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Bansho: Visually Sequencing Mathematical Ideas
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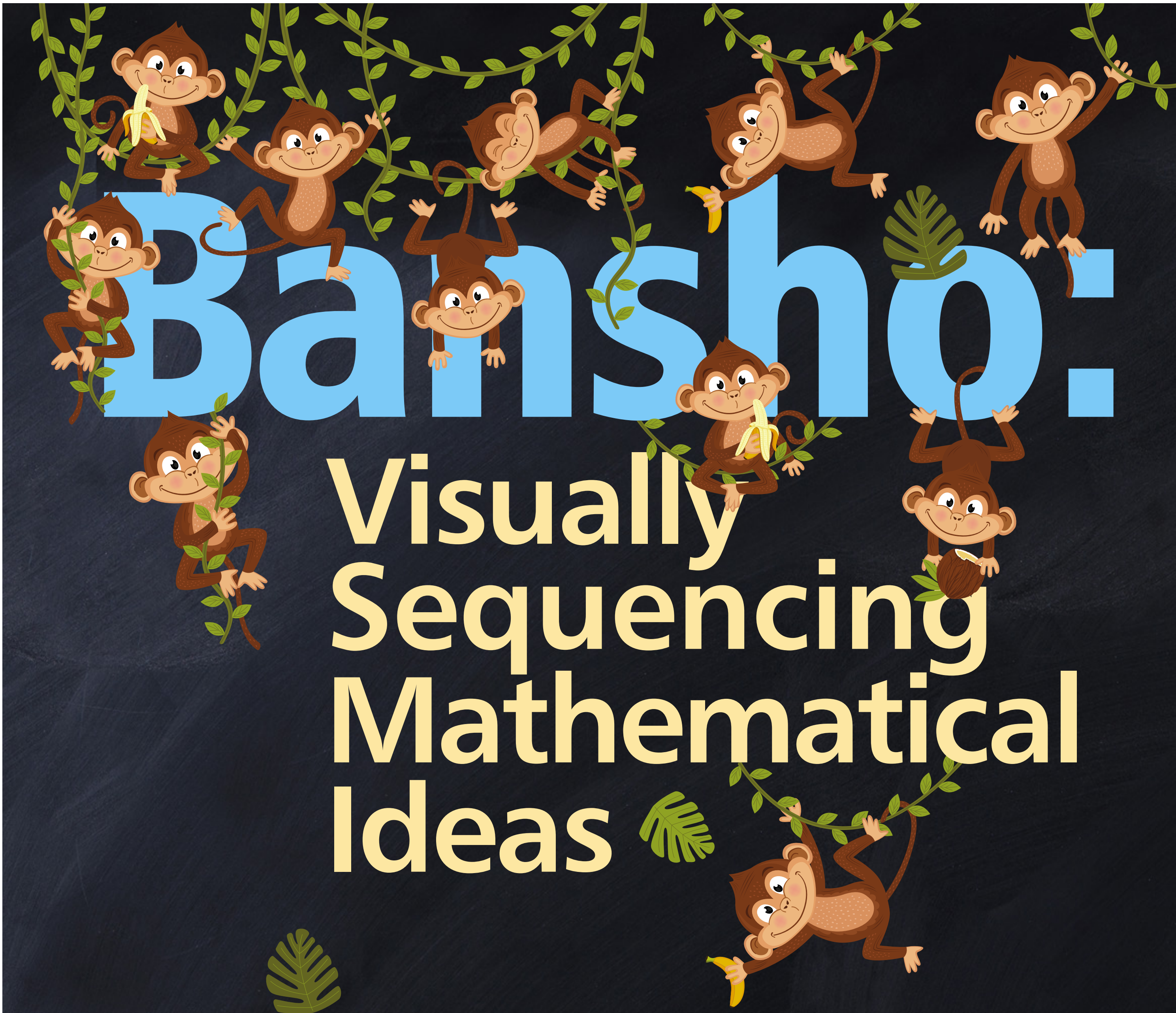

Bansho: Visually Sequencing Mathematical Ideas

Use this Japanese organizational strategy to facilitate multiple problem representations and better classroom communication.

CREATIVEIMAGES (STUDENT); NATAKA (MONKEYS)/THINKSTOCK

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Eloise R. A. Kuehnert, Colleen M. Eddy, Daphyne Miller, Sarah S. Pratt, and Chanika Senawongsa

Daphyne Miller, a first-grade teacher, poses a problem to her students and places it on the left side of the board space for students to reference throughout the lesson:

> Five monkeys are eating fruit, and six monkeys are playing. How many monkeys are there?

Why use the left side of the board? Have you given much thought to how you organize information on your board space during a lesson? In this article, we describe the Japanese term *bansho*, which refers to the intentional use of board space for facilitating student learning.

Bansho and the three-phase lesson

Bansho describes the organization of the board space as an essential "instructional tool for organizing students' thoughts" (Fernandez and Yoshida 2004, p. 235). Japanese teachers use the board to keep a record of the lesson, organize student thinking, and discover new ideas (Yoshida 2004; Ninomiya 2010). The intentional use of the board focuses students' attention on the posed problem and helps "facilitate meaningful mathematical discourse" (NCTM 2014, p. 10). Teachers anticipate student responses and plan the layout of the board. This includes recording problems, student-invented strategies, and core principles on the board. The bansho is used as a running record of student-generated ideas to connect ideas during the whole-class discussion (Yoshida 2004). Other countries, such as Thailand, have adapted bansho. In this article, we describe Miller's implementation of a lesson designed by a collaborative team of Thailand teachers and researchers (Inprasitha 2009).

Bansho divides the board space into sections to correspond to three phases in a lesson cycle: (1) activating prior knowledge, (2) exploring a problem, and (3) discussing and extending. **Table 1** outlines the purpose of each section in relation to the lesson cycle. In phase 1, the teacher introduces a prompting image to activate prior knowledge. The teacher posts keywords and mathematics vocabulary in the left-most column of the board under the image. Phase 1 concludes with the teacher presenting a problem situation connected to the image.

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TABLE 1

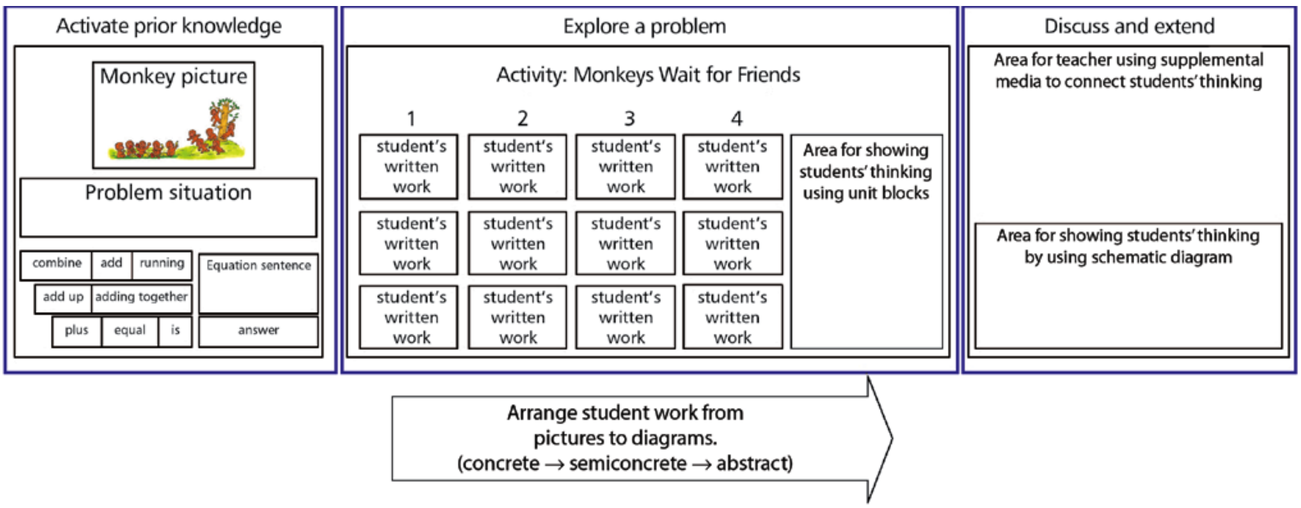
The Japanese organizational strategy of bansho divides board space into sections; this table outlines the purpose of each section in relation to the lesson cycle.

Purpose of the board space using bansho

Activate prior knowledge	Explore a problem	Discuss and extend
• Post prompting image • Post keywords, math vocabulary, and current understandings of students' • Post problem situation	• Systematically display various student-generated solutions from concrete to semiconcrete to abstract ideas	• Apply new knowledge • Display important summarizing ideas • Conclude lesson and connect to future lessons

FIGURE 1

A planned bansho for a first-grade lesson on addition shows the result of the bansho as a visual display of the collective classroom conversation. Miller adapted phase 3 using student-created word problems.



Sources: Hitotsumatsu, Okada, and Machida 2005; Inprasitha 2009

In phase 2, the teacher challenges students to explore the posed problem and document their thought processes on blank paper. Students are encouraged to represent the mathematics using various models. The teacher monitors and selects students to share their thoughts, intentionally ordering the ideas from concrete to semiconcrete to abstract. Students are challenged to describe the thinking of other students during presentations, and each presenter includes a connection to a previously presented idea. This provides all students access to the problem and fosters whole-class discussion. Similar to number talks, students communicate their thinking and justify their solutions (Parrish 2011).

In phase 3, students apply their new knowledge of the key ideas in the right-most side of the board space, with the teacher guiding them to connect to future lessons (Sisavath, Inprasitha, and Inprasitha 2012). The result of the bansho is a visual display of the collective classroom conversation (see **fig. 1**).

The math lesson

The Thai lesson focuses mathematically on surveying students' writing addition equations and methods for solving the problem. The monkey picture (see **fig. 2**) and problem situation promote mathematical communication and the use of multiple representations in solving the problem. As Miller plans the lesson for her classroom, she considers the original intent of the lesson and adapts it to her students.

Similar to Smith and Stein (2011), the lesson plan highlights the significance of *anticipating* and *sequencing* student responses, such as pictorial models, decomposing and composing numbers, and number sentences. In planning the lesson, Miller considers the anticipated student responses (see **fig. 3**). She integrates fundamental mathematical ideas, such as applying

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strategies for addition that the class studied in previous lessons; she supplies the class with counters, base-ten blocks, and linking cubes.

Miller anticipates that her students (1) will use base-ten blocks and linking cubes and (2) will produce drawings that are similar to the Thai students' drawings. However, she sees her students moving toward number sentence fact families as opposed to the schematic drawings. She plans four columns during phase 2 of the board space as follows:

1. Drawings of monkeys or other pictures
2. Drawings with numbers
3. Drawings with number sentences
4. Number sentences (fact families)

She will use these columns to direct students to place their thinking under one of them as the board space is organized with student thinking.

Bansho phases in the classroom

After anticipating solutions and planning the placement of student ideas, Miller implements the lesson. The next section outlines the lesson as it unfolds in Miller's classroom.

Phase 1: Activating prior knowledge

[*Post an image to activate prior knowledge leading to the problem situation.*] Miller begins by displaying the picture (see **fig. 2**) in the upper-left corner of the board space. In pairs, students document their initial understandings on index cards.

Miller: Think about some words. What do you see happening in the picture?

Aiden: They're running.

Evan: They're climbing.

Taylor: [They are] looking out of a tree.

Olivia: They want bananas.

[*Post student-generated words and phrases underneath the image.*] Miller tapes the phrases on the board and probes further.

Miller: Turn to your neighbor, and make a sentence [about the picture] with numbers in it. What is something your neighbor said?

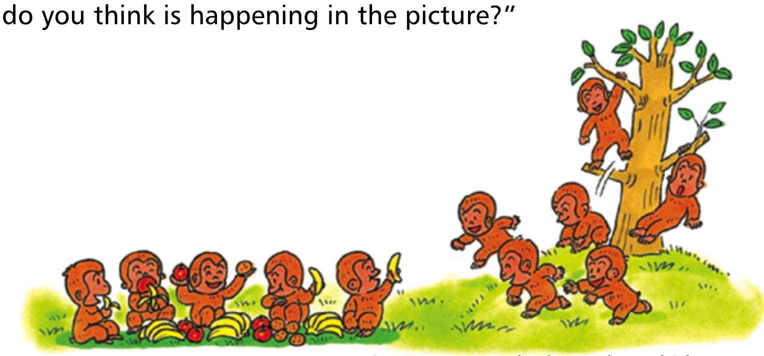
Xavier: Three of them are jumping.

Marian: There are sixteen bananas.

Wesley: Two are climbing a tree.

FIGURE 2

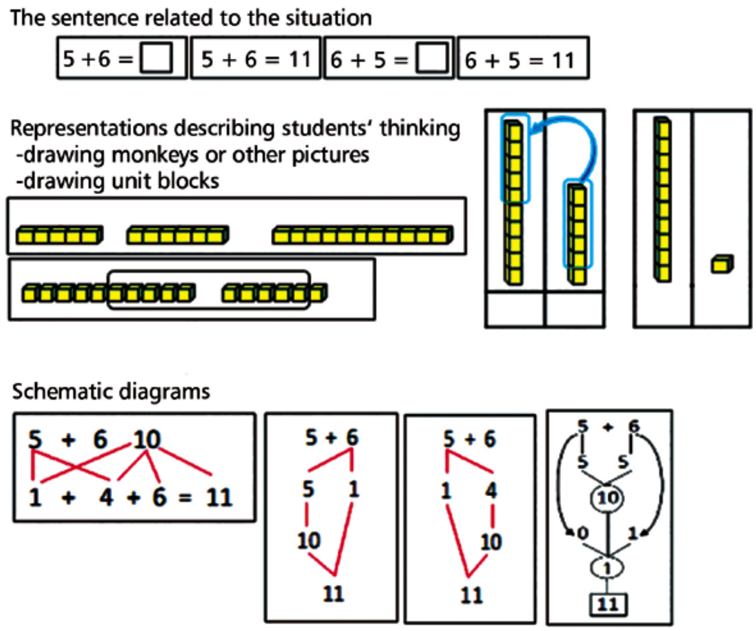
This image of monkeys is initially presented to students and placed in the left-most column of the board without the word problem. Students are asked, "What do you see?" or "What do you think is happening in the picture?"



Source: Hitotsumatsu, Okada, and Machida 2005

FIGURE 3

The original Thai bansho lesson included such anticipated responses as pictorial models, decomposing and composing numbers, and number sentences. Miller adapted this information to anticipate and sequence her student responses.



Source: Inprasitha 2009

Miller then posts the problem situation, the picture, and initial student responses in the Activate Prior Knowledge section of the board.

> Five monkeys are eating, and six monkeys are playing. How many monkeys are there?

She prompts students to write their thoughts as they read the problem.

Phase 2: Exploring and presenting student ideas

During phase 2, students are paired to document their problem-solving ideas. Miller

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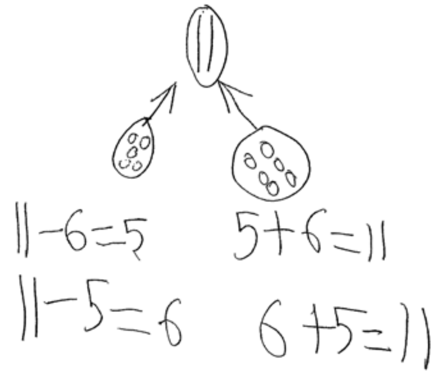
FIGURE 4

Below are examples of student work during phase 2 of the lesson.

(a) Marian drew five circles and six circles, and after the whole-group discussion, she also incorporated fact families to justify her answer.



(b) Aiden drew his own model using dots in a rectangle. He also used a doubles strategy to justify his answer.

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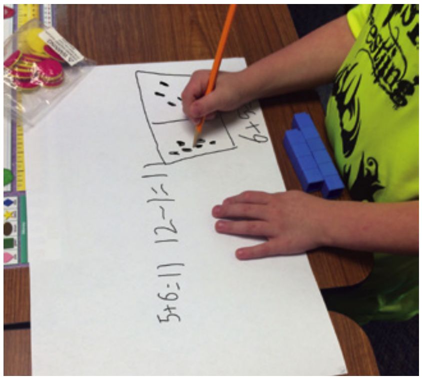
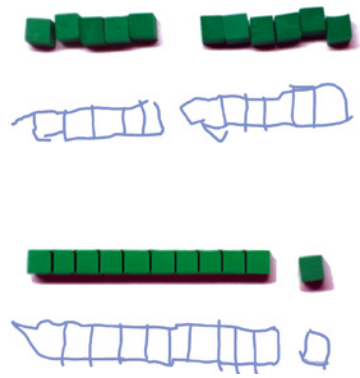


(c) Evan regrouped the five and six into ten and one using a rod and a unit.



acts as a facilitator while monitoring, sequencing, and selecting (Smith and Stein 2011) student ideas for presentations. She circulates around the room to *pose purposeful questions* (NCTM 2014) and to probe students' thinking. For example, she might ask, "What were you thinking when you wrote this?" Students write number sentences to clarify the meaning behind their drawings. Strategies range from describing the problem with linking cubes and base-ten blocks to pictorial models and fact families.

Miller: Work with your partners to name nouns and numbers.

Xavier: [*Counting*] One, two, three . . . eleven!

Samuel: Hmmm. Six and five. Eleven monkeys!

Miller: *Show* me what you think of when you read this. What numbers do you see?

Wesley: Can I have some blocks? [*Wesley uses five green blocks and six blue blocks to show the numbers he sees.*]

Samuel: Five and six.

[*Marian draws a group of five circles and then a group of six circles* (*see* **fig. 4a**).]

Aiden: I'm drawing my own models [*drawing a rectangle split in the middle with six dots on one side and five dots on another* (*see* **fig. 4b**)].

[*Evan picks up a rod and a unit and draws them on his paper* (*see* **fig. 4c**).]

[*Wesley writes, "I have 6 chocolate chip cookies, my friend has five. Count them all."*]

[*Call students forward for presentations, progressing from simple ideas to more complex, and post ideas from left to right.*] As Miller monitors student activity, she selects students to present their unique solutions to the class (Smith and Stein 2011). Moving from left to right on the board, she numbers four columns in which to sequence student ideas (see **fig. 5**) on the basis of anticipated responses. The left-most column starts with concrete student ideas; the solutions will progressively move toward pictorial models and fact families. Miller adds a fifth column to incorporate a student-generated word problem relating to the original situation—an unanticipated response. She then invites all students to sit near the front board to listen and discuss student ideas. The classroom conversation follows along a similar line of thought as number talks through student clarification of ideas leading to a "repertoire of efficient strategies" (Parrish 2011, p. 203). In addition to mental strategies, the bansho process includes students *using and connecting mathematical representations* (NCTM 2014), such as base-ten blocks, and documenting their thinking before engaging in the classroom conversation. Miller calls Marian to the board to explain her thinking and instructs the class to pay attention.

Miller: Listen so you can relate your solution.

Marian: I drew five circles for the monkeys eating and six circles for the monkeys playing [*drawing is posted in the first column*].

Miller: The next person who shares must give a connection.

Aiden: I drew six dots and then five dots instead of circles. I know six doubled is twelve, and I know twelve minus one equals eleven, so six plus five equals eleven [*drawing is posted in the third column*].

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Miller: How are [the strategies] alike? How are they different?

Aiden: They are the same because they both equal eleven.

Xavier: They are not [the same], because six plus five equals eleven, and this has, um, these two are the same.

Olivia: They are different. They have different numbers.

Samuel: But do they start the same?

Miller: Does it matter if they start the same?

Samuel: It matters how you use them.

An interesting discussion ensues as students deliberate whether six plus five and five plus six are the same, which then results in a student-led conversation about whether order matters in the case of eleven minus five and five minus eleven:

Olivia: No, you can't take the big number from the little number; it doesn't make sense!

Wesley: You can still use five minus eleven, because that makes a negative number.

Students have opportunities in this interaction to construct viable arguments and to critique others' reasoning. Wesley is the final student to explain how he changes the problem from monkeys to cookies using the same numbers. Miller places his word problem in the newly created fifth column. To ensure an understanding of the relationship among 5, 6, and 11 before moving on, Miller leads students to connect new information with Samuel's previously investigated idea of fact families.

Once the selected work is displayed, the teacher asks additional facilitation questions to help students connect with the strategies she has sequenced on the board:

Miller: Why is this picture of six plus five under column one instead of column five?

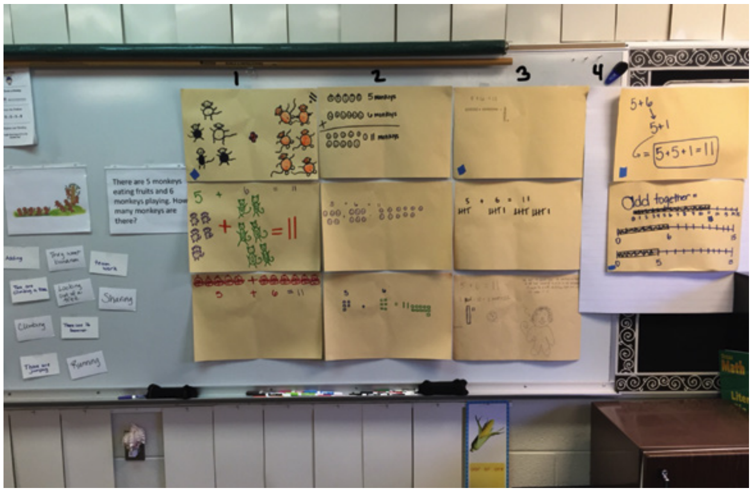
Olivia: They both have circles [*pointing to the two ideas posted in column one*].

Miller: They both have *models*.

Taylor: Oh! And that one has a sentence [*pointing to the word problem in column five*].

FIGURE 5

On the basis of responses that she anticipated from her students, Miller moved from left to right on the board, numbering four columns. She started with concrete student ideas; the solutions will progress toward pictorial models and fact families.



SARAH S. PRATT

Phase 3: Applying new knowledge and summarizing key ideas

Miller continues to give her students opportunities to connect different mathematical ideas to deepen their understandings (NCTM 2014). Curricular coherence occurs as she capitalizes on the emergent word problem in the fifth column as a springboard for the final activity. In observing Miller's use of phase 3 in her class, two strategies are noteworthy. First, before pairing students to solve each other's problem, she ensures each student has a word problem with a valid question. Her emphasis on fact families allows students to create both addition and subtraction problems (see **fig. 6**). Second, after solving his or her partner's problem, Miller asks each pair of students whether the partners solved the problems in the intended manner. Students compare their strategies to those still listed on the bansho. Many students find that their partner used a different method. Identifying similarities and differences in strategies emphasizes the connections among student-generated strategies, leading to *procedural fluency from conceptual understanding* (NCTM 2014).

The original Thai lesson plan connects base-ten blocks to schematic diagrams in phase 3. However, as a result of student engagement in sharing their ideas, combined with student interest about Wesley's self-created word problem, Miller chose to give all students the opportunity to do the same. This was an unplanned

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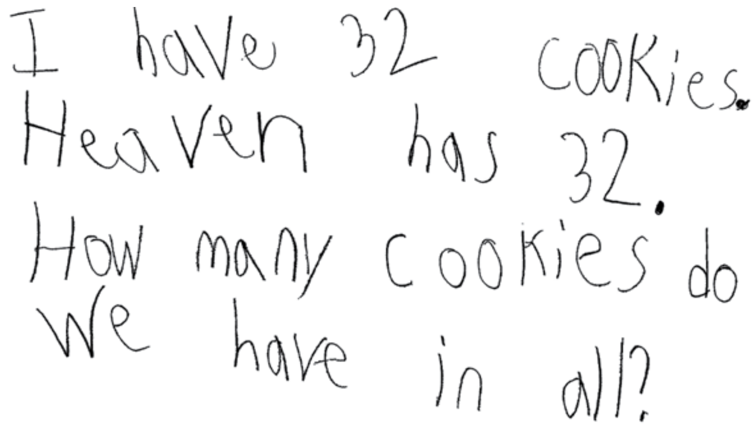
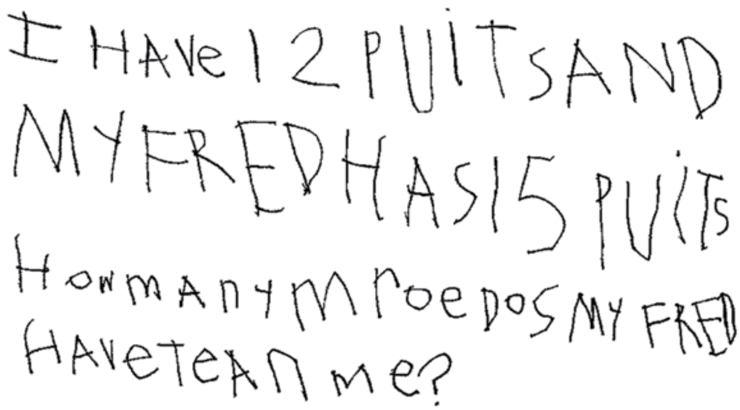
FIGURE 6

Miller ensured that each student had a word problem with a valid question. After solving his or her partner's problem, each pair of students checked the partner's solution and all compared their strategies to those still listed on the bansho. Here are examples of Taylor's and Olivia's created word problems for phase 3 of the lesson.

(a) I have 32 cookies. Heaven has 32. How many cookies do we have in all? (Taylor)



(b) I have 12 points and my friend has 15 points. How many more does my friend have than me? (Olivia)



formative adjustment from the Thai lesson plan, resulting in each student sharing ideas and incorporating what he or she had discovered about applying addition strategies in the lesson. These are both examples of transitions to phase 3 that connect to the classroom conversations. Reflecting on the lesson, Miller notes that several students (e.g., Evan, see **fig. 4c**) regrouped to create a 10 and a 1 with the rod and unit pieces of the base-ten blocks. The class did not discuss regrouping during phase 3 of the lesson. However, this is a central idea that will deepen students' thinking and will serve as a launch for the subsequent lesson on constructing and deconstructing numbers.

An intentional structure and strategy

Bansho offers a structure for sequencing mathematics visually on the board. By purposefully organizing the board space alongside the lesson, teachers can provide students with a framework showing where the lesson starts, the connections within the lesson, and where the lesson is going. Bansho is best suited for problems allowing multiple solutions or representations, and it provides an intentional strategy for fostering mathematical communication in the classroom.

Common Core Connections

1.OA.B.3	SMP 3
1.NBT.C.4	SMP 4
1.OA.C.5	SMP 5

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