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A Method for Verifying Measurements and Models of Linear and Nonlinear Systems

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Abstract— **A method has been developed to help verify system models and measurements and identify system nonlinearities. This method involves measuring the response of physical systems with a swept sine measurement device, such as the HP3562(3)A Control Systems Analyzer (CSA)[1], and simulating system models with a dynamic system simulation tool, such as SIMULINK, using swept sine input. The input-output time sequences obtained from the simulations are analyzed in a similar fashion as done by the CSA to enable more accurate comparisons. A parallelism between swept sine measurements and describing functions is exploited to allow this method to be used in identifying nonlinear systems. Using the measured and simulated frequency response functions as a guide the designer can iteratively improve the model of the system and verify the correctness of the measurements.**

I. Introduction

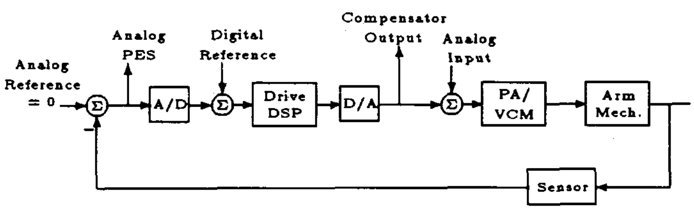

Figure 1: Typical Disk Drive Measurement Block Diagram

A typical disk drive block diagram is shown in Figure 1. This is a good metaphor for what will follow as it contains all elements of a typical SISO digital control loop. The plant consists of the portion from the input to the power amplifier to the output of the sensor. The compensator is typically a DSP chip sandwiched between an A/D and a D/A converter. The system contains both electrical and mechanical parts and the sensor (in this case the magnetic head passing over magnetic domains on the disk) can be nonlinear at either end of its range. The A/D and D/A converters typically have between 8 and 12 bit quantizers in them and their voltage range is limited, resulting in the possibility of both input and output saturation. The sample rates are high enough to require a DSP chip, yet costs must be pushed down because this is a mass production product (as opposed to a fighter aircraft). Furthermore, the packages are small enough to make test points hard to come by. Finally, the sensors can only be kept in their linear region by having the drive in some nominal feedback control loop. Not only does this create a chicken and egg type scenario for the designer, but one also must deal with trying to extract open-loop plant information from closed-loop measurements.

This is also a good metaphor[2] for yet another attempt at bridging the gap between academic and industrial control problems. In particular, this paper will deal with a new identification methodology that came about because the authors were trying to get consistent agreement between a parametric system model[3] and measurements made in the lab on physical hardware[4].

In a textbook control problem one starts with a parametric model where some features of the problem *i.e.*, some parameters may not be known. In addition there may be a nonlinearity of known character and certain noise properties are assumed. The objective from this point is to design a controller to give "good" performance along some metric. The problem discussed here starts earlier in the process. Here, the starting point is a set of electrical and mechanical parts that are accompanied by some nominal parametric models. These are used to create a nominal controller. From this point the system can be measured and an improved controller can be generated. This process is iterated until either the system meets its performance requirements or time and cost constraints dictate that the system has met its time and cost requirements.

It is important to note that in the latter problem, the measurements are not necessarily (or even often) the parametric, time-domain measurements so prevalent in the on line identification literature [1, 2, 3]. More often, nonparametric frequency domain methods are used to obtain a frequency response function (FRF) from a given system input to a given system output [4, 5, 6]. Often, control design is done strictly in the frequency domain without reducing the measurement to a parametric model [7, 8, 9]. However, in order to use the sophisticated control algorithms and CAD programs now available, or in order to deal with multivariable problems in a graceful way, a parametric model is essential. This can be obtained by curve fitting a transfer function to the frequency response function [10, 11, 12, 13, 14]. This process itself is imperfect and is one of the main difficulties in obtaining decent parametric models of industrial control problems [15, 16].

While there is considerable parallelism between time and frequency domain methods [17, 1], the latter do have some great advantages that make them hard to ignore:

- Measurements can be made on both analog and digital systems.

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[1] The HP3562A is an instrument designed for making analog measurements of dynamic systems and is known as a Dynamic Signal Analyzer (DSA). The HP3563A has logic analysis features added in, which allows it to do direct digital measurements and analysis of digital control systems as well. It contains a superset of the features found in the DSA, and is referred to as a Control Systems Analyzer (CSA). For this paper, only the DSA functionality was used.

[2] Or in the words of John Cleese, "Idiom".

[3] implemented in SIMULINK.

[4] in this case a HP 3562A.

- Measurements can be made without modifying the existing nominal controller. Making a physical measurement is not much more difficult than connecting a digital oscilloscope to the system.
- Issues of persistent excitation show themselves in different ways. For FFT based measurements, the system input is "white noise" (which is about as exciting as a signal gets). The noise spectrum is often shaped to emphasize or deemphasize some frequency bands. In this case, regions in the frequency domain where the excitation is poor are easy to spot due to the "fuzzy looking" frequency response plot and values of the coherence function [6] far below 1.

  For swept sine measurements, the system is both stimulated and measured at a single frequency providing extremely high signal to noise ratios. Much of the "fuzziness" of FFT based measurements disappears and the coherence functions are closer to 1. Since at any individual frequency only a magnitude and phase are being estimated, a single sine wave at that frequency provides enough excitation.
- ⋆ Parallels between swept sine measurements and describing functions (which will be shown) allow the designer to characterize the system including nonlinearities [4].

II. Background

II.A The Modeling Problem

A typical modeling process is shown in Figure 2. As stated

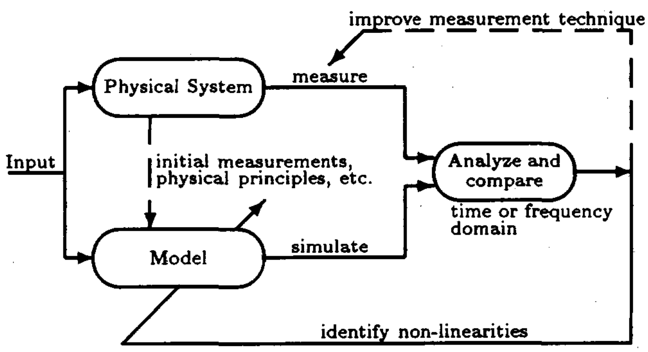

Figure 2: Block Diagram of System Modeling Procedure

earlier, this is an iterative process. While it is well known that this iteration is used to improve the system model, the system model can also be used to determine and correct flaws in the measurement procedure. The initial model is typically obtained from physical principles, component level measurements, or manufacturers' specifications. The model can take many forms, but its validity is based on how well it predicts the measurements of the physical system. Typical measurements are made in both the time and frequency domains. In comparing measured and simulated data it is important to:

- Give both the model and the physical system the same external inputs. This may mean recording the inputs given to the physical system and using these to generate the model's external inputs. This ensures that both the model and the physical system see all the nonidealities of the physical systems input.
- Take the characteristics of the physical measurement into account in the model. Just as any model is imperfect, so is any measurement scheme. It is important to account for sampling and timing issues of both the system and the measurement device (particularly in a system with a digital compensator) as well as coupling and quantization issues of the measurement device.

A useful newcomer to the modeling field is scaled 913SIMULINK from the MathWorks [18] which, like System Build from ISI and Model-C from SCT, allows for a block diagram based model and simulation of a dynamic system. An extremely important feature of this is that one can start with an idealized, linear, textbook model and add in nonideal features to make the model more closely approximate the lab setup. One can also tap into signals which may or may not be accessible on the physical plant. Furthermore, simulations can be run in "continuous-time", which will in most cases be more accurate than discrete-time simulations, and both "analog" and digital data can be readily obtained[5]. Of course, this could all be done with a FORTRAN or C program, but the intuitive ease of a graphical user interface makes this much simpler in the block diagram based tools.

An advantage SIMULINK has over other similar tools is that SIMULINK (and its base program MATLAB) can be run efficiently on a PC. This is especially convenient when one wishes to interface to other applications or instruments. For example, it is potentially possible to set up a system such that SIMULINK is used to simulate a digital compensator, and that control design can be directly downloaded to a PC based DSP system [19]. Also, a vast pool of very useful routines such as FFT already exist in Matlab, making the analysis procedure a little easier.

II.B Measurements of Dynamic Systems

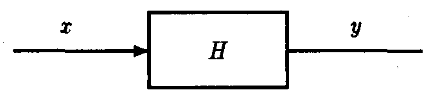

Figure 3: Open-Loop ID of a SISO System

As stated earlier, in industry much of the system measurement and identification that is done is in the frequency domain. A brief review follows. In Figure 3 the frequency response function of H is unknown. The user has access to both the injected input x and the output y. If the system is linear then x and y are related by the frequency response function, $H(f)$, which can be obtained from:

$$H(f) = \frac{Y(f)}{X(f)}, \quad \text{or} \tag{1}$$

$$H(f) = \frac{G_{yx}(f)}{G_{xx}(f)}, \tag{2}$$

where $X(f)$ and $Y(f)$ are the Fourier Transforms of $x(t)$ and $y(t)$ (DFT if x and y are discrete), and $G_{yx}(f)$ and $G_{xx}(f)$ are the one-sided cross and auto spectral density functions given by

$$\begin{aligned} G_{yx}(f) &= \begin{cases} 2E\{Y(f)X^*(f)\}, & f \geq 0 \\ 0 & f < 0 \end{cases} \quad \text{and} \\ G_{xx}(f) &= \begin{cases} 2E\{X(f)X^*(f)\}, & f \geq 0 \\ 0 & f < 0. \end{cases} \end{aligned} \tag{3}$$

Almost always the latter relation, Equation 2, is used because it produces an unbiased estimate of the true frequency response [1, 9].

Swept sine measurements stimulate the system at one frequency, f_0, at a time, producing

$$H(f_0) = \frac{G_{yx}(f_0)}{G_{xx}(f_0)}. \tag{4}$$

This allows for high signal to noise ratios, as narrow band filters can be used around f_0. In actuality, $G_{xx}(f_0)$ exists only

[5] "Continuous-time" and "analog" refer to the analog system ODE being solved numerically to high precision.

in the limit, so $H(f_0)$ is computed using Fourier series theory. The selected frequency is "swept" upwards or downwards in either a linear or logarithmic progression. Because only one frequency at a time is measured, this mode allows for some other tricks to improve the SNR, such as automatically scaling the input level to maximize the linear range of the signal. This feature must not be used when trying to characterize nonlinearities.

In general the input and the output can be any two accessible signals tapped off of the system. Often the input is chosen to be the same as the source, in which case the measurement procedure is called a two-wire measurement, as opposed to a three-wire measurement. There are advantages and disadvantages associated with either choice, but they will not be discussed in this paper.

III. Algorithm

The algorithm discussed in this paper closely follows the swept sine mode of the HP3562(3)A dynamic systems analyzer. Note that the method is not limited to this one instrument, but whichever instrument is chosen must be precisely mimicked in the simulation.

III.A Theory

A brief review of swept sine measurements follows. More detail can be found in [20].

The objective is to obtain a gain and phase relation from some input signal $x(t)$ to some output signal $y(t)$. Assuming that both signals are periodic, we can expand them into Fourier series:

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\omega nt} \quad (5)$$

$$y(t) = \sum_{n=-\infty}^{\infty} d_n e^{j\omega nt}, \quad (6)$$

where c_n and d_n are complex Fourier coefficients. The first components are

$$c_1 = \frac{1}{T}\int_0^T x(t)e^{-j\omega t}dt, \quad \text{and} \quad (7)$$

$$d_1 = \frac{1}{T}\int_0^T y(t)e^{-j\omega t}dt. \quad (8)$$

The frequency response function from $x(t)$ to $y(t)$ is then d_1/c_1, the ratio of the first harmonics.

In practice, the integration in computing c_1 (and d_1) takes place over multiple periods. This allows for more samples of $x(t)$ and therefore better resolution of c_1. The process of computing c_1 can be summarized in Figure 4.

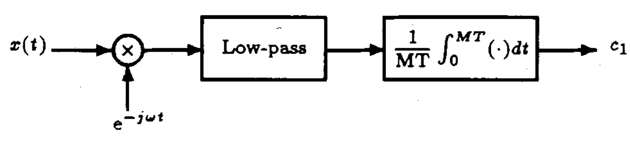

Figure 4: Theoretical Computation of First Harmonic

Note that after the signal $x(t)$ is mixed with $e^{-j\omega t}$, its fundamental component is at DC. Thus a low-pass filter is employeed. Theoretically, this low-pass operation is not needed, based on the assumption that the signals are sinusoidal. However, actual signals will be contaminated by non-periodic noise which will not integrate to zero, causing the resulting Fourier coefficients to be biased. Thus, the signals need to be passed through a low-pass filter before the integration operation.

Figure 4 cannot be implemented as shown because integration cannot be done exactly. What can be done however, is to first sample the signals, then perform a polynomial fit to the sampled data so that an approximation to the integration operation can be obtained. To more closely mimic the HP3562(3)A, the integration is approximated using a fifth-order composite quadrature formula, implemented as an FIR filter. As FIR filters are used to do both low-pass and integration, the two can be combined via convolution. The integration must be taken over an integral multiple of the signal period to properly compute the Fourier components. The process can be summarized in Figure 5 below.

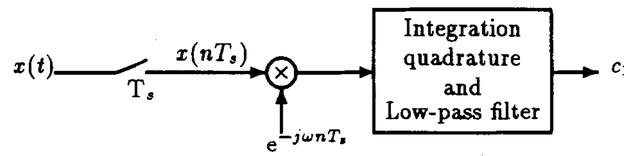

Figure 5: Implementation of Computation of First Harmonic

Much effort was made in assuring that the algorithms used in the analysis of simulated data are the same as thosed used in the HP3562(3)A. This is done to make sure that the characteristics, including the flaws, of the measurement process are taken into account in the model.

III.B Parallels Between Swept Sine and Describing Functions

One of the methods of analyzing nonlinear systems is the *describing function* method[6]. The notion here is that many nonlinear systems can be effectively analyzed by considering the effect of a sinusoidal input on the system and examining the first Fourier component. The equations used to compute a describing function of a nonlinear system are precisely Equations 5–8. In other words, for a fixed amplitude input sinusoid *the swept sine measurement of a nonlinear system measures the describing function of that system.*

The connection between describing functions and swept sine measurements has two important consequences:

- The frequency response function generated by a swept sine measurement degrades gracefully from a transfer function measurement to a describing function measurement as the system moves from linear behavior to nonlinear behavior.
- The nonlinear elements can be characterized by *simulating* a swept sine measurement in some modeling environment. In other words, one can propose a linear system model, based on either physical models or empirical measurements, and generate a swept sine frequency response function. From here nonlinear elements can be added to the model and the swept sine "measurement" can be repeated. This process is repeated until the discrepancy between the frequency response function measured in the lab and the one generated on the computer is reduced to the designer's satisfaction.

It is important to note that this method will not generate a parametric describing function. Instead it will generate exactly what we can measure with a swept sine measurement: the frequency response function of the nonlinear system. Unlike the frequency response function of a linear system, this is not independent of amplitude. In fact, generating a family of these frequency response functions can help to characterize the effect of an amplitude dependent nonlinearity.

This is a departure from previous uses of describing function analysis. In previous work describing function analysis

[6] There are many forms of describing functions. In this paper only the sinusoidal-input describing function is considered.

has largely been limited to isolated nonlinearities. Its main use was in predicting limit cycles and aiding the analysis and design of controllers for systems with some known or assumed nonlinearities. In order to use describing functions for this purpose, an assumption has been made that the higher harmonics caused by the effect of the sinusoid passing through the nonlinearity were sufficiently attenuated by the system to not have been of great importance [8].

There is some reference to using a swept sine measurement to identify nonlinearities in [21]. This is limited to nonlinearities that can be directly stimulated with a sinusoidal input and for which the output can be directly measured. Another reference to swept sine measurements of nonlinear systems is in [4]. However, there is no discussion of how to use such a measurement to determine the character of the system nonlinearity.

As used in this paper, only the first harmonic is computed at a specific frequency in the measurement from both the experiment and the model. Because this work is more concerned with matching these describing function measurements than in getting an analytical result, the assumption that the higher harmonics are attenuated is not necessary. Furthermore, since the measurement process is precisely mimicked in the model, there is no need to assume that the nonlinearity must be measured on its own. Thus, this method expands the concept of describing functions from isolated nonlinearities to overall systems. Its main use is to help identifiy nonlinearities in the system.

III.C MATLAB/SIMULINK *Implementation*

First a system model is constructed in SIMULINK using various blocks already provided. A MATLAB routine is then written to mimic the CSA. An equivalent block diagram is shown in Figure 6.

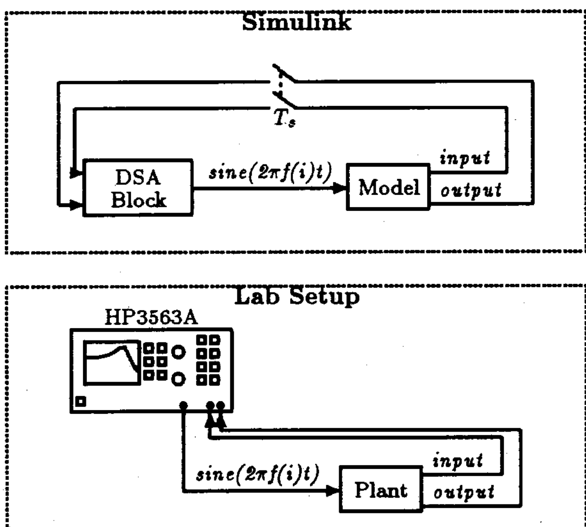

Figure 6: Simulation Implementation, Compared to Physical Setup

The MATLAB script routine performs the following:

1. Specify the frequencies to be swept. Initialize frequency and iteration counters.

2. Call a Matlab simulation routine such as rk45 (Runge-Kutta fifth-order method) to simulate the system model for a specified duration, long enough so that enough sample points can be gathered.

3. Process collected data sequences as described in [20].

4. If not done with specified number of iterations, increment iteration counter and go back to step 2. Otherwise go on.

5. If not done with all frequencies, increment frequency counter and go back to step 2. Otherwise done.

IV. Example

The swept sine algorithm is applied to a disk drive system with block diagram as shown in Figures 7–8. Note in 8 that the deadzone and preload[7] nonlinearities can be switched on or off at will.

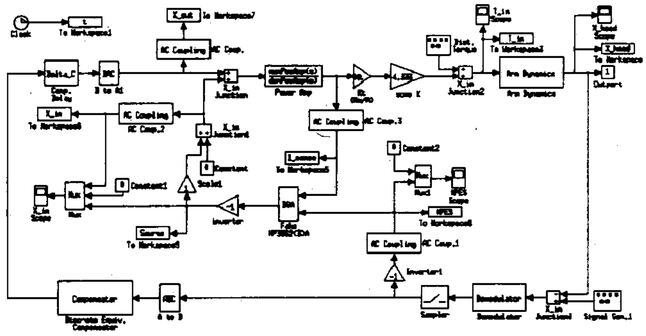

Figure 7: Disk Drive Simulation Block Diagram

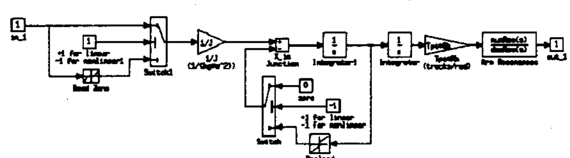

Figure 8: Arm Dynamics Block

The simulated frequency response with neither nonlinearity switched on is shown in Figure 9. The actual measured response is also plotted for comparison. Typical models for disk drive actuators have the actuator mechanics behaving as a double integrator at low frequencies. However, in this case the measured response at low frequencies at low frequencies indicate some nonlinear behavior, since the system gain tapered off and the phase rose towards zero as the frequency dropped off.

The system in Figure 7 was then simulated with the the preload switched on (in this example the dead zone is not used). The simulation results are shown in Figure 10. The magnitude plots now match nearly perfectly, and the discrepancy in the phase plots has been significantly reduced.

V. Conclusions

The above method shows considerable promise for both verifying parametric system models against non-parametric laboratory measurements and for characterizing nonlinear behaviour observed in those measurements. As it is an iterative process, with the designer supplying the candidate nonlinear elements, it does not excuse the designer from a general knowledge of nonlinear models and describing functions. Moreover, it provides an empirical tool for verifying the designer's guesses about what is actually going on in the system. That being done, the control design problem can be worked on with a much improved confidence in the system model. While the usage in this paper has been limited to the HP3562(3)A and the SIMULINK program, this method is applicable to other measurement and simulation tools. The key is to closely mimic the

[7]Preload is a velocity-dependent friction with both viscous and Coulomb components.

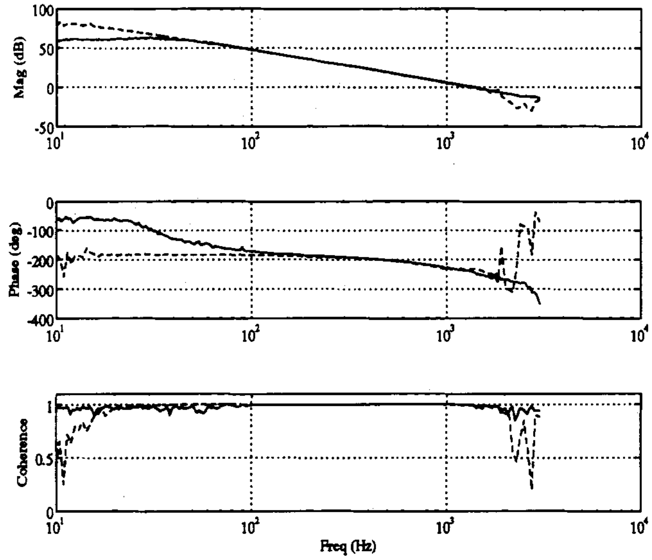

Figure 9: Simulated vs. Measured Swept Sine Frequency Response of Linear Disk Drive. Solid line: simulated, dashed line: measured.

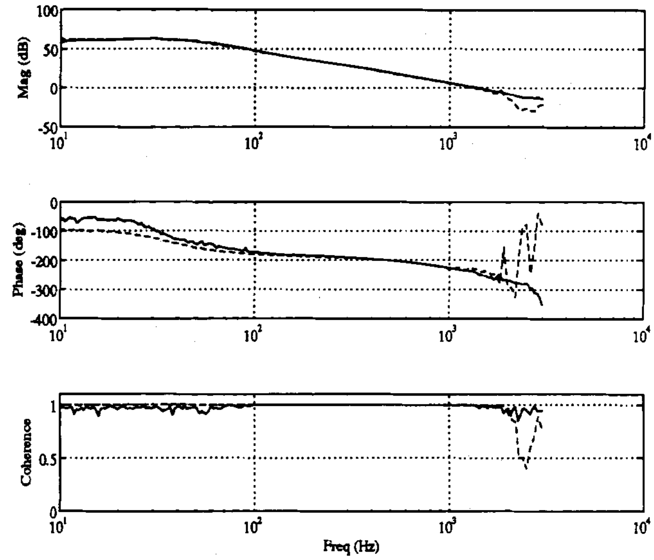

Figure 10: Simulated vs. Measured Swept Sine Frequency Response of Disk Drive with Preload. Solid line: simulated, dashed line: measured.

measurement process, complete with its flaws, in the modeling process.

VI. Acknowledgements

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