Generalised selection combining diversity performance of multi-element antenna systems via a stochastic electromagnetic-circuit methodology

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Abstract: A stochastic electromagnetic-circuit methodology for evaluating the generalised selection combining (GSC) diversity performance of coupled and arbitrarily terminated multi-element antenna (MEA) systems when operating in Rayleigh fading environments is proposed. The methodology is based on the effective length matrix, which connects the incident electric fields to the received voltage and current vectors, and on the newly introduced stochastic generation of realistic propagation environments by means of the angular power density functions and the cross polarisation power ratio of the incoming plane waves. The generated statistical ensemble of voltage and current vectors is then used to determine the receive GSC diversity performance metrics, that is mean effective gain (MEG), envelope correlation coefficient ($r_e$) and effective diversity gain (EDG). Two compact MEA configurations comprising two and four printed inverted F antennas (IFA) operating in a uniform propagation scenario at the 5.2 GHz band are investigated under various GSC schemes, which include the selection (SC) and the maximal ratio (MRC) as special cases. The simulated values of MEG, $r_e$ and EDG under the MRC scheme are in excellent agreement to those obtained from well-established closed-form methodologies.

1 Introduction

Current and future wireless/mobile communication systems require high received signal-to-noise ratios (SNRs) in order to support applications involved with high data rate transmission. This requirement can be met by using antenna diversity [1–13], which combats the multipath fading being present in the wireless communication channels and boosts the received SNR. Traditionally, the pure diversity combining schemes have been used, that is switched, selection (SC), equal gain (EGC) and maximal ratio (MRC). Although MRC is known to provide the best performance, it comes at the expense of increased hardware complexity. On the other hand, SC offers inferior than MRC performance but is much easier to implement. To overcome this trade-off and exploit simultaneously the advantages of SC and MRC, the generalised selection combining (GSC($N$, $M$)) scheme has been recently proposed and analysed, for example [14–18]. According to this scheme, the $N$ signals with the largest instantaneous SNR are selected out of the $M$ available and further combined under the MRC scheme (GSC is also referred as hybrid selection/maximal-ratio combining). Following this definition, the special cases GSC(1, $M$) and GSC($M$, $M$) are, respectively, the SC and MRC schemes. The diversity gain benefits offered by the GSC over the SC technique come at the expense of a more involved switching circuit, but this overhead is far less than the cost and complexity of the required by MRC additional analogue RF chains [14] which, moreover, are difficult to integrate on the restricted space of the user equipment.

The availability of simulation methods for evaluating and further comparing the diversity performance of different
candidate multi-element antenna (MEA) systems at the early design stage is of paramount importance from the antenna designer's perspective. Simply fulfilling the widely used criteria for achieving diversity performance [i.e. mean effective gain (MEG) ratio $\geq 1$ and envelope correlation coefficient $r_e \leq 0.5$] is not sufficient for selecting the optimum MEA system for a given application. The optimum antenna configuration can be selected according to the achieved effective diversity gain (EDG) [1] (or equivalently diversity antenna gain [3]) or the effective diversity order [11]. In [5], for example, although all the investigated dual inverted F antenna (IFA) systems satisfy the two diversity criteria, their EDG values under the MRC scheme vary in the $6$–$8.4$ dB range. EDG is calculated using the cumulative distribution function (cdf) of the combined signal, which in turn can be determined either by the covariance matrix methodology or statistically using a sufficiently large ensemble. The covariance matrix methodology, a terminology proposed in this work in order to classify the methodologies presented in a large group of studies for example [3–10], uses the closed-form cdf of the combined signal that takes into account the characteristic properties of both the actual MEA system and the operating propagation environment through the closed-form expression of the received signals' covariance matrix [19]. This method has been solely applied to the $M$-branch MRC (GSC($M$,$M$)) for example [3–10] and the two-branch SC (GSC(1,2)) [3] schemes, assuming that the received signals follow the Rayleigh distribution. In these studies a variety of realistic propagation scenarios have been simulated by means of the angular power density functions of the incoming plane waves ($p_{\theta \phi}$ and $p_{\alpha}$) and the cross polarisation power ratio (XPR) [20–22]. This approach for modelling the wireless propagation channel is practical and provides the ability to compare many candidate MEA systems at the early design stage avoiding, thus, the considerable cost and effort of measurement campaigns. These benefits have already been recognised by some popular commercial electromagnetic field software tools [23, 24], which already utilise $p_{\theta \phi}$, $p_{\alpha}$ and XPR for evaluating the diversity criteria only.

The covariance matrix methodology has not been applied, so far, to investigate the GSC($N$, $M$) diversity performance of coupled MEA structures for $N$ $<$ $M$ when $M$ $\geq$ 3. Moreover, this methodology cannot be applied for the evaluation of EDG of coupled MEA systems under reconfigurable termination conditions. This is attributed to the fact that the closed-form expression of the covariance matrix, which includes the characteristic properties of both the coupled MEA system and the operating propagation environment, can be estimated only under fixed terminations. A kind of reconfigurable termination for coupled MEA systems which is of particular practical interest is when $2$ $\leq$ $N$ $<$ $M$ and the non-selected $M$–$N$ elements are open-circuit terminated. In this case, the transferred power from the used antenna ports to the non-used ones is practically eliminated. Coupled MEA systems with reconfigurable terminations can however be treated by determining the cdf of the combined signal stochastically by means of a large ensemble. To the authors’ best knowledge, only [6, 13] determine stochastically the combined signal’s cdf but only for the special MRC and SC cases, respectively, and only for the uniform propagation scenario. As a consequence, the diversity performance of coupled MEA systems under the hybrid GSC($N$, $M$) scheme for both uniform and non-uniform propagation scenarios still remains unaddressed. An ambiguous issue in the stochastic methodologies presented in [6, 13] for evaluating the receive diversity performance concerns the calculation of the receiving voltage vector via the antenna patterns in the transmitting mode. In [6], the receiving open-circuit voltage vector is given in terms of the transmitting element patterns when all other elements are open circuited multiplied by a complex constant without, however, presenting any method for calculating this constant. In [13] on the other hand, the derivation of the Rx open-circuit voltage vector is based on the mistaken assumption that the reception relationship for a single element antenna is also valid for the elements of coupled MEA structures if one uses the active (or embedded according to [13, eqs. (9) and (12)]) element pattern (i.e. the pattern of a single element of the array when all the other elements are present and terminated to loads) instead of the element patterns when all other elements are open circuited.

The above deficiencies are treated in this study by proposing a methodology for creating stochastically the cdf of the GSC($N$, $M$) scheme for coupled and arbitrarily terminated MEA systems when operating in Rayleigh fading environments under realistic propagation scenarios. The Rx mode of a MEA system is modelled using the hybrid electromagnetic-circuit approach presented in Section 2, which is based on the effective length matrix of the MEA system. The effective length matrix is the generalisation of the vector effective length of a single element antenna [25, 26] and can be deduced by simulating the MEA system in its transmitting (Txs) mode of operation using any computational electromagnetic field solver. The equation for the received voltage vector induced at the ports of an arbitrarily terminated MEA system when excited by a uniform plane wave is then derived in terms of the effective length matrix by strictly applying the reciprocity principle [27]. Based on this compact equation and the superposition principle, the ensemble of the received voltage and current vectors at the ports of the MEA system when operating in any propagation scenario with known $p_{\theta \phi}$, $p_{\alpha}$ and XPR is generated stochastically by means of a novel methodology presented in Section 3. From this ensemble the MEA diversity performance metrics, that is MEG, $r_e$ and EDG, are then determined as shown in Section 4. The proposed methodology is applied in Section 5 to evaluate various GSC schemes on two compact MEA systems comprising two and four
2 Hybrid electromagnetic-circuit analysis of coupled MEA systems

In this section the response of a coupled MEA system under a uniform plane wave excitation is determined. Specifically, the received voltages for arbitrary termination loads are related to the effective length matrix by applying the reciprocity principle proved in [27]. Given that the effective length matrix is calculated at the MEA system’s Tx mode of operation, both the Tx and receiving (Rx) modes of operation need to be analysed as presented in the following.

2.1 Circuit representation of the transmitting and receiving modes

The Tx and Rx modes of an $M$-port MEA system are analysed using the circuit representation depicted in Fig. 1. The scalar circuit quantities involved with the Tx mode of operation are related by

$$v_S = Z_S i_T + Z_i_T v_T \quad (1)$$

where $v_S$ and $Z_S$ are the $M \times 1$ voltage vector and the $M \times M$ $Z$-parameter matrix of the source, respectively, $i_T$ and $v_T$ are the $M \times 1$ current and voltage vectors at the MEA’s input terminals and $Z$ is the $M \times M$ $Z$-parameter matrix of the MEA. Similarly, in the Rx mode of operation, the $M \times 1$ open-circuit voltage vector $v_{oc}$ is given by

$$v_{oc} = Z^T i_L + Z_i L v_L \quad (2)$$

where $i_L$ and $v_L$ are the $M \times 1$ current and voltage vectors at the MEA’s termination load, which is characterised by the $M \times M$ $Z$-parameter matrix $Z_T$, and the superscript $T$ denotes the transpose operation. It is noted that in (2) the transpose matrix $Z^T$ is used instead of $Z$, given that for a non-reciprocal MEA $Z_{Rx} = Z_{Tx}^T$ [27].

2.2 Electromagnetic analysis of the transmitting mode

To proceed with the vector electromagnetic analysis of the MEA’s Tx mode of operation, a spherical coordinate system with its origin $O$ located at the MEA’s phase centre is defined as shown in Fig. 2. The position vectors $r' = r' \hat{a}_r$ and $r = r \hat{a}_r$ point, respectively, to an arbitrary point $P$ inside the volume $V$ occupied by the antenna and to an observation point $P$ located in the far-field antenna region where the phasor of the radiated electric field is $E_T(\theta, \phi)$. To keep the mathematical expressions compact, in the rest of this section, the $(\theta, \phi)$ dependence will be suppressed but understood.

The vector electromagnetic quantities can be connected to the scalar circuit ones using the vector effective length $\epsilon_{ve} = \xi_0 \hat{a}_r + \xi_0 \hat{a}_\phi (m = 1 \text{ to } M)$, which by extending its definition [25, 26] in order to characterise coupled MEA systems, is defined by

$$\epsilon_{\theta/\phi_m} \triangleq \frac{F_{\theta/\phi_m}}{F_{\theta/\phi_m}}$$

where the slash ‘/’ stands for ‘OR’ and the subscript $m$ denotes that the $m$th port of the MEA is excited retaining the remaining open circuit. $F_{\theta/\phi_m}$ is the $\theta/\phi$ component of the far-field radiation vector defined through the current density on the MEA structure [25].

In order to determine the vector effective length $\epsilon_{ve}$, the three-dimensional power gain ($G_{\theta/\phi_m}$) and phase ($\psi_{\theta/\phi_m}$) antenna patterns are needed, which are defined by

$$G_{\theta/\phi_m} \triangleq \frac{4\pi U_{\theta/\phi_m}}{P_{in}}$$

$$\psi_{\theta/\phi_m} \triangleq \arg\left(\frac{F_{\theta/\phi_m}}{F_{\theta/\phi_m}}\right)$$

![Figure 1: Circuit representation of an M-port MEA at its modes of operation](image1)

![Figure 2: A MEA system at its Tx mode of operation](image2)
where \( \omega \) is the angular frequency, \( \mu_0 \) the free space magnetic permeability, \( k \) the wave number, \( U_m \) the radiation intensity and \( P_{in} \) the input power in the \( m \)th antenna port when excited by a current \( i_{T_m} \), the others being open circuited [28]

\[
P_{in} = \frac{1}{2} \text{Re}[i_{H} i_{T}] = \frac{1}{2} \text{Re}[Z_{nm}] |i_{T_m}|^2
\]  

where the superscript H denotes the conjugate transpose. Combining (3)–(5), the \( \theta \) and \( \phi \) components of the vector effective length \( \ell_{\theta/\phi} \), when only the \( m \)th port of the MEA is excited the remaining being open circuited, are expressed as follows

\[
\ell_{\theta/\phi} = \frac{4 \pi \text{Re}[Z_{nm}]}{\omega \mu_0 k} \exp(j \psi_{\theta/\phi})
\]  

In this case, the electric field radiated in the far-field region of the MEA can be expressed through \( \ell_{\theta/\phi} \) as follows [25]

\[
E_{T\theta/\phi} = -j \omega \mu_0 \frac{\exp(-jk r)}{4 \pi r} \ell_{\theta/\phi} e_{T_m}
\]  

The electric field \( E_T \) radiated by a fully excited MEA is then derived by applying the superposition principle using the currents at the antenna’s ports as the independent sources

\[
E_T = -j \omega \mu_0 \frac{\exp(-jk r)}{4 \pi r} L c \ell_T
\]  

where \( L_c \) is the effective length matrix, defined by

\[
L_c \triangleq \begin{bmatrix} \ell_{\theta_1} & \ell_{\phi_1} & \cdots & \ell_{\theta_1} & \ell_{\phi_1} & \cdots & \ell_{\theta_M} & \ell_{\phi_M} \end{bmatrix}^T
\]  

The effective length matrix relates the circuit to the electromagnetic quantities in a convenient mathematical way and will be valuable in the analysis of the Rx mode of coupled and arbitrarily terminated MEA systems.

### 2.3 Receiving mode under a uniform plane wave excitation

In order to determine the voltage vector at the ports of an arbitrarily terminated Rx MEA, consider that the same \( M \)-port antenna system of the previous sub-section is now illuminated by a uniform plane wave as shown in Fig. 3. The origin of the spherical coordinate system is again placed at the phase centre of the MEA, where the phasor of the incident electric field intensity is \( E_R \).

The derivation of the received voltage vector \( v_L \) is based on the electromagnetic analysis of MEA systems presented in [27]. In that work it has been proved by means of the Lorentz’s reciprocity theorem that the Tx and Rx modes of operation are connected through the following relationship

\[
v_T^T i_L + v_T^T v_L = -(j \omega \mu_0)^{-1} \frac{\exp(-jk r)}{4 \pi r} E_T^T E_R
\]  

Substituting (8) into (10) and taking into account that \( v_T = Z_T v_T \), it follows that

\[
Z^T i_L + v_L = L_c E_R
\]  

where \( I \) is the \( M \times M \) identity matrix. In the special case of an open-circuited Rx MEA, the voltage vector \( v_{oc} \) follows from the comparison of (11) with (2) to be

\[
v_{oc} = L_e E_R
\]  

which is the generalisation of the well-known formula \( v_{oc} = L_e E_R \) which holds for single-element antennas [26]. To the best of our knowledge, the characterisation of coupled MEA systems at their receiving mode through (13) has been utilised only in [29] without a proof though. Moreover, Kislinasky et al. [29] does not provide a procedure for determining the effective length matrix at the early design stage by means of any computational electromagnetics field solver.

### 3 Receiving mode of coupled MEA systems in realistic propagation scenarios

In order to evaluate the diversity performance of a coupled MEA system when operating in a Rayleigh fading environment under a realistic propagation scenario, the determination of the received voltages and currents at its termination ports for a sufficiently large number of channel
snapshots \( N_s \) is required. Each snapshot of an arbitrary propagation scenario can be synthesised by the superposition of \( L \) independent uniform plane waves [30] arriving at the Rx MEA system’s phase centre (Fig. 4). For each snapshot \( \langle \rangle \) (\( i = 1 \) to \( N_s \)) thus, the \( M \times 1 \) received voltage and current vectors for any desired termination \( Z_L \) can be calculated by applying the superposition principle to (12)

\[

\begin{align*}
\mathbf{v}^{(i)}_L &= \sum_{l=1}^{L} (I + Z^T Z_L^{-1} l L_s \mathbf{E}^{(i)}_{RL}) \\
\mathbf{i}^{(i)}_L &= Z_L^{-1} \mathbf{v}^{(i)}_L
\end{align*}
\]

(14)

(15)

where \( \mathbf{E}_{RL} \) is the electric field intensity of the \( l \)th incident plane wave (\( l = 1 \) to \( L \)). Note that the termination circuit characterised by \( Z_L \) in (14) and (15) can be different for every snapshot allowing thus the application of reconfigurable termination conditions. According to (14) and (15), the whole ensemble of the random vectors \( \mathbf{v}_L \) and \( \mathbf{i}_L \) can be created using \( N_s \) snapshots of \( L \) plane waves each. Specifically, this can be accomplished by generating random values for the incident electric field intensity \( E_R \) and the elevation and azimuth angles \( \theta, \phi \) of the direction of arrival (DoA).

The stochastic generation of the DoA elevation and azimuth angles \( \theta, \phi \) is achieved by using the angular power density functions \( p_\theta(\theta, \phi) \) and \( p_\phi(\theta, \phi) \) of the incoming plane waves which describe the propagation scenario under investigation [20]–[22]. If the power density function for the elevation angle \( p_\theta(\theta, \phi) \) is independent of that for the azimuth \( p_\phi(\phi) \), then it holds that

\[

\int_{0}^{\pi} p_\theta(\theta, \phi) \sin \theta \ d\theta \int_{0}^{2\pi} p_\phi(\phi) \ d\phi = 1
\]

(16)

where \( f_{\theta/\phi}(\theta) \) is the probability density function (pdf) of the elevation DoA angle \( \theta \) [22]. From (16) it stems out that \( p_\theta(\theta, \phi) \), apart from being a power density function, it is also the pdf of the DoA azimuth angle \( \phi \).

In order to generate the random variables \( \theta, \phi \) according to their pdf, the fundamental transformation law of probabilities [31] is employed, which states that any desired random variable may be generated through the uniform distribution. Let us thus assume that \( v \) and \( u \) are two uniformly distributed random variables in \([0,1]\)

\[

\int_{0}^{1} \rho(v)dv = \int_{0}^{1} \rho(u)du = 1
\]

(17)

where \( \rho(v) \) and \( \rho(u) \) are their pdf’s. For arbitrary \( f_{\theta/\phi}(\theta) \) and \( p_{\theta/\phi}(\phi) \), the transformation law implies that

\[

\begin{align*}
\mathbf{v}^{(i)}_l &= \int_{0}^{\phi} f_{\theta/\phi}(\theta) \ d\theta \\
\mathbf{i}^{(i)}_l &= \int_{0}^{\phi} p_{\theta/\phi}(\phi) \ d\phi
\end{align*}
\]

(18)

By generating \( NL \) samples of the random variables \( u \) and \( v \) and solving (18) for the \( \theta \) and \( \phi \) angles, analytically when possible or numerically otherwise, the desired DoA ensemble is created.

To complete the stochastic generation of a realistic propagation scenario, the complex random values of the \( \theta \)– and \( \phi \)-polarisation components of the electric field intensity \( E_R \) are generated for a specific value of the XPR, which is mathematically defined by

\[

\text{XPR} = \frac{E[(|E_R|^2)]}{E[|E_R|^2]}
\]

(19)

where \( E[\cdot] \) denotes the ensemble average operation. In order to simultaneously satisfy (19) and create a Rayleigh distributed multipath environment, the \( \theta \)– and \( \phi \)-components of the electric field \( E_R \) are expressed through the zero mean complex Gaussian random variables \( \omega \) and \( y \) by

\[

\begin{align*}
E_{RL,\theta}^{(i)} &= \sqrt{\text{XPR} \omega_l^{(i)}} \\
E_{RL,\phi}^{(i)} &= \gamma_l^{(i)}
\end{align*}
\]

(20)

More precisely, \( \omega \) and \( y \) are independent complex random variables having their real and imaginary parts independently and normally distributed with zero mean and \( \sigma \) standard deviation \( \mathcal{N}(0, \sigma^2) \). In summary, the whole ensemble of \( N_s \) vectors \( \mathbf{v}_L \) and \( \mathbf{i}_L \) can be created through (14) and (15) using the \( N_sL \) values for the electric field intensity \( E_R \) generated via (20) and the DoA angles \( \theta \) and \( \phi \) via (18).

![Figure 4 MEA system of Fig. 2 excited by L uniform plane waves that synthesize a snapshot of an arbitrary propagation scenario](image-url)
4 Receive diversity performance evaluation metrics

MEA systems can be very efficient in combating power losses because of multipath fading by exploiting their inherent spatial, pattern and polarisation diversity. In general, the amount of improvement in the power received by a MEA system, relative to that received by a single element one, depends on the two diversity criteria that is the envelope correlation coefficient \( \rho_u \) between the received voltage signals \([12, 21]\) and the logarithmic difference among the MEGs \([20]\) of the Rx antenna elements. However, the exact enhancement in received power can only be quantified through the EDG \([1]\). In the following, the diversity performance evaluation metrics \( \rho_u \), MEG and EDG are determined from the ensemble of the voltage and current vectors which are generated by the stochastic electromagnetic-circuit methodology presented in the previous two sections.

4.1 Envelope correlation coefficient

The correlation between the received voltages at the \( m \)th and \( n \)th antenna termination loads is quantified by the envelope correlation coefficient \( \rho_{en} \) which is expressed through the complex correlation coefficient \( \rho_{en} \) by \([12]\)

\[
\rho_{en} \simeq |\rho_{en}|^2 \Delta \frac{\text{cov}(v_{en}, v_{en})}{\text{stdv}(v_{en}) \text{stdv}(v_{en})}^2
\]

where \( \text{cov}(\bullet) \) and \( \text{stdv}(\bullet) \) stand for the covariance and standard deviation operations, respectively.

4.2 Mean effective gain

The MEG of the \( m \)th antenna element is defined in \([20]\) as the ratio of the mean received power to the mean incident power at the same antenna element

\[
\text{MEG}_m = \frac{\Delta E[P_m]}{E[P_{inc}]} \tag{22}
\]

The received power \( P_m \) by the \( m \)th antenna element for every snapshot \( \psi \) is given by

\[
P_m^{(\psi)} = \frac{1}{2} \text{Re}\{e^{i\psi} (v_{m}^{(\psi)} v_{m}^{(\psi)})^T_{(\psi)} \}
\]

The incident power \( P_{inc} \) at the same antenna element is calculated by means of an ideal reference antenna \([20]\) with unit \( \theta \) and \( \phi \) power gain patterns and no mismatch at its port. The \( \theta \) and \( \phi \)-polarisation components of the vector effective length of the reference antenna is found by using \([6]\) for a single element antenna with \( Z_{ref} = Z_{0}, G_{\theta,\phi}^{ref} = 1 \) and \( \theta_{\theta,\phi}^{ref} = 0 \). The incident power on the \( m \)th antenna element for every snapshot \( \psi \) is equal to the power \( P_{ref} \) received by this ideal reference antenna \([19]\) when operating in the same propagation environment. The voltage and current signals received by the reference antenna which are needed for the calculation of \( P_{ref} \) are obtained from \((14)\) and \((15)\) for a single element antenna.

4.3 Effective diversity gain

The metric that quantifies the amount of improvement obtained when using a MEA system instead of a single element one is the EDG \([1]\). EDG at a specific outage probability level \( p_{\%} \) is expressed in a linear scale as

\[
\text{EDG} = \frac{X_1}{X_2} \left[ P(P_{div} \leq x_1) = P(P_{ref} \leq x_2) = p_{\%} \right] \tag{24}
\]

where \( P(P_{div} \leq x) \) is the cdf of the received power of the combined signal and \( P(P_{ref} \leq x) \) is the cdf of the power received by the reference antenna. These cdfs are calculated statistically using the voltage and current ensembles at the RF front end. For the GSC scheme, \( P_{div} \) for each snapshot is given by

\[
P_{div}^{(\psi)} = \frac{1}{2} \text{Re}\{e^{i\psi} (v_{GSC}^{(\psi)} v_{GSC}^{(\psi)})^T_{(\psi)} \}
\]

where the whole ensemble of the \( N \times 1 \) voltage and current vectors \( v_{GSC}^{(\psi)} \) and \( r_{GSC}^{(\psi)} \) is obtained by selecting for every snapshot the \( N \) elements that provide the largest instantaneous SNR.

5 GSC diversity performance of coupled MEA systems operating in a uniform multipath environment

In this section, the simulated GSC diversity performance of two compact MEA systems when operating in a uniform multipath environment is investigated. Although other realistic propagation environments could be investigated as well, the uniform one is selected because of its simplicity and the fact that it is a fair approximation for many indoor and outdoor propagation scenarios \([9]\).

5.1 Investigated MEA systems

The geometry and dimensions of the investigated compact MEA structures printed on a wireless terminal device are depicted in Fig. 5. This device has the typical dimensions of a PC card, whereas the dimensions of the ground plane are \( 45 \text{ mm} \times 90 \text{ mm} \) and consists of two \( 35 \mu \text{m} \) thick copper layers with the antennas placed at the upper one and the ground plane at the bottom. The antenna elements used are the inverted F monopoles, which were selected because of their compact size, large bandwidth, omnidirectional radiation patterns, no additional fabrication cost, and ease of tuning \([32]\). The antenna elements are printed at the edge of the device’s ground plane on an \( 8 \text{ mils} \)-thick substrate with \( e_r = 3.38 \) and \( \tan \delta = 0.002 \) and are well tuned to the \( 5.2 \text{ GHz} \) ISM band \([5, 8, 9]\). It is worth mentioning that the two-port MEA of Fig. 5
provides the best diversity performance among many different dual IFA systems as reported in [5]. The 3D power gain and phase antenna patterns were computed using IE3D, a commercial method of moment-based EM field solver [23].

5.2 Stochastic generation of a uniform propagation scenario

The uniform propagation scenario is created as described in Section 3 using \( p_{\theta,\phi}(\theta) = 1/2, p_{\phi}(\phi) = 1/2\pi \) and XPR = 1. In this study, 20,000 snapshots \( (\mathbf{N}_s = 20,000) \) of 20 plane wave sources \( (L = 20) \) [33], which are uniformly distributed over a sphere surrounding the multi-port antenna are generated. The \( \theta \) and \( \phi \) angles of the DoA ensemble are calculated by solving (18) analytically and using the uniformly distributed in \([0, 1]\) random variables \( u \) and \( v \), as follows [34]

\[
\theta_i^{(s)} = \cos^{-1}(2v_i^{(s)} - 1) \\
\phi_i^{(s)} = 2\pi u_i^{(s)}
\]  

Since the real and imaginary parts of the \( \theta \)- and \( \varphi \)-components of the incident electric field \( \mathbf{E}_R \) are generated to follow the normal \( \mathcal{N}(0, 1/2) \) distribution, a Rayleigh multipath environment is created. In this manner, it is possible to compare the results of the proposed methodology to those obtained by the covariance matrix method for the special case of the MRC scheme.

5.3 GSC diversity performance simulation results

The diversity performance of the two MEA systems of Section 5.1 is evaluated by means of the stochastic electromagnetic-circuit methodology presented in Sections 2 and 3 under either a fixed or a reconfigurable termination scheme. In the fixed termination scheme, the termination impedances at all antenna ports are equal to the characteristic impedances of the transmission lines \( Z_0 = 50\Omega \). On the other hand, for every snapshot \( (s) \) of a reconfigurable scheme, the used antenna elements are \( Z_0 \) terminated, whereas the non-used are retained open circuited. In order to validate the proposed stochastic methodology, the simulated results of MEG, \( \rho_e \) and EDG under the MRC scheme in a uniform propagation scenario under the fixed termination scheme will be initially presented and further compared to those obtained using existing analytical methodologies. The stochastic methodology will subsequently be applied to various GSC schemes with both the fixed and the reconfigurable terminations, including cases where the covariance matrix methodology does not apply.

The MEGs of the antenna elements and the envelope correlation coefficients of the received voltage signals are compared to the results obtained by the closed-form methods presented in [20, 21], respectively. The calculated MEGs of the Rx antenna elements for the two- and the four-IFA systems are shown in Table 1. Owing to the symmetry of the antenna designs of Fig. 5, the MEG values of the symmetrical elements are the same. The envelope correlation coefficients \( \rho_e \) of the received signals calculated by the proposed methodology and the method presented in [21] are listed in Table 2. From Tables 1 and 2 it is concluded that the results from the proposed stochastic method and the widely used closed-form ones are in excellent agreement. Moreover, both MEA systems

<table>
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<th>Table 1</th>
<th>Simulated MEGs of the antenna elements</th>
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<td></td>
<td>2-IFA</td>
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<tr>
<td>MEG_(1/2)</td>
<td>MEG_(3/2)</td>
</tr>
<tr>
<td>closed-form method [20]</td>
<td>-3.7</td>
</tr>
<tr>
<td>stochastic method</td>
<td>-3.7</td>
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</table>
of Fig. 5 easily satisfy the criteria for achieving diversity gain. The EDGs obtained under the two- and four-branch MRC schemes at the 1% outage probability level as extracted from Fig. 6 and listed in Table 3 are in very good agreement with those obtained by the covariance matrix methodology presented in [9, 10]. This methodology is based on the works presented in [3, 4] which utilise MEG and ρe for evaluating the covariance matrix of a two-branch MRC diversity scheme. In Fig. 6, besides the cdf’s of the MRC combined signals against relative SNR, the cdf of the dual-isotropic reference antenna is also included which, as expected, is in excellent agreement with the Rayleigh distribution.

The EDG results of the stochastic methodology at the 1% outage probability level for the reconfigurable terminated four-IFA system under the GSC(1,4), GSC(2,4) and GSC(3,4) schemes are listed in Table 4. The corresponding cdf’s of the combined signals are shown in Fig. 6 except that of the GSC(3,4) scheme, which coincides with the cdf of the GSC(4,4). As can be seen in Table 4 the EDG benefits of the reconfigurable termination over the fixed one are 0.5, 0.7 and 0.3 dB for the three GSC schemes, respectively. All calculations needed to estimate MEG, ρe and EDG were performed using a separately created Mathcad [35] worksheet running on an Intel core quad processor at 2.5 GHz. On this computer, for example, the calculation of these metrics for the GSC(4,4) scheme required 571 MB of RAM and 1.3 min computation time.

Summarising the results, the higher and lower performance from the investigated diversity schemes is obtained by the GSC(4,4) and GSC(2,2), respectively, which offer 14.1 and 7.8 dB EDG at the 1% outage probability level over the Rayleigh reference. The results for the GSC(2,4) and GSC(3,4) schemes under the reconfigurable termination conditions cannot be evaluated using the covariance matrix method, as already explained in Section 1. As a consequence the EDGs provided by these two schemes can only be determined using the stochastic method and the results reveal that their performance is almost similar to that of the GSC(4,4) scheme. Their advantage over GSC(4,4) is that they require less analogue RF chains at the expense, however, of the extra cost and complexity involved with the required switching circuit. The GSC(1,4) scheme, on the other hand, provides lower EDG than the GSC(2,4) and GSC(3,4) schemes with the great advantage however that it can be implemented in a straightforward manner on existing transceivers with only one analogue RF chain. This scheme offers 11.3 dB EDG in the available at the RF front end SNR, which can be

Table 2 Simulated envelope correlation coefficients of the received signals

<table>
<thead>
<tr>
<th></th>
<th>2-IFA</th>
<th></th>
<th>4-IFA</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ρe1,2</td>
<td>ρe1,3</td>
<td>ρe1,4</td>
<td>ρe2,3</td>
</tr>
<tr>
<td>closed-form method [21]</td>
<td>2.30 × 10⁻⁴</td>
<td>9.30 × 10⁻⁴</td>
<td>4.39 × 10⁻³</td>
<td>2.87 × 10⁻²</td>
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<tr>
<td>stochastic method</td>
<td>1.11 × 10⁻⁴</td>
<td>8.39 × 10⁻⁴</td>
<td>4.55 × 10⁻³</td>
<td>3.07 × 10⁻²</td>
</tr>
</tbody>
</table>

Table 3 EDG at 1% outage probability level of the two- and four-IFAs under the MRC scheme

<table>
<thead>
<tr>
<th></th>
<th>GSC(2,2)</th>
<th>GSC(4,4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.i.d. Rayleigh</td>
<td>11.7</td>
<td>19.1</td>
</tr>
<tr>
<td>covariance matrix method [9, 10]</td>
<td>8.0</td>
<td>14.3</td>
</tr>
<tr>
<td>stochastic method</td>
<td>7.8</td>
<td>14.1</td>
</tr>
</tbody>
</table>

Table 4 EDG at 1% outage probability level of the four-IFA under the GSC scheme

<table>
<thead>
<tr>
<th></th>
<th>GSC(1,4)</th>
<th>GSC(2,4)</th>
<th>GSC(3,4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.i.d. Rayleigh</td>
<td>15.8</td>
<td>18.0</td>
<td>18.8</td>
</tr>
<tr>
<td>reconfigurable termination</td>
<td>11.3</td>
<td>13.6</td>
<td>14.1</td>
</tr>
<tr>
<td>fixed termination</td>
<td>10.8</td>
<td>12.9</td>
<td>13.8</td>
</tr>
</tbody>
</table>

Figure 6 cdf’s of the combined signals calculated using the proposed stochastic method
used for improving the data rate and/or for reducing the power consumption on the user equipment.

6 Conclusion

This paper has introduced a novel stochastic electromagnetic-circuit methodology for evaluating the GSC diversity performance of coupled MEA systems when operating in Rayleigh multipath environments. The method provides many degrees of freedom, since it can be applied for any desired termination conditions and arbitrary realistic propagation scenarios that are generated stochastically using the angular power density functions of the incoming plane waves and the XPR. The proposed model is based on the effective length matrix, which is used to relate the electromagnetic to the circuit quantities at the receiving mode of any MEA system. The elements of this matrix are for the first time conveniently calculated in the transmitting mode using any computational electromagnetics field solver.

The GSC diversity performance of two compact MEA systems comprising two and four IFAs when operating in a uniform propagation scenario at the 5.2 GHz band was investigated under various GSC schemes. The calculated EDG by the proposed stochastic methodology for the two MRC schemes [GSC(2,2) and GSC(4,4)] was found to agree very well with the EDG obtained using the covariance matrix methodology. A distinct advantage of the presented stochastic methodology is its ability to model coupled MEA systems under reconfigurable termination schemes where the covariance matrix methodology cannot be applied. The GSC(2,4) scheme under the specific reconfigurable termination, where the used antenna elements are 50 Ω terminated whereas the non-used are retained open circuited, provides an EDG which is very close to that obtained by the four-branch MRC but with less analogue RF hardware. The GSC(1,4) scheme, on the other hand, can be easily implemented on existing transceivers with only one analogue RF chain with its 11.3 dB EDG being available to improve the data rate and/or reduce the power consumption on the user equipment.

Future work could investigate the diversity performance of coupled MEA systems when operating in realistic non-uniform propagation scenarios under both pure and hybrid diversity combining schemes. For the cases of the GSC(1, M) with the kind of reconfigurable termination proposed in this work and the GSC(N, M) with N < M, M ≥ 3 and fixed termination, this investigation could be performed by applying either the covariance matrix or the stochastic approach. For the GSC(N, M) scheme with the proposed reconfigurable termination when 2 ≤ N < M, however, the diversity performance of coupled MEA systems could only be evaluated by means of a stochastic methodology like the presented in this work.

7 Acknowledgments

The authors are grateful to Professor Vassilios Makios for his over the years valuable support and Lecturer Panayiotis Vafeas for a fruitful discussion.

8 References


