Abstract— In this paper, we present an efficient filtering algorithm to perform accurate estimation in jump Markov nonlinear systems, in case of multi-target tracking. With this paper, we aim to contribute in solving the problem of model-based body motion estimation by using data coming from visual sensors. The Interacting Multiple Model (IMM) algorithm is specially designed to track accurately targets whose state and/or measurement (assumed to be linear) models changes during motion transition. However, when these models are nonlinear, the IMM algorithm must be modified in order to guarantee an accurate track. In this paper we propose to avoid the Extended Kalman filter because of its limitations and substitute it with the Unscented Kalman filter which seems to be more efficient especially according to the simulation results obtained with the nonlinear IMM algorithm (IMM-UKF). To resolve the problem of data association, we propose to use a Fuzzy approach. The later is then combined with the IMM-UKF algorithm; the derived algorithm is called Fuzzy-IMM-UKF.

Index Terms— Estimation, Kalman filtering, Multi-Target Tracking, Visual servoing, data association.

I. INTRODUCTION

This paper hopes to be a contribution within the field of visual-based control of robots, especially in visual-based tracking [3]; tracking manoeuvring targets, which may themselves be robots, is a complex problem, to ensure a good track when the target switches abruptly from a motion model to another is not evident. Because of the complexity and difficulty of the problem, a simple case is considered. The study is restricted to 2-D motions of a point, whose position is given at sampling instants in terms of its Cartesian coordinates. This point may be the center of gravity of the projection of an object into a camera plane, or the result of the localisation of a mobile robot moving on a planar ground.

Several of maneuvering targets tracking algorithms are developed. Among them, the interacting multiple model (IMM) method based on the optimal Kalman filter, yields good performance with efficient computation especially when the measurement and state models are linear. However, if the latter are nonlinear, the standard Kalman filter should be substituted, in our study we choose the recent Unscented Kalman Filter (UKF). The other problem treated in this paper, is about the data association. Effectively, at each sample time, the sensor (camera) present, several measures and observations, coming from different targets; the problem is how to affect each measure to the correct target, to resolve this problem we choose a fuzzy approach. The algorithm derived from the combination of the later approach and the nonlinear IMM algorithms is noted Fuzzy-IMM-UKF.

The paper is organized as follows. In section II the mathematical formulation of 2-D motion is presented. In section III we briefly present the UKF. We describe in section IV the nonlinear IMM algorithm UKF based. In section V we present the Fuzzy-IMM-UKF algorithm. In section VI we present and discuss the results of simulations. Finally in section VII we draw the conclusion.

II. MATHEMATICAL FORMULATION OF 2-D MOTION

The mathematical formulation of 2-D motion used is mainly inspired from Danes, Djouadi, and al in [4]. They make the hypothesis that the measurements are only the 2-D Cartesian coordinates of the moving point.

Let \( s(.) \) denote the curvilinear abscissa of \( M \) over time onto its trajectory, the origin of curvilinear abscissa is set arbitrarily. Functions \( x(.) \) and \( y(.) \), represent the Cartesian coordinates of \( M \). The measurement equation may be written as:

\[
\begin{bmatrix}
    x(t) \\
    y(t)
\end{bmatrix} = h(s(t), p(t))
\]  

(1)

Where \( p(.) \) is a parameter vector function of minimal size.

We can see that equation (1) is independent of the type of the motion of \( M \) onto its trajectory.

The state equation could be written as:

\[
\dot{X}(t) = AX(t)
\]  

(2)

with \( X(t) = \begin{bmatrix} s(t) \\ p(t) \end{bmatrix} \)

A equals \( \begin{bmatrix} A_n & 0 \\ 0 & 0 \end{bmatrix} \), with \( A_n \) the \( n \times n \) zero matrix with ones added on its first upper diagonal, and 0 the matrices of...
convenient sizes. The continuous time state equation (2) is linear time invariant and independent of M’s trajectory, except on the sizes of $s(\cdot)$ and $p(\cdot)$. Moreover, it may be shown that the fundamental matrix $F$ involves its exact discretization at the period $T$ takes the form
\[ F = \exp(\Delta T) = \sum_{i=0}^{n-1} \frac{(\Delta T)^i}{i!}. \]

The dynamic and measurement noises are supposed to be stationary, white and Gaussian, non inter-correlated with known covariances.

### A. Canonical motion equations

The point M is supposed to move on straight or circular trajectories at constant or uniformly time-varying speed (constant speed or constant acceleration). Those motions belong to the set of the possible behaviours of a non-holonomic robot whose wheels are driven at constant velocities or accelerations.

1) Output equations: One minimal description of a straight line is defined by the vector $p=\left(\alpha, d\right)^T$ shown in figure 1(a), which is related to Plucker coordinates. Concerning a circular trajectory one minimal description is defined by the vector $p=\left(R_0, x_0, y_0\right)^T$ shown in figure 1(b). The origin of curvilinear abscissa is uniquely defined from those parameterizations.

#### (a) Line

![Diagram of Line](image1)

#### (b) Circle

![Diagram of Circle](image2)

Fig. 1. Trajectory Parameterization

The output equations are as follows (trajectory parameter are considered time-invariant):

**Straight Line:**
\[
\Delta z(k) = \begin{pmatrix} \frac{d \cos \alpha + s(k) \sin \alpha}{d \sin \alpha - s(k) \cos \alpha} + v(k) \end{pmatrix}
\]

**Circle:**
\[
\Delta z(k) = \begin{pmatrix} x_0 + R_0 \cos s(k) \\ y_0 + R_0 \sin s(k) \end{pmatrix} + v(k)
\]

with $s(k)$ distance covered by the target and $v(\cdot)$ zero mean Gaussian noise, $E[v(k)] = 0$, $E[v(k) \cdot v^T(k)] = R \delta_{k,l}$.

2) State Equations

**Constant velocity**
\[
\begin{pmatrix} s(k+1) \\ p(k+1) \end{pmatrix} = \begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix} \begin{pmatrix} s(k) \\ p(k) \end{pmatrix} + \begin{pmatrix} w_1(k) \\ w_2(k) \end{pmatrix}
\]

**Constant acceleration**
\[
\begin{pmatrix} s(k+1) \\ p(k+1) \end{pmatrix} = \begin{pmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s(k) \\ p(k) \end{pmatrix} + \begin{pmatrix} w_1(k) \\ w_2(k) \end{pmatrix}
\]

where the random vectors $x(0)=\left[\vec{s}(0)^T, \vec{p}(0)^T\right]^T$, $w(\cdot)=\left[w_1(\cdot)^T, w_2(\cdot)^T\right]^T$ and $v(\cdot)$ are jointly Gaussian and:
\[
E\left[x(0)\right] = \hat{x}(0), \quad E\left[w(\cdot)\right] = 0, \quad E\left[v(\cdot)\right] = 0,
\]

\[
E\left[\begin{pmatrix} x(0) - \hat{x}(0) \\ w(k) \end{pmatrix} \begin{pmatrix} x(0) - \hat{x}(0) \\ w(k) \end{pmatrix}^T\right] = \text{diag}(P(0), Q \delta_{k,k'}, R \delta_{k,k'})
\]

and
\[
\vec{s}(k) = \begin{pmatrix} s(k) \\ \dot{s}(k) \end{pmatrix} \quad \text{vector of the point M dynamics}
\]

\[
\vec{p}(k) = \begin{pmatrix} p(k) \end{pmatrix} \quad \text{trajectory parameters vector}
\]

### III. THE UNSCENTED KALMAN FILTER

The basis of the UKF is the unscented transform, where a distribution is approximated using a number of vectors which are passed through the nonlinear function to determine the probability distribution of the output from the function.
A. The Unscented Transform

According to Julier and Uhlmann in [5, 6], the unscented transform is based on the intuition that it is simpler to approximate a Gaussian distribution than it is to approximate an arbitrary nonlinear function. A set of vectors, selected to be representative of the probability distribution, are chosen so that their mean and covariance are respectively \( \bar{x} \) and \( P_{xx} \).

The nonlinear function is applied to each point; the result is a set of transformed points with the statistics \( \bar{y} \) and \( P_{yy} \). The selection of these vectors is not arbitrarily done but according to a deterministic algorithm.

The n-dimensional random variable \( x \) with mean and covariance respectively \( \bar{x} \) and \( P_{xx} \) is approximated by \( 2n+1 \) weighted points given by:

\[
\chi_0 = \bar{x} \\
\chi_i = \bar{x} + \left( \sqrt{(n+\kappa)P_{xx}} \right)i, \quad W_i = \frac{1}{2(n+\kappa)} \quad i = 1, \ldots, n \\
\chi_{i+n} = \bar{x} - \left( \sqrt{(n+\kappa)P_{xx}} \right)i, \quad W_{i+n} = \frac{1}{2(n+\kappa)}
\]

Where \( \kappa \in \mathbb{R} \), \( \sqrt{(n+\kappa)P_{xx}} \) is the \( i \) th row or column of the matrix square root of \( (n+\kappa)P_{xx} \), and \( W_i \) is the weight which is associated with the \( i \) th point, note that \( \sum_{i=0}^{2n} W_i = 1 \).

The transformation procedure occurs in three steps:

1. The transformed set of vectors are: \( y_i = f(\chi_i) \) \hspace{1cm} (8)
2. The mean is given by: \( \bar{y} = \sum_{i=0}^{2n} W_i y_i \) \hspace{1cm} (9)
3. The covariance is given by:
   \[
P_{yy} = \sum_{i=0}^{2n} W_i [(y_i - \bar{y}) (y_i - \bar{y})]^T
   \]
\hspace{1cm} (10)

According to Julier and Uhlmann [5,6], many interesting properties could be noticed about this algorithm, for instance it leads to greater accuracy and permit rapid implementation, thanks to the mean and covariance which are calculated using standard vector and matrix operations.

B. The Unscented Kalman Filter

The unscented Kalman filter is obtained by occurring little modifications on standard one; this, by restructuring the state vector and process and measurement models [5, 6].

Consider the following discrete-time nonlinear dynamical system:

\[
x(k+1) = f(x(k), u(k), k) + w(k) \\
z(k) = h(x(k), u(k), k) + v(k)
\]

Where \( x(k+1) \in \mathbb{R}^n \) is the state vector, \( z(k) \in \mathbb{R}^m \) the measurement vector, \( w(k) \in \mathbb{R}^n \) and \( v(k) \in \mathbb{R}^m \) are Gaussian random vectors with zero mean and covariance matrices \( Q \geq 0 \) and \( R > 0 \), respectively.

At first the state vector is augmented with the process and noise terms this leads to an \( n' = n + q \) dimensional vector.

\[
x'(k) = \begin{bmatrix} x(k) \\ w(k) \end{bmatrix}
\]
\hspace{1cm} (11)

The process model is now a function of \( x'(k) \),

\[
x(k+1) = f[x'(k), u(k), k] \\
x(k+1) = f[x'(k), u(k), k]
\]
\hspace{1cm} (12)

The unscented transform needs to \( 2n'+1 \) vectors which are selected from:

\[
\hat{x}'(k|k) = \begin{bmatrix} \hat{x}(k|k) \\ 0_{q	imes 1} \end{bmatrix}
\]
\hspace{1cm} (13)

The Kalman filter algorithm computes on two principles phases:

1. Prediction of the state vector and its covariance matrix
2. Estimation of the state vector and its covariance matrix by updating the prediction with the current measurement.

The prediction phase using the unscented transform is as follow:

1. The set of vectors are created by applying equation (7) to the augmented system given by equation (13)
2. The transformed set of vectors are given by
   \[
   \chi_i(k+1|k) = f[\chi_i(k|k), u(k), k]
   \]
\hspace{1cm} (14)
3. The predicted mean is calculated by
   \[
   \hat{x}(k+1|k) = \sum_{i=0}^{2n} W_i \chi_i(k+1|k)
   \]
\hspace{1cm} (15)
4. The predicted covariance is calculated by
   \[
   P(k+1|k) = \sum_{i=0}^{2n} W_i [\chi_i(k+1|k) - \hat{x}(k+1|k)] [\chi_i(k+1|k) - \hat{x}(k+1|k)]^T
   \]
\hspace{1cm} (16)
5. Each prediction vector is instantiate through the measurement model
   \[
   z_i(k+1|k) = h[\chi_i(k+1|k), u(k), k]
   \]
\hspace{1cm} (17)
6. The predicted observation is calculated by
   \[
   \hat{z}(k+1|k) = \sum_{i=0}^{2n} W_i z_i(k+1|k)
   \]
\hspace{1cm} (18)
7. Assuming the fact the measurement noise is additive and independent, the innovation covariance is
   \[
   P_{yy}(k+1|k) = R(k+1|k) + \sum_{i=0}^{2n} W_i [z_i(k+1|k) - \hat{z}(k+1|k)] [z_i(k+1|k) - \hat{z}(k+1|k)]^T
   \]
\hspace{1cm} (19)
8. The cross correlation matrix is calculated by
\[ P_w(k + 1|k) = \sum_{j=0}^{2r} W_j \left[ z_j(k + 1|k) - \hat{z}(k + 1|k) \right] \tilde{z}(k + 1|k)^T \]  
(20)

Given these predicted values the state and covariance estimates are computed according to the equations:
\[ \hat{x}(k + 1|k + 1) = \hat{x}(k + 1|k) + K(k + 1) \gamma(k + 1) \]  
(21)
\[ P(k + 1|k + 1) = P(k + 1|k) - K(k + 1) P(k + 1|k) K(k + 1)^T \]
with \( \gamma(k + 1) \) the innovation and \( K(k + 1) \) the weight chosen to minimise the mean squared error of the estimate.
\[ \gamma(k + 1) = z(k + 1) - \hat{z}(k + 1|k) \]
\[ K(k + 1) = P_{x|z}(k + 1|k) P_{x|y}^{-1}(k + 1|k) \]  
(22)

IV. THE NONLINEAR INTERACTING MULTIPLE MODEL ALGORITHM

A. The IMM Algorithm

Let a system be described by the equations (notations are chosen according to [2]):
\[ x(k) = F(M(k)) x(k - 1) + w[k - 1, M(k)] \]
\[ z(k) = H(M(k)) x(k) + \nu[k, M(k)] \]  
(23)
with \( w \) (\cdot) Gaussian process noise and \( \nu \) (\cdot) measurement Gaussian noise:
\[ E\{w(k)\} = 0, E\{w(k) \cdot w^T(k)\} = Q \delta_{k,k'} \]
\[ E\{\nu(k)\} = 0, E\{\nu(k) \cdot \nu^T(k)\} = R \delta_{k,k'} \]
Where \( M(k) \) denotes the model at time \( k \). It’s a finite state Markov process taking values in \( \{M_j\}^r_{j=1} \), according to a Markov transition probability matrix \( p \) assumed to be known, \( \nu \) and \( w \) represent white Gaussian processes and are assumed to be mutually independent.

A cycle of the IMM algorithm could be summarized in four steps:

1. The mixed initial condition for filters

Starting with \( \hat{x}'(k|k - 1) \), we compute the mixed initial condition for the filter matched to \( M_j(k) \)
\[ \hat{x}'(k - 1|k - 1) = \sum_{j=1}^{r} \hat{x}'(k - 1|k - 1) \mu_j(k - 1|k - 1) \]  
(24)
\[ i, j = 1, \ldots r \]
the covariance corresponding to the above is
\[ P'(k - 1|k - 1) = \sum_{j=1}^{r} \mu_j(k - 1|k - 1) \cdot \left\{ P'(k - 1|k - 1) + \left[ \hat{x}'(k - 1|k - 1) - \hat{x}(k - 1|k - 1) \right] \left[ \hat{x}'(k - 1|k - 1) - \hat{x}(k - 1|k - 1) \right]^T \right\} \]
(25)

Where:
\[ \mu_j(k - 1|k - 1) = \frac{1}{c} \nu_j(k - 1|k - 1) \]
(26)
is the probability that model \( M_j \) was in effect at \( k - 1 \) given that \( M_j \) is in effect at time \( k \) conditioned on \( Z_{k-1} \). And where:
\[ \mu_j(k - 1) \]
the probability that the model \( M_j \) is in effect at time \( k - 1 \),
\[ c_j = \sum_{j=1}^{r} \nu_j(k - 1|k - 1) \]
the normalizing constants.

2. Model-matched filtering

The above estimate and covariance are used as input to the filter matched to \( M_j(k) \), which uses \( z(k) \) to obtain \( \hat{x}'(k|k) \) and \( P'(k|k) \).

3. Model probability update

The \( r \) model probabilities are updated from the innovation of the \( r \) Kalman filters.
The likelihood functions corresponding to the \( r \) filters are given by:
\[ \Lambda_j(k) = N\{ \hat{x}(k); \tilde{z}(k) \mid k - 1, \hat{x}(k - 1|k - 1) \}, \]
\[ S_j\{ k; P(k - 1|k - 1) \} \]
(27)
\[ \mu_j(k) = \frac{1}{c} \Lambda_j(k) c_j \]  
(28)
where \( c = \sum_{j=1}^{r} \Lambda_j(k) c_j \) is the normalization constant.

4. Estimation and covariance combination.

The output estimates and covariances are computed according to the mixture equations
\[ \hat{x}(k|k) = \sum_{j=1}^{r} \hat{x}'(k|k) \mu_j(k) \]  
(29)
\[ P(k|k) = \sum_{j=1}^{r} P'(k|k) \cdot \left\{ P'(k|k) + \left[ \hat{x}'(k|k) - \hat{x}(k|k) \right] \left[ \hat{x}'(k|k) - \hat{x}(k|k) \right]^T \right\} \]
(30)
The problem here is that this algorithm is designed with the assumption that the target motion models are linear, so, for its optimality, the use of the Kalman filter is recommended. However, in the case on which the motion models are nonlinear we propose to use the UKF detailed in section III. The modification to operate on the IMM algorithm is to use as state and covariance estimator the UKF.

Thanks to this modification we hope to track accurately targets whose models are nonlinear, the resulting algorithm is called the Nonlinear Interacting Multiple Model and noted (IMM-UKF).
V. Fuzzy-IMM-UKF Algorithm

To resolve the problem of data association, we propose to use a Fuzzy approach. Fuzzy data association (FDA) is used to evaluate the association degree between the return and the actual of interest. Inputs are the measurement innovation.

The association degree is $\beta(k) = \text{FDA}(\theta, A, \Phi)$, where FDA(.) is a fuzzy function based on the set of membership functions $A$ and the set of propositions $\Phi$ [10], using fuzzy logic methodology, the rules in the FDA rule base are evaluated. The FDA rules to determine the degree of association are provided in table 1, like in [10]. The fuzzy variable, FDA, has 49 If-Then rules; in table 1, the fuzzy term set of input variables are given by \{NL, NM, NS, Z, PS, PM, PL\}, where NL, NM NS, Z, PS, PM, and PL denote Negative Large, Negative Medium, Negative Small, Zero, Positive Small, Positive Medium, and Positive Large, respectively. The fuzzy term set of output variables are \{VL, ML, M, MH, VH\}, which denote Very Low, Medium Low, Medium, Medium High, and Very High, respectively.

<table>
<thead>
<tr>
<th>Association degree</th>
<th>X coordinate error</th>
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<td>NL</td>
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<td>Y coordinate error</td>
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<td>PL</td>
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For the FDA configurations, the sets of membership functions which fuzzify these inputs, defined over appropriate universe of discourse, are shown in Fig.1. The output of the FDA membership functions, which are used to evaluate the outputs through the application of fuzzy inference (via max-min rule) and center of gravity defuzzification [11], are shown in fig.2.

VI. Simulations and Results

In this section, we perform some simulations to evaluate our algorithm (Fuzzy-IMM-UKF).

The motion models considered are: - constant velocity on straight line (M1), -constant acceleration on straight line (M2), - constant velocity on circle (M3), - constant acceleration on circle (M4).

To explore the capability of our Fuzzy-IMM-UKF algorithm to track maneuvring targets, various scenarios are considered; among of them we select the typical case of three highly maneuvring targets with crossing trajectories.

We assume that the target is in a 2-D space and its position is sampled every $T=1$s. For all cases we assume:
- The measurement noise is zero mean, white, independent of the process noise, and with variance $\sigma_v^2=0.01$.
- The process noise is zero mean, white, independent of the measurement noise, and with variance $\sigma_w^2=0.001$.
- The initial condition with the state \(X=[s, \dot{s}, s, \alpha, d, R_0, x, y]\) is
- The probability transition matrix of four models is \(P=\begin{bmatrix} 0.97 & 0.01 & 0.01 & 0.01 \\ 0.01 & 0.97 & 0.01 & 0.01 \\ 0.01 & 0.01 & 0.97 & 0.01 \\ 0.01 & 0.01 & 0.01 & 0.97 \end{bmatrix}\)
- The initial probability of selecting a model is 0.25, that’s to say, at the start all models have the same chance to be selected.
- The curvilinear abscissa $s(.)$ remains continuous even if a trajectory jump occurs.

A. Considered scenario

We consider that we have to track simultaneously three maneuvring targets. In order to complicate the scenario, we suppose that the targets follow during there movements, crossing trajectories.
a) Target 1 (black):
The target starts moving according to model M1 until the 50\textsuperscript{th} sample when an abrupt trajectory change occurs and still moving according to this during 50 samples (switching from model M1 to M3).

b) Target 2 (blue):
The target starts moving according to model M1 until the 50\textsuperscript{th} sample when an abrupt trajectory change occurs and still moving according to this during 50 samples (switching from model M1 to M3).

c) Target 3 (green):
The target starts moving according to model M1 until the 50\textsuperscript{th} sample when an abrupt acceleration about 0.2 m/s\textsuperscript{2} occurs and still moving according to this during 50 samples (switching from model M1 to M2).

B. Results interpretation:
Figure 3 shows that the esteemed and the real trajectory for the three targets are superposable and almost identical even if an abrupt change occurs on the tracked target dynamic. This result is confirmed by the figures (4,5,6,7,8), from this we can say that the tracker based IMM-UKF algorithm is a pertinent solution to the problem of visual-based tracking highly maneuvering targets. In the other hand figure 1 shows also that the data association is correctly done even if the trajectories cross each other. This should permit us to say that the FDA algorithm computes perfectly and its combination with the IMM-UKF algorithm (Fuzzy-IMM-UKF) would be...
an efficient solution to the problem of highly maneuvering multi-target visual-based tracking.

VII. CONCLUSION

The model-based body motion estimation by using data coming from visual sensors still an open problem on which we try to provide a contribution. In this paper we presented a nonlinear algorithm which attempts to track efficiently a highly maneuvering target whose trajectory and/or dynamic could change abruptly, the algorithm proposed is noted IMM-UKF. To extend this algorithm to multi-target case, we combined the later with the FDA algorithm to ensure good data association. Simulations show that the Fuzzy-IMM-UKF is a good investment while we are asked to track a highly maneuverable targets whose measurement and/or state models present a strong nonlinearities, and when there different trajectories cross each other.

REFERENCES