Boolean Applications in Aircraft Electric Power Systems Reliability Analysis

NICOLAE JULA
Military Technical Academy
81-83 G.Cosbuc Blv.,Sector 5 Bucharest
ROMANIA
nicolae.jula@gmail.com

COSTIN CEPISCA
SC Elcos Proiect SRL
Iancului 4, s.2, Bucharest
ROMANIA
costin@wing.ro

MIRCEA COVRIG
Faculty of Electrical Engineering
Politehnica University of Bucharest
Spl. Independentei 313, s.6, Bucharest
ROMANIA
m_covrig@amotion.pub.ro

CIPRIN RACUCIU
Military Technical Academy
81-83 G.Cosbuc Blv.,Sector 5 Bucharest
ROMANIA
ciprian.racuciu@gmail.com

TUDOR URSU
Military Technical Academy
81-83 G.Cosbuc Blv.,Sector 5 Bucharest
ROMANIA
ursutudor1952@yahoo.com

Abstract: - The paper presents a concrete case of an actual aircraft electric power system analysis. Using the Boolean logical structures we define a conceptual fault tree. The fault tree will express all the combination of factors that can lead to system failure in the onboard electric system. The further on analysis rely on AND – OR logic elements, and the goal is to improve the fault-tolerance behavior of the system. The examples and numeric figures are for a c.c. electric power system of an operational aircraft.

Key-Words: - Reliability, analysis, Boolean logic, aircraft, electric power

1 Introduction

Large-scale systems reliability analysis is based on the quantification of the failure process at the structural level. Thus, any system failure is a result of a quantified sequence of states of the failure process. The quantification level is chosen in accordance with the desired goal and precision, down even to the singular components. The more detailed the quantification level gets, the more accurate are the results [1], [2].

The conceptual representation of an emergent failure state is a series of primary events, interconnected through a Boolean logical structure, which indicates the possible combination of those elements having the result of a system failure. The aircraft electric system reliability determination, using the Boolean algebra, consists in the calculus of the probability of the "failure" event.

From the structural point of view, for the reliability analysis, we will use the terms:

- Primary elements – components or blocks at the base level of the quantification;
- Primary failures – primary elements failures;
- Unwanted event – system failure state;
- Failure mode – the set of primary elements that when simultaneously in failure mode, drives to a system failure;
- Minimal failure mode – the smallest set of primary components that when simultaneously in failure mode, drive to a system failure;
- Hierarchic level – all elements that are structurally equivalent and having equivalent positions in the system failure representation.

The analysis method is based on binary logic [3], [4], [5]. Thus, a system function is equivalent with a binary function, which variables are the events (the failures). This binary function:

$$Y = f\left(X_1, X_2, \ldots, X_n\right) \quad (1)$$

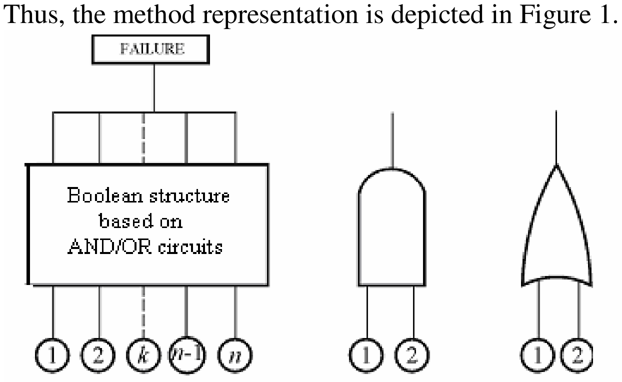
is synthesized with logical elements AND/OR, using the following symbols and states:

$\bigcup$ (reunion) for the function OR;

$\bigcap$ (intersection) for the function AND;

X_i is *1* if the primary element is good and *0* otherwise, and Y is *1* if the system is good and *0* otherwise.

Thus, the method representation is depicted in Figure 1.



a) b) c)

Fig. 1. a) The general concept of the method based on Boolean algebra
(1, 2,..., n are independent primary events);
b) the schematics of the logic function AND; c) the schematics of the logic function OR

For the reliability function indicators calculus, in the hypothesis of the failure intensity having an exponential distribution, we use the relations:

$$R(t) = \exp\left(-\sum_{i=1}^{n} \lambda_i t\right) = \exp(-\wedge t) \quad (2)$$

$$R(t) = 1 - \prod_{i=1}^{n}\left[1 - \exp(-\lambda_i t)\right] \quad (3)$$

where: $\wedge = \sum_{i=1}^{n} \lambda_i$.

Relation (2) is used for the serial connection and the relation (3) is used for the parallel connection of the elements.

2 The Analysis Method Application

In the purpose of exemplifying the method for the reliability indicators determination we will focus on the c.c. electric power supply system of an aircraft. Figure 2 depicts the electric power supply system for the aircraft.

In principle, this electric power supply system equips (as the main electric power supply system) a large number of military aircraft from the MiG family (MiG-21, MiG-23, MiG-27, etc.). The example refers only a c.c. electric power supply system, but the method can be used also for the alternative current and mixed systems. In Figure 2:

1E – starter-generator – startup time of several seconds (as starter), after a successful start (three attempts permitted) it goes to a generator regime, supplying a 28V c.c. voltage;
4E – accumulator switch;
5E – inverse polarity protection diode;
13E – accumulator;
14E – accumulator to c.c. bar switch;
24E – generator to c.c. bar coupler / de-coupler;
47E – fuse;
27E – voltage regulator.

The emerging failure state schematics using AND/OR elements is depicted in Figure 3. The failure event is the loss of voltage at the 28V bar.

For the failure intensity λ_i of the components we use the relation:

$$\lambda_i = k\lambda_0 \quad (4)$$

where:

k – maintenance and way-of-use coefficient (for aircraft components the coefficient varies between 120 and 160 [6]); λ_0 – failure intensity – manufacturer specific data.

The data relative to the electric power supply system are presented in Table 1.

In these conditions, the Boolean function associated to the logic structure depicted in Figure 3 has the following form:

$$Y = X_7 \bigcap X_{12} = \left(X_1 \bigcup X_2 \bigcup X_3 \bigcup X_4 \bigcup X_5 \bigcup X_6 \right) \bigcap \bigcap \left(X_8 \bigcup X_9 \bigcup X_{10} \bigcup X_{11} \right) \quad (5)$$

To transform the logic expression into algebraic form [3], [4] we use the following relations:

$$X_1 \bigcap X_2 = X_1 \cdot X_2, \quad X_1 \bigcup X_2 = X_1 + X_2 - X_1 \cdot X_2,$$
$$\bigcup_{i=1}^{n} X_i = 1 - \prod_{i=1}^{n} (1 - X_i) \quad (6)$$

Thus, we have:

$$Y = X_7 \cdot X_{12} = = \left[1 - (1 - X_1)(1 - X_2)(1 - X_3)(1 - X_4)(1 - X_5)(1 - X_6) \right] \cdot \left[1 - (1 - X_8)(1 - X_9)(1 - X_{10})(1 - X_{11}) \right]. \quad (7)$$

Similar with:

$$Y = X_7 \cdot X_{12} = \left[1 - \prod_{i=1}^{6} (1 - X_i) \right] \cdot \left[\left(1 - \prod_{k=8}^{11} (1 - X_k) \right) \right] \quad (8)$$

Considering the failure intensity as exponential distribution, the system failure probability:

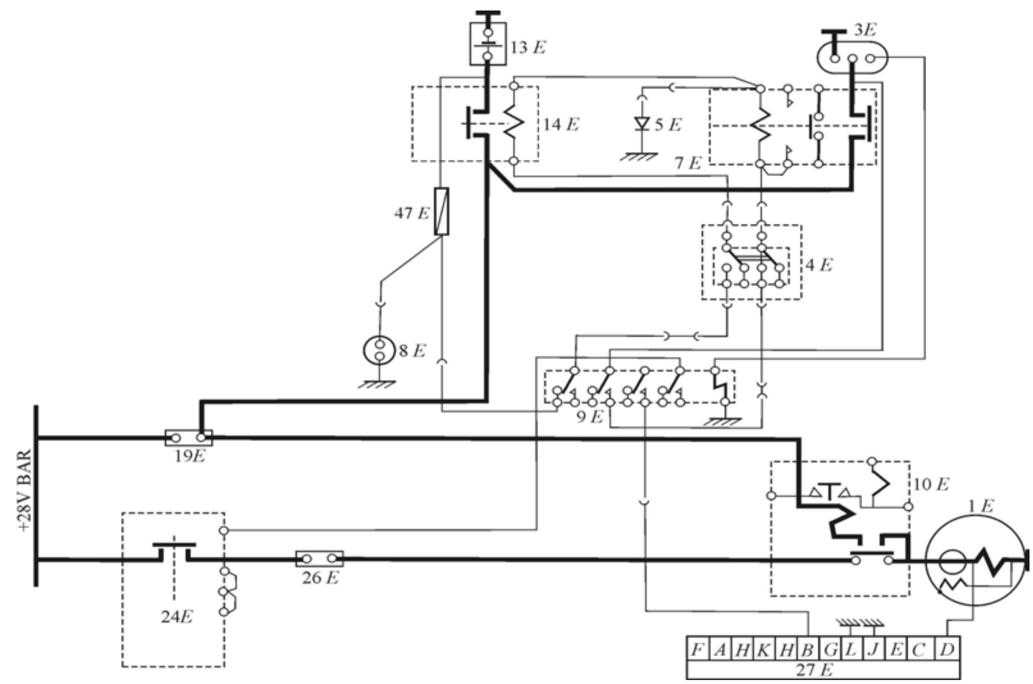


Fig.2. The electric power supply schematics for a c.c. main electric supply system aircraft (fragment)

Table 1

Symbol	Description	$\lambda_0 \left[h^{-1} \right]$	Number	k	$\lambda_i = nk\lambda_0 \left[h^{-1} \right]$	$F_i = 1 - e^{-\lambda_i t}$
4E	Switch	$0{,}12 \cdot 10^{-6}$	1	160	$\lambda_1 = 1{,}92 \cdot 10^{-5}$	$F_1 = 1 - e^{-1{,}92 \cdot 10^{5} t}$
5E	Diode	$0{,}6 \cdot 10^{-6}$	1	160	$\lambda_2 = 9{,}6 \cdot 10^{-5}$	$F_2 = 1 - e^{-9{,}6 \cdot 10^{-5} t}$
13E	Accumulator	$1{,}4 \cdot 10^{-6}$	1	160	$\lambda_3 = 22{,}4 \cdot 10^{-5}$	$F_3 = 1 - e^{-22{,}4 \cdot 10^{5} t}$
14E	Coupler	$0{,}4 \cdot 10^{-6}$	1	160	$\lambda_4 = 6{,}4 \cdot 10^{-5}$	$F_4 = 1 - e^{-6{,}4 \cdot 10^{-5} t}$
47E	Fuse	$2{,}75 \cdot 10^{-6}$	1	160	$\lambda_5 = 44 \cdot 10^{-5}$	$F_5 = 1 - e^{-44 \cdot 10^{-5} t}$

-	Contacts 1	$0{,}1 \cdot 10^{-6}$	1	160	$\lambda_6 = 16 \cdot 10^{-5}$	$F_6 = 1 - e^{-16 \cdot 10^{-5} t}$
1E	Starter-generator	$6 \cdot 10^{-6}$	1	160	$\lambda_8 = 96 \cdot 10^{-5}$	$F_8 = 1 - e^{-96 \cdot 10^{-5} t}$
24E	Coupler / De-coupler	$0{,}25 \cdot 10^{-6}$	1	160	$\lambda_9 = 4 \cdot 10^{-5}$	$F_9 = 1 - e^{-4 \cdot 10^{-5} t}$
27E	Voltage regulator	$13 \cdot 10^{-6}$	1	160	$\lambda_{10} = 208 \cdot 10^{-5}$	$F_{10} = 1 - e^{-208 \cdot 10^{-5} t}$
-	Contacts 2	$0{,}1 \cdot 10^{-6}$	10	160	$\lambda_{11} = 16 \cdot 10^{-5}$	$F_{11} = 1 - e^{-16 \cdot 10^{-5} t}$

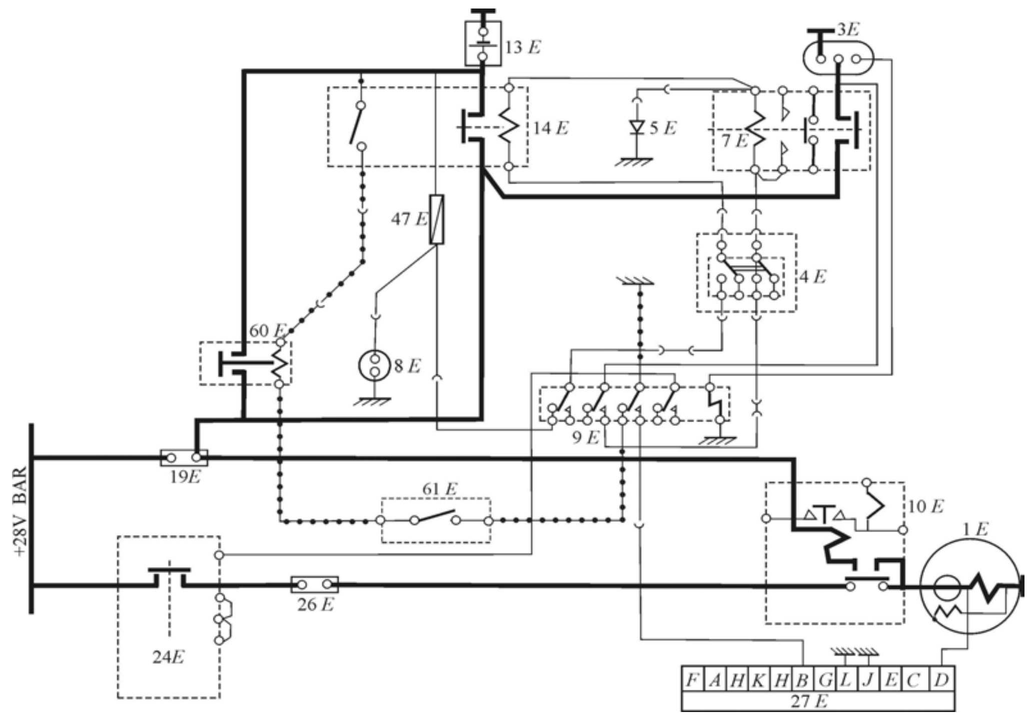


Fig. 4. Electric power supply system including the back-up subsystem (fragment)

Table 2

Symbol	Description	$\lambda_0 \left[h^{-1}\right]$	Number	k	$\lambda_i = nk\lambda_0 \left[h^{-1}\right]$	$F_i = 1 - e^{-\lambda_i \cdot t}$
60E	Coupler	$0{,}4 \cdot 10^{-6}$	1	160	$\lambda_{13} = 6{,}4 \cdot 10^{-5}$	$F_1 = 1 - e^{-6{,}4 \cdot 10^{-5} t}$
61E	Switch	$0{,}12 \cdot 10^{-6}$	1	160	$\lambda_{14} = 1{,}92 \cdot 10^{-5}$	$F_2 = 1 - e^{-1{,}92 \cdot 10^{-5} t}$
-	Contacts 3	$0{,}1 \cdot 10^{-6}$	4	160	$\lambda_{15} = 6{,}4 \cdot 10^{-5}$	$F_3 = 1 - e^{-6{,}4 \cdot 10^{-5} t}$

$$F(t)=\{1-\exp[-(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+\lambda_6)t]\}\cdot$$
$$\cdot[1-\exp(-\lambda_8-\lambda_9-\lambda_{10}-\lambda_{11})t]=1-\exp\left[-\sum_{i=8}^{11}\lambda_i t\right]-$$
$$-\exp\left[-\sum_{k=1}^{6}\lambda_k t\right]+\exp\left[-\sum_{p=1;p\neq 7}^{11}\lambda_p t\right] \quad (9)$$

Thus:

$$R(t)=1-F(t)=\exp\left[-\sum_{i=8}^{11}\lambda_i t\right]+\exp\left[-\sum_{k=1}^{6}\lambda_k t\right]-$$
$$-\exp\left[-\sum_{p=1;p\neq 7}^{11}\lambda_p t\right], \quad (10)$$

$$MTBF=\int_0^\infty R(t)dt=\frac{1}{\sum_{i=8}^{11}\lambda_i}+\frac{1}{\sum_{k=1}^{6}\lambda_k}-\frac{1}{\sum_{p=1;p\neq 7}^{11}\lambda_p}=$$
$$=\frac{1}{(96+4+208+16)\cdot 10^{-5}}+\frac{1}{(1{,}92+9{,}6+22{,}4+6{,}4+44+16)10^{-5}}+$$
$$-\frac{1}{(1{,}92+9{,}6+22{,}4+6{,}4+44+16+96+4+208+16)10^{-5}}=1069{,}78\text{ h}$$

Thus MTBF ≈ 1070 hours. (11)

3 Electric Power Supply Reliability Optimization

We can improve the electric power supply system reliability using a redundant (reserve) subsystem. The proposed improved electric power supply system, including the back-up subsystem (dotted lines) is depicted in Figure 4. Further on we will analyze the improved electric power supply system reliability, using the same method. This analysis also allows a determination of a relation between the system reliability and the system weight. Such a relation is necessary to emphasize the variation of the system reliability with the total weight of system components.

Through a compared analysis of different reliability improving variants, imposing as minimum condition the component weight, we can obtain an optimal solution. The logic structure that drives to the system failure status (for the improved system schematics) is depicted in Figure 5.

Table 2 presents the values of the failure intensity for the supplementary components from the back-up system, in the exponential distribution hypothesis.

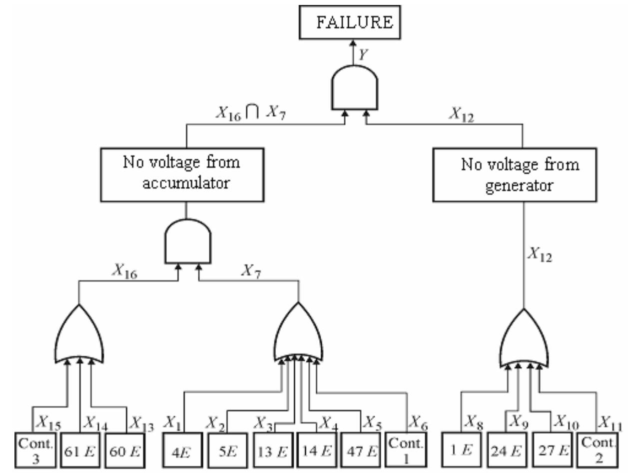


Fig.5. The logic structure that drives to the system failure status (improved system).

The Boolean function in this case is:

$$Y=(X_{16}\bigcap X_7)\bigcap X_{12}=$$
$$=(X_{13}\bigcup X_{14}\bigcup X_{15})\bigcap(X_1\bigcup X_2\bigcup X_3\bigcup X_4\bigcup X_5\bigcup X_6)$$
$$\bigcap(X_8\bigcup X_9\bigcup X_{10}\bigcup X_{11}). \quad (12)$$

Transforming in algebraic form, we have:

$$Y=[1-(1-X_{13})(1-X_{14})(1-X_{15})]\cdot$$
$$\cdot[1-(1-X_1)(1-X_2)(1-X_3)(1-X_4)(1-X_5)(1-X_6)]\cdot$$
$$\cdot[1-(1-X_8)(1-X_9)(1-X_{10})(1-X_{11})]. \quad (13)$$

$$Y=\left[1-\prod_{i=13}^{15}(1-X_i)\right]\cdot\left[1-\prod_{k=1}^{6}(1-X_k)\right]\cdot\left[1-\prod_{p=8}^{11}(1-X_p)\right] \quad (14)$$

From (14) we can determine the system failure probability $F(t)$:

$$F(t)=\left[1-\exp\left(-\sum_{i=13}^{15}\lambda_i t\right)\right]\cdot\left[1-\exp\left(-\sum_{k=1}^{6}\lambda_k t\right)\right]\cdot\left[1-\exp\left(-\sum_{p=8}^{11}\lambda_p t\right)\right]=$$
$$=1-\exp\left(-\sum_{i=13}^{15}\lambda_i t\right)-\exp\left(-\sum_{k=1}^{6}\lambda_k t\right)-\exp\left(-\sum_{p=8}^{11}\lambda_p t\right)+$$
$$\exp\left(-\sum_{\substack{i=1\\ i\neq 7}}^{11}\lambda_i t\right)+\exp\left(-\sum_{\substack{i=1\\ i\neq 7,8,9,10,11,12}}^{15}\lambda_i t\right)-\exp\left(-\sum_{\substack{i=1\\ i\neq 7}}^{15}\lambda_i t\right). \quad (15)$$

$F(t)$ and $R(t)$ are complementary functions, thus, for the electric power supply system reliability $R(t)$ we will have the following relation:

$$R(t) = \exp\left(-\sum_{i=13}^{15}\lambda_i t\right) + \exp\left(-\sum_{k=1}^{6}\lambda_k t\right) + \exp\left(-\sum_{\substack{i=1\\ i\neq 7}}^{15}\lambda_i t\right) - \exp\left(-\sum_{\substack{i=1\\ i\neq 7}}^{11}\lambda_i t\right) - \exp\left(-\sum_{\substack{i=1\\ i\neq 7,8,9,10,11,2}}^{15}\lambda_i t\right). \qquad (16)$$

For the MTBF, we will have:

$$MTBF = \int_0^\infty R(t)\,dt = \frac{1}{\sum_{i=13}^{15}\lambda_i} + \frac{1}{\sum_{k=1}^{6}\lambda_k} + \frac{1}{\sum_{p=8}^{11}\lambda_p} + \frac{1}{\sum_{\substack{i=1\\ i\neq 7}}^{15}\lambda_i} - \frac{1}{\sum_{\substack{i=1\\ i\neq 7}}^{11}\lambda_i} - \frac{1}{\sum_{\substack{i=1\\ i\neq 7,8,9,10,11,12}}^{15}\lambda_i} = 6926\,\text{h}$$

Thus, using the back-up subsystem, we increased the system reliability. The reservation efficiency [2], [6] we have:

$$\gamma = \frac{(MTBF)_r}{(MTBF)_0} = \frac{6926}{1070} = 6,5$$

4 Conclusions

From the analyzed example, we can conclude that this method can be used in the onboard electric power supply reliability determination. The MTBF influencing parameters in the main system points (power supply bars and distribution panels) can be determined and analyzed.
Through the failure related logic function analysis we can determine the circuits that can improve the system reliability. In the concrete case, through the introduction of the components 60E, 61E and corresponding contacts, we obtained a substantial increase of the reliability (approximately 6 times higher) for the 28V c.c. power supply bar.

The analyzed example is characteristic to a series of military aircraft, but the method can be also applied to other types of onboard electric power supply systems.

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