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THEORETICAL NOTE

The Weber–Fechner Law: A Misnomer That Persists But That Should Go Away

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The term “Weber–Fechner law” is arguably the most widely used misnomer in psychological science. The unification reflects a failure to appreciate the logical independence and disparate implications of Weber’s law and Fechner’s law as well as some closely aligned ones. The present statement, long overdue, is meant to rectify this situation. I discuss the roots and derivations of the relevant laws, eschewing formalism to bare essentials for sake of wider accessibility. Three of the most important conclusions are (a) Weber’s law is not indispensable for deriving Fechner’s law; (b) arguably, Fechner himself did not use Weber’s law in his original derivations; and (c) many investigators mistake the principle that subjective distance is determined by physical ratio for Weber’s law. In truth, the principle, here called the Weber principle, and Weber’s law, are different and independent. I stress the importance of drawing the distinction and illustrate confusions in the literature coming from misapplications of Weber’s law and the use of misnomer.

Keywords: Weber, Fechner, Weber principle

The goal of this comment is to mount a hopeless yet badly needed challenge against the continuous use of the misnomer, “the Weber–Fechner law.” There is no such thing as the Weber–Fechner law. So, it is quite jarring to find the misnomer as the first item in any perfunctory search on the internet or indeed in the professional literature. When Don Lewis discussed Fechner’s law 60 years ago, he was careful to note that it is “called the Weber–Fechner law by most psychologists today and correctly called Fechner’s law” (Lewis, 1960, p. 432). The situation has not much changed today, despite a pair of excellent recent analyses (Dzhafarov & Colonius, 2011; Masin et al., 2009), evincing the disparate nature of the laws.[1] This is regrettable because, in truth, as the recent studies methodically reaffirmed, Weber’s law and Fechner’s law are two separate laws. They are logically independent so that each can be true with the other being false. In fact, each law can exist without the other never been created. It is often argued that Fechner assumed Weber’s law when deriving his own logarithmic law, but this alleged history should not overshadow the logical independence of the two laws.

In this comment, I first underline the necessity of distinguishing the two laws. One notes that, over a century and a half after their discovery, Weber’s law and Fechner’s law sustain an absolutely remarkable amount of current research. Amidst this resurgence of interest, one should also take notice of recent important contributions in perception and physiological brain research. In the following section, I delineate Weber’s law, pinpointing its source in the physical properties of the stimulus. The accepted version of the derivation of Fechner’s law is presented next with Weber’s law as a basic assumption. In two succeeding sections, it is first shown that assuming Weber’s law does not necessarily mandate Fechner’s logarithmic law, and, conversely, that the logarithmic law can be derived without appeal to Weber’s law. I then mention arguments disputing that Fechner truly used Weber’s law in his original derivations. The discussion underscores the difference between (a) the Weber principle by which perceived difference is determined by stimulus ratio and (b) Weber’s law, which entails the notion of the just noticeable difference (JND) or the difference threshold or limen (DL or ΔS). Failure to distinguish between the two along with the employment of the misnomer is conductive to confusions found in the literature.

The Importance of Keeping Concepts Clear and Distinct: Imperatives for (Future) Research in Perception and Psychophysics

We can sharpen the focus of that mandate by first considering the progress made in brain research in the domain of perception during the past couple of decades. The hallmark of this research is the range of state-of-the-art electrophysiological and imaging tools deployed to unearth a small number of basic primitives or processing invariants of human perception, as well as the flexible operational characteristics under given environmental demands. Important discoveries include the presence of attentional sampling at a preferred

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[1] The unlikeness of Weber’s law to Fechner’s law is amply demonstrated in the two studies, yet they do not challenge to the ongoing use of “Weber–Fechner law”—the present argument.

frequency range (e.g., Fiebelkorn & Kastner, 2019), the interplay and difference between local versus more global judgments (e.g., Teng et al., 2016), or multiscale processing (e.g., Rammsayer & Troche, 2014) in various visual and auditory tasks. In portions of this research, psychophysical judgments align with spectrotemporal modulations to reflect the attendant neural substrate and sensitivities (e.g., Flinker et al., 2019). The cumulative work by David Poeppel and his associates (see, e.g., Kim et al., 2020; Poeppel & Assaneo, 2020) perhaps best illustrates this approach—uncovering complex information processing in the brain, mainly but not exclusively, in the domain of auditory temporal processing (e.g., Kim et al., 2020; Poeppel & Assaneo, 2020). For example, temporal integration in hearing was conceived as a fairly limited (time-wise) and unitary process (e.g., Algom & Babkoff, 1984; see also Algom & Marks, 1984, 1990; Algom et al., 1980, 1989), but Poeppel and his associates have now demonstrated the operation of multiple time scales of auditory processing at various levels of granularity (e.g., Poeppel, 2001, 2003; Teng et al., 2016). Taking a broad view of these developments,[2] one observes that Weber's law (much less Fechner's law) plays only a minor role in this literature (but see, Sierra et al., 2020).[3]

Given the limited role of Weber's law and of Fechner's law in this emerging literature on perception, it might be argued that the precise nature of these classic laws and their relation is primarily a matter of historical interest. It is not. The sheer amount of current research associated with Weber's law is staggering. Customary search targeting only the last year and a half (from 2019 to present) yields 90 studies with Weber's law in their title or with the law forming their significant component. Relaxing the criteria a bit yields 226 new studies associated with Weber's law. Within the same narrow time-frame, one finds 14 papers with Fechner in their title. Moreover, the interest in these classical laws of psychophysics transcends disciplinary boundaries, playing a pivotal role in a gamut of fields from perception (e.g., Ozana et al., 2020) to numerical cognition (e.g., Bar et al., 2019) to decision-making under uncertainty (e.g., Gilboa, 2009) to judgments within the criminal justice system (e.g., Kannai, 2001), or classic and behavioral economics (e.g., Luce, 1956; Posadas-Sánchez & Killeen, 2005). This is not history; rather, this is the present and beyond the present. More specifically, understanding the difference between Weber's law and Fechner's laws is not historical in any sense, but refers to appreciating the difference between mere detection or maximum resolution, on the one hand, and global dynamic (suprathreshold) processing of perceptual magnitude, on the other hand. As I show next, many current behavioral and imaging studies fail to distinguish these two aspects of human perception.

An unfortunate aspect of the burgeoning literature on Weber's law, and one impetus for offering this Note, is that portions of the studies rest on questionable premises and confusions with respect to the law. It must be realized that Weber's law refers to maximal resolution, to *detection*. The detection in question is that of stimulus difference, indexed by the DL or the JND. All suprathreshold discriminations are *not* Weber's law. Thus, people may well perceive stimulus ratios as subjective differences, but this is not Weber's law. I call that relation, the Weber principle, and one should be careful to keep the two separate. Mathematical psychologists Dzhafarov and Colonius (2011) have recently proved that Weber's law and the Weber principle are different and *independent*. Many studies confuse the two—testing, modeling, and referring erroneously to the principle as the law, with adverse consequences for theory and research alike. Even high-power computational research is not free from the confusion; thus Pardo-Vazquez et al. (2019) suggest that "the strong form of Weber's law states that the probability of correct discrimination between two stimuli … depends only on the ratio between their intensities" (Pardo-Vazquez et al., 2019, p. 1). The authors then proceed to model the Weber *principle*, not Weber's law (in violation of their stated goal and title).

Perhaps nowhere in the literature is the confusion more salient than in the domain of numerical cognition, particularly when examining the perception of natural numbers. Many an author does not realize that Weber's law does not apply to natural numbers, given the trivial fact that the closest possible values are already perfectly discriminable. Any young child can impeccably discriminate 3 from 4, that is, the smallest possible addendum is already 100% discriminable. The misapplication leads to absurd "findings." Thus, in a recent study with natural numbers by a respected author, it is reported that the Weber fraction was 8%. This means that the closest natural number that one can discriminate from 200 is 216! Weber's law is evoked in an inappropriate manner to account for further numerical phenomena such as the distance and the size effects (see Dehaene, 2011).[4]

For a salutary counterexample, economist Itzhak Gilboa has recently been careful to dissociate Weber's law from Fechner's law, thereby helping his discipline to resolve lingering confusions (in particular vis-à-vis Luce's semiorders and pricing; see Gilboa, 2009).

Regardless of any conjectural consideration, conceptual clarity is a sine qua non for any science, certainly for the relatively young science of psychology. Exploring the present case is well worth the effort because many long-standing confusions could be resolved if only we knew more about true nature, origin, hence the independence of the two laws. In general, we should be careful not to dismiss past efforts or we forfeit the possibility of a psychology that is truly transgenerational.

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[2] A tutorial review of these developments as well as bridge-building with mainstream (classical) psychophysics goes beyond my scope in this Note. These daunting tasks are certainly invited.

[3] Possibly due to complex information processing in the brain by which global activations impact local decisions. A regularity of this type was suggested within psychophysics by Teghtsoonian (1971, 2012), who observed the presence of a correlation between stimulus range and the Weber fraction across various sensory dimensions (noticed previously by Poulton, 1967, 1968, and even earlier by Jones & Woskow, 1962). Based on this regularity, Teghtsoonian proposed a *theory* that assumes a constant subjective range and a constant subjective JND (Ekman's $\Delta\psi$) across all sensory dimensions. This theory (to be distinguished from the observed regularity) is untestable. However, it is important to note that psychophysicists of stature, including such contributors as Baird (1997), Luce (1993), and S. S. Stevens (1975) emphatically deny the possibility of a relationship between global and local psychophysics. Their position is based on solid physical and engineering principles associated with sensors and detection devices—by which the imprecision, fluctuation, or noise inherent in the sensor has nothing to do with the sensor's operation over its dynamic range.

[4] Due to their unique cardinality representation, a symbolic number does not have a distributional spread or group of representations, so that overlap between the representations of neighboring numbers is excluded (Bar et al., 2019; Sasanguie et al., 2014, 2017). The well-documented effects of distance and size are explained by symbol-to-symbol ordinal relations, possibly governed computationally by the so-called delta-rule (Reynvoet & Sasanguie, 2016; see also, Leth-Steensen & Marely, 2000; Verguts & Van Opstal, 2005, 2014). Supporting this view is the observation on the absence of scalar variability in children's learning of numbers (Le Corre & Carey, 2007). In this context, it is important to note that, unlike symbolic numerals, Weber's law does apply in judgments of numerosity (Fitousi & Algom, 2018; Pansky & Algom, 2002). Numerosity is a standard perceptual dimension, on a par with continua such as loudness, brightness, or heaviness. It is beyond the purview of this Note to expand on these issues.

Weber's Law: What It Does and Does Not Entail

The German physiologist Ernst Heinrich Weber (1795–1878) first reported that the ability to tell the difference in intensity between a pair of physical stimuli depended on the ratio of their intensities (Weber, 1834, 1846, 1996).[5] Subsequently, Gustav Theodor Fechner (1801–1887), a former student and colleague, labeled this empirical finding as "Weber's law" (Fechner, 1860/1966), and put it in the mathematical form with which we are now familiar (Weber did not state his finding in formal terms).[6] Weber's law can be written as,

$$\Delta S = kS, \tag{1}$$

where S is stimulus magnitude, ΔS is the difference threshold or a JND in S (specified in the same physical units as S), and k is a constant fractional increment that should be added to S for the minimal change to be noticed (called the Weber fraction). In other words, the observer's resolution power decreases with stimulus magnitude: The larger the stimulus, the larger the increment that should be added in order for a minimal change to be noticed.

A remarkable feature of Weber's Law is the absence of a psychological or subjective variable. All components are physical (or mathematical): Both S and ΔS are physical stimuli (measured in the same units), and k is a (positive) constant. The sole psychological component in Weber's law is the observer's indication when two stimuli are discriminably different (i.e., it is the observer who determines ΔS). However, Weber's law is silent on a crucial question: What sensation is felt at any given ΔS.[7] Given the missing subjective variable, Weber's law is indifferent on the question of whether the JNDs in *sensation* are equal or unequal at different values of S and ΔS. Weber's law merely states a simple empirical relation: When you wish to detect a change in the stimulus, add a constant percentage to its current magnitude.[8]

Weber's law is often proclaimed a quintessential psychological law, even as *the* psychological law; it is sometimes called, "psychology's law of relativity" (e.g., S. S. Stevens, 1951; J. C. Stevens, 1971). This is probably an overstatement because the law may well be rooted in physics. In many physical domains, the variance of elemental fluctuations in the stimulus (σ_S^2) is proportional to the mean magnitude (M or S). Think for example of photons emitted in a monochromatic beam within the quantum framework of light, or of the energy in a monoatomic ideal gas as studied in statistical mechanics. Hence,

$$\sigma_S^2 \propto M, \tag{2}$$

that is, there is a necessary physical relation between stimulus variability at the quantum level and stimulus magnitude. The formula means simply that larger quantities tend to generate larger fluctuations. This relation can also result from any process wherein the total quantity (S) is the sum of all the partial quantities comprising the total. One may construe the stimulus to represent the sum of its smaller components whose variances contribute to the observed variability (σ_S^2, indeed ΔS) of the global stimulus. This idea was explored by Woodworth (1914) and more recently by Norwich (1993; see also Cattell, 1893; Solomons, 1900), who demonstrated that Weber's law emerges as the end result of such elemental processes.

Assume that the human observer is sensitive to the physical variance inherent in the stimulus (at the level of the receptor on the sensory surface, Norwich, 1993; see also Marks & Algom, 1998). Then, in Norwich's model, the receptor continuously samples the stimulus, and this process of sampling is variable, with the variance proportional to the magnitude of the global stimulus. For example, more highly concentrated solutions possess greater entropy than more dilute solutions. Sipping your coffee, your chemoreceptors in the mouth register a larger variance of the liquid when your cup contains more spoons of sugar.

The close correspondence between ΔS and stimulus variability was first stated and measured by Crozier and Holway (1937; see also Crozier, 1936), and the interchangeability of ΔS and the variance is explicit in the psychophysical method of adjustment (e.g., Baird & Noma, 1978; Gescheider, 1997; Marks & Algom, 1998).

Taking a very brief look at the empirical domain, the validity of Weber's law (over the entire stimulus range) has long been debated. For example, the Weber fraction k is often not constant, particularly at low levels of stimulus intensity (but see Holway & Pratt, 1936, who documented deviations from constant k at both low and high levels of stimulus intensity). Such deviations can be corrected by "linear generalizations" (Fechner, 1860; Miller, 1947; see also Gescheider, 1997; Marks, 1974; Marks & Algom, 1998), when, for example, ΔS is made proportional to S plus a constant (rather than proportional to S). However, the relation between ΔS and S may be nonlinear altogether. The following equation associating ΔS and S was suggested by Guilford (1932, 1954),

$$\Delta S = kS^g; \tag{3}$$

when fitting experimental data, the exponent g typically varies between 0.5 and 1.0. When $g = 1$, then this is Weber's law, and when $g = 0.5$, this is the square root law suggested by Fullerton and Cattell (1892). Being rather general, Guilford's power-law fits many

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[5] Weber's best-known research that contains Weber's law is first summarized in a collection written in Latin, *De Subtilitate Tactus* (1834) (On the sensibility of touch), usually abbreviated as *De Tactu*. Weber's further research is summarized in an article written in German for a Handbook, *Tastsinn und Gemeingefuhl* (1846) (The sense of touch and the common sensibility), usually abbreviated as *Der Tastsinn*. Both monographs were translated into English by Helen Ross and David Murray (Ross & Murray, 1996) and further comments refer to pages in that edition.

[6] It might come as a surprise realizing that what we call Weber's law is not discussed as a central or dominant notion in *Du Tactu* or in *Der Tastsinn*. The fairly limited discussions (appearing toward the end in both monographs) can be easily overlooked amid the forest of sundry experimental results and detail (Algom, 2019). It was Fechner who elevated this particular finding by Weber into a law, bequeathing it to future generations of psychologists.

[7] Further questions can be asked (Dzhafarov & Colonius, 2011): Does a pair of stimuli that differ from one another by an amount smaller than ΔS give rise to two separate sensations or just a single sensation? Is the detection of a discriminable change in S felt as a *sensation difference* between S and $S + \Delta S$ or just as a unique experience of change detection?

[8] In fairness to Weber, he was not completely oblivious to the sensation component of his finding. In his summary statements, Weber often comes close to the psychophysical views of his one time student and junior colleague, to wit, "it is shown that what is perceived is not the absolute weight difference but the ratio" (Weber, 1934/1996, p. 126), or that "people perceive relative rather than absolute differences when making discrimination" (Weber, 1934/1996, p. 127). One should be circumspect though because in such statements Weber did not mean that people actually feel or perceive subjective ratios, only that stimulus ratios determine their discrimination.

sets of data. When discriminating sound intensity (e.g., Jesteadt et al., 1977; Luce & Green, 1974; McGill & Goldberg, 1968a, 1968b), g is found approximately 0.8–0.9. The small discrepancy from Weber's law has been dubbed, "near miss to Weber's law" (McGill & Goldberg, 1968a, but see Holway & Pratt, 1936, again).[9]

To recap, it is possible that the empirical relation known as Weber's law is produced by the observer's registration of the variability inherent in the physical stimulus—not by the perception of stimulus magnitude itself. The physical reality by which stimulus variability grows with stimulus magnitude might well produce a law like that of Weber. The mechanistic foundations of Weber's law and the attendant processing is more recently supported by studies of reaction time (RT), which show a monotonic, often linear relation between the mean and the variance of time distributions (Wagenmakers & Brown, 2007). Some of the implications of the presence of variability in the responses to a nominally constant stimulus include: (a) presenting the same stimulus on different occasions will produce different responses and (b) presenting physically different stimuli on different occasions will produce (on a subset of occasions) the same response. These implications and, more generally, the notion of variability inherent in the stimulus (hence also in the responses) is captured by the d' index of signal detection theory (Green & Swets, 1966; Macmillan & Creelman, 2005; Wickens, 2002). This measure of subjective distance depends explicitly on the variance of the pair of stimuli discriminated. Cumulated d' scales can accommodate Weber's law (e.g., Laming 1997b), although one would like to see more research dedicated to the relation between Weber's law and d'-based scales.[10]

I next examine the part played by Weber's law in the derivation of Fechner's logarithmic low.

Derivation of Fechner's Law: The Role of Weber's Law

The most popular version follows Edwin Boring's early authoritative exposition (Boring, 1942, 1950).[11] Boring based his treatment on Fechner's development in his *Elemente der Psychophisik*, and his persuasive rendition has been espoused by virtually all succeeding psychophysicists (e.g., Adler, 1996; D'Amato, 1970; Gescheider, 1997; Lewis, 1960; Marks, 1974; Marks & Algom, 1998; Marks & Gescheider, 2002; see also Baird, 1997; Baird & Noma, 1978). The name of Helmut Adler on this list is notable, as Adler produced the sole translation into English to date of the *Elemente* (Fechner, 1860/1966, Vol. I). Here is the standard derivation (see, e.g., Marks & Algom, 1998).

Fechner assumed the validity of Weber's law, namely, that the amount ΔS by which stimulus intensity S must be augmented in order for the change to be detected is a constant fraction k of S (Equation 1). Notably, Fechner introduced a new variable of subjective sensation magnitude, ψ, and assumed that every ΔS or stimulus JND along the stimulus continuum produces exactly the *same* feeling of minimal subjective change $\Delta\psi$ in sensation. In other words, on a given physical continuum all *sensation* JNDs (to be distinguished from *stimulus* JNDs)[12] are equal regardless of the size of the relevant stimulus JND or ΔS. Hence,

$$\Delta\psi = c, \tag{4}$$

where c is a constant. For example, adding a small amount of light energy suffices to notice the change in brightness in a dark room, but a much larger amount is needed to detect a change in brightness in a well-lit room—and yet these vastly different physical increments are felt the same by the observer. Fechner's postulate (Equation 4) coupled with Weber's law (Equation 1) enabled him to establish a truly psychophysical function in the following way. Weber's law implies that the successive stimulus ratios, $S_2/S_1, S_3/S_2, \ldots, S_j/S_{j-1}$ *of just noticeably different stimuli*, form a geometric series. According to Fechner's postulate, these ratios produce equal increments in sensation. As a result, a geometrically spaced series of values on the physical continuum generates an arithmetically spaced series of values on the psychological continuum. This relation defines the logarithmic function,

$$\psi = m\ln(S/S_0), \tag{5}$$

where $m = c/k$ is the constant of proportionality and S_0 is the absolute threshold. The stimulus is measured as multiples of the absolute threshold at which value sensation is zero. Equation 5 was Fechner's *Massformel* or measurement formula. The label *Massformel* is substantive because Fechner realized that actual scaling requires determining ΔS and S_0 in the laboratory. Consequently, a major contribution of Fechner was the development of experimental procedures for estimating S_0 and ΔS.

The hallmark of Fechner's methods is the simplicity and naturalness of the observer's task, entailing merely the responses "smaller," "greater," or "equal." Such responses avoid many of the pitfalls associated with numerical estimates that have become popular in later-day psychophysics.[13] The probabilities associated with the "greater than" responses, for example, serve then to determine the threshold. In the general literature, Fechner's classic methods of measurement, on the one hand, and Fechner's law, on the other hand, are often treated separately (see Boring, 1961). However, there is a close connection between the two. The size of the difference

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[9] Actually, the discrepancy might be large as shown by the results reviewed by Holway and Pratt (1936) and by more recent results.

[10] One notes the contributions by Braida and Durlach and their associates on loudness identification and discrimination spanning some three decades (too many to cite here). They used the measure of d' between adjacent stimuli, but their main point of interest was (absolute) identification. There are several intricate issues involved, but their discussion will take us too far afield. I also note that in recent standard texts of detection theory (Macmillan & Creelman, 2005; Wickens, 2002) the name Weber does not occur.

[11] Edwin Boring's book, *A History of Experimental Psychology*, was originally published in 1929, so one can trace Boring's influence to this early date. It would be simplistic and misleading though to attribute Boring's impact merely to chronology. His text is informative and engaging and is marked by a painstaking effort at streamlining Fechner's less than clear text.

[12] It is important to distinguish between *stimulus* JNDs and *sensation* JNDs, a distinction that is not always respected in the general literature. The former refers to the physical increment needed to reach a just noticeable change in sensation. In other words, the stimulus JND is equivalent to the Difference Threshold or Different Limen (DL) or to ΔS. The sensation JND, by contrast, refers to the subjective experience of minimal change detection, or $\Delta\psi$. It is the increment in *sensation* felt with the addition of each stimulus JND.

[13] The merit and significance of the simple responses used by Fechner are insufficiently recognized. The observer's responses—detecting the presence of the stimulus or deciding whether one stimulus is greater than another—are about the simplest reactions a person can make. These responses are valid virtually by definition. In contrast, S. S. Stevens popularized more complex responses that involve numerical estimates by the observer. These methods are vulnerable to validity more than those devised by Fechner. Of more concern, S. S. Stevens and some of his followers considered the numbers provided by the observer to reflect directly the magnitude of her *sensation*. Stevens did not justify this crucial assumption.

threshold, as determined for example by the method of constant stimuli, can be made small (or large) by appropriate experimental convention, and these values (especially very small values) are vital for the proper derivation of Fechner's law (Dzhafarov & Colonius, 2011; Masin et al., 2009). Indeed, some consider Fechner's fundamental idea as one entailing the rate of growth of the probability-of-greater-function with subsequent accumulation of these momentary sensitivity values (Dzhafarov & Colonius, 2011).

Fechner derived his logarithmic law (Equation 5) by using his *mathematical auxiliary principle*: The properties characterizing differences as small as ΔS and $\Delta\psi$ also characterize smaller differences. Consequently, dividing Equation 4 by Equation 1 and rearranging terms, Fechner rewrote the result as a differential equation,

$$d\psi = mdS/S_0, \tag{6}$$

called the *Fundamentalformel* or the fundamental formula. Fechner then integrated Equation 6 between S_0 and S to arrive at the standard logarithmic solution given in Equation 5.

This standard story of the derivation of Fechner's law already makes it clear that Fechner's law and Weber's law are separate laws. Fechner (arguably) assumed Weber's law in his development, but this should not obscure the fact that Fechner's law includes a variable, sensation magnitude, which is not part of Weber's law. The independence of the two laws is highlighted by the possibility of accepting Weber's law (Equation 1) but rejecting Fechner's postulate (Equation 4). This avenue is conductive to a different psychophysical function, which I briefly consider next.

Weber Sans Fechner: Ekmans' Law and Stevens' Law

Gösta Ekman, the late head of the Stockholm laboratories, formalized a conjecture conceived by Brentano (1874) and others (see S. S. Stevens, 1975) on the presence of an internal counterpart to Weber's law (Ekman, 1956, 1959). Ekman posited that the *sensation* JND or $\Delta\psi$ does not remain invariant but rather grows as a constant fraction n of *sensation* magnitude ψ. Ekman's law thus states that,

$$\Delta\psi = n\psi, \tag{7}$$

where n is the Ekman fraction. Equation 7 is exactly analogous to Equation 1, Weber's law, and is inconsistent with Equation 4, Fechner's postulate. Assuming both Weber and Ekman's laws, one can divide the latter by the former and, following Fechner's logic, write then the differential equation,

$$d\psi/n\psi = dS/kS. \tag{8}$$

Integration gives,

$$\psi = \psi_0(S/S_0)^{n/k}, \tag{9}$$

where ψ_0 is the subjective counterpart of S_0. Setting a constant $M = \psi_0(1/S_0)^{n/k}$ and defining $b = n/k$ yields the familiar form of the psychophysical power function,

$$\psi = MS^b. \tag{10}$$

Equation 10 is widely recognized as Stevens' power law (S. S. Stevens 1975; see also Marks, 1974), but there is an important caveat to consider before accepting this designation. Stevens used "direct methods" of measurement where the observers provide direct numerical estimates R of the perceptual magnitudes produced by the stimuli.[14] The results of numerous experiments using such magnitude estimations revealed that the numeric responses R were a power function of stimulus magnitude S with an exponent b and constant A,

$$R = AS^b. \tag{11}$$

Note that Equation 11 is a purely empirical result, obtained by fitting a power function to the numerical estimates. However, Stevens (e.g., 1975, p. 13) and others considered R to stand for sensation ψ—without providing justification for this crucial assumption. As a result, Stevens imperceptibly replaced Equation 11 by Equation 10. However, Equation 10 can be called Stevens' power law *if and only if* R is proportional to ψ, that is, $R \propto \psi$, but this stipulation has not been adequately addressed or supported.

In summary, the present discussion makes it eminently clear that it is possible to accept the validity of Weber's law but not of Fechner's law. Based on Weber's law, it is possible to derive a psychophysical law that is different from Fechner's logarithmic law. The difference is easily seen by the derivation involving Ekman's law. In this respect, one may note that Stevens accepted the validity of Weber's law when promoting his psychophysical power law. In Stevens' view though, Weber's law is irrelevant to suprathreshold scaling of sensations.[15] The upshot is, there cannot be such thing as a "Weber–Fechner law."

Fechner Sans Weber: How Did Fechner Really Derive His Law?

Problems with the Standard Account

The legitimacy of the standard derivation appearing in textbooks was questioned early (e.g., Elsas, 1886; Müller, 1878) and more recently in the influential work done by Luce and Edwards (1958; see also Falmagne, 1971, 1985; Luce, 1993; Masin et al., 2009; but see Krantz, 1971; Laming, 1997a, 1997b). The Luce and Edwards challenge merits a brief overview (see Townsend, 1975, for a lucid development). Luce and Edwards (1958; see also Baird & Noma, 1978) suggested a general routine for deriving psychophysical functions by designating any relation between ΔS and S as a Weber function, and any relation between cumulated *sensation* JNDs and cumulated *stimulus* JNDs as a Fechner function. In this general scheme, Weber's law (Equation 1) is a specific Weber function and Fechner's postulate (Equation 4) is a specific assumption (or definition) regarding the equality or uniform size of each sensation JND. Thus specifying a Weber and a Fechner function is conductive

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[14] Concerning Stevens' "direct scaling" methods, asking observers to provide numbers to stand for their sensations, actually goes as far back as Merkel (1888) and Fullerton and Cattell (1892), and the first authors to use the method that Stevens later called magnitude estimation were Richardson and Ross (1930).

[15] Stevens distinguished between what subsequent investigators called *local* and *global* psychophysics (e.g., Baird, 1997; Luce, 1993). Local psychophysics is concerned with stimulus identification and discrimination (or absolute and difference thresholds), whereas global psychophysics is concerned with the magnitude of sensations along with the full dynamic range on a stimulus continuum (or with the psychophysical function). Significantly, the two domains are separate so that results of local psychophysics do not have bearing on global psychophysics.

to Fechner's law, and this is certainly true when using finite calculus (e.g., Lewis, 1960; Townsend, 1975).

Let us designate $S + \Delta S/S$ by z. Then, counting stimulus JNDs from the threshold, S_L (so that $S_0 = S_L$), one gets $S_N = S_L z^N$, in which the subscript on S at the left designates the number of JNDs counted above S_L. Taking logarithms, rearranging, and defining a constant $M = 1/\mathrm{Log}\, z$, we get,

$$N = M \ln(S/S_0). \qquad (12)$$

It is very important to notice the difference in the left-hand term between Equation 5 (Fechner's law) and Equation 12. In Equation 12, N specifies the number of stimulus JNDs above threshold. Only by accepting the Fechner postulate, namely, that each stimulus JND generates an equal-size sensation JND on the sensory dimension, do the two equations concur. A succinct way of getting to Equation 12 (Laming, 1997b) is by noting that if $(S + \Delta S/S)$ is constant, so too is $[\ln(S + \Delta S) - \ln S]$. Again, however, a subjective component must be added to the last term for it to be consistent with Fechner's law. It was this juxtaposition of sensation with respect to the last term that formed the heart of Luce and Edwards's (1958) critique. Does $\psi(S + \Delta S) - \psi(S)$ remain constant throughout? Or, is the equality, $\psi(S + \Delta S) = \psi(S) + \Delta\psi$, always true? Luce and Edwards (1958) showed that it is not—when one employs Fechnerian integration. The authors show that Fechner's procedure—moving from difference to differential (via the "mathematical auxiliary principle") and then integrating—leads to mathematically acceptable results with only a few Weber functions (including fortuitously Weber's law and its linear generalizations), but not with others. In general, Fechner's derivation leads to an internal contradiction: Either the sensation JND's are not equal or the specific Weber function is pathological (and not serviceable). For a single illustration, the well-known Weber function suggested by Fullerton and Cattell (1892), $\Delta S = \sqrt{S}$, does not yield equal-size sensation JNDs upon Fechnerian integration.

Appraising Luce and Edwards (1958) from the distance of 60 years, it is insufficiently recognized that theirs is *not* an attack on Fechner's law, only on a method for deriving the law (itself possibly misinterpreted, Dzhafarov & Colonius, 2011). Overlooked, too, is the recognition by Luce and Edwards (1958) that the functional-equation solution, faultless mathematically, is the same one as that obtained by Fechner's method when assuming Weber's law.[16] In practice, it is possible to bypass the difficulty by simply summing finite values of JNDs (e.g., by graphical addition). In this respect, I note that graphically summated JND scales often fail tests of consistency and additivity within and across sensory dimensions (see Marks & Algom, 1998, for a discussion of this point).

If Fechner's law endures unscathed, how was it truly derived?

Is Weber's Law Indispensable for Deriving the Logarithmic Psychophysical Law?

The answer clearly is negative, if only due to the historical fact that the logarithmic law was put forward well before the birth of Ernst Heinrich Weber. A century before Weber's findings and Fechner's work, the Swiss mathematician Daniel Bernoulli derived the logarithmic law by using alternative principles (i.e., without employing any tool remotely similar to Weber's law). A grossly underappreciated contribution of Bernoulli is his pioneering distinction between *objective-physical* variables and their *subjective-psychological* counterparts (with respect to the money, the latter is called utility). Bernoulli assumed that the perception of a small addition to one's wealth is inversely proportional to existing wealth, and specified the relation by an appropriate differential equation. Integration then resulted in a logarithmic function relating utility to wealth (Bernoulli, 1738/1954; see Masin et al. 2009, for a detailed discussion of Bernoulli's assumptions and derivation). Much later, Thurstone (1931; see again Masin et al., 2009) derived the logarithmic law by using yet other principles—again without appeal to Weber's law. These derivations are free of the problems noted with respect to the standard derivation that involves Weber law, the JND, and the Fechner postulate.[17]

The Role of Weber's Law

Did Fechner truly base his law on Weber's law, the JND, and the Fechner postulate? Dzhafarov and Colonius (2011) make a strong case that he did not. These authors argue, contrary to accepted wisdom, that "the function relating a mental continuum to its physical counterpart is explicitly assumed by Fechner to be continuous" (Dzhafarov & Colonius, 2011, p. 129). If that is the case, then, obviously, Weber's law, the postulated subjective equality of JNDs (Fechner's postulate), as well as the "mathematical auxiliary principle" are gratuitous.[18] Dzhafarov and Colonius (2011) further argue

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[16] Luce and Edwards (1958) were careful to direct their criticism at Fechner's *procedure*, but they could have been more straightforward in stating that the law stands regardless (as it is derived for example by functional-equations or by finite calculus). Donning the lenses of an amateur historian (Algom, 2019), one cannot dismiss the possibility of an effect, if tenuous and intangible, by the overwhelming personality of Luce's colleague at Harvard, S. S. Stevens, at a time when the latter's method of magnitude estimation and the attendant power-law increasingly carried the day. When discussing magnitude estimation, Luce and Edwards do not mention the method's most obvious weakness, namely, the (unproven) assumption that the "number" proffered by the observer faithfully reflects her sensation. Consider also the language: Fechner's work is invariably depicted "incorrect" or plagued by "error," whereas a certain topic is "excellently" discussed by Stevens (and others). Boring, Stevens's advisor and then colleague, is a case in point. In a paper ostensibly celebrating the centennial of the Fechner's *Elemente*, Boring writes in a condescending style, relegating Fechner's law to a historical relic, into something resolutely defeated (Boring, 1961). I believe that the current Zeitgeist is much more positive vis-à-vis Fechner.

[17] It might be of interest to note that Fechner was well aware of Bernoulli derivation, which was already a century old. Why did he insist on incorporating Weber's law? Fechner asserted that his derivation, based on Weber's law, is more general than Bernoulli's derivation, which applied only to the utility. However, as noted by Masin et al. (2009), there is no compelling reason why Bernoulli's derivation cannot be extended to all kinds of sensations. A further reason (see Dzhafarov & Colonius, 2011) might be Fechner's deference to his former teacher and colleague, immortalizing his name in psychological science. Arguably, Fechner realized that there was no real need to incorporate Weber's law (the JND and the Fechner postulate) into the derivation of the logarithmic law.

[18] Fechner's own text in the *Elemente* is admittedly less than clear in places. This much granted, one may ask again: Why did Fechner espouse Weber's law so consistently when (a) he was aware of Bernoulli's derivation and (b) he assumed continuous variables in the mental and the physical dimensions? Although the answer is not completely clear (and might never be), three reasons come to mind. First, Fechner thought the derivation entailing Weber is more general than the derivation by Bernoulli. Second, he did cling onto Weber out of respect to his teacher, colleague, and friend. Third, Fechner did not mean Weber's law in the sense in which it is currently used, but rather as the principle by which subjective differences are dependent on physical ratios.

that the standard derivation itself is mathematically correct, maintaining that the notion of the mathematical auxiliary principle (and the role Weber's law) is misinterpreted. Fechner actually entertained two derivations, both of which can be construed as a functional equation (the same equation) with a known solution. Notably, neither derivation entails Weber's law, the notion of JND, and the Fechner postulate.

Weber's Law versus the Weber Principle

A casual look at the general literature reveals that Weber's law is not always present when discussing Fechner's law. Consider for example Marks's (1974) pair of depictions: "Fechner's law can be stated as, 'Equal stimulus ratios produce equal sensation intervals'" (p. 6). Or, 'As stimulus intensity increases geometrically, sensation intensity increases arithmetically'" (p. 6). Notable in these statements is the absence of Weber's law and the JND. Although such statements in the literature are not meant to be rigorous (Marks's is), they actually convey Fechner's thinking more faithfully than the standard rendition (which entails Weber's law). As we have seen, the use of Weber's law in deriving Fechner's law is fraught with problems, even with contradictions (Luce & Edwards, 1958; Masin et al., 2009).

The idea alluded to in Marks's (1974) outflanks these problems. It says simply that the subjective interval or dissimilarity between two stimuli is determined by the ratio of their physical magnitudes. Working from the text of Fechner's *Elemente* and sustained by formal development, Dzhafarov and Colonius (2011) define the same idea as the "W-principle" and consider it to be the true Weber law—to be distinguished from Weber's law as it is conventionally documented. I here designate the principle as the "Weber principle." Fechner did not systematically distinguish in his writings between the Weber principle and Weber's law, often referring to the Weber principle as Weber's law.

It is important to recognize that Weber's law, on the one hand, and the Weber principle, on the other hand, are independent notions, with neither mandating the validity or indeed the existence of the other. However, if one accepts Weber's law (as conventionally understood) and further accepts the Fechner postulate, then (with a sufficiently small Weber fraction) the subjective difference between a pair of stimuli is approximately equal to the number of JNDs separating them. So, in the best-case scenario, Weber's law is an implication rather than the foundation of the logarithmic law (Masin et al., 2009).

To recap, Fechner's law can be derived without appeal to Weber's law, and probably was. Conceivably, Fechner used the Weber principle in his derivations, often misnamed as Weber's law.

Conclusion

Nomen est numen says the Latin adage, meaning in our case that distinct names for distinct laws best serve science by eliminating confusions. The present analysis makes it clear that (a) Weber's law, (b) the Weber principle, (c) the Fechner postulate, and eventually (d) Fechner's law, are logically independent. As a result, the term, "Weber–Fechner law," is untenable. Weber's law can be employed in deriving psychophysical laws other than Fechner's logarithmic law, and, conversely, Weber's law is not necessary for the derivation of Fechner's law. Arguably, Fechner's law itself was first derived without appeal to Weber's law.[19,20] The upshot again is, the term, Weber–Fechner law, is wrong and misleading.

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[19] Is it difficult to know what exactly went on inside Fechner's head on the faithful morning of October 22, 1850—the morning that Fechner cited later as the time when he intuited the logarithmic relation between subjective and physical measures in the universe. This nonlinearity was, on his view, the solution to the mind-body problem. It is possible that the relation, here called the "Weber-principle," is what Fechner intuited on October 22, 1850. Arguably, Weber's law was incorporated later in support of Fechner's essentially philosophical position (see also, Boring, 1961). (I thank Larry Marks for contributing this thought).

[20] One should also ponder the role of astronomers (e.g., Herschel, 1829; Steinheil, 1837) who, before Fechner, related subjective starlight intensity to photometric starlight intensity (a new invention at the time) and reported a logarithmic relation (for a review of these contributions, see, Hearnshaw, 1996; Jastrow, 1887; Masin, 2012). As shown by Jastrow (1887), Fechner was aware of these developments (even challenged some of Herschel's measurements). It is thus tempting to suggest that Fechner was influenced by this work of astronomers in deriving his own logarithmic law. Along with Boring (1950) or Marks (1974), I believe that such is not the case—Fechner's scope and interest went well beyond the measurement of stellar magnitude per se. And, along with the same scholars (and others), I do not believe that Fechner's work can be conceived as merely that of establishing the validity of one scaling procedure (category rating), the difficulties with Fechner's derivations notwithstanding.

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