.
\] (21)\( \vdots \) (22)

where the weights \( W_s \) and \( W_c \) for the state and covariance, are given by:

\[
W_s^0 = \frac{\lambda}{L+\lambda}, \quad W_s^i = \frac{\lambda}{L+\lambda} (1 - \alpha^2 + \beta), \quad W_s^{i+1} = \frac{1}{2(L+\lambda)}, \quad \text{for } i = 0, \ldots, L - 1.
\] (23)\( \vdots \) (25)

where \( \alpha, \beta, \kappa \) are noise distribution parameters, and \( \lambda \) is chosen arbitrary. There are helpful choosing during filter tuning [7]. Typical values for \( \alpha, \beta \) and \( \kappa \) for the majority of applications in which the disturbance is located in the Gaussian noise assumptions, are respectively \( 10^{-3}, 2 \) and 0. Any differences from these values can only lead to more easily tune the filter, because they add additional degrees of freedom [7].

\section{B. Correction}

The sigma points \( \chi_{k|k-1}^i \) are projected through the observation function \( H_k \):

\[
\gamma_{k}^i = H_k \chi_{k|k-1}^i, \quad i = 0, \ldots, 2L.
\] (26)

Based on weight \( W_s^i \) and \( W_c^i \) from equation (25) and observation matrix \( \gamma_k^i \) it is possible to obtain output signal:

\[
\dot{z}_k = \sum_{i=0}^{2L} W_s^i \gamma_k^i, \quad \text{and also output covariance:}
\]

\[
P_{z_k|z_k} = \sum_{i=0}^{2L} W_c^i [\gamma_k^i - \dot{z}_k] [\gamma_k^i - \dot{z}_k]^T.
\] (27)\( \vdots \) (28)

Solution of classical form of Kalman Filter is adapted in Unscented Kalman Filter. Correction \( \kappa_k \) depend directly on state covariances \( P_{k|k-1} \) and innovation of system covariances \( S_k \), so is similar to (28). So based on Kalman Correction definition can be derived expression:

\[
\kappa_k = P_{z_k|z_k}^{-1} - P_{z_k|z_k}^{-1} P_{z_k|z_k}^{-1} (2L + 1 - 2L - 1)
\]

\[
\gamma_k = H_k \dot{x}_{k|k-1} + B_k \dot{u}_{vk}.
\]

\[
\gamma_k = H_k \dot{x}_{k|k-1}.
\]

\section{V. EXPERIMENTAL RESULTS}

For experimental verification of the proposed estimation method, a laboratory setup has been constructed. It consists of the surface mounted magnets synchronous motor, supplied from the three phase power IGBT inverter. Mechanical part of laboratory setup presented on figure 2, they consist two similar motors, shaft with possible moment of inertia additional rings change. The second twin motor supplied from industrial controller (Unidrive made by Control Techniques). The voltage and currents signals are adjusted and sampled simultaneously with 12-bit A/D converters. The rotor position is measured by precision incremental encoder, only for comparing.
The presented algorithm was implemented on Analog Devices Sharc 21369 Digital Signal Processor. The code has been mainly written in C language. Field Oriented Control and control loops: speed and currents are carried out periodically by superloop with $T_s = 100 \mu s$ period, synchronised with 10-bits PWM generator. A real position is also computed using FPGA and sent to DSP by parallel memory fields.

An execution time of UKF algorithms is about $50 \mu s$ respectively, and whole such control algorithm takes $15 \mu s$.

At the first part of investigation, it was focused on controlled system behaviour by reference speed ($\omega_r^*$) excitation and next stepped load torque. This type of reference signal has such significant stages for estimator behaviour like: zero signal with no initial values in estimation vector, step demand speed signal and external load step. The maximum module of reference speed is $1000 \frac{\text{rev}}{\text{min}} = 104.72 \frac{\text{rad}}{\text{s}}$ ($\frac{1}{3}$ of maximum speed) and additional load torque is $3 \text{Nm}$. Torque is applied and next removed. Results of working are presented on figure 3. Every investigations are performed in these same ways, first non zero start-up state vector during zero real speed, but differences are additional load torque values.

For a more complete presentation of the observer behaviour rest of investigations are presented only in limited interval of time, where the speed is settled. Figure 4 shows results of impact external load value $T_{load} = 3 \text{Nm}$, and fig. 6 half – $T_{load} = 1.5 \text{Nm}$.

It is easy to note that regardless of the speed shape of...
estimated values (fig. 4, 6) and errors (5, 7) look very similar.

![Fig. 6: Load torque $T_{load} = 1.5 \text{Nm}$ response](image1)

![Fig. 7: Load torque $T_{load} = 1.5 \text{Nm}$ response – estimation errors](image2)

It can be concluded that the system behaves values estimated consistently regardless of the load torque conditions.

In the steady state the error of estimation is not great. Its maximum values depend on the dynamic of the state changes. The avoiding of the principle of constancy of the load moment causes the appearance of significant errors in dynamic states. These appears errors can be eliminated by changing the sensitivity of the observer, but changing sensitivity causes gaining the system and output noises. So in the case of cancel dynamical errors we can expect to strengthen the unknown disturbance, in most cases nonlinear.

Analysing the whole process of estimating the load torque and with those obtained from errors, it can be concluded that the load torque observer reproduces with small errors.

VI. CONCLUSION

The paper shows successful way of design of the novel Unscented Kalman Filter based estimator of mechanical quantities with disturbance for sensorless control of PMSM drive. An analytical approach for developing the algorithm was presented in detail. The task-oriented mathematical model of the system, the suitable choice of the state variables and disturbance, the adequate simplification of the procedures as well as the suitable choice of elements of the covariance matrices have decided on the success. The discrete Unscented Kalman Filter was found to be well suited to the speed, position and load torque estimation as well.

Correctness of the idea of estimator with disturbance observation was verified and validated during testing the laboratory high-dynamic PMSM drive setup. The results have shown that the proposed control strategy had a good dynamic response in a wide range of acting load.

The wide development of an experimental drive system is under way.

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