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Structural dynamic reliability analysis: review and prospects

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Abstract

Purpose – The purpose of this paper is to briefly summarize and review the theories and methods of complex structures' dynamic reliability. Complex structures are usually assembled from multiple components and subjected to time-varying loads of aerodynamic, structural, thermal and other physical fields; its reliability analysis is of great significance to ensure the safe operation of large-scale equipment such as aviation and machinery.

Design/methodology/approach – In this paper for the single-objective dynamic reliability analysis of complex structures, the calculation can be categorized into Monte Carlo (MC), outcrossing rate, envelope functions and extreme value methods. The series-parallel and expansion methods, multi-extremum surrogate models and decomposed-coordinated surrogate models are summarized for the multiobjective dynamic reliability analysis of complex structures.

Findings – The numerical complex compound function and turbine blisk are used as examples to illustrate the performance of single-objective and multiobjective dynamic reliability analysis methods. Then the future development direction of dynamic reliability analysis of complex structures is prospected.

Originality/value – The paper provides a useful reference for further theoretical research and engineering application.

Keywords Structural dynamic reliability analysis, Single-objective, Multi-objective, Surrogate model

Paper type Literature review

1. Introduction

Large-scale equipment such as aviation, machinery is usually assembled by multiple structures, and which overall reliability level is determined by the reliability of these structures (Meng *et al.*, 2020). To ensure the safe and economic operation of large-scale equipment, it is necessary to carry out a structural reliability analysis (Kim and Kang, 2013; Bagheri *et al.*, 2020). The interaction of multiphysical loads (e.g. flow field, thermal field, structural field and so forth) is often involved during the operation process of structures, and these loads have dynamic characteristics and uncertainty (Gao *et al.*, 2020a, b; Zhu *et al.*, 2021a, b). The static reliability analysis is difficult to reflect the dynamic relationship between output response and input parameters. To ensure the accuracy of reliability analysis, dynamic reliability analysis is the focus and hotspot in the field of structural reliability analysis (Nahal and Khelif, 2020; Zhi *et al.*, 2019).

The structural dynamic reliability analysis is a multidisciplinary and multiobjective problem, which has been widely concerned by many scholars (Wang *et al.*, 2009; Zhang *et al.*, 2020a, b; Shi *et al.*, 2017). In recent years, lots of research and practical application have been carried out (Jiang *et al.*, 2020; Zafar and Wang, 2020). In general, the research of structural dynamic reliability analysis includes single-objective and multiobjective dynamic reliability analysis. Single-objective dynamic reliability analysis is aimed at single component failure in structure, such as deformation, stress and strain. The research methods involved mainly

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include Monte Carlo (MC), outcrossing rate, envelope function and extreme value method. Multiobjective dynamic reliability analysis is based on single-objective dynamic reliability analysis, which refers to the same failure of multicomponents, multifailures of the same component, etc. The research methods of multiobjective dynamic reliability analysis mainly involve the series-parallel and expansion methods, multi-extremum surrogate model method and decomposed-coordinated surrogate model method.

The purpose of this paper is to study the methods of structural dynamic reliability analysis. Among them, the advantages and disadvantages of the existing theories and methods are analyzed. The structural dynamic reliability analysis development trend is summarized, which provides a reference for the development of structural reliability analysis theory.

In the remaining chapters, Section 2 reviews and summarizes the single-objective dynamic reliability analysis of structure. The multiobjective structural dynamic reliability analysis is analyzed and summarized in Section 3. In Section 4, the case studies and development prospects of structural dynamic reliability analysis are predicted. Some conclusions are summarized in Section 5.

2. Dynamic reliability analysis of single-objective structure

Structural dynamic reliability is the possibility that the structure can complete the predetermined function in the prescribed time and under the prescribed condition when the load and material characteristics change with time (Xue and Yang, 1995). The structural dynamic function G can be expressed as

$$G = g(\boldsymbol{x}, t) \tag{1}$$

where $g(\cdot)$ represents the limit state function; $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^T$ denotes n-dimensional random input variable; $x_i, i = 1, 2, \ldots, n$ is the geometric dimensions, material properties and loading conditions of the structure; t is the time variable.

In the dynamic reliability analysis problem, the dynamic reliability can be expressed as

$$R(t_0, t_s) = P\{g(\boldsymbol{x}, t) > 0, \forall t \in [t_0, t_s]\} \tag{2}$$

where $P(\cdot)$ is the probability operation, $\forall$ denotes any time t.

The dynamic failure probability $P_f(t_0, t_s)$ can be expressed as

$$P_f(t_0, t_s) = P\{g(\boldsymbol{x}, t) \leq 0, \exists t \in [t_0, t_s]\} \tag{3}$$

where $\exists$ represents the existence time t.

In this section, the dynamic reliability analysis methods of the single-objective structure are described, including MC, outcrossing rate, envelope function and extremum methods.

2.1 Monte Carlo

The MC theory is based on Chebyshev's theorem of large numbers and Bernoulli's law of large numbers (Aslett *et al.*, 2017). The sample mean converges to the parent mean and the frequency of the event converges to the probability of the event (Naess *et al.*, 2009; Cardoso *et al.*, 2008). The problem of structural reliability analysis based on the MC method is usually transformed into a probability model, and a large number of numerical simulations are carried out. Then the reliability analysis of the structure is approximately realized by the occurrence probability of events.

In the dynamic reliability analysis of structural single-failure mode based on the MC method, whether the fixed value $\boldsymbol{x}^*$ of the input variable $\boldsymbol{x}$ is safe or not in the time interval $[t_0, t_s]$, just determine if there is a time t for the $g(\boldsymbol{x}^*, t)$ to be less than or equal to 0 (Takeshi, 2013). If $\exists\, t \in [t_0,\ t_s]$, make $g(\boldsymbol{x}^*, t) \leq 0$, then the structure corresponding to $\boldsymbol{x}^*$ is the failure within the time interval $[t_0, t_s]$. Conversely, the structure is safe. The dynamic reliability solves steps by the MC method can be described as follows.

(1) The sample matrix $\boldsymbol{S}$ of the random input variable of $N \times n$ is generated from the probability density function (PDF) $f_{\boldsymbol{X}}(x)$, i.e.

$$\boldsymbol{s} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{12} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{Nn} \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_N \end{bmatrix} \tag{4}$$

(2) The time t is uniformly discretized for N^t times in the $[t_0, t_s]$, and the discrete-time vector T is obtained. T can be express

$$T = \begin{bmatrix} t^{(1)} \\ t^{(2)} \\ \vdots \\ t^{(N^t)} \end{bmatrix} \tag{5}$$

(3) The matrix $\boldsymbol{B}^{(i)} (i = 1,2,\ldots, N)$ is obtained, which is composed of the i rows of the S matrix and vector $\boldsymbol{T}$, i.e.

$$\boldsymbol{B}^{(i)} = \begin{bmatrix} B_1^{(i)} \\ B_2^{(i)} \\ \vdots \\ B_{N^t}^{(i)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_i & t^{(1)} \\ \boldsymbol{x}_i & t^{(2)} \\ \vdots & \vdots \\ \boldsymbol{x}_i & t^{(N^t)} \end{bmatrix} \tag{6}$$

(4) Substituting each row of matrix $\boldsymbol{B}^{(i)}$ into the limit state function, if $\exists t \in T, g(\boldsymbol{B}^{(i)}) \le 0$, the failure domain indicator function is $I_F(\boldsymbol{x}_i) = 1$. Otherwise, $I_F(\boldsymbol{x}_i) = 0$.

(5) The dynamic failure probability is calculated by the following formula

$$\widehat{P}_f(t_0, t_s) = \frac{\sum_{i=1}^{N} I_F(\boldsymbol{x}_i)}{N} \tag{7}$$

The MC method and its extension have been widely used in structural dynamic reliability analysis. Marseguerra *et al.* (1998) described some MC simulations for the dynamic reliability problem. Gonzalez-Fernandez and da Silva (2011) combined the MC simulation with the cross-entropy method for dynamic reliability analysis. Aabadi (2019) used the MC technique to analyze the reliability of steel moment frames. Wang *et al.* (2021a, b) introduced the multilevel MC to dynamic reliability analysis, which reduces the computational complexity of MC simulation. Luo *et al.* (2022) explored the enhanced MC simulation for improving accurate and efficient structural reliability analysis.

The MC method obtains robust reliability results, which can be solved by generating a large number of simulation samples. However, the calculation burden is large for the structural reliability problem with low failure probability, which leads to the low efficiency of reliability analysis (Gaspar *et al.*, 2014; Sun *et al.*, 2017). To improve the analysis efficiency and reduce the computational burden, many scholars have improved MC methods, such as importance sampling, subset simulation, linear sampling, directional sampling and other improved methods have been proposed. These methods can be applied not only to static reliability analysis but also to dynamic reliability analysis. To speed up the convergence speed of reliability, the importance sampling method transfers the sampling center to the design point, which makes more samples fall into the failure domain (Melchers, 1989). At the time $\bar{\boldsymbol{t}}$, the failure probability of the structure is given by

$$P_f\left(\bar{t},\bar{t}\right) = P\left\{G_{\bar{t}} \leq 0\right\} = P\left\{\boldsymbol{g}\left(\boldsymbol{x}\left(\bar{t}\right),\bar{t}\right) \leq 0\right\} = \int_{G_{\bar{t}}\leq 0} f_X(x)d\boldsymbol{x} \tag{8}$$

where $G_{\bar{t}}$ represents the limit state function of structural dynamics at the time $\bar{t}$; $f_{\boldsymbol{X}}(x)$ indicates the PDF.

Compared with the samples for the PDF values in the failure domain, the sample for the largest PDF value in the failure domain is obtained, which is used to approximate the most probable point (MPP) (Zhu *et al.*, 2020a, b). The MPP obtained by the PDF is displayed in Figure 1.

In Figure 1, the blue face denotes the limit state surface, while the green points represent the samples in the failure domain. The red point is the MPP, while x_1, x_2 and x_3 denote the components of $\boldsymbol{x}$. The β-spherical importance sampling is developed to ensure the calculation accuracy and improve the calculation efficiency (Harbitz, 1986; Yao *et al.*, 2019). In the standard normal space, the samples of β hypersphere with the radius of reliability degree are located in the safe domain. The efficiency of beta-spherical importance sampling is improved by avoiding the limit state function calculation of safety samples in the β hypersphere. Taking two-dimensional variables as an example, the schematic diagram of the β hypersphere at the time $\bar{t}$ is shown in Figure 2.

In Figure 2, the green area represents the safety domain $G_{\bar{t}} > 0$, the pink area represents the failure domain $G_{\bar{t}} < 0$, the red line is the boundary $G_{\bar{t}} = 0$, the white area is a hypersphere with the radius β and M is the MPP.

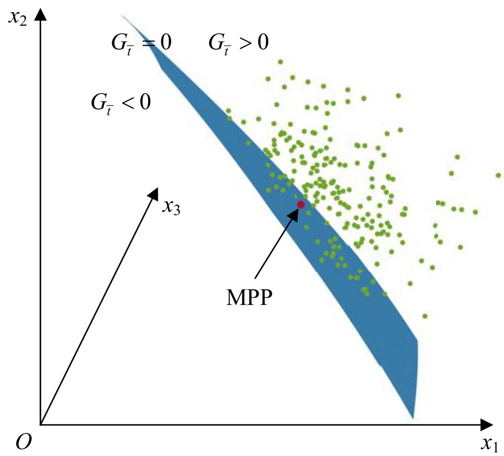


Figure 1.
MPP obtained using the PDF

Grooteman (2008) presented the adaptive radial-based importance sampling method, which gives an efficient adaptive method to determine the radius of the β hypersphere. The optimal hypersphere radius is determined by a step-by-step iterative search, which maximizes the efficiency of the β-spherical importance sampling. Taking the two-dimensional variable of U-space as an example, the adaptive radial-based importance sampling method at the time $\bar{t}$ is shown in Figure 3.

In Figure 3, β_0 is the initial hypersphere radius, β_i and β_{i-1} are the iterative processes of hypersphere radius until the optimal hypersphere radius β_{opt} is obtained.

To improve the applicability of the MC method for high dimensional and small failure probability reliability analysis, the efficiency of reliability analysis is improved in the subset simulation method (Au and Beck, 2001; Au, 2005). Linear sampling is applied to the reliability analysis of high dimensional small failure probability events, which also improved the efficiency of reliability analysis (Schueller *et al.*, 2004; De Angelis *et al.*, 2015; Papaioannou and Straub, 2021). The directional sampling method reduced the input variables dimension and improved the reliability analysis efficiency for structural reliability analysis problems with high nonlinearity and dimension (Nie and Ellingwood, 2000; Grooteman, 2011; Shayanfar *et al.*, 2018). Furthermore, some other important sampling methods have also achieved good results (Savage and Son, 2011; Singh *et al.*, 2011).

In addition, some scholars have made important contributions to the nonprobabilistic structural dynamic reliability analysis. Li *et al.* (2018) presented the nonrandom vibration

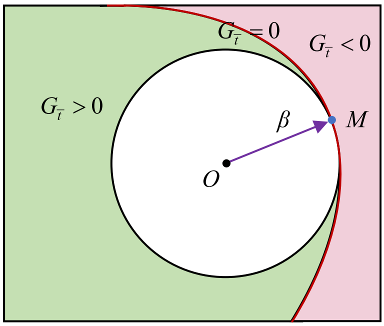


Figure 2. The schematic diagram of β-spherical importance sampling

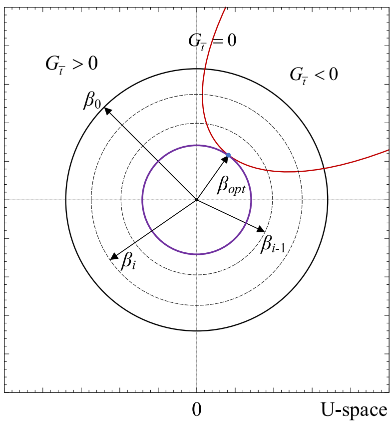


Figure 3. The schematic diagram of adaptive radial-based importance sampling method

analysis method to calculate the elastic beams dynamic displacement response bounds based on the interval process model. Jiang *et al.* (2019a, b) have made significant improvements to the interval process model and the nonrandom vibration analysis method, improving its applicability in the engineering field. Ni *et al.* (2020) developed a novel expansion method for the interval process model by reference to the Karhunen-Loève (K-L) expansion for stochastic process and random field models. The dynamic reliability analysis method based on stochastic process discretization and its improved method is explored by Jiang *et al.* (2014a, b, 2018).

The MC method can obtain robust reliability analysis results, which are often used as a reference standard to evaluate the calculation accuracy of other methods. But the MC method is not suitable for implicit function in engineering, especially when solving by the time-consuming finite element method, large-scale numerical simulation may not be affordable.

2.2 Outcrossing rate

Rice (1944) proposed the well-known first-passage formula for the first time, which lays a theoretical foundation for the research of outcrossing rate method in dynamic reliability. The Markov process is applied to the computation of first-passage probability by Siegert (1951). Wang *et al.* and Yu *et al.* described the basic theory of outcrossing rate method and proposed an improved algorithm for reliability analysis (Wang *et al.*, 2017, 2019a, b).

According to the above literature description, the failure probability in the time domain $[t_0, t_s]$ can be defined as

$$P_f(t_0, t_s) = P\left\{\{g(\boldsymbol{x}(t_0), t_0) \leq 0\} \cup \left\{N^+(t_0, t_s) > 0\right\}\right\} \tag{9}$$

where $N^+(t_0, t_s)$ represents the number of $g(\boldsymbol{x}(t_0), t_0)$ crossings from the safety state to failure state in the time domain $[t_0, t_s]$.

The classical bounds of $P_f(t_0, t_s)$ are as follows (Yu *et al.*, 2020)

$$\max_{t_0 \leq t \leq t_s} P_f(t, t) \leq P_f(t_0, t_s) \leq P_f(t_0, t_0) + P\left\{N^+(t_0, t_s) > 0\right\} \tag{10}$$

The outcrossing rate $v^+(t)$ can be expressed as

$$v^+(t) = \lim_{\Delta t \to 0, \Delta t > 0} \frac{P\left\{N^+(t, t + \Delta t) = 1\right\}}{\Delta t} \tag{11}$$

where $N^+(t, t + \Delta t)$ is the number of crossing times from the safe state to the failure state in the time domain $[t, t + \Delta t]$ for the structural system.

The $P\{N^+(\tau, \tau + \Delta\tau) = 1\}$ denotes the probability of structural system crossing from the safety state to failure state in the domain $[t_0, t_s]$, i.e. the outcrossing rate $v^+(t)$ is

$$v^+(t) = \lim_{\Delta t \to 0, \Delta t > 0} \frac{P\{g(\boldsymbol{x}(t), t) > 0 \cap g(\boldsymbol{x}(t + \Delta t), t + \Delta t) \leq 0\}}{\Delta t} \tag{12}$$

For the calculation of $v^+(t)$, Coleman (1959) derived the first-passage probability method based on the Poisson process. Crandall *et al.* (1966) used the numerical simulation method to solve the first-passage problem, which widens the application range of the outcrossing rate model. Iyengar (1973) introduced the stationary and nonstationary processes into first-passage probability for dynamic reliability analysis. The higher-order threshold crossings are used by Engelund *et al.* (1995) to introduce approximations of first-passage times for differentiable processes. Schall *et al.* (1991), Melchers and Beck (2018) and Rackwitz (1998), performed the dynamic reliability analysis for the structural load time-dependent reliability problems by the outcrossing rate method. Hagen and Tvedt (1992) developed a method for calculating the outcrossing rate of a parallel system. The PHI2 method is presented by Andrieu-Renaud *et al.* (2004), which calculated the outcrossing rates by the first-order reliability method (FORM), and the calculation of time-varying reliability is simplified (Zhu *et al.*, 2022).

The geometry explanation for solving outcrossing rates based on the first order second moment (FOSM) as shown in Figure 3.

As illustrated in Figure 4, $\beta(t)$ and $\beta(t+\Delta t)$ represent the reliability indexes of t and $t+\Delta t$ at two arbitrary times, respectively. ρ is the correlation coefficient (Andrieu-Renaud *et al.*, 2004). The formula for calculating the outcrossing rate by the PHI2 method is

$$v^{+}(t)=\frac{\Phi_2[\beta(t),-\beta(t+\Delta t);\rho(t,t+\Delta t)]}{\Delta t} \tag{13}$$

where Φ_2 denotes the binormal cumulative distribution function.

The PHI2 method has high calculation efficiency for outcrossing rate calculation, but the improper selection of Δt can affect the calculation results accuracy of outcrossing rate. Sudret (2008) proposed the improved PHI2 method realized the analytic solution of outcrossing rate, which has higher accuracy than the PHI12 method. Singh *et al.* (2011) introduced an important sampling technique to calculate the outcrossing rate for dynamic reliability analysis. Mejri *et al.* (2011) discussed the application of outcrossing rate in nonlinear systems. Hu and Du (2012) performed a time-dependent reliability analysis for the hydrokinetic turbine blade by employing the upcrossing rate method. Hu and Du (2013a, b) studied the time-dependent reliability analysis with joint upcrossing rate method, and the FOSM is used to derive the upcrossing rate, which shows good accuracy when the probability of failure is small and the dependency between failures is strong. Yan *et al.* (2017) discussed the application of FORM combined with outcrossing rate in ship structure reliability evaluation. Zhao *et al.* (2021) improved the solution method for outcrossing rate and avoided using a numerical scheme to calculate the outcrossing rate. Qian *et al.* (2020) provided a modified outcrossing rate method for time-dependent reliability analysis by avoiding assumptions in Rice's formula to improve computational accuracy. The applications of outcrossing rate method in nonprobabilistic dynamic reliability analysis are discussed (Jiang *et al.*, 2014a, b, 2017a, b). Outcrossing rate method currently has obtained a series of theoretical achievements and has been successfully applied to varieties of industrial departments (Soltanian *et al.*, 2018; Jia and Moan, 2009; Barbato and Conte, 2011).

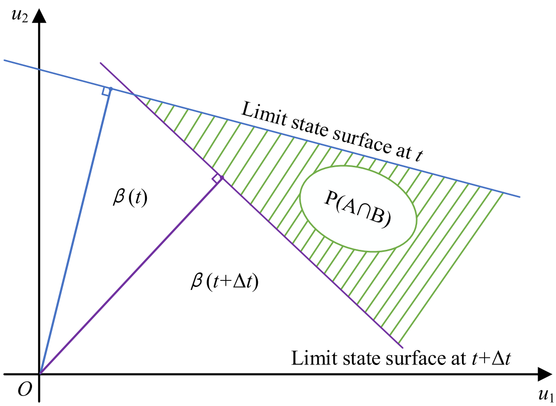


Figure 4.
Geometry explanation for solving outcrossing rates based on FOSM

2.3 Envelope functions

The envelope functions can transform a large number of constraints into a single or few constraints. Du *et al.* introduced the envelope surface of the failure probability maximum point on the failure boundary to approximate the failure domain, which transformed the structural dynamic reliability problem into the static reliability at the expansion point (Du, 2014). The steps of solving dynamic reliability by envelope function method can be described as

(1) The first-order Taylor expansion of limit state function $g(\boldsymbol{x},t)$ at the mean point can be obtained

$$g(\boldsymbol{x},t) \approx a_0(t) + \boldsymbol{a}(t)^{\mathrm{T}}(\boldsymbol{x}-\mu) \tag{14}$$

where $\mu = (\mu_1, \mu_2, \cdots \mu_n)^{\mathrm{T}}$, $a_0(t) = g(\mu, t)$, $\boldsymbol{a}(t) = \left(\frac{\partial g(\boldsymbol{x},t)}{\partial x_1}\Big|\mu, \cdots, \frac{\partial g(\boldsymbol{x},t)}{\partial x_n}|\mu\right)^{\mathrm{T}}$, $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)^{\mathrm{T}}$, $\boldsymbol{x}-\mu = (x_1-\mu_1, \cdots, x_n-\mu)^{\mathrm{T}}$.

To simplify the calculation, the random input variable $\boldsymbol{x}$ is transformed into the standard normal variable $\boldsymbol{u}$, i.e.

$$\boldsymbol{u} = (u_1, \ldots, u_n)^{\mathrm{T}} = \left(\frac{x_1-u_1}{\delta_1}, \ldots, \frac{x_n-u_n}{\delta_n}\right)^{\mathrm{T}} \tag{15}$$

where σ_i $(i = 1, 2, \ldots, n)$ is the standard deviation of x_i, then formula (16) can be expressed as

$$g(\boldsymbol{x},t) \approx L(\boldsymbol{u},t) = b_0(t) + \boldsymbol{b}(t)^{\mathrm{T}}\boldsymbol{u} \tag{16}$$

where $b_0(t)$ and $\boldsymbol{b}(t)^{\mathrm{T}}$ can be denoted as

$$\begin{aligned} b_0(t) &= a_0(t) \\ \boldsymbol{b}(t) &= (b_1(t), \ldots, b_n(t))^{\mathrm{T}} = (a_1(t)\delta_1, \ldots, a_n(t)\delta_n)^{\mathrm{T}} \end{aligned} \tag{17}$$

The envelope function of $L(\boldsymbol{u},t)$ can be obtained by

$$\left.\begin{aligned} L(\boldsymbol{u},t) = b_0(t) + \boldsymbol{b}(t)^{\mathrm{T}}\boldsymbol{u} = 0 \\ L'(\boldsymbol{u},t) = b_0'(t) + \boldsymbol{b}'(t)^{\mathrm{T}}\boldsymbol{u} = 0 \end{aligned}\right\} \tag{18}$$

in which $b_0(t)$ and $\boldsymbol{b}'(t)$ represent the partial derivatives of $b_0(t)$ and $\boldsymbol{b}(t)$ for parameter t, respectively.

(2) Assuming that $\boldsymbol{u}^*$ is the extension point on the envelope function, then $\boldsymbol{u}^*$ is on the linear function $L(\boldsymbol{U}, t) = 0$ and has the maximum probability density on the failure boundary, i.e. the point closest to the origin on $L(\boldsymbol{U}, t) = 0$ is $\boldsymbol{u}^*$. Therefore, $\boldsymbol{u}^*$ and $L(\boldsymbol{U}, t) = 0$ are perpendicular to each other. In the two-dimensional state space, the estimated envelope and real envelope are shown in Figure 5.

In Figure 5, the security domain and the failure domain are shown in green and pink, respectively. The vertical feet of $\boldsymbol{u}^*$ and $L(\boldsymbol{U}, t) = 0$ are denoted by A and B. The estimated envelope surface is composed of multiple $L(\boldsymbol{U}, t) = 0$ lines, and the purple line is the real envelope surface.

Then $\boldsymbol{u}^*$ can be described as

$$\boldsymbol{u}^* = \frac{-b_0(t)\boldsymbol{b}(t)}{\boldsymbol{b}(t)^{\mathrm{T}}\boldsymbol{b}(t)} \tag{19}$$

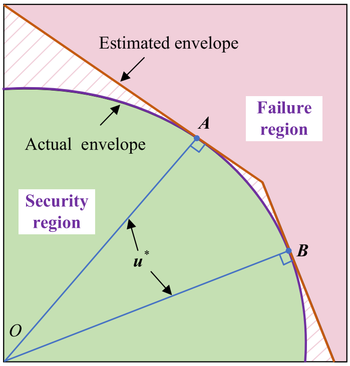


Figure 5. Relationship between estimated envelope and real envelope

And because $\boldsymbol{u}^*$ satisfies $L'(\boldsymbol{u}, t) = 0$, i.e.

$$b_0'(t) - b_0(t)\frac{\boldsymbol{b}'(t)^{\mathrm{T}}\boldsymbol{b}(t)}{\boldsymbol{b}(t)^{\mathrm{T}}\boldsymbol{b}(t)} = 0 \tag{20}$$

A set of solutions of the parameter t as $t_i^+ = (i = 1, 2, \cdots, p^+)$ can be obtained by solving formula (20). The two endpoints t_0 and t_s of the time interval are also considered, i.e. $p = p + 2$, $t_i \in (t_0, t_s, t_1^+, \cdots, t_{p^+}^+)(i = 1, 2, \cdots, p^+)$. Then the true failure envelope surface can be approximated by $\cup_{i=1}^{p} L(\boldsymbol{u}, t_i) = 0$, as shown in Figure 5. The dynamic failure domain $F = \{g(\boldsymbol{x}, t) \le 0, \exists t \in [t_0, t_s]\}$ can be replaced by the failure domain $\{\cup_{i=1}^{p} L(\boldsymbol{u}, t_i) \le 0\}$, which is composed of the failure envelope surface.

(3) The failure probability can be calculated as

$$\begin{aligned} P_f(t_0, t_s) &= P\{g(\boldsymbol{x}, t) \le 0, \exists t \in [t_0, t_s]\} = P\left\{\bigcup_{i=1}^{q} L(\boldsymbol{u}, t_i) \le 0\right\} \\ &= 1 - P\left\{\bigcap_{i=1}^{q} L(\boldsymbol{u}, t_i) > 0\right\} \\ &= 1 - \int_0^{+\infty} \cdots \int_0^{+\infty} \frac{1}{(2\pi)^{q/2}|\Sigma_{\boldsymbol{L}}|^{1/2}} \exp\left\{-\frac{1}{2}(\boldsymbol{x} - \mu_{\boldsymbol{L}})^T(\Sigma_{\boldsymbol{L}})^{-1}(\boldsymbol{x} - \mu_{\boldsymbol{L}})\right\} d\boldsymbol{x} \end{aligned} \tag{21}$$

where the mean value is $\mu_{\boldsymbol{L}} = (\mu_{L_i})_{i=1,2,\ldots,p} = (b_0(t_i))_{i=1,2,\ldots,p}$, $\Sigma_{\boldsymbol{L}} = (\delta_{L_i}\delta_{L_j})_{i,j=1,2,\ldots,p} = (\boldsymbol{b}(t_i)^{\mathrm{T}}\boldsymbol{b}(t_j))_{i,j=1,2,\ldots,p}$ denotes the covariance matrix; μ_{L_i} and δ_{L_i} (i = 1, 2, …, p) are the mean and standard deviation of L_i.

In recent years, some scholars have explored the application of envelope function method in structural dynamic reliability analysis. Wang *et al.* (2019a, b) proposed the general framework and corresponding methods based on envelope function and vine-copula function, which extends the distribution type of envelope function to general distribution. Zhang and Du (2015) combined the envelope function method and hybrid dimension reduction method is used for interval reliability analysis with clearances. The envelope function method combined with the first-order approximation of the motion error function is developed for efficiently estimating the time-variant global reliability sensitivity (Wei *et al.*, 2016, 2017). Zhang *et al.* (2020a, b) introduced a new envelope function method for the time-dependent mechanism reliability and sensitivity analysis with imprecise probability distributions, which has achieved better results.

In the envelope function method, the dynamic limit state function is expanded linearly at the random variables mean point, and the approximate envelope is formed by the piecewise linear hyperplane of the expanded point. Then the dynamic reliability problem is transformed into a static reliability problem, which improves the efficiency of dynamic failure probability calculation. Because the dynamic limit state function is expanded linearly at the mean point of random variables, this method is only suitable for problems with low nonlinearity or small variation coefficients.

2.4 Extreme value method

By analyzing and solving the minimum value of the dynamic limit state function in the time interval, the extreme value method transforms the structural dynamic reliability problem into a static reliability analysis problem. When the input variable $\boldsymbol{x}$ is a fixed value $\boldsymbol{x}^*$, the dynamic limit state function $g(\boldsymbol{x}^*,t)$ is the function of the parameter t, which extreme value and endpoints on the time interval $[t_0, t_s]$ are shown in Figure 6.

In Figure 6, A and B are the extreme value points, and C and D are the endpoints values. The minimum value of $g\,(\boldsymbol{x}^*, t)$ in the time interval $[t_0, t_s]$ is

$$\min_{t\in[t_0,t_s]} g(\boldsymbol{x},t) = \min\{g(\boldsymbol{x}^*,t_0), g(\boldsymbol{x}^*,t_A), g(\boldsymbol{x}^*,t_B), g(\boldsymbol{x}^*,t_s)\} \tag{22}$$

The dynamic failure probability $P_f(t_0, t_s)$ can be denoted as

$$P_f(t_0,t_s) = P\left\{\min_{t\in[t_0,t_s]} g(\boldsymbol{x}^*,t) \le 0\right\} \tag{23}$$

Then the problem of dynamic reliability analysis is transformed into static reliability analysis by analyzing the minimum value of $g(\boldsymbol{x}^*,t)$ in the time interval $[t_0, t_s]$. The estimated value of the dynamic failure probability $\widehat{P}_f(t_0,t_s)$ is obtained by judging the relationship between the extreme value of input variable $\boldsymbol{x}_i (i = 1, 2,\ldots,n)$ and 0, i.e.

$$\widehat{P}_f(t_0,t_s) = \frac{\sum_{i=1}^{n} I_F\left(\min_{t\in[t_0,t_s]} g(\boldsymbol{x}_i,t)\right)}{n} \tag{24}$$

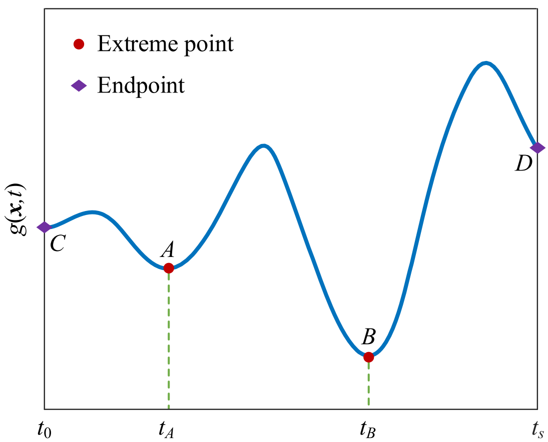


Figure 6.
Extreme value and endpoints of $g\,(\boldsymbol{x}, t)$

in which $I_F(\bullet)$ is the failure domain indication function corresponding to $\boldsymbol{x}_i$, if $\min_{t\in[t_0,t_s]} g(\boldsymbol{x}_i,t) \le 0$, then $I_F\left(\min_{t\in[t_0,t_s]} g(\boldsymbol{x}_i,t)\right) = 1$, otherwise, $I_F\left(\min_{t\in[t_0,t_s]} g(\boldsymbol{x}_i,t)\right) = 0$.

Many scholars have explored the application of extremum value and its extension method in structural dynamic reliability analysis and obtained many research results. The application of extreme value method is discussed in nonlinear structures with uncertain parameters (Radhika *et al.*, 2008; Chen and Li, 2007). Zhang *et al.* (2021a, b) studied an efficient method for nonlinear structures dynamic reliability analysis by the linear moments. To efficiently obtain the extreme value distribution, Hu and Du (2013a, b) employed a saddle point approximation to estimate the distributions of the extreme value. Yu *et al.* (2018) used the extreme value moment method and the improved maximum entropy method to approximate the PDFs of responses. To improve the efficiency of extremum method, Meng *et al.* (2021) developed a semi-analytical extreme value method for improving the computational efficiency of extreme value methods.

Many scholars combined the surrogate model with the extreme value method and proposed the extreme value surrogate model method to reduce the amount of calculation. The schematic diagram of the principle is shown in Figure 7.

From Figure 7, the extreme value of the output response (e.g. stress, deformation, strain and so forth) is determined by multiple dynamic deterministic analyses based on the material property, structure loads and dimensions, and the samples of input variables and output responses are obtained. Then the limit state function is approximated by the response surface method (RSM) (Lu *et al.*, 2020; Zhang *et al.*, 2017; Kaymaz and McMahon, 2005), support vector regression (SVM) (Feng *et al.*, 2019; Chen *et al.*, 2022; Hariri-Ardebili and Pourkamali-Anaraki, 2018; Keshtegar *et al.*, 2021), artificial neural network (ANN) (Li *et al.*, 2021; Cherid *et al.*, 2021; Peng *et al.*, 2019) and Kriging (Teng *et al.*, 2022; Zhang *et al.*, 2021a, b; Jiang *et al.*, 2019a, b) surrogate model, and the reliability is analyzed by combining the allowable value.

Zhang and Bai (2012) presented the extremum RSM used for the reliability analysis of a two-link flexible robot manipulator. As the core of aircraft operation safety, the aero-engine turbine is often regarded as a typical case of reliability analysis (Niu *et al.*, 2021; Zhu *et al.*, 2018). Similarly, the extremum RSM is applied to the reliability analysis of aero-engine casing

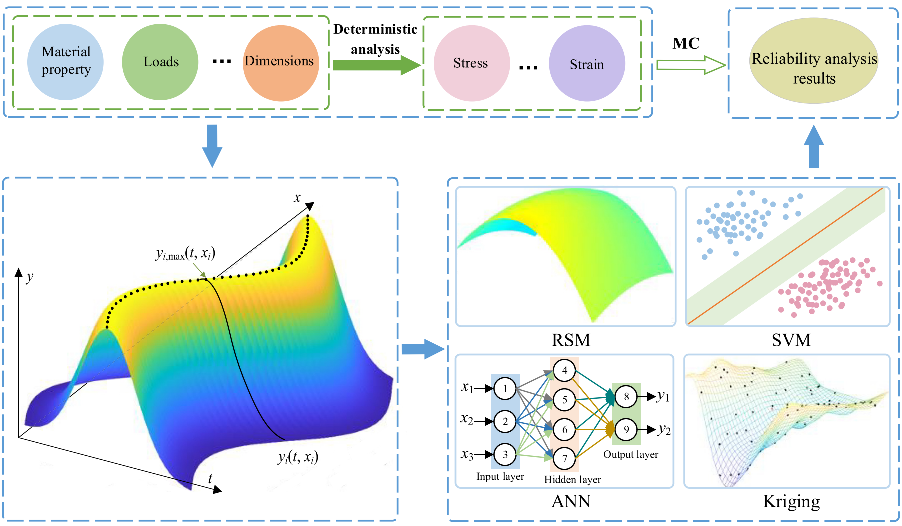


Figure 7.
Schematic diagram of the extreme value surrogate model method

and bladed disks (Bai and Bai, 2014). The application of extremum RSM and support vector machine are explored in the nonlinear dynamic reliability analysis of turbine disk-radial deformation (Fei *et al.*, 2015a, b). The neural network method is combined with extremum RSM to analyze the structural dynamic reliability (Song *et al.*, 2017, 2018). Zhao *et al.* (2020) extended the application of extremum neural networks in dynamic reliability analysis of flexible mechanisms. Lu *et al.* (2018a, b) developed the Kriging with extremum RSM for structural dynamic reliability. In the process of approaching the limit state function, the least-squares are usually used to model, which cannot make use of the known samples information effectively, and affects the precision of modeling and simulation. Then, the modified Kriging-based moving extremum framework based on extremum response surface is proposed for structural dynamic reliability analysis, which combined the moving modeling method with the surrogate model to improve the modeling and analysis accuracy of structural reliability (Lu *et al.*, 2021).

In addition, some scholars have studied the approximation method of extreme value function because getting the extreme value is time-consuming by multiple dynamic deterministic analyses. Wang and Wang (2013) proposed the nested extreme response surface (NERS) to handle the time dependency issue in the dynamic reliability analysis, the extreme value of the limit state function can be obtained by employing the Kriging model. The efficient global optimization (EGO) technique is integrated with the NERS approach to extract the extreme time responses of the limit state function for any given system design (Wang and Wang, 2012). Wang and Chen (2017) developed the adaptive extreme response surfaces that can handle both random variables and random processes as input uncertainty by introducing the gaussian process. A mixed EGO method is proposed to improve the efficiency of building such a surrogate model of the extreme response (Hu and Du, 2015). In the above double-loop procedure, it is necessary to use the extremum of inner loop to build the surrogate model in outer loop, and the optimization process of getting the extremum in inner loop will reduce the computational efficiency. Hu *et al.* presented a single-loop Kriging surrogate modeling method for dynamic reliability analysis to improve the efficiency of dynamic reliability analysis, which removes the optimization process of the inner loop and generates the random variables and time samples in the single-loop (Hu *et al.*, 2021; Hu and Mahadevan, 2016). Fang *et al.* (2018) and Jiang *et al.* (2017a, b) studied the dynamic reliability-based design optimization problems, and the computational cost is effectively reduced.

The extreme value method transforms the dynamic reliability analysis to the static by finding the dynamic limit state function minimum value in the time interval, which reduces the calculation burden in the dynamic reliability analysis. To further improve computational efficiency, the combination of extreme value and surrogate model method has become a research hotspot. However, the extreme value method based on the surrogate model is still in the development stage from the perspective of ensuring model credibility.

3. Dynamic reliability analysis of multiobjective structure

The dynamic reliability analysis of multiobjective structure is developed from the dynamic reliability analysis of a single objective structure. This section mainly reviews and summarizes the series-parallel and expansion method, multi-extremum surrogate model method and decomposed-coordinated surrogate model method.

3.1 Series-parallel and expansion methods

The relationship of multiple failure modes of structures includes series, parallel and compound. The failure modes of series connection and parallel connection are taken as examples, as shown in Figures 8 and 9. The reliability analysis general method for structural multifailure modes is to establish the limit state function of each failure mode separately and to carry out reliability analysis. Then the reliability of the structure is obtained by the relationship analysis of multiple failure modes.

From Figure 8, we can see that the failure probability with multiple parallel failure modes can be expressed as

$$P_{f_{parellel}} = P\left\{ \bigcap_{i=1}^{k} g_i(\boldsymbol{x}) < 0 \right\} = P\left\{ \max_{i=1}^{k} g_i(\boldsymbol{x}) < 0 \right\} \tag{25}$$

where k indicates the number of failure modes; $g_i(x)$ is the limit state function corresponding to the ith failure mode. It is worth noting that the structure is a failure when all parallel modes fail.

In Figure 9, the failure probability with multiple series failure modes can be described as

$$P_{f_{Series}} = P\left\{ \bigcup_{i=1}^{k} g_i(\boldsymbol{x}) < 0 \right\} = P\left\{ \min_{i=1}^{k} g_i(\boldsymbol{x}) < 0 \right\} \tag{26}$$

The structure is in the failure state when a mode fails. Conversely, the structure is in a safe state if all modes do not fail.

Many studies of the series-parallel and expansion methods are investigated by scholars. Savari *et al.* (2021) used the first passage probability method for composite repaired pipes multiple failure modes probability analysis. The joint upcrossing rates method is applied to calculate the multifailure probability of composite hydrokinetic turbine blades (Hu *et al.*, 2013). Hagen and Tvedt (1991) combined outcrossing rate method and combinations of bivariate responses to multiobjective dynamic reliability, especially for Gaussian and non-Gaussian vector processes. The joint first-passage probability method is presented based on the conditional distribution analysis (Song and Der Kiureghian, 2006). Jiang *et al.* (2017a) extended the application of outcrossing rate model in multiobjective failures. For application in general problems with random variables, stationary and nonstationary stochastic processes, Hu *et al.* (2018) developed an improved upcrossing rate method. The MC and envelope function methods are rarely used in multifailure modes, and the representative results are as follows. Qian *et al.* (2021a, b) explored a new multiobjective dynamic reliability method based on the multiple-response Gaussian process and subset simulation, which is used to solve a small failure probability problem. Wu *et al.* (2021) developed a new dynamic

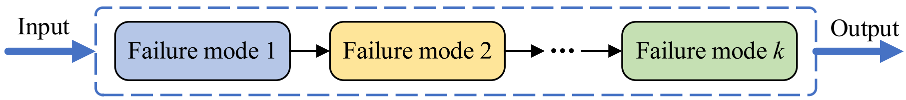


Figure 8.
Series of multiple failure modes

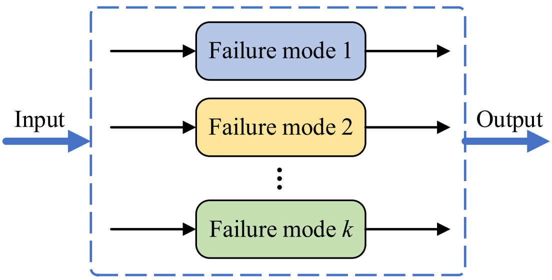


Figure 9.
Parallel of multiple failure modes

reliability method based on the envelope method and second-order reliability method, which is used for series failure modes.

Many scholars have carried out more research on the application of extreme value methods in multifailure modes. Li *et al.* (2007) elaborated the equivalent extreme-value event approach to evaluate the structural system reliability. This method can convert complex random processes into a series of random events. The PDF of the random events equivalent extreme value is calculated by the probability density evolution method, and integrates the PDF in the time domain to obtain reliability. To avoid the correlation information problem, Wang *et al.* (2021a, b) proposed the generalized equivalent extreme-value event for the reliability problem of multicomponent simultaneous failure. Qian *et al.* (2021a, b) developed a single-loop strategy based on the multiple response Gaussian process and the Kriging model, which improved the efficiency of multifailures reliability analysis. In addition, some scholars have explored a new idea of using the maximum entropy method to solve multifailure modes. For example, Yu *et al.* (2018) combined the extreme value moment method and improved maximum entropy method is used to multifailure modes dynamic reliability assessment.

3.2 Multi-extremum surrogate model method

Some scholars put forward the multi-extremum surrogate model method for multiobjective dynamic reliability analysis. The extremum response surface is established for each failure mode, which combines extreme values with the surrogate model. Then the structural dynamic reliability analysis of multiple failure modes is realized according to the relationship between failure modes. The RSM in surrogate model is taken as an example, the multi-extremum RSM is illustrated in Figure 10.

In Figure 10, for the *k* failure modes of the research object, *k* extremum response surface models are established by using the extreme value method and RSM respectively, and the limit state function of *k* failure modes is determined. Then the linkage sampling technology is used to analyze the multifailure modes reliability at the same time, and the reliability of each failure mode is obtained. The structural reliability is calculated with the relation of several failure modes, to improve the efficiency of reliability analysis.

Zhang *et al.* (2016) studied the advanced multiple RSM by integrating particle swarm optimization, ANN and multiple response surface theory, which is verified by the reliability analyses of turbine blisk multifailure modes (e.g. deformation, stress, strain and so forth). Subsequently, the multi-extremum surrogate model method is developed based on the quadratic polynomial function to evaluate the reliability of an aero-engine turbine blisk with

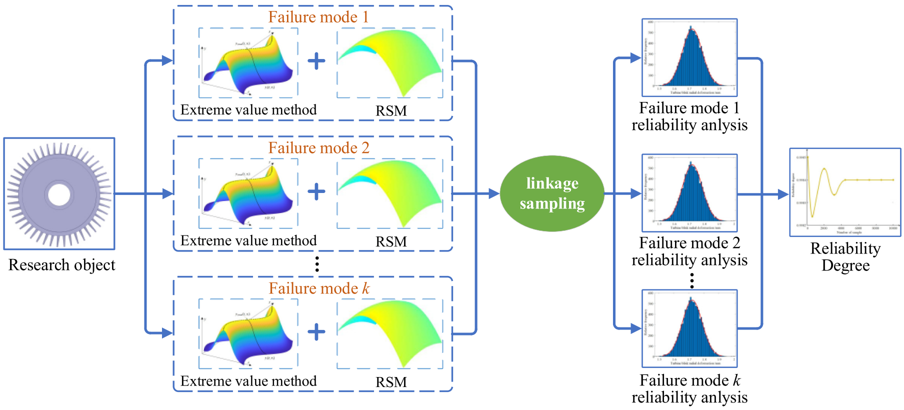


Figure 10.
Schematic diagram of multiple extremum RSM

two components (Zhang *et al.*, 2018). The fuzzy multi-extremum surrogate model method is presented to deal with the reliability analysis of turbine bladed disks under multifailure modes (Zhang *et al.*, 2019a, b). Lu *et al.* (2018a, b) extended the multi-extremum surrogate model method for the nonlinear transient structural reliability analysis with multifailure modes and two-way fluid-thermal-solid coupling. Qi *et al.* (2020) used the multiple response surface and coupled thermal-structural finite element model for the reusable rocket engine dynamic reliability calculation under multifailure modes.

3.3 Decomposed-coordinated surrogate model method

The decomposed-coordinated surrogate model method is to introduce the decomposed-coordinated strategy into the extreme value surrogate model for realizing multiobjective structural dynamic reliability analysis.

Multiple components of the structure are decomposed into single-component based on a decomposed-coordinated strategy, and the mathematical models of each component are established with the extreme value surrogate model. Subsequently, the relationship between the total output response and the input variables is coordinated based on the relationship among each component output response. Then the overall mathematical model of output response is established, and the limit state function is determined by the allowable value for the dynamic reliability analysis. The principle of decomposed-coordinated strategy is shown in Figure 11.

From Figure 11, four layers of the structure are taken as the research objects (structure layer, first substructure, second substructure layer and variable layer) to illustrate the decomposed-coordinated strategy. The output response relationship between the structure layer and the first substructure layer is expressed by $f(\cdot)$; $f^{(m)}(\cdot)$ denotes the output response relationship between the mth first substructure layer and nth second substructure layer; $f^{(mn)}(\cdot)$ represents the output response relationship between the nth second substructure layer of the mth first substructure layer and the variable layer.

The structure layer function is given by:

$$Y = f\left(y^{(1)}, y^{(2)}, \ldots, y^{(m)}\right) \tag{27}$$

where m is the number of output responses in the first substructure layer; and $y^{(i)}(i = 1, 2, \ldots, m)$ is the decomposed subfunctions of the ith first substructure layer. This is represented by:

$$y^{(i)} = f^{(i)}\left(y^{(i1)}, y^{(i2)}, \ldots, y^{(in)}\right) \tag{28}$$

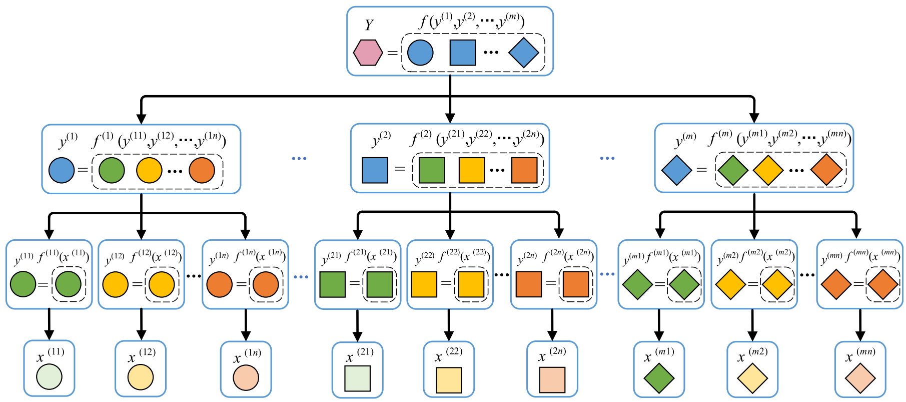


Figure 11.
Schematic diagram of decomposed-coordinated strategy

In which n is the number of output responses in the second substructure layer; and $y^{(i\ j)}$ ($j = 1,2,\ldots,m$) indicates the decomposed subfunctions of the jth second substructure layer in the ith first substructure layer. This is represented by:

$$y^{(i\ j)} = f^{(i\ j)}\left(x^{(i\ j)}\right) \tag{29}$$

where $x^{(i\ j)}$ is the jth decomposed subfunction in the second substructure layer of the ith first substructure layer.

Many scholars have carried out a lot of research on the decomposed-coordinated surrogate model method. Bai and Fei (2013) proposed a distributed collaborative RSM based on the quadratic response surface function to improve the accuracy and efficiency of dynamic assembly relationship reliability analysis. Fei and Bai (2014) discussed a decomposed-coordinated probabilistic design method-based support vector machine of regression for the probabilistic design of aero-engine high-pressure turbine blade-tip radial running clearance. Gao *et al.* (2020a, b) employed the decomposed-coordinated Kriging method to evaluate the probability of turbine blades. Zhang *et al.* (2019a, b) applied the decomposed-coordinated extremum neural network method to address dynamic uncertain loads in time-dependent reliability problems. Later, to further improve the decomposed-coordinated surrogate model method in dynamic reliability evaluation, Lu *et al.* (2019) adopted an improved decomposed-coordinated Kriging modeling strategy to evaluate the dynamic probabilistic analysis of multicomponent structures, by integrating decomposed-coordinated strategy, extremum RSM, genetic algorithm and Kriging surrogate model. Teng *et al.* (2021) developed the novel Kriging-based decomposed-coordinated approach for the reliability analysis of assembled structures, by introducing the Kriging model and important sampling-based Markov chain technique. Meng *et al.* (2019) studied collaborative design and optimization with the surrogate model for turbine blade reliability analysis. Fei *et al.* (2019) presented the Decomposed-coordinated surrogate model method (DCSMM) based on the mixture of quadratic polynomial and Kriging for the structural reliability analysis, which is demonstrated the effectiveness of turbine blisk with multifailure modes dynamic reliability analysis.

4. Case studies and further research of the structural dynamic reliability analysis

In this section, the numerical example and high-pressure turbine blisk with single-failure (deformation failure) and multifailure modes (deformation failure, stress failure and stain failure) are used to illustrate the performance of the above dynamic reliability analysis methods. Then the development trend of structural dynamic reliability analysis technology is combed based on the preliminary research and tracking of relevant literature.

4.1 Case studies

4.1.1 Numerical example. The numerical example is used to analyze the precision and efficiency of MC, outcrossing rate, envelope function and extreme value method. The dynamic limit state function can be expressed as

$$g(\boldsymbol{x}, t) = x_1^2 x_2 - 5x_1 t + (x_2 + 1)t^2 - 20 \tag{30}$$

where t denotes a time variable, and the time domain is [0, 5]; The input variables x_1 and x_2 obey normal distribution, i.e. $x_1 \sim N(3.5, 0.3^2)$ and $x_2 \sim N(3.5, 0.3^2)$. The dynamic limit state function is solved by the MC, outcrossing rate, envelope function and extreme value method, respectively. The solution conditions of each method are as follows.

(1) MC method: 32,768 samples of input variables are extracted by the Sobol sequence, and the time domain is discretized into 1,001 equidistant time points.

(2) Outcrossing rate method: assuming that the crossing times obey Poisson distribution, the crossing rate of dynamic limit state function is calculated by Advanced first-order second moment (AFOSM).

(3) Envelope function method: the solution of dynamic failure probability is transformed into the union of failure domains that a set of linear limit state functions subject to normal distribution.

(4) Extreme value method: the dynamic limit state function output response is converted into the extreme value of time domain [0, 5], and 32,768 samples of input variables are taken.

The dynamic reliability analysis results of four single-objective methods are listed in Table 1.

In the case of retaining four decimal places, the dynamic reliability analysis results from four single-objective methods are listed in Table 1.

As shown in Table 1, taking MC as a reference, the calculation efficiency of envelope function (69) is higher than that of outcrossing rate (459), extreme value (163,833) and MC method (32,800,768); The precision of extreme value (100%) is higher than that of envelope function (99.88%) and outcrossing rate (97.60%).

4.1.2 Turbine blisk dynamic reliability analysis. To further explore the performance of extreme value surrogate model, multi-extremum surrogate model, decomposed-coordinated surrogate model methods and the dynamic reliability analysis of an aero-engine high-pressure turbine blisk deformation, stress and stain are taken as the case studies.

To reduce the computational burden, the tenons, pin holes and cooling holes of blisk are ignored, and the simplified 1/46 blisk is selected as the research object with fluid-structural interaction. The three-dimensional (3-D) model, finite element model and finite volume (FV) model are shown in Figure 12.

The material of turbine blisk is the nickel-based superalloy GH4133, which elastic modulus, Poisson ratio and density are 1.61×10^{11} Pa, 0.3224, and 8.56×10^{3} kg/m^3, respectively (Lu *et al.*, 2018a, b). The input variables contain density, inlet pressure (2×10^6 Pa), outlet pressure (5.88×10^5 Pa), inlet velocity and angular speed (Fei *et al.*, 2022). The inlet velocity and angular speed are regarded as changes in the time domain [0, 215 s], which can be seen in Figure 13.

The maximum deformation, stress and stain is obtained in the climb phase, and the time domain is [165s, 200s] (Lattime and Steinetz, 2004). Combined with the idea of extreme value method, t = 180s is selected for turbine blisk dynamic reliability analysis. The input variables obey normal distribution, which distribution characteristics are shown in Table 2 (Fei *et al.*, 2019).

(1) Turbine blisk deformation failure

The deformation of turbine blisk is shown in Figure 14 when the time point is 180s.

The 200 input samples are extracted by the Latin hypercube method according to the distribution of input variables, and the corresponding output responses are determined by

Methods	Failure probability	Reliability	Precision	Number of model calls
MC	0.1845	0.8155	–	32,800,768
Outcrossing rate	0.1649	0.8351	97.60%	459
Envelope function	0.1855	0.8145	99.88%	69
Extreme value	0.1845	0.8155	100%	163,833

Table 1. Dynamic reliability analysis results of the numerical example

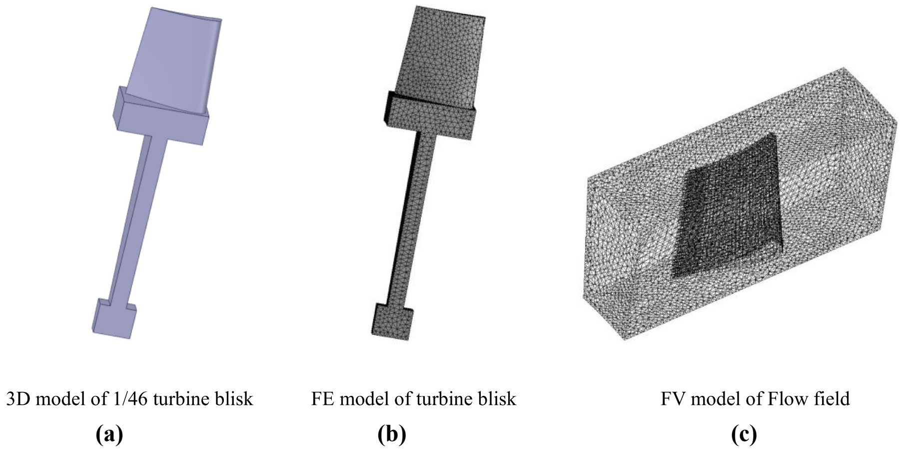


Figure 12.
3D/FE/FV models of turbine blisk

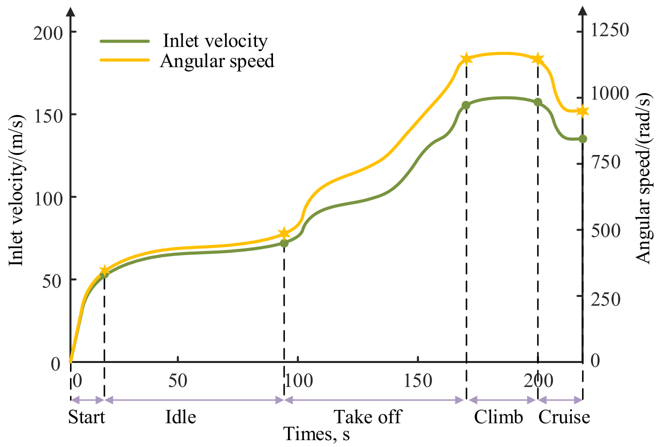


Figure 13.
Dynamic curves of angular speed and inlet velocity within [0 s, 215 s]

Table 2.
Distribution features of input variables

Parameter	Mean	Standard deviation
v, m/s	160	3.2
p_{in}, Pa	2 000 000	60 000
p_{out}, Pa	588 000	17 600
ρ, kg/m^3	8 560	171.2
ω, rad/s	1 168	23.36

dynamic deterministic analysis and extreme value method. Then 100 samples are used as training samples to establish RSM, SVM, ANN and Kriging models, and the remaining 100 samples are used as testing samples to verify the precision and efficiency of the established model. The modeling properties of these methods are evaluated by absolute error (E_{a_error}) and average absolute error (E_{aa_error}), which are displayed in Table 3 and Figure 15.

As illustrated in Table 3 and Figure 15, the descending order of average relative error is SVM (2.599×10^{-3}m), RSM (0.823×10^{-3}m), ANN (0.609×10^{-3}m) and Kriging (0.558×10^{-3}m); Compared with RSM and SVM, the Kriging and ANN have less relative error fluctuation and better robustness. It is important to note that the SVM hyperparameter greatly affects the prediction precision.

The simulation performance of four extreme value surrogate models is compared based on the dynamic reliability degree of turbine blisk. All computations are completed on the

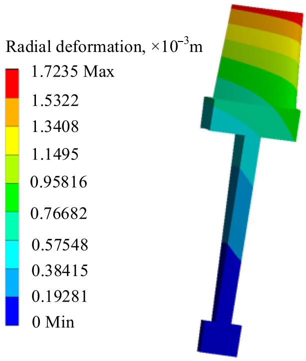


Figure 14.
Distribution of the deformation of turbine blisk

Table 3.
Average absolute errors and prediction precision of surrogate models

Methods	E_{aa_error}, $\times 10^{-3}$m
RSM	0.823
SVM	2.599
ANN	0.609
Kriging	0.558

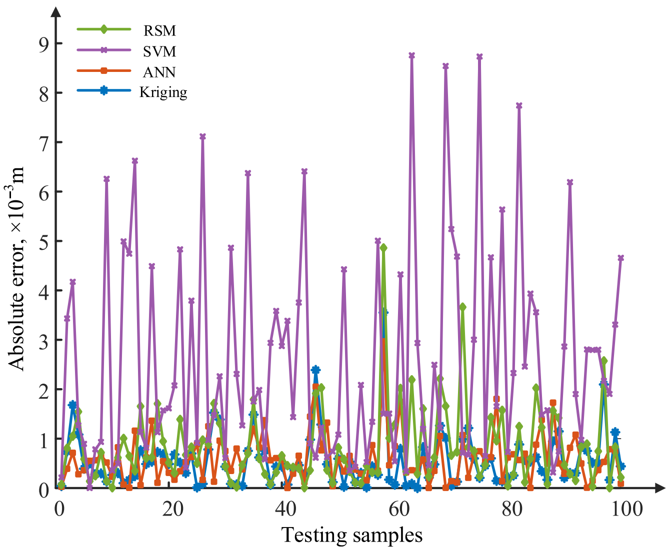


Figure 15.
Absolute errors of surrogate models

computer with Advanced Micro Devices (AMD) Ryzen 74800H of 2.9 GHz central processing unit (CPU) and 16 GB random access memory (RAM). The simulation efficiency and precision are shown in Tables 4 and 5.

As illustrated in Table 4, the extreme value surrogate model methods have less calculation time compared with direct simulation. This is because extreme value surrogate model methods reduce the process of deterministic analysis of finite element (FE) model. Among the extreme value surrogate model methods, ANN and SVM are better than the RSM and Kriging in computing time. As shown in Table 5, the direct simulation is used as a reference standard to explain the precision of extreme value surrogate model methods. The simulation precision of Kriging and ANN are higher than RSM and SVM. When the simulation times are 5×10^4, the simulations precision of Kriging, ANN, SVM and RSM are 99.98, 99.97, 99.93 and 99.96%, respectively.

(2) Turbine blisk multifailure modes

The deformation, strain and stress are involved in turbine blisk multifailure modes, which are shown in Figures 14 and 16, respectively at 180s.

The 100 training samples and 100 testing samples of multiple failure modes are obtained by the Latin hypercube sampling and dynamic deterministic analysis. To study the performance of the multi-extremum surrogate model method, the training samples are used to establish multi-extremum RSM (MERSM), multi-extremum SVM (MESVM), multi-extremum ANN (MEANN) and multi-extremum Kriging (MEKriging), respectively, and the testing samples are used to verify the precision and efficiency of the established model. The simulation efficiency and precision of four multi-extremum surrogate models are shown in Tables 6 and 7.

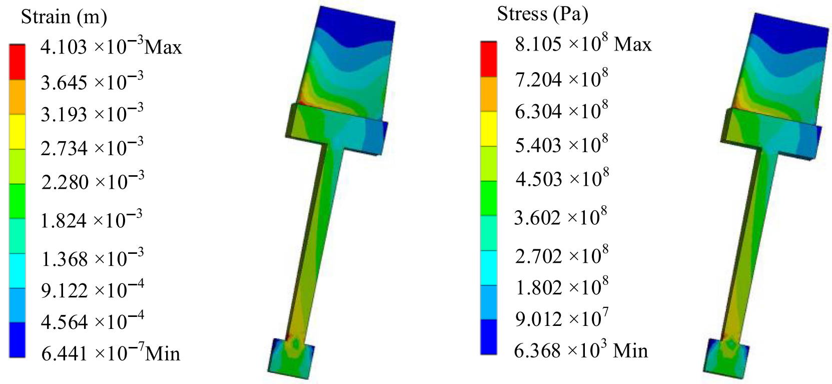
The results are tabulated in Table 6, the simulation time of multi-extremum surrogate model method is much less than that of direct simulation, which effectiveness is more obvious with the increase in simulation times. The simulation time of MEANN and MESVM is less than that of MERSM and MEKriging, which have relatively high simulation efficiency. As shown in Table 7, the MEKriging and MEANN are close to the analysis results of MC and better than the precision of MERSM and MESVM.

Table 4. Simulation efficiency with three surrogate models

Methods	Sampling number 10^2	10^3	5×10^3	10^4
Direct simulation	90,765s	915,680s	4,613,960s	–
RSM	1.45s	1.58s	1.65s	1.89s
SVM	0.42s	0.53s	0.58s	0.65s
ANN	0.26s	0.33s	0.41s	0.57s
Kriging	1.38s	1.42s	1.77s	1.89s

Table 5. Simulation precision with the three surrogate models

	Reliability degree					Precision/%			
Sampling number	MC	RSM	SVM	ANN	Kriging	RSM	SVM	ANN	Kriging
100	0.99	1	1	1	1	98.99	98.99	98.99	98.99
1,000	0.998	0.999	0.996	0.997	0.997	99.90	99.80	99.99	99.99
5,000	0.9982	0.9986	0.9989	0.9985	0.9980	99.96	99.93	99.97	99.98
10,000	–	0.9987	0.9980	0.9986	0.9984	–	–	–	



Turbine blisk strain

(a)

Turbine blisk stress

(b)

Figure 16. Distribution of the strain and stress of turbine blisk

Table 6. Simulation efficiency with four multi-extremum surrogate models

Methods	Sampling number 10^2	10^3	5×10^3	10^4
Direct simulation	272,456s	2,752,345s	138,678,943s	–
MERSM	1.56s	1.72s	1.85s	1.98s
MESVM	0.68s	0.74s	0.88s	0.97s
MEANN	0.51s	0.65s	0.71s	0.78s
MEKriging	1.62s	1.86s	1.95s	2.32s

Table 7. Simulation precision with the four multi-extremum surrogate models

Sampling number	Reliability degree MC	MERSM	MESVM	MEANN	MEKriging	Precision/% MERSM	MSVM	MEANN	MEKriging
100	0.97	0.96	0.98	0.97	0.97	98.97	98.97	100	100
1,000	0.996	0.993	0.992	0.994	0.997	99.70	99.60	99.98	99.99
10,000	–	0.9972	0.9969	0.9976	0.9977	–	–	–	

According to the mean and standard deviation of deformation, stress and strain of multifailure modes, the corresponding PDF can be formed. Then the comprehensive reliability of multifailure modes is calculated, it is worth noting that only the series relationship of multiple failure modes is considered. The turbine blisk deformation, stress and strain (i.e. decomposed surrogate model input samples) are obtained by deterministic analysis of the input samples (i.e. decomposed surrogate model output samples and coordinated surrogate model input samples). The comprehensive reliability (i.e. coordinated surrogate model output samples) can be taken by PDF and allowable value (Fei *et al.*, 2019). Therefore, the decomposed-coordinated surrogate model can be established. The decomposed-coordinated RSM (DC-RSM), decomposed-coordinated SVM (DC-SVM), decomposed-coordinated ANN (DC-ANN) and decomposed-coordinated Kriging (DC-Kriging) are established with 100 training samples. The 100 testing samples are used to illustrate the absolute error, average absolute error and simulation efficiency of the established model, which are displayed in Figure 17 and Table 8.

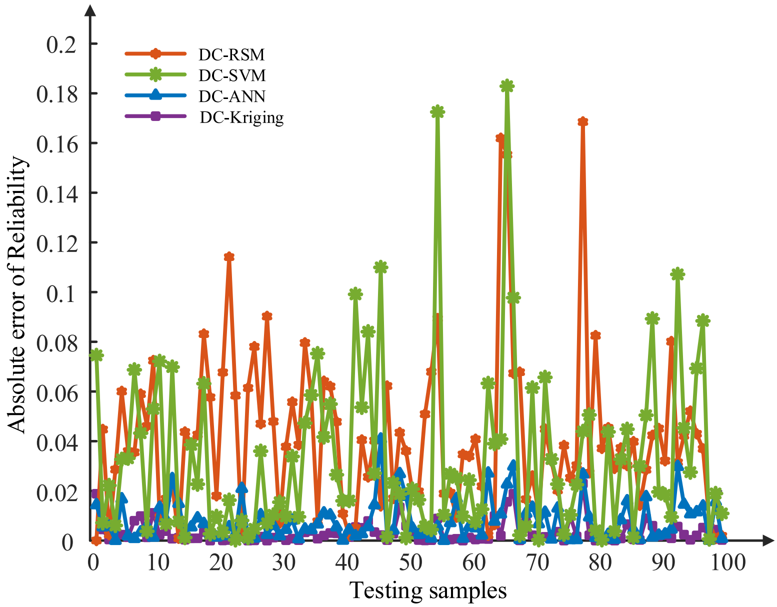


Figure 17.
Absolute errors of decomposed-coordinated surrogate models

Table 8.
Simulation efficiency with four decomposed-coordinated surrogate models

Methods	E_{aa_error}	Times
Direct simulation	–	272,456s
DC-RSM	0.0413	0.42s
DC-SVM	0.0335	0.56s
DC-ANN	0.0078	0.24s
DC-Kriging	0.0032	0.45s

As can be seen from Figure 17, the test values of DC-Kriging are close to the real values by direct simulation calculation. In addition, the absolute error curves of DC-Kriging are less volatile than those of DC-RSM, DC-SVM and DC-ANN for different testing samples. Therefore, DC-Kriging has better robustness and economy in modeling. As shown in Table 8, the average absolute error of DC-Kriging (0.0032) is less than that of DC-RSM (0.0413), DC-SVM (0.0335) and DC-ANN (0.0078). The simulation efficiency of decomposed-coordinated surrogate models is much higher than that of the direct simulation method, in which the simulation efficiency advantage of DC-ANN is more obvious than that of DC-SVM, DC-Kriging and DC-RSM.

4.2 Further research on structural dynamic reliability analysis

To ensure the safety of large equipment and measure its safe operation probability, the structural dynamic reliability analysis is of great significance. The development trend of structural dynamic reliability analysis technology is combed based on the preliminary research and tracking of relevant literature.

4.2.1 Single-objective structure. The MC method is often used as an effective means to verify the accuracy of other methods. Reducing the reduction of MC calculation and improving the calculation efficiency is still the research focus, especially for high-dimensional small probability events. The correlation of crossing events and the applicability of crossing rate method in various distribution types are follow-up research important directions for the crossing rate method. The application of envelope function in input variables large variability or limit state function high nonlinearity is the next research focus, for instance, increasing more expansion points. The efficiency of structural dynamic reliability analysis is improved by combining extreme value method and surrogate model. However, how to obtain effective samples beneficial to modeling and dealing with the highly nonlinear relationship between variables effectively is an important research hotspot, which realizes the high-precision reliability analysis of structure single-failure mode. For example, the moving modeling technology and intelligent algorithm are used to establish models with higher accuracy and efficiency.

4.2.2 Multiobjective structure. In recent years, with the development of structural dynamic reliability analysis, the method and theory of structures multiobjective dynamic reliability analysis involving multiple components and multiple failures are developing. But most scholars mainly focus on the multicomponent single-failure mode and single-component multifailure mode. The coupling between multiple parameters and the correlation between multiobjective are weakened by establishing multiple limit function models. Therefore, it is an important development direction that further explores the application of the multi-extremum surrogate model method and decomposed-coordinated strategy to realize multiobjective integrated reliability analysis of multicomponents with multifailures.

5. Conclusions

In this paper, the development status of structural dynamic reliability analysis is studied, and some main conclusions are summarized as follows:

(1) The reliability analysis methods of single-objective structure application scenarios are analyzed and summarized: the MC method is mainly used as a reference standard to evaluate the calculation accuracy of other methods; The outcrossing rate method can estimate outcrossing rate efficiently with the FOSM, which is not suitable for the highly nonlinear limit state function; The envelope function method is suitable for reliability analysis with low nonlinearity or the small coefficient of variation; The combination of extreme value method and surrogate model is suitable for almost all dynamic reliability analysis, which greatly improves the efficiency of dynamic reliability analysis and reduces the calculation cost of dynamic reliability analysis.

(2) The reliability analysis methods of multiobjective structure application scopes are summarized as follows: the series-parallel and expansion method is applied to the situation with the requirements for calculation efficiency is not high, and can also be used as a reference standard for other methods; The multi-extremum surrogate model method can carry out the collaborative modeling of single-component multifailure modes to realize the structural dynamic reliability analysis; The decomposed-coordinated surrogate model method can quickly realize the highly nonlinear structural dynamic reliability analysis multifailure modes.

(3) Combining moving modeling, intelligent algorithm and other technologies to obtain efficient modeling samples, and establish an approximation function model with higher efficiency and accuracy is an important research direction for single-objective structure dynamic reliability analysis. The multiobjective integrated dynamic reliability analysis of multicomponent with multifailure modes needs to be further studied by combining with the multi-extremum surrogate model method and decomposed-coordinated strategy.

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