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Application of calculating the reverberation time from room impulse responses without using regression

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Summary
This paper presents the application and a method that calculates the reverberation time based on Lp-norms, generalized measures of the room impulse response (RIR), without using regression on decay curves. The reverberation time in this approach is a function of a parameter, and is constant only in a perfectly diffuse space; therefore the method may present useful information beyond the decay time itself. Properties of this method using theoretical and real-life RIRs are examined in cases of wide-band and sub-octave-band calculations. Correction methods for the finite support of the RIR, as well as for the effects of stationary background noise are presented and the connection to previous methods is shown.

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1. Introduction

Reverberation time is one of the objective room acoustic parameters of key interest [1]. Since its introduction, various measurement methods have been developed for its calculation. Although the traditionally defined reverberation time does not reflect all important information of a real space for a thorough assessment but acts as an overall measure, it is still an important parameter both in room evaluation and design, and it also correlates with subjective experiences such as sound coloration. Room acoustic measurements are mostly based on the evaluation of room impulse responses (RIRs). The traditional calculation method is based on forming a decay curve [2] of the RIR for the purpose of applying regression [3] or a decay model fitting [4] on this curve to obtain the reverberation time. The regression process or the model fitting serve two purposes; firstly, to mitigate bias due to background noise in measured data, and secondly, to numerically express deviations from the perfectly diffuse decay process, which can be characterized by a single time-constant. To provide a smooth decay curve, Schroeder proposed a backward integrated method, the energy decay curve (EDC). This curve has excellent [4] but not maximum possible smoothing features [5]. Physically, all these methods share the idea of using the energy of the impulse response, or mathematically, the (squared) L_2 norm. L_2 norm-based calculations are convenient because additive white Gaussian noise can be handled analytically by exploiting its independence from the signal. The main idea of the present paper is to apply a new method of calculating reverberation time that diverts from the physically interpretable energy quantities, and is based on lower valued norms that provide smoothing for RIRs. The present paper focuses on the applicability of using this forward integration calculation method. The calculation method has two main differences compared to the backward integrated approach: firstly, the finite length bias is corrected in the present approach, whereas the EDC curves are often treated for noise bias only; and secondly, with the help of the proposed smoothing method, no regression is necessary, which has further advantageous consequences – among them the ability to calculate the reverberation time in the presence of a strong direct sound or extreme background noise levels. Furthermore, the proposed method provides results supporting a consistent interpretation of non-exponential multiple-decay curves or bended decay shapes.

2. Calculating the reverberation time using forward integration

2.1. Definition of the reverberation time

First, a new practical definition for the reverberation time is introduced. Beranek defines the reverberation time as "the length of time required for the reverberant sound to decay 60 decibels" [6]. However, in a real non-diffuse space, the decay does not usually follow a perfect exponential decrease, and for coupled spaces, bent tunnels or large corridors, "the reverberation time is difficult to define" and "the measured reverberation time has little meaning" according to [7]. Extrapolation from different reference levels on a decay curve to a 60 dB decrease in these cases has often no further meaning than to provide a qualitative comparison.

In this paper, the term 'reverberation time' is defined slightly differently. Instead of fixing the reverberation time to a given level decrease, the definition is as follows: *the reverberation time is the generalized central gravity point of the decay envelope multiplied with a constant corresponding to the length of time required for a 60 dB decrease in the perfectly diffuse space*. This definition is consistent with Beranek's original definition for the diffuse case but it is different for the non-diffuse case. In the present definition, the reverberation time does not correspond to a fixed decay level; instead, the decay process is compared to the diffuse-case decay, and this comparison is achieved by introducing a smoothing parameter p. The perfect exponential decay process has a reverberation time with no dependence on p; however, a sparse, non-diffuse reverberation will show p-dependence. The degree of p-dependence can therefore become a measure related to the sparsity or deviance of a given RIR compared to a perfectly diffuse one.

2.2. RIR model

The room impulse response for a perfectly diffuse case $h(t)_i$ is modeled as the multiplication of a given sample of a white Gaussian random process $n(t)_i$ and an envelope $e(t)$ which is characterized by the reverberation time R and a constant $k = 3\ln(10)$ that supports a 60 dB level decrease on the envelope for the diffuse case.

$$\begin{aligned} e(t) &= \exp\left(-k\frac{t}{R}\right) \\ h(t)_i &= n(t)_i \cdot e(t) \end{aligned} \tag{1}$$

The $e(t)$ envelope, as an upper bound for this RIR model, is used in the forthcoming analytic equations.

A second, non-diffuse model is simply assumed to only have a single Gaussian-based reflection at a time point R, where the shape control parameter of the reflection is a:

$$\begin{aligned} h(t)_{\mathrm{nd}} &= \lim_{a\to 0}\frac{1}{a\sqrt{\pi}}\exp\left(-\frac{(t-R)^2}{a^2}\right) = \\ &= \delta_a(t-R) \end{aligned} \tag{2}$$

All RIR models are assumed to have 1-limited amplitude.

2.3. Introduction of the smoothing parameter

The present approach aims at the smoothing of strong echoes and other abrupt changes in general. A strong direct sound for example often influences the reliability of current methods using fixed reference levels [8] on a decay curve. Therefore, measurement standards include operating constraints to avoid these reflections to occur. Smoothing is achieved based on L_p norms, by modifying the RIR as

$$h(t,p) \doteq |h(t)|^p \tag{3}$$

where $0 < p \leq 1$. This method suppresses outliers because differences between smaller and larger values in the RIR become smaller as p decreases. For a discrete RIR, this modification is related to its Lebesgue norm L_p.

2.4. Direct calculation of the reverberation time

Based on the smoothed RIR of (3), the reverberation time is defined by the following improper integral

$$R(p) = kp \cdot \frac{\int_0^\infty t \cdot h(t,p)dt}{\int_0^\infty h(t,p)dt} \tag{4}$$

The first term kp of (4) assures a p-independent result for the pure exponential decay $e(t)$, while the second term, containing the improper integrals, is called the generalized dispersion around zero (GDZ), which can be interpreted as the generalized gravity point of the smoothed RIR (3); or in other words, the generalized center time. The word generalized is used because not $h(t)$ directly, but its p-th power is involved. Interestingly, a similar term appeared in the calculation of the room acoustic parameter 'echo criterion' [9] but with a different purpose, integration intervals and exponents. Note that in this calculation there is no regression involved but the reverberation time is directly obtained by a forward integration from the impulse response. The obtained value is a single, unambiguous number for any given p.

2.5. Theoretical validity

Let $\hat{R}$ denote the calculated reverberation time at a given p. From a mathematical point of view, it is true that (4) yields a p-independent reverberation time result for the diffuse space envelope $e(t)$ because:

$$\hat{R} = kp \cdot \frac{\int_0^\infty t \cdot \left|\exp\left(-k\frac{t}{R}\right)\right|^p dt}{\int_0^\infty \left|\exp\left(-k\frac{t}{R}\right)\right|^p dt} = R \tag{5}$$

However, for a single reflection case (3), the result becomes p-dependent:

$$\hat{R}(p) = \lim_{a\to 0}\left(kp \cdot R + \frac{ak\sqrt{p}}{\sqrt{\pi}\exp\left(\xi^2\right)\left(\operatorname{erf}(\xi)+1\right)}\right) = $$
$$= kp \cdot R \tag{6}$$
$$\xi = \frac{R\sqrt{p}}{a}$$

where ξ is introduced for shorter notation. This supports the assumption that the dependency of p provides additional information on 'diffuseness', 'reflection density' or 'irregularities' within the RIR. Therefore, for a given non-exponential decay, the reverberation time is calculated for a (small) set of p values, and the results will reveal how non-exponential the decay is.

3. Bias and its correction

3.1. Finite length bias correction

In real-life calculations, the improper integral of (4) cannot be evaluated as measurement lengths are finite. This problem also exists for the energy decay curve but the problem is often not mitigated there. In the current method, using the exponential envelope model and a given available finite length L, the calculated reverberation time becomes

$$\hat{R}_L(p) = R - \frac{Lkp}{\exp\left(\frac{Lkp}{R}\right) - 1} \tag{7}$$

where the second term is the upper integration limit (UIL) underestimation bias. Unfortunately (7) has no analytic closed form solution with respect to R; however, the following recursion provides the UIL bias correction:

$$\hat{R}(p)_{i+1} = \hat{R}_L(p)_0 + \frac{Lkp}{\exp\left(\frac{Lkp}{\hat{R}(p)_i}\right) - 1} \tag{8}$$

with the starting condition

$$\hat{R}_L(p)_0 = kp \cdot \frac{\int_0^L t \cdot |h(t)|^p dt}{\int_0^L |h(t)|^p dt} \tag{9}$$

so the recursion process is simply initiated by calculating the reverberation time at the available finite length, then (8) is used for i times, and finally the result is obtained at a given accuracy. From a mathematical point of view, it can be seen that the iteration (8) is convergent and its limit is $R(p)$. As a practical consideration, the required number of iterations is defined as follows: the UIL bias should be within 1% for the $\frac{R}{L} = 15$ ratio, meaning that a nearly bias-free result is required even if only $\frac{1}{15}$-th of the reverberation time was measured. This criterion is not meaninglessly strict, but is useful for the noise bias mitigation discussed in the following section, as that process further limits the available length by defining a truncation point that is usually earlier than the available measured length. The aforementioned criterion yields approximately $N = 10^5$ iteration steps. This process is easy to implement and consumes few resources thus can be used effectively in practice.

3.2. Noise bias correction

The presence of additive stationary background noise in a measured RIR introduces over-estimation bias both in the traditional regression based methods and in the present method as well, although slightly differently. Mitigating this bias is possible by appropriately truncating the RIR. A perfect mitigation requires the assumption of a well-behaved noise (stationary, ergodic) and reverberation (having more than one reflection), and the *a priori* knowledge of both a descriptive quantity of the background noise (such as its variation) and the reverberation time. Since this latter requirement is too strict to be useful, a blind method is needed. One of the blind noise mitigation methods for traditional reverberation time calculation, proposed by Chu [10], assumes a low R/L ratio, and works by deducting the root mean square (RMS) value of a noise sample from the RIR before calculating the Schroeder curve. Other methods are also available, such as an iterative fitting method by Lundeby et al [11] using a smoothed ETC to iteratively determine the knee point where the noise begins to dominate the signal. It is common in these methods that they work with the energy of the RIR, or in other words, they are L_2 optimized. Therefore, they cannot be directly used in the current calculation method.

The presently proposed noise mitigation method relies only on the assumption that the measurement is noisy. To formulate a blind noise mitigation method it is first shown that there exists a truncation point that eliminates the effect of noise, for the analytically manageable case of $p = 1$. Assume the RIR envelope

$$h(t) = \exp\left(-k \cdot \frac{t}{R}\right) + N \tag{10}$$

where N denotes the offset caused by the additional stationary noise. The reverberation time calculated at a finite length using this noisy RIR then becomes

$$\hat{R} = k \cdot \frac{\frac{L^2 N}{2} + \frac{R^2}{k^2} - \frac{R(R+Lk)}{k^2 \exp\frac{Lk}{R}}}{LN - \frac{R\left(\frac{1}{\exp\frac{Lk}{R}} - 1\right)}{k}} \tag{11}$$

and it is true that $\hat{R} = R$ stands if and only if

$$L = \frac{R\left(W_0\left(\frac{2\exp(-2)}{N}\right) + 2\right)}{k} \quad (12)$$

where W_0 is the positive (single-valued) branch of the Lambert W function. This solution shows that there exists a definite truncation point for any finite N that eliminates the effect of noise, but this truncation point requires an accurate noise estimate and also the *a priori* knowledge of the reverberation time. This latter one is not available, and it is sometimes also inconvenient to assume the availability of a proper noise estimate. Therefore a blind method is proposed as follows. The mitigation is possible blindly exploiting the qualitative fact that the function constructed as the reverberation time of a noisy RIR evaluated at different lengths has a certain shape: before it starts to monotonically increase due to background noise, an inflection can be seen which corresponds to the optimum truncation length and can therefore be automatically detected. In detail, firstly, the reverberation time is calculated for different finite lengths by employing the UIL bias correction, for a given fixed p. For example, the L length is split to 100 equidistant intervals or to 10 ms intervals, whichever is more applicable. Other values may also work. This ensures that the calculation will be fast and that a proper time-averaging is used. The selection of this time interval may affect the variation in the results, but practical values such as the above yield useful results. The obtained curve is then differentiated with respect to length, and the minimum is found using the median of a set of candidate values. This ensures some stability or robustness in the determination of the inflection time point in the presence of random noise. Other noise mitigation methods may also be possible but not used here.

4. Simulation results

4.1. Double decay with and without noise

Bended, multiple or double decays can be observed for example in coupled spaces, bended cavities or rooms with highly non-uniformly distributed absorption. Traditional decay curves may reveal such a decay if the noise conditions allow; however, the reverberation time for those cases is difficult to be determined and interpreted. In the present example, a double decay artificial RIR was simulated. Two decays were mixed at a given mixing level, and the reverberation time determined. Both a noiseless and a noisy decay process was examined. Results show that for the noiseless case, the traditional method yields different reverberation times corresponding to their reference levels, and the results are between the two time-constants. However, when there is noise, even if noise correction is employed, the results will not necessarily be bounded between the given time constants; which is unsatisfactory (Fig. 1). However, the new method, with the applied noise correction, yields consistent results with the noiseless case (Fig. 2). It is also shown that different p values reveal the nature of the decay process, as a p-dependency shows that there is deviation from the perfect exponential decay.

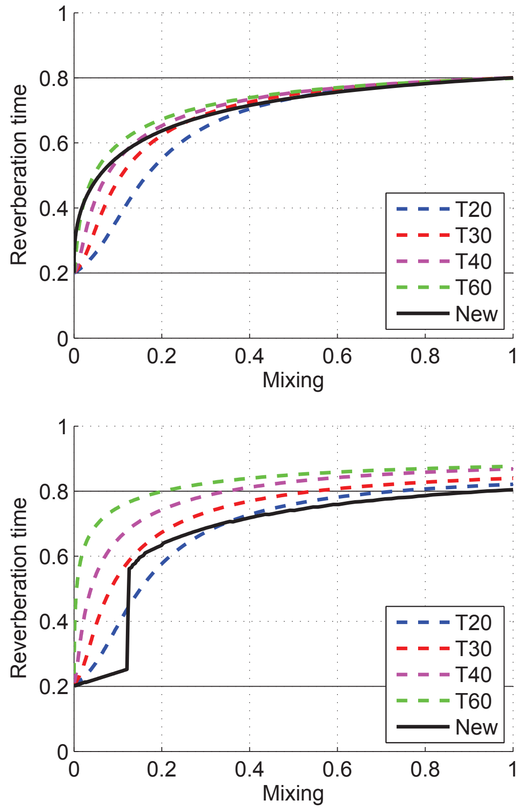

Figure 1. Reverberation times using the traditional and the new method on simulated double decay RIRs. Two decays of $R_1 = 0.2$ and $R_2 = 0.8$ were mixed at mixing levels between 0 and 1. The sample length was 1 second and the sampling rate 48000 Hz. Top: noiseless case. Bottom: noisy case. SNR was set to a fixed 40 dB and Chu's method was used as a noise correction for the traditional calculations. The new method was plotted at an arbitrary $p = 0.55$.

4.2. Sub-band filtering

Fractional octave-band filtering is a widely used method for obtaining frequency-dependent reverberation times. To compare the traditional and the new method, a selection of 1/3 octave band filter bank types were used on a 2-second white random sample IR with a reverberation time of 1 second. The filter banks were a zero-phase Butterworth multirate filter bank, a complex Morlet wavelet filter bank, and a complex Q transformation (CQT) representation [13], which is similar to a DFT representation but works

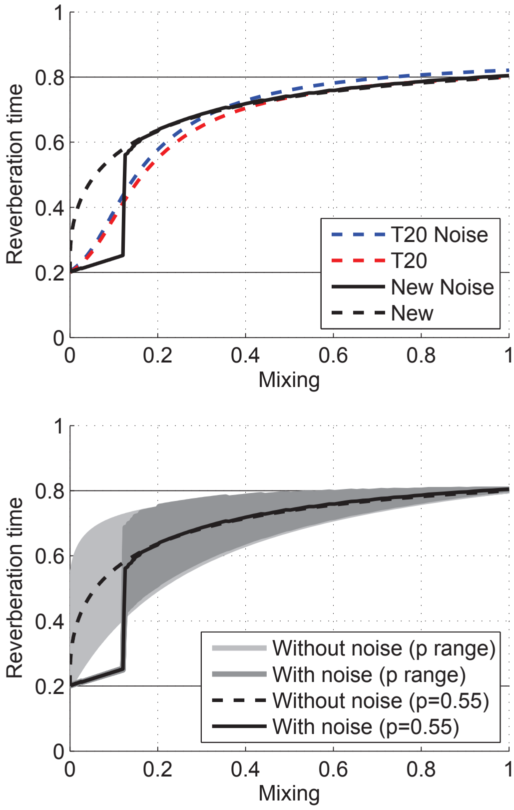

Figure 2. Comparison of determined reverberation times. Top: consistency of the determined values for noiseless and noisy RIRs by the new method, and inconsistency by the traditional method. Bottom: p values obtained with the new method. The jump corresponds to the level where the secondary decay becomes visible above noise.

on a logarithmic frequency scale. Therefore, this latter representation reduces the time resolution according to the number of points used in the transformation. The other filter types maintain time resolution. The traditional calculation method uses a regression on the Schroeder curve. For the new method, p was evaluated at 0.1 steps from 0.1 to 1. Results show a good agreement and a comparable result. The mean reverberation time of all band obtained with the new method in the presented example were similar to that of the traditional method, but the standard deviation was lower (Fig. 3). Considering the average of 10000 artificial RIRs using CQT, the new method shows more accurate results than the traditional method (Fig. 3).

5. Measurement results

A measurement database with more than 1700 RIRs of 15 spaces, including small rooms, concert halls and cathedrals was used that cover reverberation times of approximately 1 to 10 seconds. Since reverberation times using different definitions were compared, the results are expected to show some differences. The traditional wide-band T_{30} reverberation time values

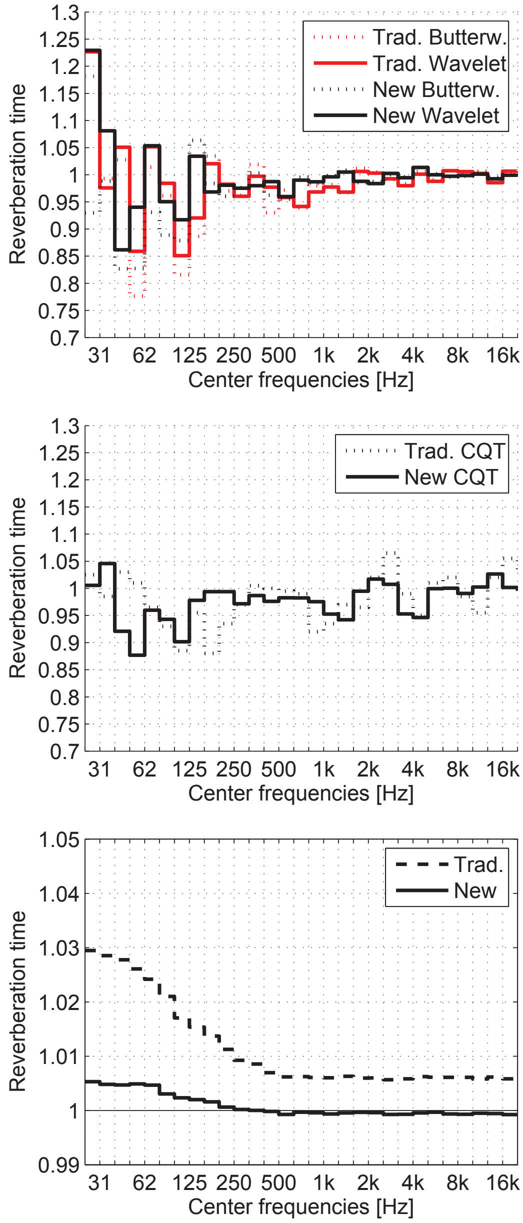

Figure 3. 1/3-octave-band calculation comparison of the traditional and new reverberation time methods on RIRs having 2-second decaying random noise with 1-second reverberation time in all bands. Top: a single RIR comparison using Butterworth multirate and complex Morlet Wavelet filter banks. Mean / Std: 0.9805 / 0.06878 (Trad. Bw.), 0.9885 / 0.06256 (Trad. Wavelet), 0.9711 / 0.05338 (New Bw.), 0.9955 / 0.05867 (New Wavelet). Middle: the same RIR compared using CQT. Mean / Std: 0.9818 / 0.04563 (Trad.), 0.9774 / 0.03622 (New). Bottom: average of 10000 RIRs using CQT.

were compared to the new method R, evaluated at different smoothing parameter values p, and the their correlation was plotted (Fig.4). Results show a good agreement verifying the applicability of the proposed method.

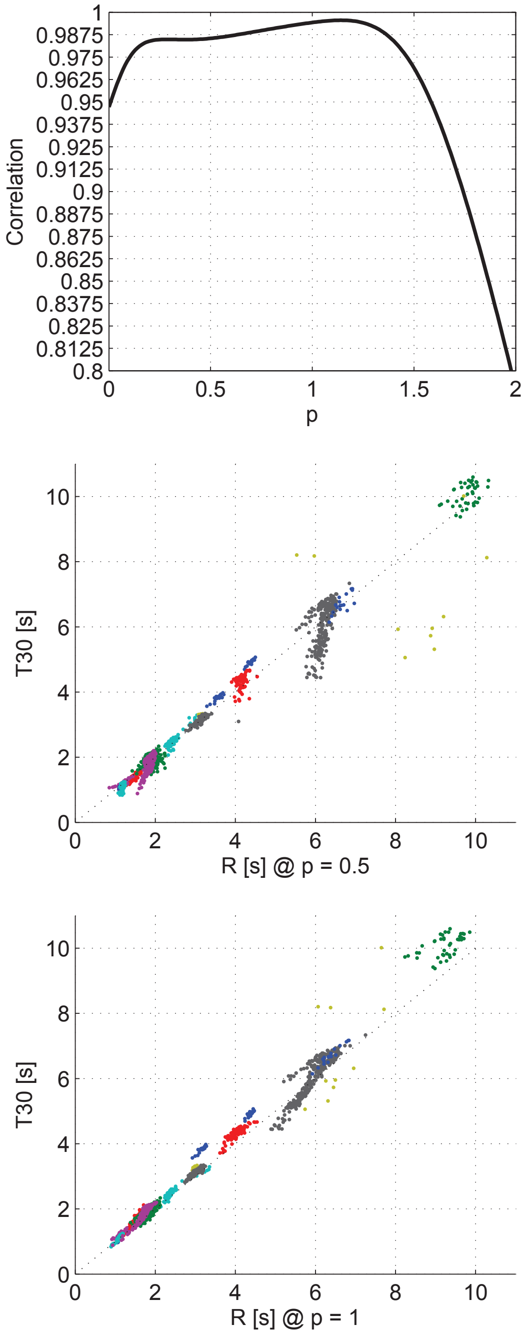

Figure 4. Top: correlation of the wide-band reverberation time obtained with the traditional T_{30} and the new method at different smoothing parameter values of p. A measurement database of 1769 RIRs of 15 spaces was used. The RIRs contained background noise which was corrected with Chu's method for the traditional calculation; for the new calculation method, both the UIL and noise bias were corrected. Bottom: reverberation times. Colors represent different rooms, points represent results of a given RIR.

6. Conclusions

In this paper the application of a new method of calculating the reverberation time without using regression on decay curves was presented. Along with the proposal of a new practical definition of the reverberation time, it was shown that the proposed method can be used to consistently handle the reverberation time of double decays. It was also shown that traditional calculation methods can be outperformed by the proposed method in terms of accuracy for sub-octave band filtered results when the constant Q transformation is used. The present method was applied to a large measurement database and it was shown that the method is applicable for real-life RIRs.

References

[1] W. Sabine, *Collected papers on acoustics*. Harvard University Press, 1922.

[2] M. R. Schroeder, "New method of measuring reverberation time," *The Journal of the Acoustical Society of America*, vol. 37, pp. 409–412, Mar. 1965.

[3] N. Xiang, "Evaluation of reverberation times using a nonlinear regression approach," *The Journal of the Acoustical Society of America*, vol. 98, no. 4, pp. 2112–2121, 1995.

[4] M. Karjalainen, P. Antsalo, A. Makivirta, T. Peltonen, and V. Valimaki, "Estimation of modal decay parameters from noisy response measurements," *Journal of the Audio Engineering Society*, vol. 50, no. 11, pp. 867–878, 2002.

[5] C. Huszty and S. Sakamoto, "Generalized backward integrated decay curves with controllable smoothing to detect echoes in impulse responses," in *Proc. 2011 Spring Meeting of the Acoustical Society of Japan*, 2-12-4, (Tokyo, Japan), Acoustical Society of Japan, Mar. 2011.

[6] L. L. Beranek, *Concert halls and opera houses: music, acoustics, and architecture*. Springer Verlag, 2nd ed., 2005.

[7] Z. Maekawa and P. Lord, *Environmental and architectural acoustics*. Taylor & Francis, 1994.

[8] ISO, "3382-1:2009 - Acoustics – Measurement of room acoustic parameters – Part 1: Performance spaces," *International Organization for Standardization, Geneva, Switzerland*, 2009.

[9] L. Dietsch and W. Kraak, "Ein objektives kriterium zur erfassung von echostörungen bei musik- und sprachdarbietungen," *Acustica*, vol. 60, pp. 205–216, 1986.

[10] W. T. Chu, "Comparison of reverberation measurements using schroeder's impulse method and decay-curve averaging method," *The Journal of the Acoustical Society of America*, vol. 63, no. 5, pp. 1444–1450, 1978.

[11] A. Lundeby, T. E. Vigran, H. Bietz, and M. Vorländer, "Uncertainties of measurements in room acoustics," *Acustica*, vol. 81, no. 4, pp. 344–355, 1995.

[12] J. Brown, "Calculation of a constant q spectral transform," *Journal of the Acoustical Society of America*, vol. 89, no. 1, pp. 425–434, 1991.

[13] J. Brown and M. Puckette, "An efficient algorithm for the calculation of a constant q transform," *Journal of the Acoustical Society of America*, vol. 92, no. 5, pp. 2698–2701, 1992.