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Image formation in the scanning microscope

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Abstract. Fourier imaging in the scanning microscope is considered. It is shown that there are two geometries of the microscope, which have been designated Type 1 and Type 2. Those of Type 1 exhibit identical imaging to the conventional microscope, whereas those of Type 2 (confocal microscopes) display various differences. Imaging of a single point object, two-point resolution and response to a straight edge are also considered. The effect of various arrangements using lenses with annular pupil functions is also discussed. It is found that Type 2 microscopes have improved imaging properties over conventional microscopes and that these may be further improved by use of one or two lenses with annular pupils.

1. Introduction

In the conventional transmission microscope the resolution is primarily determined by the numerical aperture and aberrations of the objective lens, whereas the condenser lens controls the degree of coherence in the imaging process and has only a secondary effect on the resolution. The performance of a scanning transmission microscope, on the other hand, depends on the geometry of the optical system: the microscopes in figure 1 are all equivalent to one another (Type 1), whereas the same is also true for the microscopes in figure 2 (Type 2). These two types of scanning microscope have, however, completely different imaging properties. Those of Type 1 have been shown by reciprocity [1, 2] to be equivalent to a conventional microscope, providing that the properties of the objective lenses are identical, and that the collector lens of the scanning microscope and the condenser lens of the conventional microscope have the same pupil function. This is only true if the effective source in the scanning microscope is infinitesimal, that is if the radiation incident on the objective is coherent and this will indeed be the case for laser illumination. The collector lens of these microscopes is only a light-gathering device, the aberrations of which are unimportant. By changing the angle of the cone of collection the imaging properties of the scanning microscope may be altered from being identical to the coherent imaging of the conventional microscope, through partially coherent imaging to incoherent imaging, even though the scanning probe is always coherent. Although the concept of partial coherence is central to the theory of the conventional microscope, the imaging of the scanning microscope may be explained without reference to this property. In this way there is some advantage in developing a theory of the scanning microscope and then using reciprocity to explain the performance of the conventional microscope.

The microscopes of Type 2 are different from those of Type 1 in that the performance of the former is affected by the aberrations in both lenses. The

imaging has similarities to that of the incoherent conventional microscope [3] even though the incident radiation and the detection are coherent.

In this paper we consider the imaging properties of both types of scanning microscope for objects which are plane structures. We then discuss the special cases of the performance of microscopes with lenses with circular and annular pupil functions.

2. The equivalence of scanning microscopes of Type 1 and conventional microscopes

Let us first consider the scanning microscope of figure 1 (*a*). The scanning probe formed by coherent laser light is assumed stationary, the object being moved mechanically in the focal plane to build up the image point by point. The probe formed in the focal plane of the objective lens is $h_1(x_0, y_0)$, the impulse response

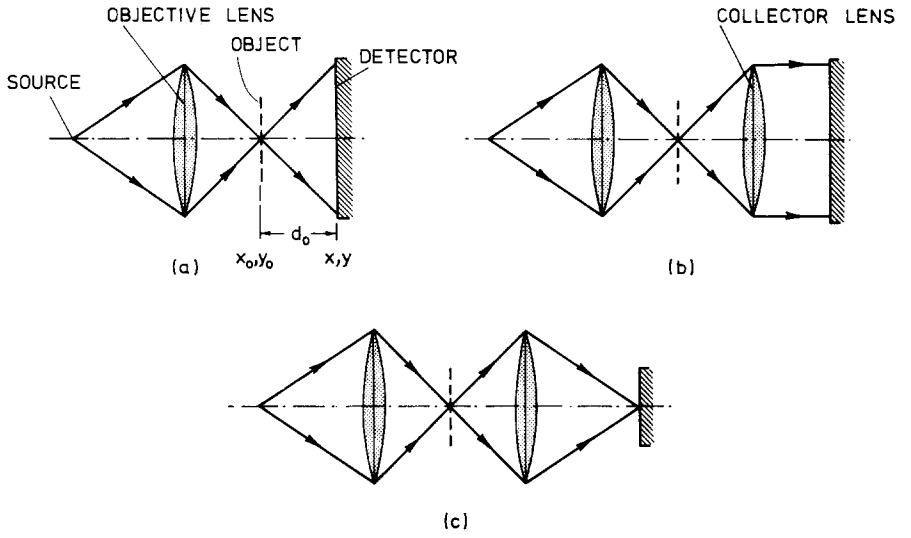

Figure 1. Scanning microscopes of Type 1.

function of that lens, and if $t_0(x_s-x_0,\ y_s-y_0)$ is the complex amplitude transmission of the object, $x_s y_s$ being its displaced position, the amplitude distribution at the detector is given by

$$U_d(x,y)=\int\int_{-\infty}^{+\infty}\frac{1}{jd_0}\exp\left\{-\frac{jk}{d_0}(xx_0+yy_0)\right\}P_2(x,y)\exp\left\{\frac{jk}{2d_0}(x_0^2+y_0^2)\right\}$$
$$\times h_1(x_0,y_0)t_0(x_s-x_0,y_s-y_0)\,dx_0\,dy_0, \qquad (1)$$

where $P_2(x,y)$ is the pupil function of the detector and d_0 is the distance of the detector from the focal plane. As the distribution at the detector in figure 1 (*a*) is the same as that at the collector exit pupil and hence also the detector in figure 1 (*b*), these two collection systems are identical if $P_2(x,y)$ is the same for each.

Let us assume that $x_0/d_0, y_0/d_0 \ll 1$. Then the total intensity detected is

$$I(x_s, y_s) = \text{const.} \int\int_{-\infty}^{+\infty} |P_2(x,y) \int\int_{-\infty}^{+\infty} t_0(x_s - x_0, y_s - y_0) h_1(x_0, y_0)$$
$$\exp\left\{-\frac{jk}{d_0}(xx_0 + yy_0)\right\} dx_0\, dy_0|^2\, dx\, dy, \tag{2}$$

and this is allocated to the picture point x_s, y_s. We can now see that by conservation of energy, or Parseval's theorem, the systems of figure 1 (*b*) and 1 (*c*) are equivalent, and indeed the same is true for any intermediate case.

We now normalize the coordinates to $u = x/\lambda d_0$, $v = y/\lambda d_0$ and follow the procedure adopted by Hopkins [4]. Using the generalization of the convolution theorem to three functions, we obtain

$$I(x_s, y_s) = \text{const.} \int\int\int\int_{-\infty}^{+\infty} C(m, n; p, q) T_0(m, n) T_0^*(p, q)$$
$$\exp\left(2\pi j\{(m-p)x_s + (n-q)y_s\}\right) dm\, dn\, dp\, dq, \tag{3}$$

where $T_0(m, n)$ is the Fourier transform of the object amplitude transmission $t_0(x_s, y_s)$, and with $\tilde{m} = \lambda d_0 m$ and so on,

$$C(m, n; p, q) = \int\int_{-\infty}^{+\infty} P_2(-x, -y) P_1(x + \tilde{m}, y + \tilde{n}) P_1^*(x + \tilde{p}, y + \tilde{q})\, dx\, dy. \tag{4}$$

Turning now to the conventional transmission microscope, the intensity distribution in the image is [4]

$$I(x_s, y_s) = \text{const.} \int\int_{-\infty}^{+\infty} \left| \int\int_{-\infty}^{+\infty} h_1(x_s - x_0, y_s - y_0) t_0(x_0, y_0) P_2(x, y) \right.$$
$$\left. \times \exp\left\{-\frac{jk}{d_0}(xx_0 + yy_0)\right\} dx_0\, dy_0 \right|^2 dx\, dy, \tag{5}$$

where suffices 1, 2 refer to the objective and condenser respectively. Using a similar method to that used for the scanning microscope, we again find that this may be written as in equation (3), but now $C(m, n; p, q)$ has the value

$$C(m, n; p, q) = \int\int_{-\infty}^{+\infty} P_2(x, y) P_1(x + \tilde{m}, y + \tilde{n}) P_1^*(x + \tilde{p}, y + \tilde{q})\, dx\, dy, \tag{6}$$

which differs from equation (4) in the signs of the arguments of the pupil function P_2. If this function is an even function of x and y, $C(m, n; p, q)$ is identical in the two cases and the imaging is thus equivalent. This will be true if P_2 is an axially symmetric function, but the condition is also true in more general cases.

3. The imaging performance of scanning microscopes of Type 2

We consider the scanning microscope of figure 2 (*a*). The signal amplitude in the detection plane is

$$U(x_i, y_i) = \int\int_{-\infty}^{+\infty} h_2(x_i, y_i; x_0, y_0) t_0(x_s - x_0, y_s - y_0) h_1(x_0, y_0)\, dx_0\, dy_0, \tag{7}$$

where $h_2(x_i, y_i; y_0, y_0)$ is the impulse response of the collector lens [5, p. 92]. In this microscope a point detector is placed at $x_i = 0$, $y_i = 0$, and this gives, when the lens law is satisfied and again taking x_0 and y_0 as small

$$h_2(0, 0; x_0, y_0) = \text{const.} \int\int_{-\infty}^{+\infty} P_2(x, y) \exp\left\{ -\frac{jk}{d_0} (x_0 x + y_0 y) \right\} dx\, dy. \qquad (8)$$

Lemons [3] obtained the same result for the microscope in figure 2 (*b*) in his work on the scanning acoustic microscope.

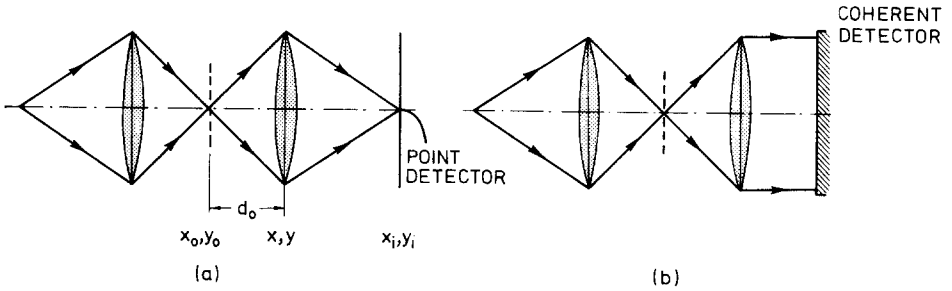

Figure 2. Scanning microscopes of Type 2.

For simplicity let us assume that the object plane is a distance d_0 from the objective lens as well as from the collector lens. If this is not so, it results in a scaling factor in the relevant pupil function. The signal intensity may be expressed in terms of the function $T_0(m, n)$, and it is found that equation (3) is still valid if $C(m, n; p, q)$ is now given by

$$C(m, n; p, q) = \{P_1(\tilde{m}, \tilde{n}) \otimes P_2(\tilde{m}, \tilde{n})\}\{P_1^*(\tilde{p}, \tilde{q}) \otimes P_2^*(\tilde{p}, \tilde{q})\}, \qquad (9)$$

and this may be separated into the product of two functions, one in m, n and one in p, q. Here $\otimes$ represents the convolution operation.

4. Image evaluation for line structures with Type 1 scanning microscopes

Hopkins [4] shows that if $C(m, n; p, q) = 1$ for all values of $m, n; p, q$ present in the object, a perfect image will be formed. We thus can think of $C(m, n; p, q)$ as being the transfer function of the imaging system. For a line structure, if spatial frequencies m and p in the amplitude transmittance of the object are present, then $C(m, p)$ gives the transfer function, resulting in a spatial frequency $m - p$ being present in the intensity image. Thus the imaging properties of a system may be studied by plotting $C(m, p)$ as a function of m and p. This method avoids difficulties which arise from the fact that coherent imaging results in amplitude imaging, whereas incoherent imaging is effectively intensity imaging.

For conventional microscopes or scanning microscopes of Type 1, the transfer function is given by equation (4) or (6). If P_2 is axially symmetric, then these two equations are identical. If the lenses have pupil functions which are unity in the apertures and zero elsewhere we may evaluate the transfer function as the area in common to the two pupil functions P_1 centred on $\tilde{m}$ and $\tilde{p}$ which are also

within the aperture of the pupil function P_2. This region is illustrated in figure 3. It is also desirable to normalize the transfer functions so that $C(0,0)$ is unity, so that the uniform intensity in the image of an object of uniform intensity is itself unity.

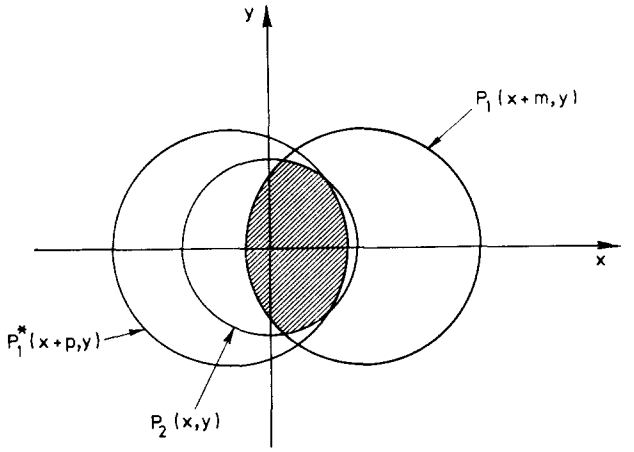

Figure 3. Region of integration for $C(m, p)$. (m, p should read $\tilde{m}$, $\tilde{p}$.)

Let us consider the case when P_1 and P_2 are both circular apertures. There are then two limiting cases as the diameter of P_2 is altered. If P_2 is a circle of vanishingly small diameter, the transfer function reduces to the product of a function of m and a function of p and we have amplitude imaging. This corresponds to the coherent imaging of the conventional microscope. If, on the other hand, P_2 is always unity, then the transfer function is a function of $(m-p)$ only, and we have intensity imaging, which corresponds to the incoherent imaging of the conventional microscope. All intermediate values for the diameter of P_2 result in imaging identical to partially coherent imaging in the conventional microscope. The form of the $C(m,p)$ surface for two circular pupil functions is shown in figure 4 for various degrees of coherence. The two limiting cases, coherent and incoherent, are shown in figures 4 (a) and 4 (e). Figure 4 (a) shows that the coherent transfer function is unity if the spatial frequency in both m and p directions is less than the frequency $f_0 = a_1/\lambda d_0$, where a_1 is the radius of the objective pupil function. Figure 4 (e) shows that the incoherent transfer function is a function of $m-p$ only. When $p=0$, that is the amplitude transmittance consists of one complex exponential component and a constant component, the transfer function falls from unity at $m=0$ to zero at $m=2f_0$. In between it is given by [5, p. 120]

$$C(m,0) = \frac{2}{\pi}\left[\cos^{-1}\left|\frac{m}{2f_0}\right| - \left|\frac{m}{2f_0}\right|\sqrt{\left(1-\left(\frac{m}{2f_0}\right)^2\right)}\right] \tag{10}$$

or as a function of $(m-p)$

$$C(m-p) = \frac{2}{\pi}\left[\cos^{-1}\left|\frac{m-p}{2f_0}\right| - \left|\frac{m-p}{2f_0}\right|\sqrt{\left(1-\left(\frac{m-p}{2f_0}\right)^2\right)}\right]. \tag{11}$$

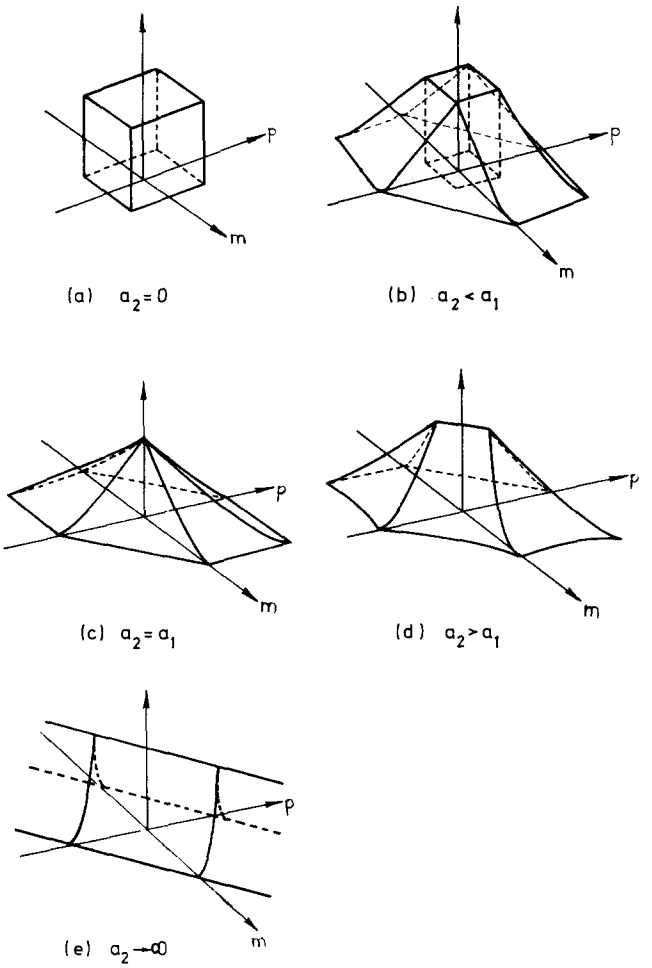

Figure 4. The transfer function $C(m, p)$ for scanning microscopes of Type 1 with different collector lens apertures a_2.

An important intermediate case is that shown in figure 4 (c) when the apertures P_1 and P_2 are equal. We see that when $p=0$, the transfer function is the same as that for the incoherent case. If m and p are of opposite sign, the transfer function is also identical to that for incoherent imaging, but if they are of the same sign the symmetry is that of the coherent case, that is the transfer function is a product of functions of m and p.

We also show two further intermediate cases. When a_2, the radius of the aperture P_2, is less than a_1, the radius of the aperture P_1, figure 4 (b) shows that when m and p have the same sign there is coherent symmetry. For $p=0$ the maximum spatial frequency detected is now $(a_1+a_2)/\lambda d_0$, and the transfer function is unity if both spatial frequencies are less than $(a_1-a_2)/\lambda d_0$. When the aperture P_2 is greater than the aperture P_1, if m and p are of opposite sign, imaging is as for the incoherent case. The fact that the section through the $C(m,p)$ surface at $p=0$ gives the response to an amplitude transmission made up of one complex exponential component and a constant component has been mentioned. Let us

now consider some other transmission functions. If the amplitude transmittance is given by

$$t_0(x_s) = \cos 2\pi f x_s \tag{12}$$

(this not very practical transmittance was considered by both Hopkins [4] and Goodman [5, p. 128]) it consists of two exponential frequencies, $\pm f$. We are thus interested in the values of the transfer functions $C(f,f)$, $C(-f,-f,)$, $C(f,-f)$ and $C(f,-f)$, which are on the lines $m=\pm p$. By considering the regions in m,p space where the transfer function is non-zero (figure 5) we can see that for any degree of coherence the transfer function is non-zero if $f<f_0$, and since the coherent imaging condition gives unity for all the relevant values of $C(m,n)$ it provides the most perfect image.

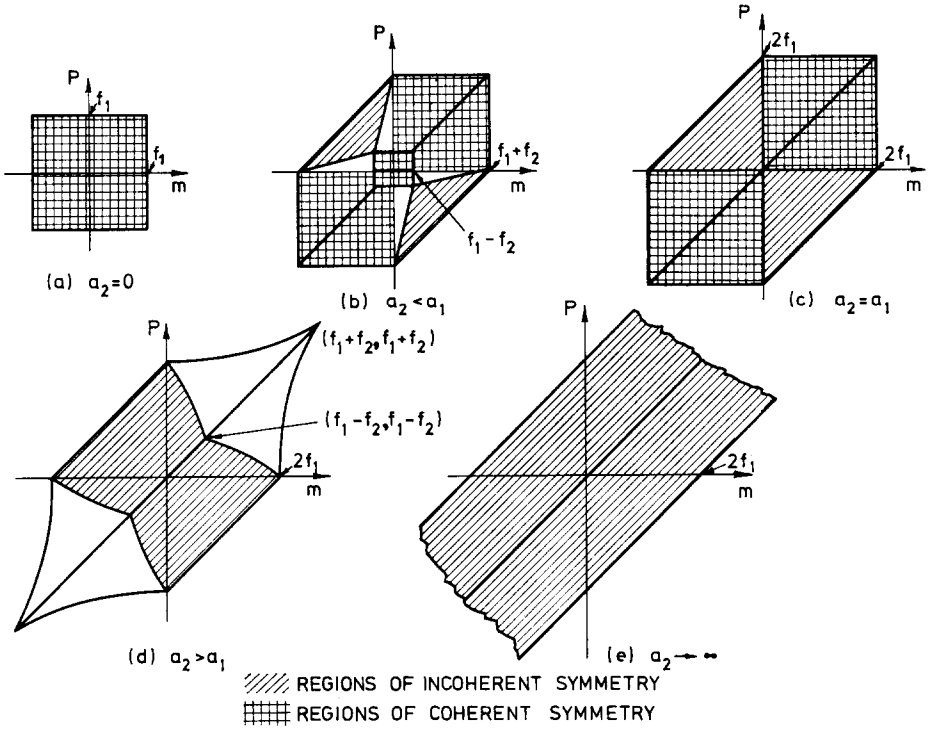

Figure 5. The regions of m, p space for $C(m, p)$ to be non-zero in a Type 1 scanning microscope.

For a transmittance

$$t_0(x_s) = 1 + b\cos 2\pi f x_s \tag{13}$$

we must consider extra values of the transfer function $C(0,0)$, $C(0,\pm f)$ and $C(\pm f,0)$. If, furthermore, b is small, that is we have a small variation in a large background, the terms in $C(\pm f,\pm f)$ are all negligible, and the important terms are those in $C(0,\pm f)$ and $C(\pm f,0)$, which are on the lines $m=0$, $p=0$. As the relative size of the two pupil functions is altered the cut-off frequency increases

from f_0 when imaging is coherent to $2f_0$ when the pupil functions are equal. Further increase in the collector aperture produces no change in the image. Although coherent imaging has a lower cut-off frequency, the transfer function is, of course, unity for all spatial frequencies less than this value.

If we now consider an object with amplitude transmittance containing all frequencies up to a frequency f, the relevant values of the transfer function are those contained in a square of side $2f$ in m, p space symmetrically placed relative to the axes. Now in principle it is possible to correct the transfer function by image processing providing the transfer function is non-zero. The largest square which may be fitted in the non-zero region of the transfer functions in figure 5 has side $2f_0$ for any size of collector aperture, that is any degree of coherence. Finally we may note that the greatest frequency which can be present in the image is $2f_0$, and this too is independent of the collector aperture.

5. Image evaluation for line structures with Type 2 scanning microscopes

We now consider imaging with a scanning microscope of Type 2 using lenses with circular pupil functions. Figure 6 shows the transfer function for three sizes of collector pupil function. If P_2 is very small, the transfer function is identical to that for a coherent scanning microscope of Type 1. As P_2 increases in diameter, the value of the transfer function for $p=0$ is identical to that for the Type 1 microscope, until, when $P_2=P_1$, it is the same as that for the incoherent microscope of Type 1. Unlike microscopes of Type 1, the aberrations of the collector lens are important, and hence there is a limit to the size of P_2 which may be employed. Optimum results are obtained if both lenses have the same

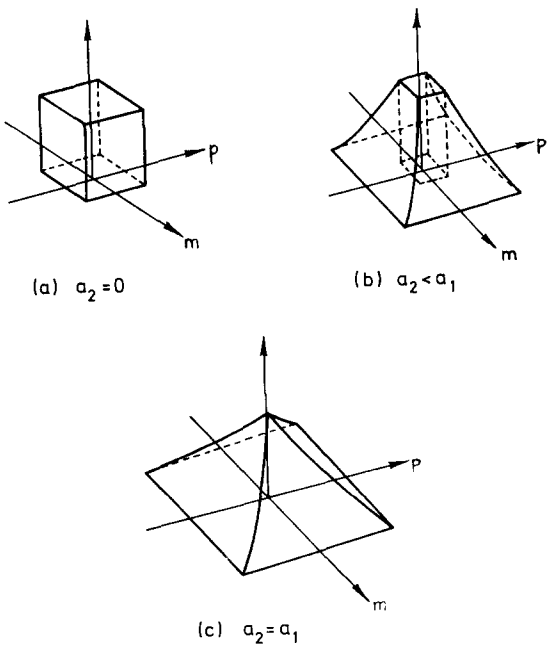

Figure 6. The transfer function $C(m, p)$ for scanning microscopes of Type 2 with different collector lens aperture a_2.

aperture. Although the distribution in the transfer function for $p=0$ is as for the Type 1 microscope, the transfer function now exhibits coherent symmetry for all quadrants.

An object with a single sinusoidal amplitude transmittance (equation (12)) gives a non-zero transfer function for $f<2f_0$ and this is thus superior to all microscopes of Type 1.

If the object has a transmittance given by equation (13) with small b, that is small sinusoidal variation on a large background, imaging is identical to the incoherent microscope of Type 1.

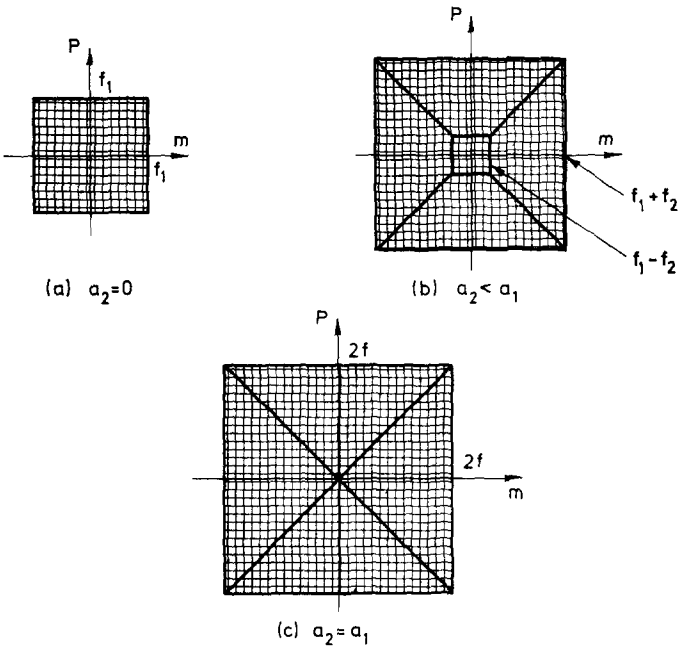

Figure 7. The regions of m, p space for $C(m, p)$ to be non-zero in a Type 2 scanning microscope.

If we now consider non-zero values of the transfer function, we see that the region in figure 7 (c) can contain a square of side $4f_0$, which is twice the size of that for microscopes of Type 1. Imaging of an object with all spatial frequencies up to a frequency f present is thus superior in Type 2 microscopes. The greatest frequency which can be present in an image with this type of microscope is $4f_0$, that is twice as high as that for a Type 1 microscope.

6. Imaging with lenses with annular aperture

Let us consider the convolution with itself of an annular aperture with outer and inner radii a and ϵa respectively. This has been calculated by O'Neill [6] and McCrickerd [7] and is plotted in figure 8 for various values of ϵ. In this figure the convolution is normalized to unity for zero displacement between the centres. For values of ϵ near to unity the convolution exhibits a substantially flat region over most of the range, with two peaks, one at the origin and one near to the cut-off.

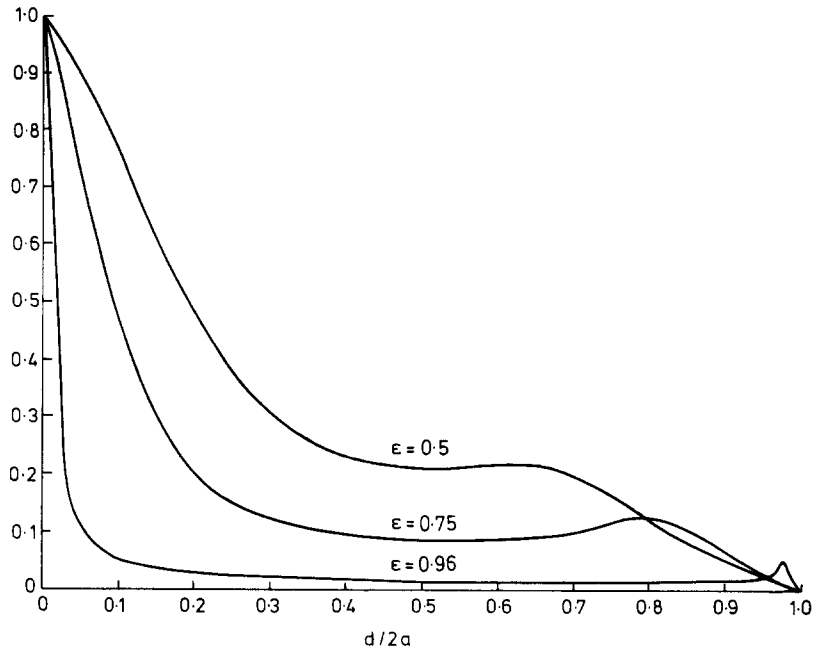

Figure 8. The convolution with itself of an annular aperture with outer and inner radii a and ϵa respectively [6, 7].

As the width of the annulus tends to zero, the height of the central region tends to zero with respect to the heights of the peak. If the distance between the centres is d, the convolution in this case is given by

$$C\left(\frac{d}{2a}\right) = \tfrac{1}{2}/\left\{\frac{d}{2a}\left[1-\left(\frac{d}{2a}\right)^2\right]^{1/2}\right\}, \tag{14}$$

where we have renormalized so that now the minimum is unity. This is plotted in figure 9.

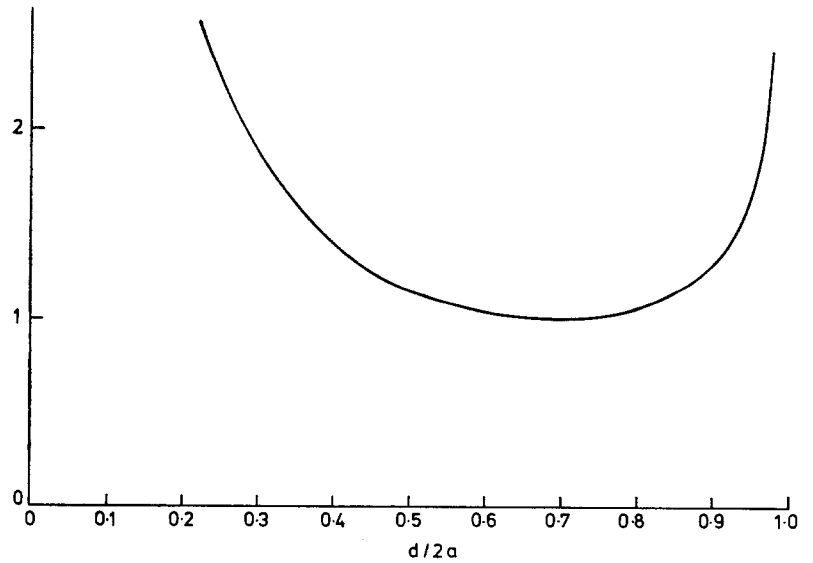

Figure 9. The convolution with itself of an annular aperture in the limiting case at the width of the annulus tends to zero.

We now consider the convolution of a thin annulus with a full circle of radius equal to the average radius of the annulus. The convolution is given by

$$C\left(\frac{d}{2a}\right) = \frac{1}{\pi}\cos^{-1}\left(\frac{d}{2a}\right) \tag{15}$$

which is shown in figure 10.

We are now in a position to sketch the transfer function for various scanning microscope systems employing annular apertures. First we consider a scanning microscope of Type 1 with an annular collector and a circular objective of the same numerical aperture. The transfer function is shown in figure 11. For an object amplitude transmittance consisting of a single cosinusoidal variation (equation (12)), this microscope gives very poor results because the transfer function is zero if m and p have opposite sign. For an object transmittance consisting of a small cosinusoidal variation on a large background (equation (13)), however, the transfer function is given by the convolution of an annulus and a

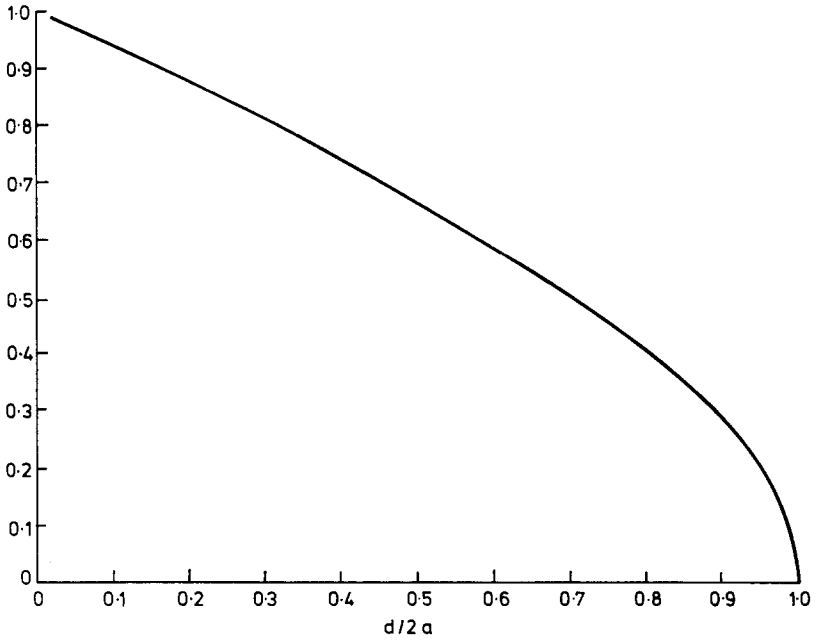

Figure 10. The convolution of a thin annulus with a full circle equal to the outer radius of the annulus.

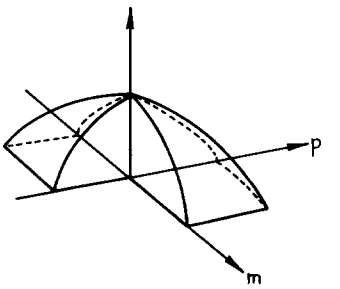

Figure 11. The transfer function (m, p) for a Type 1 scanning microscope with annular collector and a circular objective of the same numerical aperture.

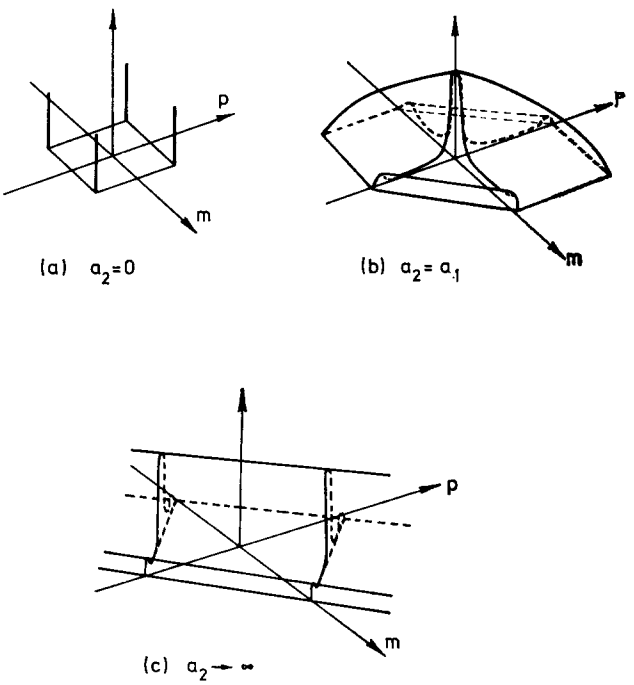

Figure 12. The transfer function $C(m, p)$ for a Type 1 scanning microscope with an annular objective and a full circular collector of various sizes.

circle, which means that the high frequency response is improved over that of a microscope with two circular pupil functions.

Figure 12 shows the transfer function for a Type 1 scanning microscope with an annular objective and a circular collector. If the collector has a small numerical aperture, that is the microscope is coherent, the transfer function consists only of spikes (figure 12 (*a*)) and hence is not useful for examining arbitrary specimens. If the lenses are of equal size, the transfer function (figure 12 (*b*)) is given by the convolution of an annulus and a circle for $m=p$, and by the convolution of two annuli for $m=0$ or $p=0$. If the collector becomes very large (incoherent microscope, figure 12 (*c*)) the transfer function has incoherent symmetry and again is given by the convolution of two annuli for $m=0$ or $p=0$. Compared with the incoherent microscope with a circular objective, this microscope gives an enhanced high frequency response. However, the large low frequency response would be expected to result in low contrast on extended objects. We also have a peak near cut-off which could result in oscillations, that is fringes, in the image.

We now consider Type 2 scanning microscopes. If both lenses are of annular aperture the transfer function is as shown in figure 13. The transfer function is comparatively flat over a large area of the figure and the high spatial frequency response is improved. However, we still have the peak near the cut-off frequency and also a large low frequency response which would again be expected to reduce contrast.

If a Type 2 scanning microscope has one lens with an annular aperture and the other a circular aperture, the transfer function is as shown in figure 14. This

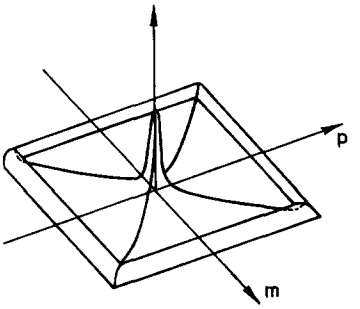

Figure 13. The transfer function $C(m, p)$ for a Type 2 scanning microscope with two equal lenses with an annular aperture.

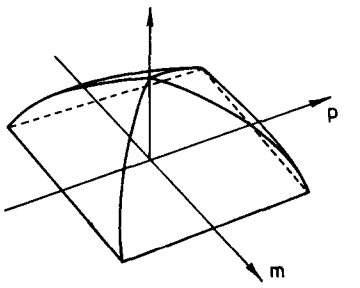

Figure 14. The transfer function $C(m, p)$ for a Type 2 scanning microscope with one lens with an annular aperture and one lens with a circular aperture of the same size.

microscope also has a good high frequency response, and also does not have the peak near the cut-off frequency or the large relative low frequency component.

7. Images of non-periodic structures

Equation (3) is suitable for determining the image of a periodic structure, but if the object is non-periodic it is more convenient to utilize equation (2) or (7). For a single point object, for example, equation (2) gives for the Type 1 scanning microscope

$$I(x_s, y_s) = \text{const.}[h_1(x_s, y_s)h_1^*(x_s, y_s)] \tag{16}$$

which, apart from the value of the constant, is independent of the size of the collector lens. For the same object in a Type 2 microscope, equations (7) and (8) give

$$I(x_s, y_s) = \text{const.}[h_1(x_s, y_s)h_2(x_s, y_s)h_1^*(x_s, y_s)h_2^*(x_s, y_s)]. \tag{17}$$

If the objective of a Type 1 microscope has a circular aperture the impulse response for $y_s = 0$ is thus

$$I(u_s) = \text{const.}\, J_1^2(2\pi a u_s)/(2\pi a u_s)^2 \tag{18}$$

where J_1 is a Bessel function of order 1, whereas if it is annular, the impulse response is

$$I(u_s) = \text{const.} J_0^2(2\pi a u_s) \tag{19}$$

with J_0 a Bessel function of order zero. These functions are plotted in figure 15. It is seen that the annular aperture gives an impulse response which is narrower —the radius of the first dark ring is reduced by 37 per cent. However, the maximum intensity in the first side-lobe is increased from 0·02 to 0·16 times the intensity at the focus.

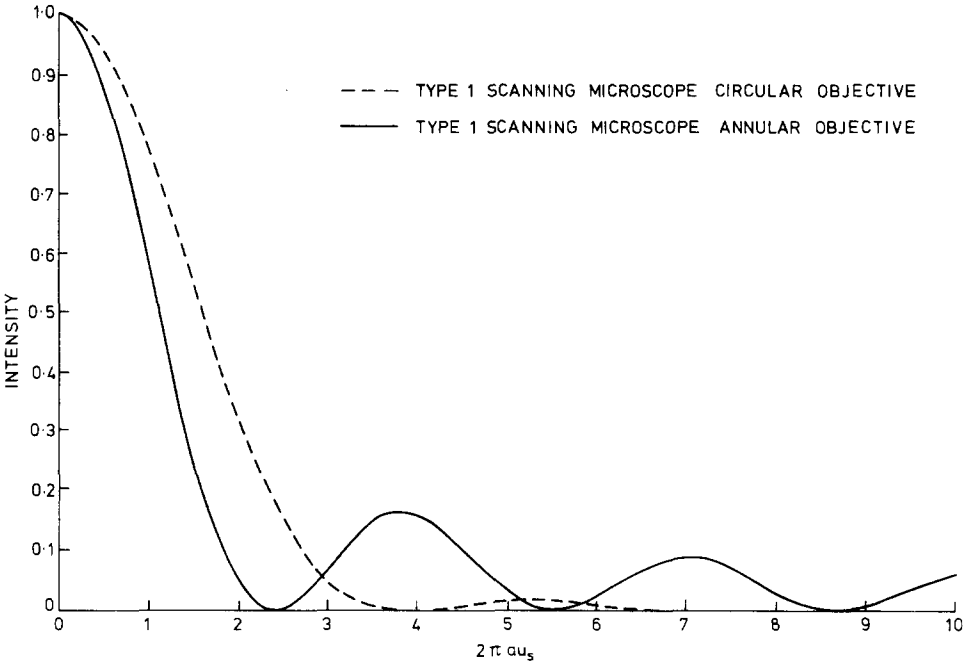

Figure 15. Image of a single point in a Type 1 scanning microscope.

For a Type 2 microscope with two lenses with circular aperture the impulse response is given by

$$I(u_s) = \text{const.} J_1^4(2\pi a u_s)/(2\pi a u_s)^4. \tag{20}$$

This is a sharper response than for the Type 1 microscope (figure 16) but the first zero is of course at the same radius. With two annular lenses, the impulse response is

$$I(u_s) = \text{const.} J_0^4(2\pi a u_s) \tag{21}$$

which is narrower than that for two circular apertures. It should be noted that now the first side-lobe is nearly as small as for the Type 1 microscope with circular pupil function. Figure 16 also shows the impulse response for a Type 2 microscope with one lens with circular aperture and one with annular aperture, which is

$$I(u_s) = c\text{const.} J_0^2(2\pi a u_s) J_1^2(2\pi a u_s)/(2\pi a u_s)^2 \tag{22}$$

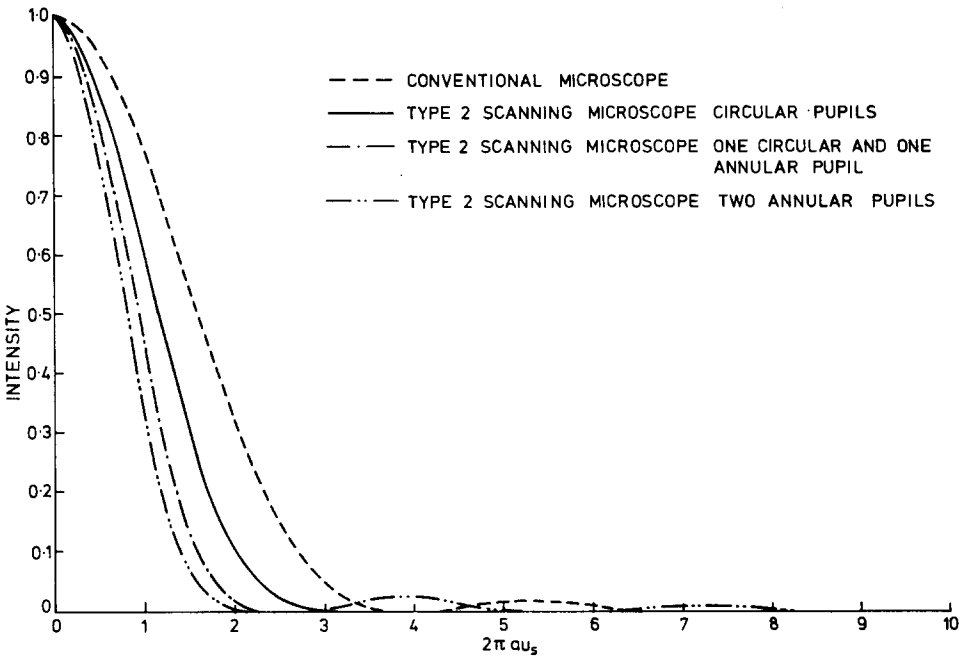

Figure 16. Image of a single point in a Type 2 scanning microscope.

This also shows an improvement over the conventional microscope, and in this case the side-lobes are very much reduced.

If the object consists of two points, each point gives rise to an amplitude distribution in the image identical to that for a single point image. However, with a Type 1 microscope the images of these points may add either coherently or incoherently, whereas in the Type 2 microscope they always add coherently. Using the results of McKechnie [8], it may be shown that the image of two points at $(\pm x^1, 0)$ in a Type 1 microscope is given by

$$I(x_s, 0) = h_1(x_s - x')h_1^*(x_s - x') + h_1(x_s + x')h_1^*(x_s + x') + 2\text{Real}\{h_2(2x')\}h_1(x_s - x')h_2(x_s + x'). \tag{23}$$

It is conventional to take two points as being just resolved when the intensity in the image midway between them is equal to 0·735 times that at the points. This is a generalization of the Rayleigh criterion, which takes two points as just resolved in an incoherent system with circular aperture when one point is placed at the first minimum in the image of the second. If the ratio of the apertures a_2/a_1 is denoted by s, the normalized distance between the points is given by

$$2u' = L(s)/a_1. \tag{24}$$

The function $L(s)$ for a microscope with two circular pupils [9] is shown in figure 17. It is a result of the particular form of resolution criterion that the imaging of two just-resolved points is incoherent when $s = 1$. This does not imply that imaging of a general object is incoherent for $s = 1$, and moreover imaging of two

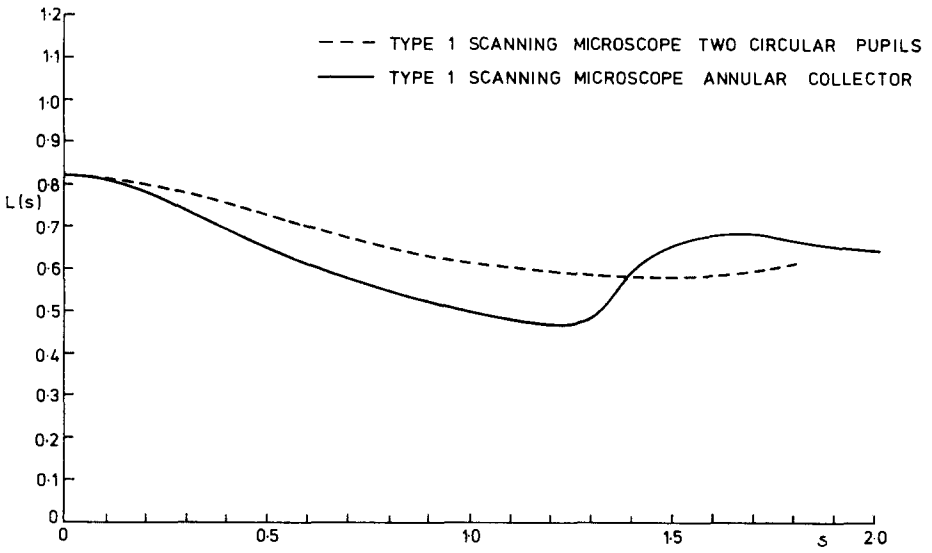

Figure 17. Two-point resolution in a Type 1 scanning microscope.

just-resolved points is not incoherent if the pupil functions are of a different form, for example annuli. Also shown in figure 17 is the function $L(s)$ for a microscope with a circular objective and an annular collector [8]. This shows that for this particular object, the limiting resolution is improved by employing an annular collector.

In a Type 2 scanning microscope, the image of a two-point object is given by

$$I(x_s, 0) = |h_1(x_s - x')h_2(x_s - x') + h_1(x_s + x')h_2(x_s + x')|^2. \tag{25}$$

We consider three particular cases. First, we take a microscope with two circular apertures. As shown in figure 18 the function $L(s)$ continues to decrease monotonically for all values of s. However, this method of representing the results is a little confusing, as the aberrations of the collector lens are important in a Type 2 microscope (it is used so as to conform with other publications on two-point resolution), and hence the best resolution is obtained when both apertures are as large as possible consistent with aberrations being negligible, that is for $s = 1$. For this value of s, $L(s)$ has a value 0·56, which compares with the minimum value achievable with a conventional microscope of 0·57, and a value of 0·61 at the frequently used ratio $s = 1$. For a Type 2 microscope with two annular pupils, $L(s)$ is reduced to 0·39 for $s = 1$, an improvement of about a factor of $1\frac{1}{2}$ compared with the conventional microscope. If the microscope has one lens with a circular aperture and one with an annular aperture, two curves for $L(s)$ are given according as to which lens is the objective. This does not imply that the system is not reversible, but occurs because $L(s)$ is defined in terms of the objective aperture. When $s = 1$, L(s) has a value 0·44, which is a factor of 1·4 better than that obtained with a conventional incoherent microscope.

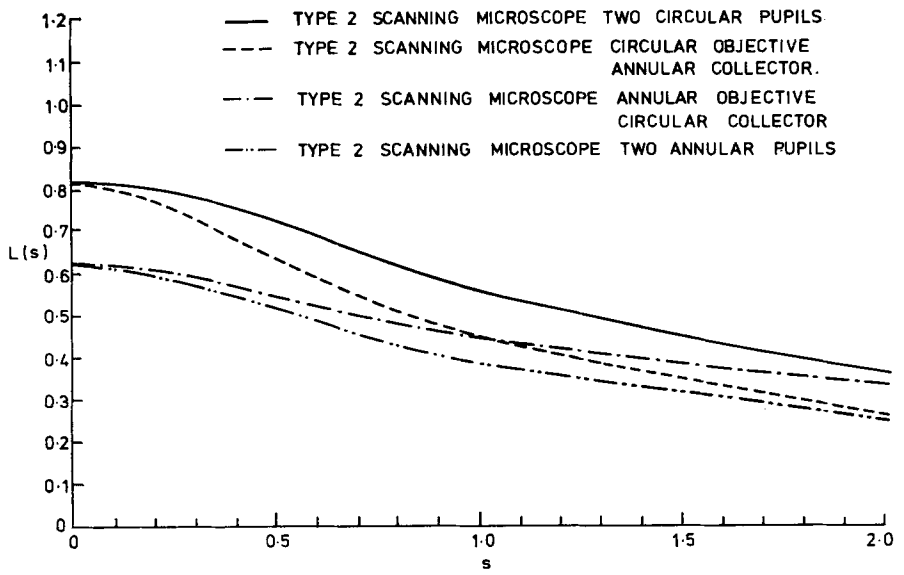

Figure 18. Two-point resolution in a Type 2 scanning microscope.

Finally, we consider imaging of a straight edge. It may be shown [10, p. 322] that the response of a coherent microscope with a lens of circular symmetry to an object which varies only in one direction (i.e. is independent of y) is identical to that produced by a rectangular pupil, the pupil function of which varies in the direction of variation of the object in the same manner as the circular symmetric pupil function varies across its diameter. We obtain for the coherent microscope with a circular pupil a response to a step object, or straight edge,

$$I(u_s) = \left[\tfrac{1}{2} + \frac{1}{\pi}\,\mathrm{Si}\,(2\pi a u_s)\right]^2 \tag{26}$$

where Si is a sine integral. This response is plotted on a logarithmic scale in figure 19. For an incoherent microscope, the response may be calculated by integrating across a cross-section of the intensity impulse response and then convolving with the step to give [10, p. 167]

$$I(u_s) = \frac{1}{\pi}\int_{2\pi a u_s}^{\infty} \frac{H_1(2z)}{z^2}\,dz. \tag{27}$$

$$= \frac{1}{\pi}\left\{\frac{H_1(4\pi a u_s)}{4\pi a u_s}\right\} + \frac{1}{\pi}\int_{4\pi a u_s}^{\infty} \frac{H_0(z)}{z}\,dz \tag{28}$$

Here H_0 and H_1, are Struve functions and we have made use of Struve's integral [11, p. 497]. This again is plotted in figure 19. The coherent microscope exhibits a sharper response to the straight edge than the incoherent microscope, but also gives large fringes. The image produced by a conventional microscope with various degrees of coherence has been calculated by Watrasiewicz [12]. It is found that the response of a conventional microscope or Type 1 scanning microscope with two lenses of equal numerical aperture behaves in a very similar fashion to the incoherent microscope.

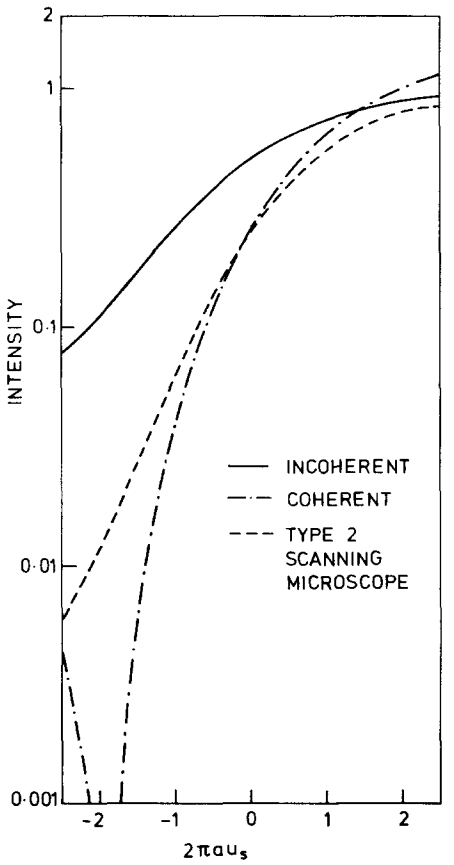

Figure 19. Straight-edge response of microscopes with circular pupils.

Let us now turn to the scanning microscope of Type 2. We have from (7)

$$I_2 = |h_1 h_2 \otimes t_0|^2, \tag{29}$$

whereas for a coherent Type 1 microscope

$$I_{1\,coh.} = |h \otimes t_0|^2, \tag{30}$$

from (2) with P_2 replaced by a δ-function, and for an incoherent Type 1 microscope

$$I_{1\,inc.} = (hh^*) \otimes t_0 t_0^*, \tag{31}$$

from (2) with P_2 replaced by a constant. Comparison of (29) with (30) shows that the Type 2 microscope behaves as a coherent microscope with an effective impulse response $h_1 h_2$, which is a coherent microscope with an effective pupil function given by the convolution of the pupil functions of the two lenses. This allows us to calculate the straight-edge response of the Type 2 microscope, but an alternative method follows from comparison of (29) and (31). If the two lenses of the Type 2 microscope are equal and free from aberrations, and if the object consists only of regions which are perfectly transmitting or perfectly absorbing (as is the

case for a straight edge), the response of the Type 2 microscope is given simply as the square of the response of an incoherent microscope. The response of a Type 2 microscope with two equal circular pupils is given in figure 19. It is nearly as sharp as that for the conventional coherent microscope, without having the large fringes and hence is an improvement over conventional microscopes, either coherent or incoherent. This result may be explained by the fact that the effective impulse response for the Type 2 microscope has extremely small side-lobes.

Let us now consider the effect of using annular lenses in microscopes. In a conventional coherent microscope, if a thin annulus is used the amplitude response to a straight-edge is cosinusoidal and hence this type of microscope would not be useful. This response could be predicted from the discussion in § 6 which shows that only one spatial frequency is transmitted by the system. For a conventional incoherent microscope with a thin annulus we come to the conclusion that the straight-edge response would be just a constant, that is there is no resolution. The fact that such a microscope would not be useful for extended objects has been discussed by Welford [13].

Turning now to the Type 2 scanning microscope, we may calculate the response of a microscope with one annular pupil and one circular pupil by finding the

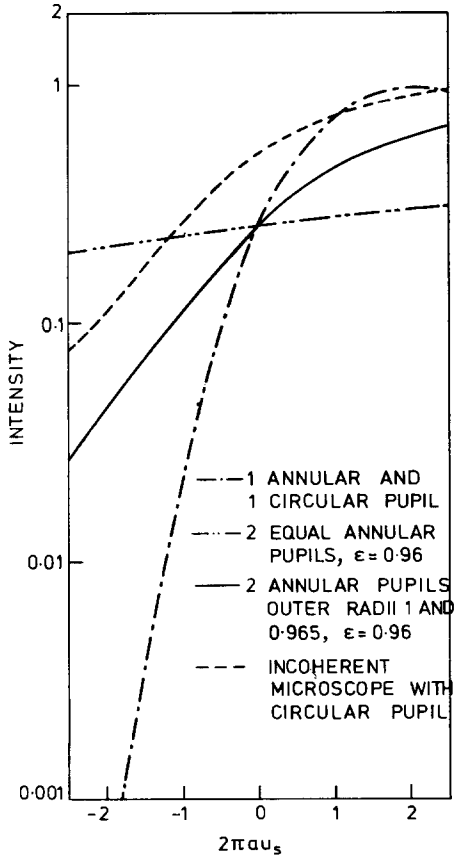

Figure 20. Straight-edge response of scanning microscopes of Type 2 with one or two lenses with annular pupil.

one-dimensional Fourier transform of the radial distribution of the effective pupil function, convolving with a step and squaring to give

$$I(u_s)=\left[\frac{1}{\pi}\int_{4\pi a u_s}^{\infty}\frac{H_0(z)}{z}\,dz\right]^2 \tag{32}$$

This is plotted in figure 20 and shows an extremely sharp response with no fringes.

For a Type 2 microscope with two equal annular lenses, we must similarly find the one-dimensional Fourier transform of the radial pupil function given in figure 8, convolve with a step, and square. This has been done numerically for the case of two annuli with $\epsilon=0{\cdot}96$, and the result is shown in figure 20. It is seen that there is very little contrast in the image, and we hence come to the conclusion that such a microscope would be useless for study of extended objects.

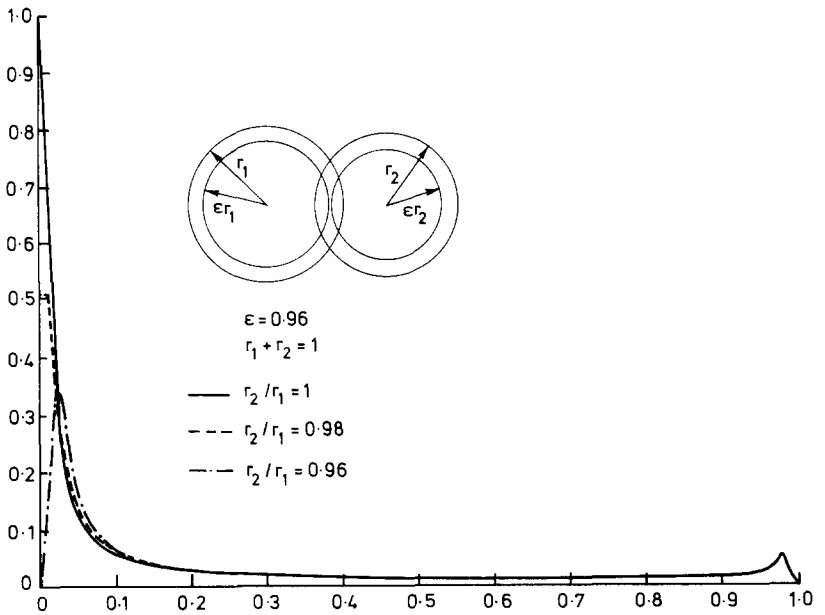

Figure 21. The convolution of two annuli of differing radii.

The low contrast is a result of the large zero spatial frequency value of the transfer function, and also the large proportion of the area under the transfer function which is at low spatial frequencies. One way of reducing both these values is to use two annuli of slightly differing radii. The convolution of two such annuli has been computed (figure 21) and from this the straight-edge response for a microscope of Type 2 with two annular pupils, one with outer radius 0·965 times that of the other has been calculated. This is plotted in figure 20, and shows that the straight-edge response has been much improved over that for two equal annuli.

8. Discussion

It has been shown that there are two distinct forms of scanning microscope, which have been designated Type 1 and Type 2. Type 1 scanning microscopes employ an incoherent detector and are equivalent to conventional microscopes providing that the source in the scanning microscope may be considered as a point. The pupil function of the objective is thus primarily responsible for the resolution obtainable, but the form of the transfer function is dependent on the size of the collector pupil function. The two limiting cases of very large or very small collector pupil function may be termed ' incoherent ' or ' coherent ' respectively, in analogy with conventional microscope imagery. Relative imaging performance depends on the structure of the specimen. The aberrations of the collector lens are unimportant; thus we may replace it by an equivalent incoherent detector. This is analogous to replacing the condenser lens in a conventional microscope by an effective source [14, p. 510].

Type 2 scanning microscopes employ a coherent or point detector. Imaging is identical to that of a coherent microscope with effective pupil function given by the convolution of the pupil functions of the two lenses. As the performance of Type 2 microscopes depends on the aberrations of both lenses, optimum results are achieved if the two lenses are identical, and then the transfer function exhibits a similarity to that of the incoherent microscope of Type 1, but more so to that of a Type 1 microscope with pupils of equal size. Response to different objects is always as good as the incoherent microscope, but often better. In particular, the upper spatial frequency in the amplitude transmission of an object for which the transfer function is non-zero for any resultant spatial frequency in the intensity distribution of the image is twice as high for a Type 2 microscope. Furthermore, the maximum spatial frequency which may be present in an image is also twice as high for a Type 2 microscope.

An alternative way of regarding a Type 2 microscope with two equal pupils with no aberrations is as giving the same amplitude image of amplitude variations in the object as the intensity image of similar intensity variations in the object produced in an incoherent microscope with a similar lens. In this case, if the object consists of only regions of black and white, the intensity in the image with the Type 2 microscope is just the square of that produced by an incoherent microscope.

The Type 2 microscope shows an improvement of about a factor of one and a half over the conventional microscope in the image of a single point, the two point resolution is slightly improved, and the response to a straight edge is sharper than that given by an incoherent microscope. We conclude, therefore, that this type of scanning microscope has advantages over Type 1 scanning microscopes and therefore also over conventional microscopes.

Use of an annular collector in a Type 1 microscope gives a spatial frequency response which gives an improvement in the imaging of some objects, but an inferior image for others. Single point resolution is unchanged over the normal microscope but two-point resolution is improved. A Type 1 microscope with an annular objective has large side-lobes and these result in a very poor straight edge response.

A Type 2 scanning microscope with one annular pupil has a frequency response which is improved over and above that of a similar microscope with two

circular pupils. The single-point resolution is improved by a factor of 1·7 in relation to the conventional microscope, and the side-lobes are very small. Compared with a conventional microscope with two equal circular pupils the two-point resolution is improved by about a factor of 1·4, and the response to a straight edge is greatly improved. This microscope thus is superior to a conventional microscope in its imaging properites, and the increased depth of focus of the annular lens means that the adjustment of the spacing between the two lenses is not critical.

If a Type 2 scanning microscope has two annular pupils, the high frequency response is also improved but there is a reduction in the mid-frequency response which reduces contrast in the image of an extended object. The straight-edge response may be improved by employing two annuli of slightly differing radii. The response to a single point object is now twice as good as in the conventional microscope, and the two-point resolution is improved by a factor of about one and a half. The depth of focus is also increased with this type of microscope [15] and a further advantage is that a much simpler lens may be used for a given amount of spherical aberration. Experiments are under way to study the performance of these microscopes with a wide variety of objects.

9. Conclusions

A Type 1 scanning microscope gives identical imaging of a plane object to a similar conventional microscope. The former type of microscope has certain other advantages [16]. A Type 2 scanning microscope has, in addition, improved imaging properties, and these may be further improved by use of one or two lenses with annular pupils.

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