This paper presents the extension of the Spherical-Polygon Search technique, to accomplish the star pattern recognition, to the recently proposed multiple field-of-view star trackers DIGISTAR II and III which observe star fields in orthogonal directions, thus providing substantial gain in both the attitude estimation accuracy and in the operating time. The main idea of the proposed algorithm, which extensively uses the \( k \)-vector technique (a new range searching approach which does not require of searching phases), is based on the fact that any star direction can always be expressed as a linear combination of two star directions (star pair basis) together with their vector cross product. Using this property, which does not depend on the used system of coordinates, the problem of accessing candidate stars is then transformed into one of accessing the stars falling within a cone about a given direction. The cone observed surface is approximated herein as a spherical polygon, and its aperture is set to be \( h \) times of the star image centroiding accuracy standard deviation \( \sigma \). Linear Error Theory is then applied for to establish an analytical approximation of \( h \) while, for fast applications, an empirical formula, is also presented. The resulting algorithm is very fast and has a high probability of quickly identifying the imaged stars. This we validate by numerical tests reported herein. This approach has the additional capability to identify and discard spurious images and we have validated that it is a suitable and reliable concept to perform star pattern recognition in the most general lost-in-space case, that is, when no attitude information is available.

INTRODUCTION

Spacecraft attitude, estimated using only a wide Field-Of-View (FOV) Charged-Coupled-Device (CCD) star tracker, is of the highest importance mainly because these sensors at present provide observed star directions with the highest precision and because they allow inherently enable autonomous attitude navigation. The latter is a very important aspect, especially when the spacecraft attitude is completely unknown (the lost-in-space case). Therefore, even though wide FOV star sensors have the disadvantage of requiring a star pattern recognition process, they can be used in a stand alone mode to estimate spacecraft (S/C) attitude, thus not requiring any additional less-precise attitude sensors. This implies the need to input to the attitude estimation algorithm a high precise data set and, consequently, to obtain high attitude estimation accuracy. High precision CCD star direction measurement is obtained, thanks to the well-known defocusing technique which, actually, allows to evaluate the star directions with uncertainties of few arc-seconds (\( 1\sigma \)), and typically correspond to about 1/20 of a pixel accuracy.
However, to increase the attitude estimation accuracy, not only the measured direction precision is important, but also the spatial distribution of the several required star direction measurements. Unfortunately, a standard fixed head star tracker (with one FOV) cannot provide directions with interstar angle greater than the instrument FOV. Moreover, using the same CCD size, a narrow FOV star tracker is, usually, more precise than a wide FOV star tracker (except with regard to the estimated rotation about the boresight). With the objective to more nearly measure star directions with the best spatial distribution (orthogonal), the multiple FOVs star trackers DIGISTAR II and III, have been recently suggested. These instruments, which provide actual and substantial improvement in the attitude data set precision, use one/two mirrors deflecting the sensor FOV to two/three orthogonal directions. While our concept is that only one focal plane will be used to image the several sub-fields of view, the methodology for star pattern identification and attitude determination applies equally well to the case of separate star trackers with each having independent focal planes and processing electronics. However obtained, we seek to estimate the S/C attitude with an optimal condition data set of two or more fields of near-orthogonal observed star directions, thus obtaining a substantial gain in the attitude estimation precision (up to 28 times with respect to an equivalent-FOV star tracker). An additional important feature of the DIGISTAR II and III sensors lies in redundancy/robustness, since we can still estimate the S/C attitude when one of the FOVs is not operating (because of proximity to the sun, the moon, the earth), that is, using only the stars observed in the FOVs able to image stars. This fact yields also a clear gain in terms of the instrument operating time and reliability.

Figure 1 shows a DIGISTAR III sketch, while Fig. 2 highlights a DIGISTAR II concept design, and Fig. 3 illustrates the DIGISTAR II and III FOVs by dividing a DIGISTAR I circular-FOV.

Multiple FOVs star sensors have been proposed (and are under development) by Texas A&M University together with the University of Rome “La Sapienza,” within the DIGISTAR project, mainly devoted to the development of small star tracker technologies aimed at the coming generation of small low cost spacecraft. Studies and researches for the data processing scheme to be used with these sensors have also been performed, including methods for estimating the mirror misalignment as well as for star identification. The present paper extends the analytical and computational methods for attitude estimation using multiple FOV star sensors, in particular, we report here a unique new method to identify stars observed by multiple FOVs star trackers.
In the last two decades, a variety of different approaches have been proposed to identify the stars observed by a wide-FOV star sensor (star pattern recognition process), the best of which do not require any (accurate or not) a priori attitude information, thus solving the lost-in-space case. Some of these approaches use angular separations between stars, others use angular separation along with knowledge of an initial attitude estimate, others use star triangles, fuzzy logic and neural networks, and also stochastic approaches. However, all of these methods have to solve (in one or another way) the common problem of a range searching, that is, they need to identify only the star data belonging to a given data range within long catalogs of stored or measured data vectors. Up to now, the range searching problem is usually solved using binary searching technique (or similar methods) which first requires to sort a data vector and then to answer \(2 \log_2 n\) questions for each range search process, where \(n\) is the number of data.

A solution to minimize the computer time consuming associated with this problem (thus, making a faster range searching procedure), resulted from the introduction of the \(k\)-vector technique, a method which does not require searching phases which, in turn, represent the heaviest computational loads of the existing star identification procedures. A range searching problem means to identify the indices of all the data \(x_i\) satisfying a given requirement \([x_{\text{min}} \leq x_i \leq x_{\text{max}}]\), within a \(n\)-long data vector \(x\). The \(k\)-vector technique was first introduced as a part of the Search Less Algorithm (SLA), and then, also utilized in the Spherical Polygon Search (SPS) technique. These two methods do not require any (accurate or not) initial guess of the spacecraft attitude (thus are applicable to the lost-in-space general case), they do not use the magnitude information (because instrument magnitude is typically inaccurate and may vary with time), and they are capable to identify and discard spikes (spurious images due to electronics noise, dust, reflection, etc). In particular, the SPS algorithm uses a reference observed star pair, based on which the star identification process is accomplished for all of the observed stars. Now, since the performance of the proposed method increases with the angular separation of the basis star pair, it is clear that the SPS approach should be better suitable for the multiple FOVs star trackers, such as the DIGISTAR II and III sensors.

This paper, by adapting the SPS approach to the multiple FOVs star trackers, provides a solution tool to accomplish the star pattern recognition process for these instruments. With the only exception of the SLA algorithm (which was adapted successfully for these sensors), no other competitive methods are available (or even proposed) to accomplish this task.

In the next section the SPS algorithm, which extensively uses the \(k\)-vector technique and for three
different tasks, will be briefly summarized.

**SPHERICAL POLYGON SEARCH ALGORITHM**

Let $s_i$, $s_j$, and $s_k$, be three different observed star directions (unit-vectors). Unless $s_i$ and $s_j$ are not parallel (i.e. double stars), it always be possible to set

$$s_k = a s_i + b s_j + c(s_i \times s_j)$$

so that the $a$, $b$, and $c$ coefficients can be evaluated as

$$\{a \ b \ c\}^T = [s_i \ s_j \ (s_i \times s_j)]^{-1} s_k = [A]^{-1} s_k; \quad [A] \equiv [s_i \ s_j \ (s_i \times s_j)]$$

these coefficients represent an invariant set of parameters with respect the used system of coordinates. This means that there exist, in the star catalog, three stars $(v_i, v_j, v_k)$ such that the condition (1) holds as

$$v_k = a' v_i + b' v_j + c' (v_i \times v_j)$$

where the $a'$, $b'$, and $c'$ coefficients will differ from the $a$, $b$, and $c$ coefficients because of the sensor’s limited precision, that is, because the observed stars directions do not perfectly overlap with the catalog star directions. However, the error expectation vector $\varepsilon^T = E\{ (a' - a), (b' - b), (c' - c)\}^T$ has zero mean value. Based on these facts, the SPS algorithm follows the following logical steps:

1. by using the $k$-vector technique, all of the catalog star pairs admissible with the observed pair basis $(s_i, s_j)$, are evaluated. The $k$-vector technique outputs the range limits $k_{\text{start}}$, and $k_{\text{end}}$ which allow the admissible star pairs identification through the integer vectors $I$, and $J$. In fact, the admissible star pairs are identified as $\{v_i, v_j, v_k\}$, where $k_{\text{start}} \leq m \leq k_{\text{end}}$, and the integer vectors $I$, and $J$, are built in advance, just once, and they depend on the instrument FOV and on the set magnitude threshold.

2. Since there is an ambiguity (in fact, it is unknown which of the following equations holds: $v_{I(m)} = v_i$ and $v_{J(m)} = v_j$, or $v_{I(m)} = v_j$ and $v_{J(m)} = v_i$), two different solutions, associated with the considered star pair, can be evaluated:

$$\begin{cases} w_k^{(1)} = a v_{I(m)} + b v_{J(m)} + c (v_{I(m)} \times v_{J(m)}) \\ w_k^{(2)} = a v_{I(m)} + b v_{J(m)} + c (v_{J(m)} \times v_{I(m)}) \end{cases}$$

where $a$, $b$, and $c$ are given by Eq. (2). This means that the searched true $v_k$ star direction, which is associated with the observed $s_k$ star, must be very close (no more than $h \sigma$ far) to one of the $2n_{ij} = 2(k_{\text{end}} - k_{\text{start}} + 1)$ inertial directions provided by Eq. (4) and obtained with $k_{\text{start}} \leq m \leq k_{\text{end}}$.

3. Associated with each computed $w_k$ there is an expected admissible cone with aperture $h \sigma$ where to direct any search for the true inertial star $v_k$. To identify the catalog stars falling within this cone, the $k$-vector technique can still be used. This is accomplished by first approximating the observed cone area as the area covered by a spherical polygon, which is identified as the intersection of six cones, as shown in Fig. 4. The ranges of this spherical polygon (that is the six cones) are given in Ref. 19, where a uniform CCD centroid error distribution were considered (with a maximum value of $\beta$). For a Gaussian centroid error distribution, it is possible to use the same formulation by replacing $\beta$ with $3 \sigma$ (or, better, with $4 \sigma$).
4. The SPS procedure has a successful end if only one star falls within the $2(k_{\text{end}} - k_{\text{start}} + 1)$ cones of aperture $h\sigma$. When this occurs (almost 100% of the times), not only the $s_k$ star is identified, but also the star pair basis used ($s_i$, $s_j$).

5. Finally, when the star triad ($s_i$, $s_j$, and $s_k$) is identified, this information is then used to identify the remaining $(n - 3)$ stars, which becomes an easy task to be accomplished.

SPS star pattern recognition technique, can be considered a method belonging to the triangle identification philosophy approaches, even though it may appear that we are looking for a single star.

It is clear, now, that the problem to quantify how far $w_k$ is from $v_k$, may become critical for the success of the method. Therefore, the quantification of the $h$ parameter is here accomplished in two ways: analytically, using the Linear Error Theory, and numerically, using a fitting empirical formula. Both of these approaches are explained in the following two sections.

**EVALUATION OF $h$ BY LINEAR ERROR THEORY**

The $h$ parameter is defined as the ratio between the maximum aperture of the cone angle $[\xi = \max\{\cos^{-1}(w_k^T v_k)\}]$, and the CCD centroiding accuracy standard deviation $\sigma$. Under the hypothesis that $\xi$ has a Gaussian error distribution, an approximated value for $\xi$ can be three or four times the standard deviation of the distribution of the cone angle $\cos^{-1}(w_k^T v_k)$ apertures, which is the definition adopted in this section.

For simplicity, let us assume that the measured vectors ($s_i$, $s_j$, and $s_k$) are parameterized as a function of right ascensions ($\alpha_i$, $\alpha_j$, $\alpha_k$) and declinations ($\delta_i$, $\delta_j$, $\delta_k$) as

$$s_i = \begin{cases} \cos \alpha_i \cos \delta_i \\ \sin \alpha_i \cos \delta_i \\ \sin \delta_i \end{cases} ; \quad i \rightarrow j, k$$
Let \( \alpha = \{\alpha_i, \delta_i, \alpha_j, \delta_j, \alpha_k, \delta_k\}^T \). Suppose that we know the assumed Gaussian uncertainty of the measurements, and this uncertainty is represented by the zero mean errors with the covariance matrix having the structure:

\[
E\{\Delta \alpha \Delta \alpha^T\} = \sigma^2 \text{diag} \left\{ \frac{1}{\cos^2 \delta_i}, 1, \frac{1}{\cos^2 \delta_j}, 1, \frac{1}{\cos^2 \delta_k}, 1 \right\}
\]

Question: Assuming small errors in \((\alpha_i, \delta_i, \alpha_j, \delta_j, \alpha_k, \delta_k)\), characterized by the above covariance matrix, how does this uncertainty (covariance) propagate into the covariance of the errors all the derived quantities, i.e., \((\Delta a, \Delta b, \Delta c)\), and functions of \((a, b, c)\)? The answer follows from Linear Error Theory\(^{20-22}\) as the similarity transformation:

\[
E\left\{ \begin{pmatrix} \Delta a \\ \Delta b \\ \Delta c \end{pmatrix} \right\} = \begin{pmatrix} \sigma^2_{\Delta a} & E\{\Delta a \Delta b\} & E\{\Delta a \Delta c\} \\ E\{\Delta a \Delta b\} & \sigma^2_{\Delta b} & E\{\Delta b \Delta c\} \\ E\{\Delta a \Delta c\} & E\{\Delta b \Delta c\} & \sigma^2_{\Delta c} \end{pmatrix} = BE\{\Delta \alpha \Delta \alpha^T\}B^T
\]

where \(B\) is the 3 \times 6 locally evaluated Jacobian matrix:

\[
B = \begin{bmatrix} \frac{\partial}{\partial \alpha_i} \begin{pmatrix} a \\ b \\ c \end{pmatrix} & \frac{\partial}{\partial \delta_i} \begin{pmatrix} a \\ b \\ c \end{pmatrix} & \frac{\partial}{\partial \alpha_j} \begin{pmatrix} a \\ b \\ c \end{pmatrix} & \frac{\partial}{\partial \delta_j} \begin{pmatrix} a \\ b \\ c \end{pmatrix} & \frac{\partial}{\partial \alpha_k} \begin{pmatrix} a \\ b \\ c \end{pmatrix} & \frac{\partial}{\partial \delta_k} \begin{pmatrix} a \\ b \\ c \end{pmatrix} \end{bmatrix}
\]

The above partial derivatives can all be taken analytically, for example:

\[
\frac{\partial}{\partial \alpha_{i,j}, \delta_{i,j}} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = -A^{-1} \begin{bmatrix} \frac{\partial A}{\partial \alpha_{i,j}, \delta_{i,j}} \end{bmatrix} A^{-1} s_k \quad \frac{\partial}{\partial \alpha_k, \delta_k} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = A^{-1} \begin{bmatrix} \frac{\partial s_k}{\partial \alpha_k, \delta_k} \end{bmatrix}
\]

where

\[
\frac{\partial A}{\partial \alpha_i, \delta_i} = \begin{bmatrix} \frac{\partial s_i}{\partial \alpha_i, \delta_i} & 0 & 0 \\ 0 & \frac{\partial s_i}{\partial \alpha_i, \delta_i} \times s_j \end{bmatrix} \quad \frac{\partial A}{\partial \alpha_j, \delta_j} = \begin{bmatrix} 0 & 0 & s_i \times \frac{\partial s_j}{\partial \alpha_j, \delta_j} \end{bmatrix}
\]

and

\[
\frac{\partial s_{i,j,k}}{\partial \alpha_{i,j,k}} = \begin{cases} -\sin \alpha_{i,j,k} \cos \delta_{i,j,k} \\ \cos \alpha_{i,j,k} \cos \delta_{i,j,k} \\ 0 \end{cases} \quad \frac{\partial s_{i,j,k}}{\partial \delta_{i,j,k}} = \begin{cases} -\cos \alpha_{i,j,k} \sin \delta_{i,j,k} \\ -\sin \alpha_{i,j,k} \sin \delta_{i,j,k} \\ \cos \delta_{i,j,k} \end{cases}
\]

Thus all of the partial derivatives and therefore the covariance matrix can be derived from the uncertainty in the direction angles \((\alpha_i, \delta_i, \alpha_j, \delta_j, \alpha_k, \delta_k)\) of the measured vectors. The remaining elements of \(B\) are not presented, to minimize the length of this discussion.

Equation (4) can be written as

\[
w_k^{(1)} = D_k^{(1)} \begin{pmatrix} a \\ b \\ c \end{pmatrix}; \quad 1 \rightarrow 2
\]

so that the covariance matrix associated with errors in the \(w_k\) vectors is

\[
E\{\Delta w_k^{(1)} \Delta w_k^{(1)T}\} = D_k^{(1)} B E\{\Delta \alpha \Delta \alpha^T\} B^T D_k^{(1)T}, \quad 1 \rightarrow 2
\]
In order to think of the errors in the $w_k$ vectors as an equivalent angle, consider the $w_k$ vectors to be parameterized as a function of right ascension and declination as

$$w_k^{(1)} = \begin{bmatrix} x_k^{(1)} \\ y_k^{(1)} \\ z_k^{(1)} \end{bmatrix} = \begin{bmatrix} \cos \alpha_k^{(1)} \cos \delta_k^{(1)} \\ \sin \alpha_k^{(1)} \cos \delta_k^{(1)} \\ \sin \delta_k^{(1)} \end{bmatrix}, \quad \alpha_k^{(1)} = \tan^{-1}\left( \frac{y_k^{(1)}}{x_k^{(1)}} \right), \quad \delta_k^{(1)} = \sin^{-1}(z_k^{(1)})$$

So that the errors in $w_k$ are related to those in $\alpha, \beta$ by

$$\Delta w_k^{(1)} = \begin{bmatrix} -\sin \alpha_k^{(1)} & -\cos \alpha_k^{(1)} \sin \delta_k^{(1)} & 0 \\ \cos \alpha_k^{(1)} & -\sin \alpha_k^{(1)} \sin \delta_k^{(1)} & \cos \delta_k^{(1)} \\ 0 & \cos \alpha_k^{(1)} \sin \delta_k^{(1)} & \cos \delta_k^{(1)} \end{bmatrix} \begin{bmatrix} \Delta \alpha_k^{(1)} \cos \delta_k^{(1)} \\ \Delta \alpha_k^{(1)} \sin \delta_k^{(1)} \\ \Delta \delta_k^{(1)} \end{bmatrix} \equiv \Delta \gamma_{w_k}^{(1)}$$

These equations can be inverted for the vector $\Delta \beta_{w_k}^{(1)}$ as

$$\Delta \beta_{w_k}^{(1)} = \begin{bmatrix} \Delta \alpha_k^{(1)} \cos \delta_k^{(1)} \\ \Delta \alpha_k^{(1)} \sin \delta_k^{(1)} \\ \Delta \delta_k^{(1)} \end{bmatrix} = \begin{bmatrix} -\sin \alpha_k^{(1)} & \cos \alpha_k^{(1)} & 0 \\ \cos \alpha_k^{(1)} & 0 & \cos \delta_k^{(1)} \\ 0 & \cos \alpha_k^{(1)} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1/\cos \delta_k^{(1)} \end{bmatrix} \Delta w_k^{(1)} \equiv E_k^{(1)} \Delta w_k^{(1)}$$

So that we see the angular uncertainty of the vectors is captured analytically by the $2 \times 2$ covariance matrix:

$$E\{\Delta \theta_{w_k}^{(1)} \Delta \beta_{w_k}^{(1)}^\tau\} = (E_k^{(1)} D_k^{(1)} B) E\{\Delta \alpha \Delta \alpha^\tau\} (E_k^{(1)} D_k^{(1)} B)^\tau, \quad 1 \rightarrow 2$$

The trace of this matrix provides a 1σ estimate of the cone angle of uncertainty centered on $v_k$

$$\sigma^2 = \text{trace} \left[(E_k^{(1)} D_k^{(1)} B) E\{\Delta \alpha \Delta \alpha^\tau\} (E_k^{(1)} D_k^{(1)} B)^\tau\right], \quad 1 \rightarrow 2$$

If we wish to view this as an amplification $h$ of the centroiding accuracy standard deviation $\sigma$, so

$$h = \frac{\sigma \Delta \theta_{w_k}^{(1)}}{\sigma} = \left( \frac{1}{\sigma} \right) \sqrt{\text{trace} \left[(E_k^{(1)} D_k^{(1)} B) E\{\Delta \alpha \Delta \alpha^\tau\} (E_k^{(1)} D_k^{(1)} B)^\tau\right]}, \quad 1 \rightarrow 2$$

The described analytical approach certainly presents the disadvantage of requiring a heavy computational load that could not be accepted when fast star pattern identification process is required, which is the practical case. Therefore, in the following section, to maximize the speed in evaluating $h$, some formulae fitting the maximum values for $h$ obtained in several numerical test, are provided.

**EVALUATION OF $h$ BY EMPIRICAL FORMULA**

Some easy expressions for $h$ (to be used in fast codes), are here given as a function of the star-pair basis interstar-angle $\theta_{ij} = \cos^{-1}(s_i^T s_j)$. These expressions are obtained by fitting the maximum values for $h$ recorded in numerical tests.

To this end, the DIGISTAR II/III multiple FOV star sensors, with CCD precision of $10''$ (3σ), and constituted with two/three orthogonal observation FOVs (cones), equal in size (semiaperture $\theta_{\text{fov}}$),
are considered. The sky surface observed by either of these instruments is equal to that observed by a standard one-circular FOV star tracker (DIGISTAR I), that can be described by a cone of 10 deg of aperture.

The selected star distribution \((s_i, s_j, \text{and } s_k)\) leads to the following three different cases:

1. SINGLE case: when all the stars belong to the same FOV, which may occur for all the DIGISTAR sensors (I, II, and III),
2. DUAL case: when the stars \(s_i, s_j,\) and \(s_k\) (star pair basis) belong to different FOVs while the third star \(s_k\) belongs to the FOV of \(s_i\) (or of \(s_j\)), which occurs only for DIGISTAR II and III, and
3. FULL case: when all the three stars belong to different FOVs, which occur only for the DIGISTAR III.

Numerical tests show that the values for \(h\) are highly depending on the star-pair basis inter-star-angle \(\vartheta_{ij}\), and weakly depending on the \(s_k\) spatial direction.

It is possible to show that in the SINGLE case the numerical maximum values obtained for \(h\), plotted as a function of the star-pair basis inter-star angle \(\vartheta_{ij}\), are approximately distributed, in a bi-logarithmic scale, with a negative linear slope. This yields to propose the following empirical formula

\[
h = d \vartheta_{ij}^p \tag{5}\]

where \(d\) and \(p\) are numerical constants which depend on the FOV aperture and, thus, they need to be computed just once. For the DIGISTAR sensors, the following Table 1 provides the values of these constants.

<table>
<thead>
<tr>
<th>DIGISTAR</th>
<th>FOV (deg)</th>
<th>(d)</th>
<th>(p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1 \times 10.00</td>
<td>1.748183</td>
<td>-1.018506</td>
</tr>
<tr>
<td>II</td>
<td>2 \times 7.067</td>
<td>1.479391</td>
<td>-0.900371</td>
</tr>
<tr>
<td>III</td>
<td>3 \times 5.769</td>
<td>1.606613</td>
<td>-0.84149</td>
</tr>
</tbody>
</table>

For the DUAL case, the obtained numerical test results suggest the use of a constant \(h\) value. In this case, it is possible to adopt \(h = h_{II}^{(2)} = 6.1385\) for DIGISTAR II and \(h = h_{III}^{(2)} = 5.5057\) for DIGISTAR III. Finally, when all of the \(i, j, k\) stars belong to different FOVs, numerical tests show that \(h = h_{III}^{(3)} = 6.0025\) is a very reliable maximum value.

Figure 5 plots with marks, and for the DIGISTAR III sensor, the maximum values obtained in extensive numerical tests when all the \(s_i, s_j,\) and \(s_k\) stars belong to the same FOV (SINGLE case) and the values provided using the Eq. (5) (continuous line). Figure 6 shows, for the same sensor and for the FULL case, the results obtained when a constant value for \(h\) is adopted (\(h = h_{III}^{(3)} = 6.0025\)).

**NUMERICAL TESTS**

The proposed extension of the SPS algorithm to the multiple FOVs star trackers is numerically tested by 1000 random tests for all the DIGISTAR sensors (I, II, and III). The successful percentage
obtained was 100% in all the 3000 random tests. Random test implies random choice of both the spacecraft attitude and the orientation of the sensor’s optical axes.

For each test, the SPS program, which is presently written in MATLAB\textsuperscript{23}, considers as the star pair basis, the first two stars belonging to different FOVs (if any). Using only this star pair basis, the SPS code tries to identify the $s_k$ star as one of the remaining $(n-2)$ observed stars. If this is successfully achieved, then the remaining $(n-3)$ stars are straightforwardly identified using the known star indices, otherwise (that is, when for each $w_k$ direction more than one $v_k$ star fall within the $2n_{ij} = 2(k_{end} - k_{start} + 1)$ cones) the star identification is considered as failed. This is obviously too restrictive and, also, subject to future improvements. In fact, it is likely that the choice of another star pair basis could lead to a successful identification, even in the unlucky case of only three observed stars. However, even under these restricted conditions, a complete star identification failure never occurred. However, a modest level of what if logic should suffice to eliminate negative consequences from these occasional events, especially since star cameras with high frame rates provide new measured star fields several times per second, and an occasional data dropout due to failed pattern identification is usually acceptable. Certainly occasional data dropout or delays are more acceptable than an incorrect star pattern identification. It should be noted, however, that following an initial lost-in-space attitude estimation, thereafter an a priori attitude estimate will be available, and this information can be used subsequently in a variety of ways to accelerate the process and improve confidence in the results. For example, to delete infeasible cataloged stars identified by the $k$-vector approach, and to provide a feasibility check whenever only a small number of stars are imaged. The presence of the double stars (interstar angle less then 0.005 deg) from the mission catalog have been deleted. In any case, the SPS program is capable to flag the presence of double stars. However, since their identification is not possible, the program will discard them (as if they were noisy spikes).

In the numerical tests performed, the visual magnitude threshold was set to 4.2 because with this value (associated with the area covered by the instrument FOVs), the case of a three stars observation has only 1% to occur (in random observations). In spite of not using the star magnitude information, the three stars case represents also the minimum star number to accomplish reliable star pattern recognition process, using the interstar angles only.

The tests have been performed for all of the DIGISTAR star sensors (I, II, and III). In particular, DIGISTAR I has one circular FOV with aperture of 10 deg, its CCD provides data with Gaussian er-
ror distribution with $3\sigma = 10''$, and the magnitude threshold is set to 4.2 (that is, magnitude threshold set to 4.0 with uncertainty of ±0.2).

Figure 7. $n_{ij}$ and $P_{\text{success}}$ values for DIGISTAR I (star pair basis in one FOV)

The great percentage of success of this star identification approach clearly appears from the following.

With a magnitude threshold 4.2, the observable stars are $N = \ldots$. Therefore, the star density is $\rho = N/(4\pi)$. With a star pair basis having an interstar angle $\vartheta_{ij}$, a number of $n_{ij}$ catalog star pairs are admissible with the pair basis (note that the values of $n_{ij}$ are evaluated using a real star catalog). These are all the catalog star pairs with interstar angle ranging from $\vartheta_{ij} - 2(4\sigma)$ to $\vartheta_{ij} + 2(4\sigma)$. Now for any observed star $s_k (k \neq i, j)$, a number of $2n_{ij}$ cones (each one covering an area of $A_{\text{cone}} = 2\pi[1 - \cos(\vartheta_{ij})]$, with $h$ given by Eq. (5)), has to be considered. Therefore, the number of extraneous stars falling within these $2n_{ij}$ cones is $n_{ext} = 2n_{ij}A_{\text{cone}}\rho$. Finally the success probability percentage, for each $s_k$ star, is $P_{\text{success}} = 100(1 - n_{ext})$.

Figure 7 shows the $n_{ij}$ and the $P_{\text{success}}$ values as a function of the star-pair basis interstar-angle $\vartheta_{ij}$ for the DIGISTAR I sensors. For this sensor the star pair basis always belong to the same unique FOV. The values for $n_{ij}$ increase linearly with $\vartheta_{ij}$ and the result for $P_{\text{success}}$ is really high. The overall star pattern identification success probability is higher than that depicted in Fig. 7, since the latter only represents the successfull probability for each $s_k$ considered, and the average number of these $s_k$ stars is 6.8.

Figures 8 and 9 provide the test results for DIGISTAR II multiple FOVs sensors, and for the two cases of star pair basis falling in one or two fields of view, respectively. The same test results
obtained for DIGISTAR III are given in Figs. 10 and 11.

The performance results of the proposed star identification algorithm when applied to DIGISTAR II and III are just a little bit lower than those obtained with a single FOV star tracker. This, which was not expected when the SPS approach was presented and proposed, is caused by the fact that the reduced value for the $h$ parameter, which suggested the application of the proposed method for multiple FOVs star sensors, cannot compensate the increasing of $n_{ij}$ with the star-pair basis interstar-angle $\vartheta_{ij}$. In fact, the number of extraneous stars is $n_{ext} = 2n_{ij}A_{cone}\rho$, that is, a number directly proportional to $n_{ij}$, and the values for $n_{ij}$, when the star pair basis falls in two FOVs is approximately four times the highest $n_{ij}$ values obtained when the star pair basis falls in one FOV.

To sum up: numerical results clearly show that the Spherical-Polygon Search Algorithm provides very high percentage of success (practically the 100%) when applied to both the single and the multiple FOV star trackers.

CONCLUSIONS

In this paper the adaptation of the Spherical-Polygon Search (SPS) algorithm to the multiple fields-of-view (FOVs) DIGISTAR II and III star sensors, is presented to accomplish the identification of the observed stars. Basic algorithms, error analysis, and a simulation studies are presented which support the validity and utility of this modified SPS approach. By observing stars in orthogonal directions, the CCD multiple FOVs star trackers provide substantial gains in the attitude estimation accuracy and in the operating time as well. The main idea of the SPS algorithm is that any star direction can always be linearly expressed in terms of two star directions (star pair basis)
Figure 9. $n_{ij}$ and $P_{\text{success}}$ values for DIGISTAR II (star pair basis in one FOV)

Together with their vector cross product. This property does not depend on the used system of coordinates. Using this property, the problem of accessing candidate stars is then mapped into a new one: to find all the stars falling within cones of aperture $h\sigma$, where $\sigma$ represents the centroiding accuracy standard deviation and $h$ a constant quantified in this paper. The SPS algorithm approximates the cones’ observed surfaces by spherical polygons. If only one star falls within these cones, then the identification of this star, together with the identification of the star pair basis, is accomplished. Based on this knowledge, the remaining stars are then easily and fast identified.

Analytical developments and numerical tests show how the apertures $h\sigma$ of the cones and how the number of the admissible catalog star pairs $n_{ij}$ depend on the star-pair basis interstar-angle. These dependences are such that single FOV star sensors present very small advantages in the SPS application with respect to a multiple FOVs star trackers. This difference is, however, practically negligible.

The Linear Error Theory is here applied to evaluate analytically the $h$ values as a function of the angle between the FOV boresights and, for fast applications, a more efficient empirical formula is established, which fits well numerical maximum values.

Numerical tests show the SPS as having a high probability to identify stars. The method is capable to identify and discard double stars as well as spikes (due to spurious images), it does not use the stars’ magnitude information, and does not requires any (accurate or not) a-priory attitude knowledge.

The SPS algorithm extensively uses the $k$-vector technique, a method that presents the advantage
of performing range searching without any searching phases, that is, in a much faster way than the methods using binary searching techniques. This makes the SPS very fast.

These characteristics allow us to propose the application of the SPS algorithm for both the single and the multiple FOVs star trackers, as a suitable and reliable solution tool for the star pattern recognition process in the lost-in-space general case. We believe this to be the most efficient approach available to address the lost-in-space problem.

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REFERENCES

Figure 11. $n_{ij}$ and $P_{\text{success}}$ values for DIGISTAR III (star pair basis in one FOV)


