An Adaptive Bilateral Filter For Inpainting

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Abstract— An adaptive model of bilateral filter is presented for digital inpainting. The model works by transforming inpainting into an equivalent energy condition minimization and generation of patches for missing areas by interpolating within working frame. It combines knowledge of local structure by bilateral filter and intensive value. Bilateral filter is adapted to missing regions to check similarity of regions to fill-in. Standard deviation in range kernel of the filter is regulated by total variation. This helps to create patches that keep edges. Total variation is also efficient for detection of missing pixels which possibly stay on edges. Benefit of the model was demonstrated in experiment of inpainting for gray and color images.

Keywords— Inpainting, total variation, bilateral filter, similarity, edge-preserving filter

I. INTRODUCTION

Image inpainting play a fundamental role in improvement for unity and completeness of image. It is still open research topic in image analysis. Typical algorithm would take missing areas and use information of good areas to restore the missing areas.

The conventional methods for solving this problem use Partial Differential Equations (PDE) in iterative algorithms. Pixels of missing area in the border with good areas are checked with their neighbor pixels in good area by PDE based functions. This runs in each iterative cycle until all missing pixels were recovered.

Exemplar based methods, similar to PDE approach, also run around the border of missing area, but they check each small region of pixels but not a single pixel. Exemplars with specific structure are created by small regions in known area and they are used like example to fill-in missing area.

Bilateral filter introduced firstly into image denoising application. This non-linear filter uses Gaussian in the spatial domain where weight of each pixel is influenced by the intensity domain. Its advantage is to eliminate noise but keep edges. This paper work follows concept of the exemplar based methods, incorporating with total variation (TV) and bilateral filter to check similarity of structure. In adopting bilateral filter concept, incomplete bilateral filter was introduced in optimal condition, presenting similarity of intensive value and structure between missing and good areas of image.

II. OUTLINE OF PAPER

Related work contributions and notation for this work are presented in section 2, 3, 4, 5. In section 6, major concept of adaptive bilateral filter for inpainting is formulated. The section explains how bilateral filter can be transformed into suitable form for checking similarity regions in image. Section 6 presents the experimental evaluation of the total variation bilateral filter for inpainting gray and color images.

Before describing paper work in details, let’s see figure 1 for visual display of working concept. Fig.1a shows input image. Fig.1b draws full Gaussian net in left and partial net in right. The partial net is an example of incomplete bilateral filter for a pixel on border of paint mask, that produces result in fig.1c.

III. PREVIOUS AND RELATED WORK

Image inpainting has practical application and several researches addressed to the topic. PDE based algorithm is proposed by Marcelo Bertalmio et al [1]. PDE based methods found success in iterative algorithms. Pixels of missing area in the border with good area are checked with their neighbor pixels in good area by PDE based functions. This is run in each iterative cycle until all missing pixels were recovered.

Exemplar based methods, similar to PDE, also run around the border of missing area. It checks each small region of pixels but not a single pixel. Exemplars with specific structure
are created by small region in good area and they are used like a template to fill-in missing region.

Criminisi et al in [2] proposed exemplar-based texture synthesis process that propagates image texture by direct sampling of source region. Distance between two generic patches is defined as sum of squared differences of the already filled pixels in the two patches. Texture-based methods use textures as samples to recover each small missing regions.

Ashikhmin in [3, 4] presented a simple texture synthesis algorithm. The algorithm starts from a sample image and generates a new image of arbitrary size the appearance of which is similar to that of the original image. Texture-based approach was found in inpainting application. Dong Liu et al [5] proposed a method of edge-based inpainting and texture synthesis.

Bertalmío et al had ideas from classical fluid dynamics to propagate isophote lines continuously from the exterior into missing region [6]. The approach is to think of the image intensity as a stream for a two-dimensional incompressible flow. The method is directly based on the Navier-Stokes equations. There are other inpainting approach like Sub-Riemannian minimizers [7], super-resolution [8], median diffusion [9], total variation [10, 22].

Bilateral filter was introduced by Tomasi et al in [11] and has wide applications in image deblurring and de-noising. Bilateral filter consists from Gaussian space filter and a partial differential filter. Thanks to the second filter results keep edges.

Several applications of bilateral filter were studied [13, 14, 21]. Authors of [15] proposed image interpolation technique using the small-kernel bilateral filter, so that it is suitable for real-time image zoom, video up-scaling. Bilateral filter was jointly used with integral histograms in [16]. Bilateral filter applied for noise reduction [17], stereo matching [18], image decomposition [19].

Noori et al in [20] proposed an iterative algorithm with bilateral filter for inpainting. It substitutes the difference between two gray level values in the range filter by multiplication of two vectors: direction between two pixels and gradient direction of known pixel in the neighborhood of damaged pixels.

IV. CONTRIBUTIONS

This work is inspired by comprehensive study of Bilateral Filter in [14] and Total Variation in [12]. Contribution of the work is a novel inpainting model with TV-based bilateral filter. The model checks similarity of small regions like exemplar based methods [2]. It combines knowledge of intensive value and local structure by TV-based bilateral filter to check similarity of regions to fill-in missing areas.

Bilateral filter in the inpainting model was adapted to missing regions. Total variation was implemented in our work to find the missing pixels that possibly stay on edge. Standard deviation in range kernel of the filter is regulated by total variation. This helps to define similarity of local structure and to generate patches that keep edges.

V. NOTATION AND PRELIMINARIES

Assume that input image is given by a M-dimension function of space $\Omega$.

$$ u : \Omega \rightarrow \mathbb{R}^M, u(x) = (u_1(x), \ldots, u_M(x)) $$

$$ u_i : \Omega \rightarrow \mathbb{R}, i = 1, \ldots, M $$

Let $\nabla$ represent divergent operator:

$$ \nabla u : (\nabla u_1, \ldots, \nabla u_M) : \Omega \rightarrow \mathbb{R}^{M \times N} $$

Euclidean scalar product is denoted by equation:

$$ \langle u, v \rangle := \sum_{i=1}^{M} u_i v_i $$

$L$ Euclidean norm is represented by $\|u\|^L$, so $\|u(x)\|^L$ is $L$ Euclidean norm of $u$:

$$ \|\nabla u(x)\|^L = \sqrt{\sum_{i=1}^{M} u_i^2}, u \in \mathbb{R}^M $$

VI. TV BILATERAL FILTER

A. Inpainting

This section formulates TV adaptive bilateral filter for inpainting. Input image is supposed to have missing areas. Let $\Phi$ stands for known areas and $\Psi$ - for the missing areas.

$$ \Omega = \Phi \cup \Psi, \Phi \cap \Psi = \emptyset $$

$\Psi^c$ stands for border of missing areas and good areas:

$$ \Psi^c \subset \Psi, \forall x \in \Psi^c, \exists y \in \Phi, \text{neighbor}(x, y) $$

B. Adaptive bilateral filter for inpainting

The Gaussian function is recalled:

$$ G_\sigma : S \rightarrow [0,1] $$

$$ G_\sigma(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right), x \in \Omega $$

The filter domain is a frame that may contain missing pixels. By denoting $S^*$ as the good area of the frame, the Gaussian filter turns to a function of $S^*$:

$$ G_\sigma^* : S^* \rightarrow [0,1] $$

Original bilateral filter in [11] now has form:

$$ B^* (x) = \frac{1}{W} \sum_{y \in S^*} G_\sigma^*(\|x - y\|) G_\sigma^*(\|u(x) - u(y)\|) u(y) $$

Here $S_k$ is a small frame around $x$. Due to the fact that $u(x)$ may miss value at $x$, it’s necessary to get representative intensive value $p(x)$ for $u(s)$ in (10). $p(x)$ can be calculated by Gaussian function:
\[ p_i(x) = \int_{y \in S_x^+} u_i(y) G'_x(x) dy, x \notin \Phi, i = 1, \ldots, M \]
\[ u(x), x \in \Phi \]

Where \( S_x^+ \) denotes region of known pixels around \( x \).

\[ p(x) \text{ from (11) then is used to formulate total variation } t(x): \]
\[ t(x) = \frac{1}{W_v} \sum_{y \in S_x^+} \| p(y) - p(x) \|^2, x \in \Psi^c \]
\[ W_v \text{ in (12) is normalization parameter,} \]
\[ W_v = \sum_{x \in \Psi^c} \sum_{y \in S_x^+} \| p(y) - p(x) \|^2 \]

Standard deviation in range kernel \( \sigma_r^x \) in (10) is global and its local version is regulated by total variation \( t(x) \) from (12):
\[ \sigma_r^x = \frac{\sigma_r(x) = \sigma_r^x(t(x))}{W_v} \]

Where \( W_v \) is normalization weight to keep mean \( \sigma_r^x(t(x)) = \sigma_r^x \).

Replace \( u(x) \) in (10) by \( p(x) \) from (11), and use \( \sigma_r^x \) from (15), TV-based bilateral filter for inpainting is specified:

\[ B^x(x) = \frac{1}{W_v} \sum_{y \in S_x^+} G_{x,v}(\| x - y \|^2) G_{x,u}(\| p(x) - u(y) \|^2) u(y) \]

Here, \( W_v \) is normalization parameter at \( x \),
\[ W_v = \sum_{y \in S_x^+} G_{x,v}(\| x - y \|^2) G_{x,u}(\| p(x) - u(y) \|^2) \]

Condition for choosing the optimal solution of inpainting is based on two factors: difference of self-function \( u \) and difference of bilateral filter function \( B^u \). The following energy function expresses the condition:
\[ E^*(y) = \lambda \| u(y) - p(x) \|^2 + (1 - \lambda) \| B^x(y) - B^u(x) \|^2 \]

Where \( \lambda \in [0,1] \) is a weighting parameter, \( L \) can be 1 or 2.

Optimal solution for (18) is formulated as follows:
\[ y^* = \arg \min_{y \in \Phi} E^*(y) \]

Solution \( y^* \) is essential to find coordinates that products similar visual effect for a predefined space position. And this is a major step for inpainting. The energy function (18) is regularized by total variation (12). This helps to measure local structure better than energy function by self bilateral filter and thus find the best similar pixel in the known area corresponding to the unknown boundary pixel.

Example in figure 2d is the result of regulation by TV (18), that gives Structural SIMilarity (SSIM) better than regulation by only bilateral filter (fig. 2c).

![Example of inpainting gray image](https://via.placeholder.com/150)

Fig. 2. Example of inpainting gray image
C. Adaptive bilateral filter for inpainting gray image

Formula (1) now has a simple form for gray image:
\[ u : \Omega \rightarrow \Re \]

Representative function (11) becomes (21):
\[ p(x) = \int_{\Re} u(y) * G_{\sigma}^{\tau}(x)dy \cdot S_{\tau}^{\tau} \subseteq \Phi, x \in \Omega \]

With bilateral filter (16), energy function (15) has its form for gray image:
\[ E_{u}^{\tau}(y) = \lambda \|u(y) - p(x)\|^2 + (1 - \lambda)\|B^{\tau}(y) - B^{\tau}(x)\|^2 \]

Algorithm TVBF-G in section VII-A will need these formulas for inpainting gray images.

D. Adaptive bilateral filter for inpainting color image

Color image is presented by 3D function \( u : \Omega \rightarrow \Re^3 \), \( u(x) := (u_r(x), u_g(x), u_b(x)) \).

With RGB channel \( u_i : \Omega \rightarrow \Re, i \in (R,G,B) \)

Each channel has its representative function:
\[ p_{i}(x) = \int_{\Re} u(y) * G_{\sigma}^{\tau}(x)dy \]

\( L_2 \) Euclidean norm is used in for color image, and energy function (18) has a new form:
\[ E_{u}^{\tau}(y) = \lambda \|u(y) - p(x)\|^2 + (1 - \lambda)\|B^{\tau}(y) - B^{\tau}(x)\|^2 \]

In section VII-C, above functions will be called in inpainting algorithm TVBF-C for color image.

VII. IMPLEMENTATION

A. Inpainting gray image

ALGORITHM TVBF-G FOR INPAINTING GRAY IMAGE

Given: Painted gray image \( u(x) \), its paint mask \( \mu(x) \), initial \( \sigma_{\tau}, \sigma_y, \) weight \( \lambda \), frame size \( \tau \).

Output: Inpainted image.

Step 1: Select a \( \tau \times \tau \)-size frame that contains an area of missing edge \( \Psi^e \) by condition (13). Store position of center of the frame (mark by \( x \)).

Step 2: Define \( \tau \times \tau \)-size mask frame \( \mu' \)(\( x \)) basing on above selected frame.

Step 3: Calculate representative value by partial Gaussian filter (21) with \( \sigma_y \) and mask frame \( \mu' \)(\( x \)).

Step 4: Apply TV-Bilateral Filter (12) with \( \sigma_{\tau}, \sigma_y, \mu'(x) \) and calculate energy function by (18).

Step 5: Find inpainting solution \( y^* \) for \( x \) by (19): missing area of frame at \( x \) will be full filled by corresponding area from frame centered at \( y^* \).

Step 6: Turn to step 1 if still exists unfilled missing area.

B. Example of inpainting gray image

Implementation of the algorithm for “trui” image is started by inputting a painted image (fig.2a). Assume \( \tau = 7 \). Border of painting mask for the image is in fig.2b.

Step 1 will find out position \( x = (122,33) \) in the image 2a. 7x7 frame at the position marked by red line has detailed view in fig.2f. Step 2 creates a mask with 7x7 size.

Step 3 products partial Gaussian filter, see fig.2b, flat area in the fig.2b is corresponding to the missing area in the frame.

Next, step 4 estimates energy function in fig.2e.

Step 5 will find out \( y^* = (97,29) \) and according 7x7 frame is fig.2g. Frame in fig.2f now is inpainted, see fig.2h.

Final result of the algorithm is fig.2d. Fig.2c is alternative result of the algorithm in case of using self bilateral filter for deviation regulation but not with TV.

C. Inpainting color image

Algorithm TVBF-C for inpainting color image (23) is similar to TVBF-G, where step 1 uses the same condition (13), step 3 has specific partial Gaussian filter (24), step 4 takes energy function by (25).

D. Example of inpainting color image

A set of images from Berkeley Segmentation Dataset and Benchmark were tested with the TVBF-C algorithm (fig.3a) and results are in fig.3d.

VIII. DISCUSSION

A set of test images (fig.3a) are checked with Exemplar based method (fig.3b) and Navier-Stokes (fig.3c) and TVBF-C (fig.3d). Experiment shows that speed of TVBF is faster than others because it does not take iterative loops. Quality of the inpainting methods are variable for different inputs, depending structure of missing regions. SSIM index is metrics selected for experiment in this work. The first row of figure 3 gives good results for all methods where the missing region is not well structured. In this case, it’s hard to find defect in inpainting result.

In the second row first two methods have defects. Result of exemplar based method shows wrong texture around collar, but Navier-Stokes example displays wrong texture of cross background.
Fig. 3. Effect of Exemplar-based method, Navier Stokes and Adaptive Bilateral Filter for color image.

Fig. 4. Effect of Exemplar-based method of [23], [24], [25] and Adaptive Bilateral Filter for color image.
The third row show small defects created by all methods but results are visually acceptable, thought TVBF gives the best result. An example from [23] is tested with TVBF in figure 4. Fig 4a is original image, the base for SSIM calculation. Mask is fig.4d. Results of methods [24], [25] referred in [23] and [23] are fig 4b, 4c, and 4e correspondingly. Fig. 4f is result of TVBF. SSIM index is enhanced by TVBF.

Let’s review computing cost for algorithm TVBF-G. Denote \( n \) is number of pixels, step 1 takes cost of \( O(n \tau r) \) or \( O(kn) \) for searching a frame with missing areas. Step 3 takes the same \( O(kn) \) to get representative value by partial Gaussian filter. Step 4 runs Bilateral Filter for frame of \( \tau \times \tau \) size for all pixels, so it takes the cost nearly \( O(kn \log n) \). \( O(kn) \) is for step 5. Finally, the cost of TVBF-I is \( O(kn \log n) \), where \( k \) depends on size of working frame and size of region of missing information. Cost for algorithm TVBF-C is the same.

As big size of working frame, as long algorithm takes time for checking information inside the frame. Thought, it leads to fill missing areas by far neighbor regions. Depending on concrete situation, far neighbor areas could give good or bad inpainting results. Algorithms TVBF-G and TVBF-C use Gaussian parameters \( \sigma_1, \sigma_2, \lambda \), frame size \( \tau \). Changes of these parameters impact performance and quality of painting. By default, \( \sigma_1 = 5, \sigma_2 = 1, \lambda = 7 \), frame size \( \tau = 7 \). The TVBF-G and TVBF-C can run well with thin or thick missing regions, as its first step selects only a frame with predefined missing percentage.

IX. CONCLUSIONS AND FUTURE WORK

In this paper an inpainting model with total variation based bilateral filter is presented for gray and color image. The model combines knowledge of intensive value and local structure to fill missing areas. Good accuracy over challenge examples was demonstrated. The novel contribution of the work is an adaptive bilateral filter approach for inpainting.

We regard experimental results as promising application in future and it would be desirable to extend the model to incorporate advantages of seeking global and local similarity to improve performance of inpainting.

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XI. REFERENCES