A multi objective solid transportation problem in fuzzy, bi-fuzzy environment via genetic algorithm

Sutapa Pramanik
Deulia Balika Bidyamandir, Deulia, Purba Midna Pur-721154, West Bengal, India
E-mail: sutapapramanik12@gmail.com

D.K. Jana*
Department of Applied Sciences, Haldia Institute of Technology, Haldia, Purba Midna Pur-721657, West Bengal, India
Fax: 03224-252800
E-mail: dipakjana@gmail.com
*Corresponding author

K. Maity
Department of Mathematics, Mugberia Gangadhar Mahavidyalaya, Mugberia, Purba Medinipur-721425, West Bengal, India
Fax: 03224-252800
E-mail: kalipada_maity@yahoo.co.in

Abstract: In this paper, we concentrate on developing a bi-fuzzy multi objective transportation problem (MOSTP) according to bi-fuzzy expected value method (EVM). In a transportation model, the available discount is normally offered on items/criteria, etc., in the form all unit discount (AUD) or incremental quantity discount (IQD) or combination of these two. Here transportation model is considered with fixed charges and vehicle costs where AUD, IQD or combination of AUD and IQD on the price depending upon the amount is offered and varies on the choice of origin, destination and conveyance. To solve the problem, multi objective genetic algorithm (MOGA) based on Roulette wheel selection, arithmetic crossover and uniform mutation has been suitably developed and applied. To illustrate the models, numerical examples have been presented. Here, two types of problems are introduced and the corresponding results are obtained. To provide better customer service, the entropy function is considered.

Keywords: bi-fuzzy; genetic algorithm; solid transportation problem; STP; entropy.

Biographical notes: Sutapa Pramanik is an Assistant Teacher in Deulia Balika Bidyamandir, Deulia, Purba Medinipur, India. She received her MSc in Applied Mathematics from Vidyasager University. She also qualified NET, GATE, SET examinations. Her research interests are transportation problem in fuzzy, fuzzy rough and uncertain environments. She has published many research papers in reputed international journals.

D.K. Jana is an Assistant Professor in Haldia Institute of Technology, Haldia, Purba Medinipur, and West Bengal India. He received his MSc in Applied Mathematics from Vidyasager University. His research interests are in inventory and optimal control of production systems in fuzzy, fuzzy rough and uncertain environments. He published many research papers in reputed international journals such as JOS, IJOR, OPSEARCH, etc.

K. Maity received his PhD in Applied Mathematics from Vidyasagar University in 2006. He is a Lecturer in the Department of Mathematics, Mugberia Gangadhar Mahavidyalaya, Purba Medinipur. His research and development efforts focus on operational research, optimal control theory, fuzzy mathematics and fuzzy logic. He published many research papers in reputed international journals such as EJOR, MCM, FODM, AJMMS, Information Sciences, etc.

1 Introduction

Generally speaking, uncertainty is common to all real-life problems for example randomness, fuzziness and roughness. Since Zadeh (1965) introduced the fuzzy set in 1965, fuzzy set theory has been well developed and applied to a wide variety of real problems. After that, Liu and Liu (2003) have developed a class of fuzzy random optimisation: expected value models. Maity et al. (2008) have developed a production recycling model with imprecise holding cost. Production of defective units is a natural phenomenon in a production process. Bi-fuzzy sets were originally presented by Zadeh (1971) and were further elaborately by Gottwald (1979), Mendel and John (2002), Pandian and Anuradha (2010), others. But, till now, none has considered MOSTP problem with fuzzy fixed charge, vehicle cost and price discounted varying charge and bi-fuzzy total sources, destinations and conveyances.

The solid transportation problem (STP) was first introduced by Haley (1962) in 1962, in which three kinds of constraints are taken into consideration, that is, source constraint, destination constraint and conveyance capacity constraint. The STP degenerates into the classical transportation problem as the number of conveyance is only one. In recent years, there have been numerous papers in this area. Some papers only minimise the total transportation cost. For example, Ojha et al. (2010) and Tao and Xu (2012) considered a STP for an item with fixed charge, vehicle cost and price discounted varying charge. To solve the problem, the genetic algorithm (GA), which is based on Roulette wheel selection, arithmetic crossover and uniform mutation, was suitably developed and
A multi objective solid transportation problem

applied. However, in practical programming problems, the decision maker (DM) usually needs to optimise several objectives. Unfortunately, the objectives are often conflicting and incommensurable. Thus, the DM cannot obtain the optimal values of all the objectives simultaneously. The growing literature of STP focuses on multiple objective problems, that is, multiple objective solid transportation problems (MOSTPs). For example: Bit et al. (1993) used a fuzzy programming approach to solve a MOSTP; Ida et al. (1996) presented a neural network method to solve a MOSTP. Tao and Xu (2012) have developed a class of rough multiple objective programming and its application to STP.

In today’s highly competitive market the pressure on organisations to find the better ways to create and deliver values to customers is increasing more and more. How and when to send the products to the customers in the quantities they want in a cost-effective manner has become more challenging in terms of price discount [all unit discount (AUD) and/or incremental quantity discount (IQD)] on the unit transportation cost. There is a similarity to economic ordered quantity model for such a discount (cf. Maiti and Maiti, 2008).

To solve the STP, the GA, which is based on Roulette wheel selection, arithmetic crossover and uniform mutation, was suitably developed and applied. However, in practical programming problems, the DM usually needs to optimise several objectives. Unfortunately, the objectives are often conflicting and incommensurable. Thus, the DM cannot obtain the optimal values of all the objectives simultaneously. A growing body of literature of STP focuses on multiple objective problems, that is, MOSTPs. For the solution of decision-making problems, there are some inherent difficulties in the traditional direct and gradient-based optimisation techniques used for this purpose. Normally, these methods

1. are initial solution dependent
2. get stuck to a sub-optimal solution
3. are not efficient in handing problems having discrete variables
4. cannot be efficiently used on parallel machines
5. are not universal, rather problem dependent.

To overcome these difficulties, recently GAs are used for optimisation of decision making problems. GAs (Goldberg, 1989; Vignaux and Michalewicz, 1991) are adaptive computational procedures modelled on the mechanics of natural genetic systems. They exploit the historical information to speculate on new offspring with expected improved performance. These are executed iteratively on a set of coded solutions (called population) with three operators – selection/reproduction, crossover and mutation. One set of these three operators is known as a generation in the parlance of GA Since a GA works simultaneously on a set of solutions, it has very little chance to get stuck at local optimum. Here, the resolution of the possible search space is increased by operating on potential solutions and not on the solutions themselves. Further, this search space need not be continuous. Recently, GAs have been applied in different areas like neural network, travelling salesman, scheduling, numerical optimisation (Vignaux and Michalewicz, 1991), inventory (Mandal and Maiti, 2000; Maiti and Maiti, 2008; Pramanik and Roy, 2008), etc. Recently, Ojha et al. (2010) have developed a STP for an
item with fixed charge, vehicle cost and price discounted varying charge using GA. But none can consider MOSTP in imprecise environment.

In this paper, a multi objective STP with discounted costs, fixed charges and vehicle costs are formulated as a linear programming problem. The discounted costs are in the form of AUD, IQD and combination of AUD and IQD. Here AUD, IQD or combination of these two discounts on purchasing price with two price breaks are considered. Here, the transportation costs, resources, demands at various centres and conveyance capacities, times for different modes of transport between origins and destinations are bi-fuzzy. The STP has been formulated in two different ways with and without entropy function following Shannon’s measure of entropy. Then the multi objective STPs are solved using MOGA and the results of these two types of problems are tabulated.

2 Necessary knowledge about fuzzy, bi-fuzzy sets and bi-fuzzy EVM

In this section, we recall some basic knowledge of fuzzy set theory, bi-fuzzy and bi-fuzzy EVM.

2.1 Fuzzy set

A fuzzy set is a class of objects in which there is no sharp boundary between those objects that belong to the class and those that do not. Let \( X \) be a collection of objects and \( x \) be an element of \( X \), then a fuzzy set \( \tilde{A} \) in \( X \) is a set of ordered pairs \( \tilde{A} = \{ (x, \mu_{\tilde{A}}(x)) / x \in X \} \) where \( \mu_{\tilde{A}}(x) \) is called the membership function or grade of membership of \( x \) in \( \tilde{A} \) which maps \( X \) to the membership space \( M \) which is considered as the closed interval \([0, 1]\).

Fuzzy number: A fuzzy number \( \tilde{A} \) is a convex normalised fuzzy set on real line \( \mathbb{R} \) such that

1. it exists exactly one \( x_0 \in \mathbb{R} \) with \( \mu_{\tilde{A}}(x_0) = 1 \) (\( x_0 \) is called the mean value of \( \tilde{M} \))
2. \( \mu_{\tilde{A}}(x) \) is piecewise continuous.

Example: In particular if \( \tilde{A} = (a_1, a_2, a_3) \) be a triangular fuzzy number (TFN) (cf. Figure 1) then \( \mu_{\tilde{A}}(x) \) is defined as follows

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x - a_1}{a_2 - a_1} & \text{for } a_1 \leq x < a_2 \\
\frac{a_3 - x}{a_3 - a_2} & \text{for } a_2 < x \leq a_3 \\
0 & \text{otherwise}
\end{cases}
\]

where \( a_1, a_2, \) and \( a_3 \) are real numbers.
Lemma 1: Let $\tilde{a} = (a_1, a_2, a_3)$ be a triangular fuzzy number and $\rho$ is a crisp number. The expected value of $\tilde{a}$ is

$$E[\tilde{a}] = \frac{1}{2}[(1-\rho)a_1 + a_2 + \rho a_3], \quad 0 < \rho < 1.$$  

(1)

where each ordinary fuzzy set $V$ is defined by

$$V = \{(x, \mu_V(x)) \mid \forall x \in U : \mu_V > 0\}.$$  

(2)

For convenience, the membership grades $\mu_V(\tilde{V})$ of the fuzzy sets $\tilde{V} \in \tilde{\Gamma}(U)$ are called ‘outer-layer’ membership grades, whereas the membership grades $\mu_V(\tilde{x})$ of the elements $x \in U$ are called inner-layer membership grades. Since level 2 fuzzy sets are still fuzzy sets, their mathematical behaviour is defined by the fuzzy set operators.

Type 2 fuzzy sets were introduced by Zadeh (1975) as another extension of the concept of an ordinary fuzzy set, and it was elaborated by Mendel and John (2002). Such
sets are fuzzy sets whose membership grades them as ordinary fuzzy sets. They are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set. Normally speaking, a Fu-Fu variable (bi-fuzzy) is a fuzzy variable under fuzzy environment.

Example: \( \tilde{\xi} = (s_L, \tilde{s}, s_R) \) with \( \rho = (\rho_L, \rho_M, \rho_R) \) is called Fu-Fu variable (cf. Figure 2), if the outer-layer and inner-layer membership functions are as follows

\[
\mu_{\tilde{s}}(x) = \begin{cases} 
\frac{x - s_L}{\tilde{\rho} - s_L} & \text{if } s_L \leq x \leq \tilde{\rho} \\
0 & \text{otherwise} \\
\frac{s_R - x}{s_R - \tilde{\rho}} & \text{if } \tilde{\rho} \leq x \leq s_R 
\end{cases}
\]

and

\[
\mu_\rho(x) = \begin{cases} 
\frac{x' - \rho_L}{\rho_M - \rho_L} & \text{if } \rho_L \leq x' \leq \rho_M \\
0 & \text{otherwise} \\
\frac{\rho_R - x'}{\rho_R - \rho_M} & \text{if } \rho_M \leq x' \leq \rho_R 
\end{cases}
\]

where \( \tilde{\rho} \) is the centre of \( \tilde{\xi} \), which is a triangular fuzzy variable, \( s_L \in R \) and \( s_R \in R \) are the smallest possible value and the largest possible value of \( \tilde{\xi} \), \( \rho_L \in R \), \( \rho_M \in R \) and \( \rho_R \in R \) are the smallest possible value, the most promising value and the largest possible value of \( \tilde{\rho} \) respectively.

Figure 2: Regular bi-fuzzy variable (see online version for colours)

Lemma 2: The expected value for the bi-fuzzy variable \( \tilde{c} = (l, \tilde{c}, r) \) with \( \tilde{c} = (l_2, c, r) \) we obtain that
A multi objective solid transportation problem

\[ E[\tilde{z}] = c + \frac{(n + r_2 - (h + l_2)}{4} \]  

(4)

\( \text{Proof:} \) The proof of the Lemma 2 is in Xu and Zhou (2009) in Section 4, p.276.

Lemma 3: Assume that \( \xi \) and \( \eta \) are fuzzy/bi-fuzzy variables with finite expected values. Then for any real numbers \( a \) and \( b \), we have

\[ E[a\xi + b\eta] = aE[\xi] + bE[\eta] \]  

(5)

\( \text{Proof:} \) The proof of the Lemma 3 is in Xu and Zhou (2009).

2.3 General model for bi-fuzzy EVM

First we give the general model of Fu-Fu multi-objective decision making model as follows,

\[
\begin{align*}
\begin{cases}
\text{Min } & f_1(x, \tilde{\xi}), f_2(x, \tilde{\xi}), \ldots, f_m(x, \tilde{\xi}) \\
& g_r(x, \tilde{\xi}) \leq 0, \quad r = 1, 2, \ldots, p \\
& x \in X
\end{cases}
\end{align*}
\]  

(6)

If \( \tilde{\xi} \) is a Fu-Fu vector, \( x = (x_1, x_2, \ldots, x_n) \) is decision vector, then the objective function \( f_i(x, \tilde{\xi}) \) and constraint functions \( g_r(x, \tilde{\xi}) \) are also Fu-Fu variables, \( i = 1, 2, \ldots, m, \)

\( r = 1, 2, \ldots, p \). In order to rank Fu-Fu objective \( f_i(x, \tilde{\xi}) \), we may employ the expected value operator to deal with the objective functions and constraints, and we can get the following model (6). For the expected value of the objective \( E[f_i(x, \tilde{\xi})] \), \( i = 1, 2, \ldots, m \), it means that the larger the expected returns \( E[f_i(x, \tilde{\xi})] \), the better the decision \( x \). The first type of Fu-Fu decision-making model is expected value multi-objective decision-making model in which the underlying philosophy is based on selecting the decision with maximum expected objective values.

\[
\begin{align*}
\begin{cases}
\text{Min } & E[f_1(x, \tilde{\xi})], E[f_2(x, \tilde{\xi})], \ldots, E[f_m(x, \tilde{\xi})] \\
& E[g_r(x, \tilde{\xi})] \leq 0, \quad r = 1, 2, \ldots, p \\
& x \in X
\end{cases}
\end{align*}
\]  

(7)

Theorem 1: If \( \alpha_{i1}^\ell, \alpha_{i2}^r, \beta_{i1}^l, \beta_{i2}^r \) are left and right spreads of \( \tilde{b}_{ij}(\theta) \) and \( \tilde{b}_{ij}(\theta) \), \( \alpha_{i2}^\ell, \beta_{i2}^r \) are left and right spreads of \( \tilde{b}_{ij}(\theta) \) and \( \tilde{b}_{ij}(\theta) \), \( r = 1, 2, \ldots, p, j = 1, 2, \ldots, n \), the basis function \( L, R : [0, 1] \rightarrow [0, 1] \) are monotone decreasing continuous function, and it satisfies \( L(1) = R(1) = 0 \), \( L(0) = R(0) = 1 \) and the LR fuzzy variable is specified as the triangular fuzzy variable and \( R^{-1}(\theta_i) = 1 - \theta_i, R^{-1}(\eta_i) = 1 - \eta_i \). For any \( j = 1, 2, \ldots, n \), and if \( \tilde{c}_{ij}(\theta) \) and \( \tilde{b}_{ij}(\theta) \) are independent fuzzy variables. Then
\[
\text{Pos}\left\{ \theta \mid \text{Pos}\left\{ \hat{\epsilon}_{ij}^{\theta}(\theta) x \leq \hat{b}_{j}(\theta) \right\} \geq \eta_{r} \right\} \geq \eta_{r}
\]

is equivalent to

\[
R^{-1}(\theta_{r}) \beta_{i}^{b} + L^{-1}(\theta_{r}) \alpha_{i}^{r} x - \epsilon_{i}^{r} x + b_{j} + \eta_{r} \left( \alpha_{i}^{r} x + \beta_{i}^{b} \right)
\]

**Proof:** The proof of the Theorem 1 is in Xu and Zhou (2009) in p.254.

**Theorem 2:** Assume that the Fu-Fu variable \( \hat{\epsilon}_{ij} \) and \( \hat{b}_{j} \) is as same as the assumption in Theorem 1, \( i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \). For confidence level \( \delta_{i}, \gamma_{i} \in [0, 1], i = 1, 2, \ldots, m \). Then

\[
\text{Nec}\left\{ \delta \mid \text{Nec}\left\{ \hat{\epsilon}_{ij}(\delta) x \leq \hat{b}_{j}(\delta) \right\} \geq \delta_{i} \right\} \geq \gamma_{i}
\]

is equivalent to

\[
b_{j} - \epsilon_{j}^{r} x - L^{-1}(1 - \gamma_{i}) \left( \alpha_{i}^{r} x + \beta_{i}^{b} \right) - L^{-1}(1 - \delta_{i}) \alpha_{i}^{r x} - R^{-1}(\delta_{i}) \beta_{i}^{b x} x \geq 0
\]

**Proof:** The proof of the Theorem 1 is in Xu and Zhou (2009) in p.257.

### 3 Assumptions and notation

#### 3.1 Notation

In this STP, the following notation

1. \( m \) = number of sources of the transportation problem
2. \( n \) = number of demands of the transportation problem
3. \( K \) = number of conveyances, i.e., different modes of the transportation problem
4. \( O_{i} \) = origins of the transportation problem
5. \( D_{j} \) = destination of the transportation problem
6. \( E_{k} \) = conveyances of the transportation problem
7. \( \tilde{a}_{i} \) = bi-fuzzy amount of a homogeneous product available at \( i^{th} \) origin
8. \( \tilde{b}_{j} \) = bi-fuzzy demand at \( j^{th} \) destination
9. \( \tilde{e}_{k} \) = bi-fuzzy amount of product which can be carried by \( k^{th} \) conveyance
10. \( f_{jk} \) = the fixed charge of the transportation problem
11. \( \tilde{C}_{ij}^{a} \) = fuzzy unit transportation cost from \( i^{th} \) origin to \( j^{th} \) destination by \( k^{th} \) conveyance
12. \( \tilde{C}_{ij}^{a} \) = fuzzy transportation time which amount transported from \( i^{th} \) origin to \( j^{th} \) destination by \( k^{th} \) conveyance
A multi objective solid transportation problem

3.2 Assumptions

In this STP, the following assumptions are made.

1. The total vehicle cost
   
   \[
   G(x_{ijk}) = \begin{cases} 
   s'V & \text{if } s'V = x_{ijk} \\
   (s + 1)V & \text{Otherwise} 
   \end{cases}
   \]  
   
   (8)

2. \(s = [x_{ijk} / V_c]\), \(V_c = \) vehicle capacity and \(V = \) vehicle cost.

3. The unit transformation cost under AUD scheme:

   \[
   \tilde{C}_{ijk}^s = \begin{cases} 
   \tilde{p}_{ijk} & \text{if } 0 < x_{ijk} < R_1 \\
   \tilde{p}_{2ijk} & \text{if } R_1 \leq x_{ijk} < R_2 \\
   \ldots & \ldots \\
   \tilde{p}_{tijk} & \text{if } R_{(t-1)} \leq x_{ijk} < R_t \\
   \tilde{p}_{(t+1)ijk} & \text{if } R_t \leq x_{ijk} 
   \end{cases}
   \]  
   
   (9)

   where \(\tilde{p}_{ijk}, \tilde{p}_{2ijk}, \ldots, \tilde{p}_{tijk}, \tilde{p}_{(t+1)ijk}\) are all fuzzy unit cost.

4. IQD: The unit-transportation cost under IQD system is \(\tilde{p}_{ijk}\) for \(0 < x_{ijk} < R_1\), \(\tilde{p}_{2ijk}\) for an additional quantity over \(R_1\) but less than \(R_2\) and so on, lastly \(\tilde{p}_{(t+1)ijk}\) for any additional quantity over \(R_t\). Thus, unit transportation cost becomes

   \[
   \tilde{C}_{ijk} = \begin{cases} 
   \tilde{p}_{ijk} & \text{if } 0 < x_{ijk} \leq R_1 \\
   \left[\tilde{p}_{ijk}R_1 + \tilde{p}_{2ijk}\left(x_{ijk} - R_1\right)\right]/x_{ijk} & \text{if } R_1 < x_{ijk} \leq R_2 \\
   \left[\tilde{p}_{ijk}R_2 + \tilde{p}_{3ijk}\left(R_2 - R_1\right) + \tilde{p}_{5ijk}\left(x_{ijk} - R_2\right)\right]/x_{ijk} & \text{if } R_2 < x_{ijk} \leq R_3 \\
   \ldots & \ldots \\
   \left[\tilde{p}_{ijk}R_t + \tilde{p}_{2ijk}\left(R_t - R_1\right) + \ldots + \tilde{p}_{t+1}\left(x_{ijk} - R_{t-1}\right)\right]/x_{ijk} & \text{if } R_{t-1} < x_{ijk} \leq R_t \\
   \left[\tilde{p}_{ijk}R_t + \tilde{p}_{2ijk}\left(R_t - R_1\right) + \ldots + \tilde{p}_{(t+1)}\left(x_{ijk} - R_t\right)\right]/x_{ijk} & \text{if } R_t < x_{ijk} 
   \end{cases}
   \]  
   
   (10)

4. Formulation of a STP with discount cost, fixed charge and vehicle cost

We assume that \(m\) origins (or sources) \(O_i (i = 1, 2, \ldots, m)\), \(n\) destinations (i.e., demands) \(D_j (j = 1, 2, \ldots, n)\) and \(K\) conveyances \(E_k (k = 1, 2, \ldots, K)\). \(K\) conveyances, i.e., different modes of transport may be trucks, cargo flights, goods trains, ships, etc. Let \(a_i\) be the amount of a homogeneous product available at \(i^{th}\) origin, \(b_j\) be the demand at \(j^{th}\) destination and \(e_k\) represents the amount of product which can be carried by \(k^{th}\) conveyance. \(C_{ijk}\) be the cost under AUD, IQD or combination of these two systems associated with transportation of a
unit product from $i^{th}$ source to $j^{th}$ destination by means of the $k^{th}$ conveyance. The variable $x_{ijk}$ represents the unknown quantity to be transported from origin $O_i$ to destination $D_j$ by means of $k^{th}$ conveyance.

Furthermore, the modelling analyst sets the system parameters according to statistics data and information from the DM. The parameters may not be perfectly precise. Thus, the ‘feasible region’ is not fixed any more. Then the programming problem is not perfectly precise but flexible instead. The bi-fuzzy MOSTP with bi-fuzzy resources, demands, conveyances and fuzzy cost coefficients and under AUDs, IQD and combination of these two systems can be represented as:

$$\min \bar{Z}_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \left[ \tilde{C}_{ijk} x_{ijk} + f_{ijk} + G(x_{ijk}) \right]$$  \hspace{1cm} (11)

$$\min \bar{Z}_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \tilde{C}_{ijk}^2 y(x_{ijk})$$  \hspace{1cm} (12)

where

$$y(x_{ijk}) = \begin{cases} 1 & \text{for } x_{ijk} > 0 \\ 0 & \text{for } x_{ijk} = 0 \end{cases} \hspace{1cm} (13)$$

subject to

$$\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq \tilde{a}_i \hspace{1cm} i = 1, 2, 3, \ldots, m \hspace{1cm} (14)$$

$$\sum_{i=1}^{m} x_{ijk} \leq \tilde{b}_j \hspace{1cm} j = 1, 2, 3, \ldots, n \hspace{1cm} (15)$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq \tilde{e}_k \hspace{1cm} k = 1, 2, 3, \ldots, K \hspace{1cm} (16)$$

where $\tilde{C}_{ijk}$’s are given by (9) for AUD, by (10) for IQD and by (9) and (10) for combination of AUD, IQD.

5 Reduced crisp model

Let the bi-fuzzy numbers $\tilde{a}_i, \tilde{b}_j$ and $\tilde{e}_k$ are approximated to $\tilde{a}_i = (a_{i1}, \bar{a}_i, a_{i2})$ with $\bar{a}_i = (a_{i3}, a_{i}, a_{i4})$, $\tilde{b}_j = (b_{j1}, \bar{b}_j, b_{j2})$ with $\bar{b}_j = (b_{j3}, b_{j}, b_{j4})$ and $\tilde{e}_k = (e_{k1}, \bar{e}_k, e_{k2})$ with $\bar{e}_k = (e_{k3}, e_{k}, e_{k4})$ respectively. Then the earlier transportation model takes the following form:

$$\min E[\bar{Z}_1] = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \left[ E[\tilde{C}_{ijk}] x_{ijk} + f_{ijk} + G(x_{ijk}) \right]$$  \hspace{1cm} (17)
A multi objective solid transportation problem

\[ \min E[\tilde{Z}_2] = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} E\left[ \tilde{C}_{ijk}^2 \right] y(x_{ijk}) \]  

(18)

subject to

\[ \text{Pos} \left\{ \theta \mid \text{Pos} \left( \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq \bar{u}_i \right) \right. \geq \theta_i, \quad i = 1, 2, 3, \ldots, m \]  

(19)

\[ \text{Nec} \left\{ \delta \mid \text{Nec} \left( \sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk} \leq \bar{b}_j \right) \right. \geq \delta_j, \quad j = 1, 2, 3, \ldots, n \]  

(20)

and

\[ \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq E[e_k], \quad k = 1, 2, 3, \ldots, K \]  

(21)

Using Lemmas 1 and 2, and Theorems 1 and 2, the above equations reduces to

\[ \min E[\tilde{Z}_1] = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} E\left[ \frac{C_{ijk}^1 + 2C_{ijk}^2 + C_{ijk}^3}{4} \right] x_{ijk} + f_{ijk} + G(x_{ijk}) \]  

(22)

\[ \min E[\tilde{Z}_2] = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} E\left[ \frac{C_{ijk}^2 + 2C_{ijk}^2 + C_{ijk}^3}{4} \right] y(x_{ijk}) \]  

(23)

subject to

\[ (1 - \theta_i) a_i + (1 - \theta_i) \sum_{k=1}^{K} x_{ijk} - \sum_{k=1}^{K} x_{ijk} + a_i + (1 - \eta_i) \left( \sum_{k=1}^{K} x_{ijk} + a_i \right) \geq 0, \quad i = 1(m) \]  

(24)

\[ b_j - \sum_{k=1}^{K} x_{ijk} - \gamma_j \left( b_{jk} + \sum_{k=1}^{K} x_{ijk} \right) - \delta_j b_{jk} - (1 - \delta_j) \sum_{k=1}^{K} x_{ijk} \geq 0, \quad j = 1, 2, 3, \ldots, n \]  

(25)

\[ \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq e_k + \frac{(e_{2k} + e_{4k}) - (e_{3k} + e_{3k})}{4}, \quad k = 1, 2, 3, \ldots, K \]  

(26)

5.1 Entropy function

Let \( T \) be the transported amount, i.e., \( T = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \). Consider a function \( F(X) \) which represents the number of possible assignment for the state \( X = (x_{ijk}) \).

The (Shannon) entropy of a variable \( X \) is defined as
\[ F(X) = \text{the number of ways selecting } x_{11}, \text{ from } T, \text{ multiplied by the number of ways selecting } x_{112}, \text{ from } T - x_{11}, \ldots, \text{ multiplied by the number of ways selecting } x_{mK}, \text{ from } T - x_{111} - x_{112} - \ldots - x_{mK}. \]

\[ = T \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} C_{n_{ij}} (T - x_{i1}) C_{n_{ij2}} (T - x_{i1} - x_{i2}) C_{n_{ij3}} (T - x_{i1} - x_{i2} - x_{i3}) \cdots C_{n_{mK} (T - x_{m1} - x_{m2} - \ldots - x_{mK})} \]

\[ = \frac{T!}{\prod_{i=1}^{m} \prod_{j=1}^{n} \prod_{k=1}^{K} x_{ijk}!} \]

\[ = \ln(T!) - \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \ln(x_{ijk}!) \]

\[ = \ln(e^{T^T}) - \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \ln(e^{-x_{ijk}^2}) \]

\[ = T \ln(T) - \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \ln(x_{ijk}) \]

[by using Stirling’s approximation formula]

Here the entropy function \( En(x) = \frac{\ln(F(X))}{T} \).

The function (Shannon) entropy can be expressed as

\[ En(X) = -\sum_{x} f(x) \]

where

\[ f(x) = \begin{cases} h(x) \ln h(x) & \text{if } h(x) \neq 0 \\ 0 & \text{if } h(x) = 0 \end{cases} \]

\( h(x) \) being the probability that \( X \) is in the state \( x \).

In transportation problem, normalising the trip number \( x_{ijk} \) by dividing the total number of trips \( T \left( \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \right) \), a probability distribution, \( h_{ijk} = \frac{x_{ijk}}{T} \) is formulated.

Therefore,

\[ En(X) = -\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \left( \frac{x_{ijk}}{T} \right) \ln \left( \frac{x_{ijk}}{T} \right) \]

\[ = \ln(T) - \frac{1}{T} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \ln \left( x_{ijk} \right) \]

(27)

In transportation problem, this entropy function acts as a measure of dispersal of trips among origins, destinations and conveyances. It becomes more useful, if we would like to have minimum transportation costs as well as maximum entropy amount.

Taking entropy function as an additional objective function, the final multi objective problem takes the following form:
A multi objective solid transportation problem

- **Problem 1**: objective function with entropy

\[
\begin{align*}
\text{Minimise} & \quad E[\hat{Z}_1], \ E[\hat{Z}_2] \\
\text{Maximise} & \quad En(\chi) \\
\text{subject to} & \quad (24) \text{ to } (26)
\end{align*}
\]

\[(28)\]

- **Problem 2**: objective function without entropy

\[
\begin{align*}
\text{Minimise} & \quad E[\hat{Z}_1], \ E[\hat{Z}_2] \\
\text{subject to} & \quad (24) \text{ to } (26)
\end{align*}
\]

\[(29)\]

6 Solution approaches for MOGA

In this paper, the proposed solid transportation Problem 1 (28), Problem 2 (29) are solved by multi objective GA. A GA is a heuristic search process for optimisation that resembles natural selection. GAs has been applied successfully in different areas. GA for the linear and non-linear transportation problem develop by Vignaux and Michalewicz (1991). As the name suggests, GA is originated from the analogy of biological evolution. GAs consider a population of individuals. Using the terminology of genetics, a population is a set of feasible solutions of a problem. A member of the population is called a genotype, a chromosome, a string or a permutation. A GA contains three operators – reproduction, crossover and mutation. The MOGA is illustrated as follows.

We assume that there are \(M\) objective functions. In order to cover both minimisation and maximisation of objective functions, we use the operator between two solutions and as to denote that solution is better than solution on a particular objective. Similarly, for a particular objective implies that solution is worse than solution on this objective. For example, if an objective function is to be minimised, the operator would mean the \(<\) operator, whereas if the objective function is to be maximised, the operator would mean the \(>\) operator. The following definition covers mixed problems with minimisation of some objective functions and maximisation of the rest of them.

6.1 Reproduction

Parents are selected at random with selection chances biased in relation to chromosome evaluations. Next to initialise the population, we first determine the independent and dependent variables from all (here 12) variables and then their boundaries. This problem \(x_{221}, x_{231}, x_{122}, x_{132}, x_{212}, x_{222}\) and \(x_{322}\) are independent variables and \(x_{111}, x_{121}, x_{131}, x_{211}, x_{112}\) are the dependent variables. The independent variables

\[
\begin{align*}
\ x_{221} \in (0, \min(a_2, b_2, e_1)); \ x_{231} \in (0, \min(a_2, b_3, e_1)); \ x_{122} \in (0, \min(a_1, b_2, e_2)); \\
\ x_{322} \in (0, \min(a_1, b_3, e_2)); \ x_{212} \in (0, \min(a_2, b_1, e_2)); \ x_{222} \in (0, \min(a_2, b_2, e_2))
\end{align*}
\]

and \(x_{322} \in (0, \min(a_1, b_3, e_2))\).
6.2 Crossover

Crossover is a key operator in the GA and is used to exchange the main characteristics of parent individuals and pass them on the children. It consists of two steps:

1. **Selection for crossover:** For each solution of $P^1(T)$ generate a random number $r$ from the range $[0..1]$. If $r < p_c$ then the solution is taken for crossover, where $p_c$ is the probability of crossover.

2. **Crossover process:** Crossover takes place on the selected solutions. For each pair of coupled solutions $Y_1$, $Y_2$ a random number $c$ is generated from the range $[0..1]$ and $Y_1$, $Y_2$ are replaced by their offspring's $Y_{11}$ and $Y_{21}$ respectively where $Y_{11} = cY_1 + (1 - c)Y_2$, $Y_{21} = cY_2 + (1 - c)Y_1$, provided $Y_{11}$, $Y_{21}$ satisfied the constraints of the problem.

6.3 Mutation

The mutation operation is needed after the crossover operation to maintain population diversity and recover possible loss of some good characteristics. It is also consist of two steps:

1. **Selection for mutation:** For each solution of $P^1(T)$ generate a random number $r$ from the range $[0..1]$. If $r < p_m$ then the solution is taken for mutation, where $p_m$ is the probability of mutation.

2. **Mutation process:** To mutate a solution $X = (x_1, x_2, \ldots, x_K)$ select a random integer $r$ in the range $[1..K]$. Then replace $x_r$ by randomly generated value within the boundary of $r^{th}$ component of $X$.

**Definition:** A solution $X_1$ is said to dominate the other solution $X_2$, if the following both conditions 1 and 2 are true:

1. The solution $X_1$ is no worse than $X_2$ in all objectives, or for all $j = 1, 2, \ldots, M$
2. The solution $X_1$ is strictly better than $X_2$ in at least one objective, or for at least one $j = 1, 2, \ldots, M$.

If any of the above condition is violated, the solutions $X_1$ does not dominate the solution $X_2$. If $X_1$ dominates the solution $X_2$, it is also customary to write any of the following:

1. $X_2$ is dominated by $X_1$
2. $X_1$ is non-dominated by $X_2$
3. $X_1$ is non-inferior to $X_2$.

It is intuitive that if a solution $X_1$ dominates another solution $X_2$, the solution $X_1$ is better than $X_2$ in the parlance of multi-objective optimisation. Since the concept of domination allows a way to compare solutions with multiple objectives, most multi-objective optimisation methods use this domination concept to search for non-dominated solutions.
6.4 Crowding distance

Crowding distance of a solution is measured using the following rule.

Step 1 Sort the population set according to every objective function values in ascending order of magnitude.

Step 2 For each objective function, the boundary solutions are assigned an infinite distance value. All other intermediate solutions are assigned a distance value equal to the absolute normalised difference in the function values of two adjacent solutions. This calculation is continued with other objective functions.

Step 3 The overall crowding distance value is calculated as the sum of the individual distance values corresponding to each objective.

Each objective function is normalised before calculating the crowding distance. Following algorithm is used for this purpose.

\[
\text{set } k = \text{number of solutions in } F \\
\text{for each } k \\
\quad \{ \\
\quad \quad \text{set } F[k]_{\text{distance}} = 0 \\
\quad \} \\
\text{for each } m \\
\quad \{ \\
\quad \quad \text{sort } F, \text{ in ascending order of magnitude of } m^{\text{th}} \text{ objective} \\
\quad \quad \text{set } F[1]_{\text{distance}} = F[m]_{\text{distance}} = M \text{ where } M \text{ is a large number} \\
\quad \quad \text{for } i = 2 \text{ to } k - 1 \\
\quad \quad \quad \{ \\
\quad \quad \quad \quad F[i]_{\text{distance}} = F[i]_{\text{distance}} + (F[i + 1]_m - F[i - 1]_m) / (f_m^{\text{max}} - f_m^{\text{min}}) \\
\quad \quad \quad \} \\
\} \\
\]

Here, \( F[i]_m \) refers to the \( m^{\text{th}} \) objective function value of \( F[i] \). \( f_m^{\text{max}} \) and \( f_m^{\text{min}} \) are the maximum and minimum values of the \( m^{\text{th}} \) objective function.

6.5 Non-dominated sorting of a population

In this case, first, for each solution we calculate two entities:

1. domination count \( n_p \), the number of solutions which dominate the solution \( p \)
2. \( S_p \), a set of solutions that the solution \( p \) dominates.

All solutions in the first non-dominated front will have their domination count as zero. Now, for each solution \( p \) with \( n_p = 0 \), we visit each member \( (q) \) of its set \( S_p \) and reduce its domination count by one. In doing so, if for any member \( q \) the domination count becomes zero, we put it in a separate list \( Q \). These members belong to the second non-dominated
front. Now, the above procedure is continued with each member of $Q$ and the third front is identified. This process continues until all fronts are identified.

6.6 Parameters

Firstly, we set the different parameters on which this GA depends. These are the number of generation ($MAXGEN$), population size ($POPSIZE$), probability of crossover ($PXOVER$), probability of mutation ($PMU$). There is no clear indication as to how large a population should be. If the population is too large, there may be difficulty in storing the data, but if the population is too small, there may not be enough string for good crossovers. In our problem, $POPSIZE = 100$, $PXOVER = 0.7$, $PMU = 0.3$ and $MAXGEN = 5,000$.

6.7 Chromosome representation:

An important issue in applying a GA is to design an appropriate chromosome representation of solutions of the problem together with genetic operators. Traditional binary vectors used to represent the chromosome are not effective in many non-linear physical problems. Since the proposed problem is non-linear, hence to overcome this difficulty, a real-number representation is used in this problem.

6.8 Evaluation

Evaluation function plays the same role in GA as that which the environment plays in natural evolution. To this problem, the evaluation function is

$$eval(V_i) = \text{objective function value}$$

By Roulette wheel selection method the better chromosome are selected from the population to generate the next the improved chromosomes. Now new chromosomes are produced by arithmetic crossover and uniform mutation. The general outline of the algorithm is following:

begin
  $t \leftarrow 0$
  initialize Population($t$)
  evaluate Population($t$)
  while (not terminate-condition)
    {
      $t \leftarrow t + 1$
      select Population($t$) from Population($t - 1$)
      alter(crossover and mutate) Population($t$)
      evaluate Population($t$)
    }
  Print Optimum Result
end.
6.9 Procedure of MOGA

Step 1 Generate initial population $P_1$ of size $N$.
Step 2 $i \leftarrow 1$ [$i$ represent the number of current generation].
Step 3 Select solution from $P_i$ for crossover.
Step 4 Made crossover on selected solution to get child set $C_1$.
Step 5 Select solution from $P_i$ for mutation.
Step 6 Made mutation on selected solution to get solution set $C_2$.
Step 7 Set $P'_i = P_i \bigcup C_1 \bigcup C_2$
Step 8 Partition $P'_i$ into subsets $F_1, F_2, \ldots, F_k$, such that each subset contains non-dominated solutions of $P'_i$ and every solutions of $F_i$ dominates every solution of $F_{i+1}$ for $i = 1, 2, \ldots, k - 1$.
Step 9 Select largest possible integer $l$, so that no of solutions in the set $F_1 \bigcup F_2 \bigcup \cdots \bigcup F_l \leq N$.
Step 10 Set $P_{i+1} = F_1 \bigcup F_2 \bigcup \cdots \bigcup F_l$.
Step 11 Sort $F_{i+1}$ in decreasing order by crowding distance.
Step 12 Set $M =$ number of solutions in $P_{i+1}$.
Step 13 Select first $N - M$ solutions from set $F_{i+1}$.
Step 14 Insert these solutions in solution set $P_{i+1}$.
Step 15 Set $i \leftarrow i + 1$.
Step 16 If termination condition does not hold, go to Step 3.
Step 17 Output $P_i$.
Step 18 End.

7 Numerical experiment

To illustrate the problems, we consider a STP with two origins, three destinations and two types of conveyances. So $m = 2$, $n = 3$, $K = 3$ and $l = 1, 2, 3$. Values of the corresponding origins (i.e., resources), destination (i.e., demands), maximum amount to be transported by a particular conveyances, total vehicle costs and fixed charges are assumed as follows.

7.1 Input data

To give the 90% priority of amount of availability, 70% of demands, we consider the confidence level of the parameters as $\theta_1 = \theta_2 = 0.9$, $\eta_1 = \eta_2 = 0.9$, $\delta_1 = \delta_2 = \delta_3 = 0.7$, ...
$\gamma_1 = \gamma_2 = \gamma_3 = 0.70$, and other cost $V = 5.5$; $V_e = 7$; $f_{111} = 12$; $f_{221} = 10$; $f_{311} = 15$; $f_{411} = 9$; $f_{221} = 11$; $f_{231} = 16$; $f_{122} = 14$; $f_{222} = 16$; $f_{122} = 15$; $f_{222} = 120$; $f_{222} = 11$; $f_{232} = 11$. Hence, total fixed charge $= \sum f_{ijk} = 169$. The unit transportation cost under AUD, IQD and combination of these two discount systems are as follows.

### Table 1: Input data for fuzzy unit transportation cost

<table>
<thead>
<tr>
<th>$C_{ijk}$</th>
<th>$p_{ijk}$</th>
<th>AUD</th>
<th>IQD</th>
<th>$C_{ijk}$</th>
<th>$p_{ijk}$</th>
<th>AUD</th>
<th>IQD</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>(4, 5, 6)</td>
<td>0 &lt; $x_{ijk} &lt; 10$</td>
<td>0 &lt; $x_{ijk} \leq 10$</td>
<td>12</td>
<td>(7, 8, 9)</td>
<td>0 &lt; $x_{ijk} &lt; 10$</td>
<td>0 &lt; $x_{ijk} \leq 10$</td>
</tr>
<tr>
<td></td>
<td>(2, 4, 6)</td>
<td>10 \leq $x_{ijk} &lt; 20$</td>
<td>10 \leq $x_{ijk} \leq 20$</td>
<td></td>
<td>(6, 7, 8)</td>
<td>10 \leq $x_{ijk} &lt; 20$</td>
<td>10 \leq $x_{ijk} \leq 20$</td>
</tr>
<tr>
<td></td>
<td>(1, 2, 3)</td>
<td>$x_{ijk} \geq 20$</td>
<td>$x_{ijk} &gt; 20$</td>
<td></td>
<td>(3, 4, 5)</td>
<td>$x_{ijk} \geq 20$</td>
<td>$x_{ijk} &gt; 20$</td>
</tr>
<tr>
<td>11</td>
<td>(4, 6, 8)</td>
<td>0 &lt; $x_{ijk} &lt; 10$</td>
<td>0 &lt; $x_{ijk} \leq 10$</td>
<td>12</td>
<td>(5, 7, 9)</td>
<td>0 &lt; $x_{ijk} &lt; 10$</td>
<td>0 &lt; $x_{ijk} \leq 10$</td>
</tr>
<tr>
<td></td>
<td>(4, 5, 6)</td>
<td>10 \leq $x_{ijk} &lt; 20$</td>
<td>10 \leq $x_{ijk} \leq 20$</td>
<td></td>
<td>(5, 6, 7)</td>
<td>10 \leq $x_{ijk} &lt; 20$</td>
<td>10 \leq $x_{ijk} \leq 20$</td>
</tr>
<tr>
<td></td>
<td>(2, 3, 4)</td>
<td>$x_{ijk} \geq 20$</td>
<td>$x_{ijk} &gt; 20$</td>
<td></td>
<td>(3, 4, 5)</td>
<td>$x_{ijk} \geq 20$</td>
<td>$x_{ijk} &gt; 20$</td>
</tr>
<tr>
<td>11</td>
<td>(7, 9, 11)</td>
<td>0 &lt; $x_{ijk} &lt; 10$</td>
<td>0 &lt; $x_{ijk} \leq 10$</td>
<td>12</td>
<td>(8, 10, 12)</td>
<td>0 &lt; $x_{ijk} &lt; 10$</td>
<td>0 &lt; $x_{ijk} \leq 10$</td>
</tr>
<tr>
<td></td>
<td>(7, 8, 9)</td>
<td>10 \leq $x_{ijk} &lt; 20$</td>
<td>10 \leq $x_{ijk} \leq 20$</td>
<td></td>
<td>(6, 8, 10)</td>
<td>10 \leq $x_{ijk} &lt; 20$</td>
<td>10 \leq $x_{ijk} \leq 20$</td>
</tr>
<tr>
<td></td>
<td>(4, 6, 8)</td>
<td>$x_{ijk} \geq 20$</td>
<td>$x_{ijk} &gt; 20$</td>
<td></td>
<td>(5, 6, 7)</td>
<td>$x_{ijk} \geq 20$</td>
<td>$x_{ijk} &gt; 20$</td>
</tr>
<tr>
<td>11</td>
<td>(5, 8, 9)</td>
<td>0 &lt; $x_{ijk} &lt; 10$</td>
<td>0 &lt; $x_{ijk} \leq 10$</td>
<td>12</td>
<td>(7, 9, 11)</td>
<td>0 &lt; $x_{ijk} &lt; 10$</td>
<td>0 &lt; $x_{ijk} \leq 10$</td>
</tr>
<tr>
<td></td>
<td>(5, 7, 9)</td>
<td>10 \leq $x_{ijk} &lt; 20$</td>
<td>10 \leq $x_{ijk} \leq 20$</td>
<td></td>
<td>(7, 8, 9)</td>
<td>10 \leq $x_{ijk} &lt; 20$</td>
<td>10 \leq $x_{ijk} \leq 20$</td>
</tr>
<tr>
<td></td>
<td>(4, 5, 6)</td>
<td>$x_{ijk} \geq 20$</td>
<td>$x_{ijk} &gt; 20$</td>
<td></td>
<td>(3, 5, 7)</td>
<td>$x_{ijk} \geq 20$</td>
<td>$x_{ijk} &gt; 20$</td>
</tr>
<tr>
<td>11</td>
<td>(4, 5, 6)</td>
<td>0 &lt; $x_{ijk} &lt; 10$</td>
<td>0 &lt; $x_{ijk} \leq 10$</td>
<td>12</td>
<td>(5, 7, 9)</td>
<td>0 &lt; $x_{ijk} &lt; 10$</td>
<td>0 &lt; $x_{ijk} \leq 10$</td>
</tr>
<tr>
<td></td>
<td>(1, 3, 5)</td>
<td>10 \leq $x_{ijk} &lt; 20$</td>
<td>10 \leq $x_{ijk} \leq 20$</td>
<td></td>
<td>(5, 6, 7)</td>
<td>10 \leq $x_{ijk} &lt; 20$</td>
<td>10 \leq $x_{ijk} \leq 20$</td>
</tr>
<tr>
<td></td>
<td>(1, 2, 3)</td>
<td>$x_{ijk} \geq 20$</td>
<td>$x_{ijk} &gt; 20$</td>
<td></td>
<td>(1, 3, 5)</td>
<td>$x_{ijk} \geq 20$</td>
<td>$x_{ijk} &gt; 20$</td>
</tr>
<tr>
<td>11</td>
<td>(4, 7, 8)</td>
<td>0 &lt; $x_{ijk} &lt; 10$</td>
<td>0 &lt; $x_{ijk} \leq 10$</td>
<td>12</td>
<td>(7, 8, 9)</td>
<td>0 &lt; $x_{ijk} &lt; 10$</td>
<td>0 &lt; $x_{ijk} \leq 10$</td>
</tr>
<tr>
<td></td>
<td>(4, 5, 6)</td>
<td>10 \leq $x_{ijk} &lt; 20$</td>
<td>10 \leq $x_{ijk} \leq 20$</td>
<td></td>
<td>(5, 6, 7)</td>
<td>10 \leq $x_{ijk} &lt; 20$</td>
<td>10 \leq $x_{ijk} \leq 20$</td>
</tr>
<tr>
<td></td>
<td>(2, 3, 4)</td>
<td>$x_{ijk} \geq 20$</td>
<td>$x_{ijk} &gt; 20$</td>
<td></td>
<td>(3, 4, 5)</td>
<td>$x_{ijk} \geq 20$</td>
<td>$x_{ijk} &gt; 20$</td>
</tr>
</tbody>
</table>

### Table 2: Time required for transportation (in hrs.)

<table>
<thead>
<tr>
<th>Conveyance 1</th>
<th>Conveyance 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10, 12, 14)</td>
<td>(13, 14, 19)</td>
</tr>
<tr>
<td>(6, 7, 8)</td>
<td>(13, 15, 17)</td>
</tr>
<tr>
<td>(12, 13, 14)</td>
<td>(12, 13, 14)</td>
</tr>
<tr>
<td>(6, 7, 8)</td>
<td>(13, 15, 19)</td>
</tr>
<tr>
<td>(12, 13, 14)</td>
<td>(13, 15, 19)</td>
</tr>
<tr>
<td>(6, 7, 8)</td>
<td>(9, 11, 13)</td>
</tr>
</tbody>
</table>

With the above input data, we solve the problem as stated earlier using above mentioned MOGA. The optimum results are presented below. Here, the bi-fuzzy total resources, bi-fuzzy total demands at various centres and bi-fuzzy conveyance capacities $\tilde{a}_1$, $\tilde{a}_2$, $\tilde{b}_1$, $\tilde{b}_2$, $\tilde{b}_3$, $\tilde{e}_2$ are approximated given by $\tilde{a}_1 = (0.75, \tilde{a}_1, 0.95)$ with $\tilde{a}_1 = (110, a_1, 120)$, $\tilde{a}_2 = (0.75, \tilde{a}_2, 0.95)$ with $\tilde{a}_2 = (105, a_2, 115)$, $\tilde{b}_1 = (0.75, \tilde{b}_1, 0.85)$ with $\tilde{b}_1 = (110, b_1, 115)$, $\tilde{b}_2 = (0.65, \tilde{b}_2, 0.85)$ with $\tilde{b}_2 = (125, b_2, 130)$ and $\tilde{e}_2 = (0.55, \tilde{e}_2, 0.65)$ with $\tilde{e}_2 = (115, e_2, 120)$, $\tilde{e}_2 = (0.75, \tilde{e}_2, 0.95)$ with $\tilde{e}_2 = (110, e_2, 120)$, $\tilde{e}_3 = (0.55, \tilde{e}_3, 0.95)$ with $\tilde{e}_3 = (125, e_3, 130)$. 


### 7.2 Optimum result for Problem 1 (with entropy function)

Here the optimum solutions are obtained using GA presented in and given below.

**Table 3**  
Near optimum transported amounts and min. cost, min. time and max. entropy, total vehicle

<table>
<thead>
<tr>
<th>Discount system</th>
<th>Transported amounts $x_{111}$ $x_{121}$ $x_{131}$ $x_{211}$ $x_{221}$ $x_{231}$ $x_{112}$ $x_{122}$ $x_{132}$ $x_{212}$ $x_{222}$ $x_{232}$</th>
<th>Min. cost</th>
<th>Min. time</th>
<th>Max. entropy</th>
<th>Total vehicle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(8.7836, 8.5376, 10.1649, 10.2322, 2.6137, 4.668, 5.8664, 9.3706, 7.2768, 5.1178, 4.4781, 2.8902)</td>
<td>857.6556</td>
<td>14.5466</td>
<td>2.4021</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>(8.5263, 10.6266, 8.161, 10.1899, 2.3994, 5.0968, 7.1426, 8.178, 7.3655, 4.1412, 3.796, 4.3767)</td>
<td>838.1154</td>
<td>14.2150</td>
<td>2.408</td>
<td>95</td>
</tr>
<tr>
<td>IQD AUD</td>
<td>(9.5734, 8.8967, 8.8386, 7.6364, 4.5801, 5.4748, 8.3684, 7.7592, 6.5637, 4.4217, 3.764, 4.1229)</td>
<td>847.1358</td>
<td>14.5474</td>
<td>2.4375</td>
<td>90</td>
</tr>
</tbody>
</table>
7.3 Optimum result for Problem 2 (without entropy function)

Table 4 Optimum transported amounts, min. cost, min time, total vehicle

<table>
<thead>
<tr>
<th>Discount system</th>
<th>Transported amounts $x_{111}, x_{121}, x_{131}, x_{211}, x_{221}, x_{231}, x_{112}, x_{122}, x_{132}, x_{212}, x_{222}, x_{232}$</th>
<th>Min. cost</th>
<th>Min. cost</th>
<th>Total vehicle cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td>7.012458, 0.214578, 2.241543, 2.214596, 0.142514, 3.0 7.082682, 5.51745, 0.8, 0.026248, 2.865958, 1.511511</td>
<td>880.214578</td>
<td>14.241587</td>
<td>79</td>
</tr>
<tr>
<td>IQD</td>
<td>6.816023, 6.214578, 3.321457, 6.0457895, 3.214578, 0.214578 3.589106, 5.548839, 0.1214578, 0.451452, 1.214574, 0.2145784</td>
<td>891.682922</td>
<td>14.1458</td>
<td>84</td>
</tr>
<tr>
<td>AUD IQD</td>
<td>3.124157, 5.315749, 9.621457, 0.321454, 2.321454, 2.338087 2.351649, 5.149052, 2.4644, 0.44681, 0.673212, 0.098881</td>
<td>889.071625</td>
<td>14.2145</td>
<td>88</td>
</tr>
<tr>
<td>IQD AUD</td>
<td>5.603525, 1.824369, 3.668098, 3.197157, 4.631457, 5.2754.321457, 4.3214578, 0.214578, 0.713853, 4.694244, 0.654214</td>
<td>899.398926</td>
<td>14.321454</td>
<td>85</td>
</tr>
</tbody>
</table>

7.4 Optimum result with different change of parameters in MOGA

Table 5 Optimum cost for different values of population, generation and probability of crossover and mutation with AUD discount system

<table>
<thead>
<tr>
<th>Population</th>
<th>Generation</th>
<th>Probability crossover</th>
<th>Probability mutation</th>
<th>Minimum cost</th>
<th>Minimum time</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>1,000</td>
<td>0.9</td>
<td>0.3</td>
<td>871.5645</td>
<td>14.5789</td>
</tr>
<tr>
<td>110</td>
<td>800</td>
<td>0.9</td>
<td>0.3</td>
<td>869.5412</td>
<td>14.5478</td>
</tr>
<tr>
<td>100</td>
<td>600</td>
<td>0.7</td>
<td>0.4</td>
<td>875.364</td>
<td>14.3145</td>
</tr>
<tr>
<td>100</td>
<td>400</td>
<td>0.7</td>
<td>0.3</td>
<td>879.538</td>
<td>14.4574</td>
</tr>
<tr>
<td>110</td>
<td>200</td>
<td>0.9</td>
<td>0.3</td>
<td>881.4578</td>
<td>14.4175</td>
</tr>
<tr>
<td>120</td>
<td>1,000</td>
<td>0.9</td>
<td>0.3</td>
<td>876.102</td>
<td>14.5415</td>
</tr>
<tr>
<td>115</td>
<td>1,000</td>
<td>0.7</td>
<td>0.4</td>
<td>870.102</td>
<td>14.5414</td>
</tr>
<tr>
<td>110</td>
<td>1,000</td>
<td>0.7</td>
<td>0.3</td>
<td>870.102</td>
<td>14.5454</td>
</tr>
<tr>
<td>100</td>
<td>1,000</td>
<td>0.7</td>
<td>0.3</td>
<td>871.386</td>
<td>14.129</td>
</tr>
<tr>
<td>95</td>
<td>1,000</td>
<td>0.8</td>
<td>0.3</td>
<td>879.617</td>
<td>14.2144</td>
</tr>
<tr>
<td>80</td>
<td>1,000</td>
<td>0.7</td>
<td>0.4</td>
<td>873.524</td>
<td>14.4446</td>
</tr>
<tr>
<td>70</td>
<td>1,000</td>
<td>0.7</td>
<td>0.3</td>
<td>872.483</td>
<td>14.5444</td>
</tr>
<tr>
<td>60</td>
<td>1,000</td>
<td>0.7</td>
<td>0.3</td>
<td>877.52</td>
<td>14.1468</td>
</tr>
<tr>
<td>100</td>
<td>1,000</td>
<td>0.8</td>
<td>0.3</td>
<td>881.00</td>
<td>14.129</td>
</tr>
<tr>
<td>100</td>
<td>1,000</td>
<td>0.3</td>
<td>0.4</td>
<td>862.525</td>
<td>14.5416</td>
</tr>
<tr>
<td>100</td>
<td>1,000</td>
<td>0.4</td>
<td>0.3</td>
<td>868.328</td>
<td>14.5415</td>
</tr>
<tr>
<td>100</td>
<td>1,000</td>
<td>0.5</td>
<td>0.3</td>
<td>874.026</td>
<td>14.2415</td>
</tr>
<tr>
<td>100</td>
<td>1,000</td>
<td>0.6</td>
<td>0.3</td>
<td>875.834</td>
<td>14.2145</td>
</tr>
<tr>
<td>100</td>
<td>1,000</td>
<td>0.7</td>
<td>0.3</td>
<td>871.386</td>
<td>14.8958</td>
</tr>
</tbody>
</table>
Table 5

<table>
<thead>
<tr>
<th>Population</th>
<th>Generation</th>
<th>Probability crossover</th>
<th>Probability mutation</th>
<th>Minimum cost</th>
<th>Minimum time</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1,000</td>
<td>0.8</td>
<td>0.3</td>
<td>874.289</td>
<td>14.1457</td>
</tr>
<tr>
<td>100</td>
<td>1,000</td>
<td>0.8</td>
<td>0.22</td>
<td>841.792</td>
<td>14.129</td>
</tr>
<tr>
<td>100</td>
<td>1,000</td>
<td>0.7</td>
<td>0.4</td>
<td>871.386</td>
<td>14.3894</td>
</tr>
<tr>
<td>100</td>
<td>1,000</td>
<td>0.7</td>
<td>0.4</td>
<td>871.495</td>
<td>14.7129</td>
</tr>
<tr>
<td>100</td>
<td>1,000</td>
<td>0.7</td>
<td>0.5</td>
<td>872.023</td>
<td>14.1241</td>
</tr>
<tr>
<td>100</td>
<td>1,000</td>
<td>0.7</td>
<td>0.62</td>
<td>872.786</td>
<td>14.2145</td>
</tr>
</tbody>
</table>

8 Discussion

From Tables 3 and 4, it is observed that AUD system gives minimum transportation cost whereas other systems provide more costs, of which IQD is the highest. Comparing the Tables 3 and 4, it is revealed that consideration of entropy in STP forces more number of cell-allotments than the case without entropy. It gives uniform distribution of units in the cells and demands at destinations are better satisfied which are desirable for a real-life STP, though the corresponding transportation costs and times are more. The results are calculated (in Table 5) by using different GA parameters, which implies that the algorithm is robust to the GA parameters setting, and effective in solving the bi-fuzzy multiple objective programming problems. Figures 3–4 depicts the convergence of the MOGA against generation number and crossover probability.

Figure 3 Probability of crossover vs. costs (see online version for colours)
9 Conclusions and future research work

For the first time, a multi-objective STP is considered with generalised fuzzy transportation costs and bi-fuzzy demand, supply and conveyances. We analysed a multi-item transportation problem with AUD, IQD and combination of these two price breaks, linear cost function, fixed charge and vehicle cost and solved by GA. Here the transportation model is more realistic and flexible in nature. In the proposed problem the constraint functions are expressed in bi-fuzzy possibility and necessity measure. The entropy function was considered as an additional objective function. The entropy function is constructed from the concept of Shannon’s measure of entropy. It acts as a measure of dispersal of trips among the origins, destinations and conveyances of the transportation model. The concept of entropy presented here is quite general in nature and can be extended to other fields of operation research like supply chain model, market research, etc., in fuzzy rough, uncertain environment.

Although the general bi-fuzzy multiple objective programming problem and its application for a MOSTP are discussed in this paper, more detailed analysis for this class of problems and more practical applications should be discussed in the further research. This method can also be used in other different areas such as portfolio distribution, urban and regional planning, etc.
References


