Adaptive Control of Rotor Vibration Using Compact Wavelets

This paper investigates the use of dyadic wavelets for the control of multifrequency rotor vibration. A scheme for real-time control of rotor vibration using an adaptive wavelet decomposition and reconstruction of time-varying signals is proposed. Quasi-periodic control forces are constructed adaptively in real-time to optimally cancel vibration produced by nonsmooth disturbance forces. Controller adaptive gains can be derived using a model-based synthesis or from system identification routines. The controller is implemented on a flexible rotor system incorporating two radial magnetic bearings, with standard proportional-integral-derivative control employed in a parallel feedback loop for rotor levitation. An experimental investigation of controller performance is used to deduce the best choice of wavelet basis for various operating conditions. These include steady synchronous forcing, step changes in synchronous forcing and multifrequency forcing produced by a rotor impact mechanism. [DOI: 10.1115/1.2203352]

Keywords: wavelets, adaptive vibration control, repetitive control, magnetic bearings, flexible rotor, impact disturbance

1 Introduction

Vibration in rotating machinery is often dominated by a sinusoidal component at the rotational frequency caused by mass eccentricity of rotating components. Methodology for the active control of such synchronous vibration is now highly developed. Numerous strategies and algorithms have been proposed that are based on the application of sinusoidal control forces having a frequency matched to the rotational frequency and with amplitude and phase chosen to minimize selected signal amplitudes. This concept was first proposed over 20 years ago and demonstrated on an experimental flexible rotor using a magnetic bearing as a control actuator [1]. This basic technique has been further enhanced with closed loop and adaptive schemes that can provide improved performance during transient conditions [2–5]. Recent developments have led to strategies that can cope with auxiliary bearing contact, changing rotor dynamics, nonconstant speeds, and multiple disturbance frequencies [6–8].

Significant nonsynchronous vibration components can arise in rotor systems for a number of reasons, including:

1. nonlinear bearing characteristics
2. rotor-stator rub or other contact phenomena
3. aerodynamic or fluid dynamic interaction forces
4. external vibration sources

When such phenomena generate multifrequency vibration signals of a nonstationary nature, modern multivariable controller designs are the obvious choice for vibration suppression if the system dynamics are reasonably linear. When multifrequency vibration is dominated by stationary components, extensions of established adaptive and closed loop strategies can be employed [7,8]. These techniques are based on signal analysis and synthesis using harmonic functions, therefore their effectiveness is reliant on having a finite number of cyclic disturbance components that are smooth and periodic in character. This raises the question: what are the alternative methods for signal analysis and synthesis that can cope more effectively with discontinuous or non-smooth signal characteristics? A possible candidate considered in this paper is the wavelet transform [9]. Wavelet analysis is a recent development, which involves decomposing a signal into components having different scale levels and thus frequency content.

The wavelet transform has numerous applications, including signal compression, signal analysis, noise reduction, and vibration analysis. Although the wavelet transform has previously been proposed as a method to analyze transient vibration signals [10], including those that arise in rotor systems [11], its application in the control of vibration has received little attention. In the context of rotating machinery equipped with magnetic bearings, vibration may be associated with rotor displacement and force transmission through the bearings to other system components. Control strategies may be developed to minimize rotor vibration or force transmission, either individually or in a weighted combination. In principle, the control methodology developed in this paper may be generalized to cover all cases. However, for validation purposes the focus is towards the attenuation of rotor vibration alone through the use of wavelet coefficients of measured signals in the feedback loop. The controller structure is derived for an experimental rotor/magnetic bearing system. Rotor vibration is excited by a variety of means and the controller is applied to attenuate wavelet coefficients, hence, real-time signals.

2 Vibration Control Problem

2.1 Rotor System Dynamics. During operation, rotating machinery may experience phases of near periodic vibration, possibly with a changing fundamental frequency (due to rotor speed changes), and/or slowly changing vibration amplitudes (due to mass deposition/erosion, wear, etc.). Steady operation may be interspersed with periods of transient excitation caused by sudden events such as a change in load, rotor impact, motion of the machine base, rotor mass loss, or any other system fault. Consequently, there is a range of vibration sources and characteristics that must be considered in any vibration control strategy. Moreover, if a strategy is to be based on measurement signal decomposition and control signal superposition, the best choice of basis will depend on the expected vibration characteristics.
Consider the structural dynamics of a flexible rotor system, subjected to arbitrary disturbance signals, which may be described by linear state space equations of the form

\[ \dot{x} = Ax + Bu + Ed \]

The system inputs are a vector of time-varying control forces \( u(t) \) acting at discrete locations and a vector of disturbance inputs \( d(t) \) which may include unbalance forces, electrical noise, and external vibration sources. Such a model is commonly derived using a finite element discretization of the rotor and machine foundation so that the vector \( x(t) \) includes displacement and velocity states at selected positions [12]. The system output vector \( y(t) \) could constitute any weighted combination of machine vibration signals that must be minimized, including rotor displacements, stator accelerations, and bearing/actuator forces. It is assumed that all components of the vector \( y(t) \) are available to the controller and can thus be used for control force synthesis.

Consider a periodic response of the system produced by a periodic disturbance signal and/or control force. The Fourier transforms of these signals satisfy

\[ \hat{x}_n = (io_0I - A)^{-1}(B\hat{u}_n + E\hat{d}_n) \]

\[ \hat{y}_n = C\hat{x}_n + D\hat{d}_n \]

where \( \hat{x}_n, \hat{u}_n, \hat{x}_n, \) and \( \hat{d}_n \) are the signal Fourier coefficients for the frequency \( o_n = 2\pi n/T \) and \( T \) is the period. Therefore

\[ y(t) = \sum_{n=-\infty}^{\infty} \left[ G_{ya}(o_n)\hat{u}_n + G_{yd}(o_n)\hat{d}_n \right] \exp(io_n t) \]

Equation (7) implies that the smoother the signal \( y(t) \) (in the sense of Sobolev regularity), the more rapid the convergence of the Fourier coefficients to zero must be, with \( e(R) = o(R^{-2s}) \) as \( R \to \infty \).

A discrete wavelet basis, referenced by two indices \( (j,k) \), is formed by rescaling and translating the "mother wavelet" \( \psi(t) \)

\[ \psi_{jk}(t) = 2^{j/2} \psi(2^j t - k) \]

The basis is thus formed from finite duration pulses scaled such that the characteristic frequencies of the wavelet at different scale levels \( j \) form a dyadic series. The approximation of a signal using a wavelet basis is

\[ y(t) = \sum_{k=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} b_{jk} \psi_{jk}(t) \]

where the wavelet coefficients are

\[ b_{jk} = \int_{-\infty}^{\infty} y(t) \psi^{*}_{jk}(t) dt \]

The regularity condition, equivalent to Eq. (7), but in terms of an orthonormal wavelet basis is [9]

\[ \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} 2^{-2j}|b_{jk}|^2 < \infty \]

where the wavelet has \( p > s \) vanishing moments. Equations (11) and (12) imply that the smoother the signal \( y(t) \), the more quickly the wavelet coefficients will converge to zero with \( j \) as the scale decreases (\( j \) increases). A wavelet basis is not always the best choice of basis for approximating very smooth signals, such as sinusoids. However, the Daubechies family of wavelets (Fig. 1) has been explicitly designed with a maximum number of vanishing moments and can be used to efficiently approximate smooth signals such as polynomials [14].

A dyadic wavelet reconstruction of a signal \( y(t) \) in \( L^2 \) over the interval \( t \in [0,1] \) can be made using only \( R \) of the largest scale wavelets. If the signal has a uniform Sobolev regularity of at least \( s \) and the wavelet is selected to have \( p > s \) vanishing moments then the error in the reconstructed signal is \( e(R) = o(R^{-2s}) \) [9]. It is apparent that in general this type of simple wavelet approximation
The next subspace where the indices in the set \( V \) space scale of \( 2^{-j} \) is considered then, to complete these subspaces in a scale \( 2^j \), i.e., there exist coefficients \( b_n \) that satisfy the dilation equation

\[ \phi(t) = \sqrt{2} \sum_{k=0}^{K} c_k \phi(2t-k) \]  

(15)

This dilation equation, which implies the scaling function must have compact support, leads to a wavelet and scaling function with length of support \( K \). With an appropriate choice of coefficients \( c_k \), Eq. (15) can be solved, usually by an iterative method, to obtain the exact form of the scaling function \( \phi(t) \). Equation (14) also implies that the subspace \( V_j \) is formed from a combination of the wavelet subspace \( W_j \) and the scaling function subspace \( V_0 \). Therefore, the wavelet at level 0 (scale 2) can also be constructed from translates of the scaling function at a finer scale through the wavelet equation

\[ \psi(t) = \sqrt{2} \sum_{k=0}^{K} d_k \phi(2t-k) \]  

(16)

Conditions for orthogonality of the wavelet to the scaling function impose restrictions on the choice of coefficients \( c_k \) and \( d_k \). Further restrictions are imposed by the need to obtain a stable basis, i.e., one that produces finite wavelet coefficients for a finite signal. The exact properties and form of the wavelet depend entirely on the choice of coefficients \( c_k \) and \( d_k \).

3 Wavelet Fundamentals

An overview of the relevant aspects of wavelet analysis is presented to explain how the multiresolution capabilities may be incorporated into a control scheme for machine vibration suppression.

3.1 Multiresolution Analysis. The formulation of a multiresolution analysis requires the introduction of a scaling function \( \phi(t) \), which can be combined with its associated wavelet function \( \psi(t) \) to form a complete basis for the function space \( L^2 \). The wavelets at level \( j \) form a basis for the subspace \( W_j \) of \( L^2 \) at a scale of \( 2^{-j} \). If the union of all subspaces \( W_j \) down to a scale level \( J \) is considered then, to complete these subspaces in \( L^2 \), the subspace \( V_j \) is introduced:

\[ V_j \oplus \bigoplus_{j=0}^{J} W_j = L^2 \]  

(14)

The basis functions for \( V_j \) are translates of the scaling function at a scale \( 2^{-j} \) \((\phi_{j0}(t) = 2^{j/2} \phi(2^j t - k); k = 0, \pm 1, \pm 2, \ldots)\).

Equation (14) implies that each subspace \( V_j \) is contained within the next subspace \( V_{j+1} \). Therefore, the scaling functions at level 0 can be formed from translates of the scaling functions at level 1, i.e., there exist coefficients \( c_n \) that satisfy the dilation equation

\[ a_{j+1,k} = \sqrt{2} \sum_{n=0}^{K} c_n a_{j,2n+k} \]  

(18)

\[ b_{j+1,k} = \sqrt{2} \sum_{n=0}^{K} d_n a_{j,2n+k} \]  

(19)

Equations (18) and (19) are equivalent to filtering the coefficients \( a_{j,2n+k} \) using finite impulse response (FIR) filters followed by down-sampling, i.e., discarding every other output value. This means a wavelet transform can be performed efficiently in real time by a cascade of digital filters, where the filter transfer func-
tion coefficients are $\sqrt{2}c_n$ and $\sqrt{2}d_n$ [15]. At each stage of the process the stream of coefficients $a_{j+1}$ are filtered to produce two streams of coefficients $a_j$ and $b_j$ at half the original sample rate (Fig. 2). The duration, or length of support, of the wavelet and scaling function is equal to the order of the filter transfer functions $K$. For real-time processing the filter transfer functions $H(z)$ and $G(z)$ must be causal. This requires multiplying each filter transfer function by $z^{-K}$, thereby introducing a delay that is equal to the filter order $K$. Consequently, the more compact the wavelet the less the time lag in generating the coefficients. This will have important consequences for rotor vibration control. The inverse wavelet transform can be performed by the reverse process of filtering followed by up-sampling.

3.3 Control Implications. Writing the measured vibration in terms of the discrete wavelet transform

$$y(t) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} a_{jk} \phi(t) + \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} b_{jk} \psi(t)$$

Equating Eqs. (3) and (20), multiplying by $\psi_{lm}(t)$ and integrating over time gives

$$b_{lm} = \int_{0}^{T} \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \left[ G_{jm}(\omega_{m}) \hat{\psi}_{lm} + G_{jm}(\omega_{m}) \hat{d}_{lm} \right] 2^{j/2} \psi(2^j t - m) \exp(i\omega_{m} t) dt$$

Thus

$$b_{lm} = b_{lm}^0 + 2^{-j/2} \sum_{m=-\infty}^{\infty} G_{jm}(\omega_{m}) \hat{u}_{lm} \exp(i\omega_{m} m/2) \hat{\psi}(\omega_{m}/2)$$

where $\hat{\psi}(\omega)$ is the Fourier transform of the mother wavelet and $b_{lm}^0$ are the wavelet coefficients for the uncontrolled response with $u(t)=0$. This equation expresses the wavelet coefficients for the measured vibration in terms of the disturbance signal, control force and other time-invariant parameters.

Since disturbances in rotor systems are often periodic, a control scheme can be devised that repeatedly updates and refines the control force on a cycle-by-cycle basis in order to minimize the magnitude of the measured vibration signals $y(t)$. The basic concept is to construct the control force $u(t)$ using a wavelet series and refine the wavelet coefficients each cycle according to the values of the largest wavelet coefficients of the measured vibration signal from previous cycles. The controller will then cancel the effects of a periodic, but possibly irregular, disturbance signal $d(t)$.

A periodic control force may be constructed using an infinite series of wavelets, possibly with a different mother wavelet $\psi^j$ to the analysis wavelet:

$$u(t) = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} b_{jk}^j \psi^j(2^j t - k)$$

If the time base of the signals is rescaled so that the period becomes unity, the Fourier transform of the control force is then given by

$$\hat{u}_n = \int_{0}^{1} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} b_{jk}^j 2^{j/2} \psi^j(2^j t - k) \exp(-i\omega_{n} k/2)$$

Periodicity of the signal means that only a finite number of wavelets contribute to the integral, i.e., wavelets for which $k = 0, \ldots, 2^{j-1}$:

$$\hat{u}_n = \sum_{j=0}^{\infty} \sum_{k=0}^{2^{j-1}} b_{jk}^j 2^{j/2} \psi^j(2^j t - k) \exp(-i\omega_{n} k/2)$$

Therefore, from Eq. (22)

$$b_{lm} = \sum_{j=0}^{\infty} \sum_{k=0}^{2^{j-1}} T_{jklm} b_{jk}^j$$

where

$$T_{jklm} = 2^{-(l+j)/2} \sum_{m=-\infty}^{\infty} \exp[i\omega_{m}(m/2^j - k/2^j)]$$

$$\times \hat{\psi}(\omega_{m}/2) \hat{\psi}(\omega_{m}/2) G_{jm}(\omega_{m})$$

The objective of control will be to minimize the root-mean-square (rms) vibration signal. This implies minimization of the rms value of the wavelet coefficients, as in the steady state

$$\int_{0}^{1} |d|^2 dt = \sum_{j=0}^{\infty} \sum_{k=0}^{2^{j-1}} |b_{jk}|^2$$

If the controller is designed to operate on the measured wavelet coefficients, a compromise is required between accuracy, which determines steady state controller performance, and the speed of vibration attenuation. Optimum vibration cancellation requires that many coefficients be used in the control force calculation, while a fast response requires that as few coefficients as possible
are involved in the control force calculation. The trade-off will depend on the speed of the digital processor performing the calculations, as well as the sample rate of the controller. Other issues include signal noise levels and delays introduced by the decomposition and synthesis filters used to process the wavelet coefficients. If the number of wavelet coefficients used in the control algorithm is to be limited then a method is needed to select the most important coefficients (the best basis) for vibration control. Care needs to be taken to ensure that the processing costs involved in adaptively selecting a basis do not outweigh the benefits. However, the selection of the best basis is not as time critical as the control itself and can therefore be performed as a background task by the controller microprocessor, or even by a separate microprocessor.

4 Dyadic Wavelet Based Control Scheme

4.1 Control Algorithm. An ideal controller would measure the vibration response of the rotor during one cycle and then produce the required control force to minimize the response during the following cycle. If the signal coefficients for a translation \( k \) at level 0 are combined, Eq. (26) can be rewritten as

\[
\mathbf{z}_{q+1} = \mathbf{z}_q + \mathbf{Rw}_{q+1}
\]

(29)

where \( \mathbf{R}_{j2^{k+1}2^{j+1}} = T_{j0_{2^{k+1}}} \), \( \mathbf{x}_{j2^{k+1}} = \mathbf{b}_j \), and \( \mathbf{w}_{j2^{k+1}} = \Delta \mathbf{b}_{j}^T \) are the change in the control force coefficients from the previous cycle. The index \( q \) corresponds to the cycle number and is thus also the index for control signal updates. An algorithm that can select only the most significant basis functions and operate on the corresponding coefficients is required. The algorithm that was adopted performs updates of the control coefficients in order of diminishing effect on the vibration signal. The algorithm is not time critical in that it uses the time available between control updates to calculate as many modified control coefficients as possible thus circumventing the need to find the inverse of a large matrix.

Renormalization of the control synthesis coefficients can ensure that each coefficient has the same influence, in terms of energy contribution, on the measured vibration signals. Thus a diagonal matrix \( \mathbf{\Sigma} \) is defined

\[
\mathbf{\tilde{w}}_q = \mathbf{\Sigma} \mathbf{w}_q \quad \text{and} \quad \mathbf{\tilde{R}} = \mathbf{R} \mathbf{\Sigma}^{-1}
\]

(30)

such that

\[
\mathbf{z}_{q+1} = \mathbf{z}_q + \mathbf{\tilde{R}} \mathbf{\tilde{w}}_{q+1} \quad \text{and} \quad (\mathbf{\tilde{R}}^T \mathbf{R})_{j,j} = 1 \quad \text{for all} \quad j
\]

(31)

i.e., the columns of \( \mathbf{\tilde{R}} \) have been normalized to unity magnitude. To find the most significant coefficient within \( \mathbf{\tilde{w}}_q \) for vibration minimization, the inner product of the vibration vector \( \mathbf{z}_q \) with the columns of \( \mathbf{\tilde{R}} \) is calculated

\[
\mathbf{\tilde{x}}_q = \mathbf{\tilde{R}}^T \mathbf{z}_q
\]

(32)

If only one of the control coefficients, say the \( p \)th element of \( \mathbf{\tilde{w}}_q \), is to be updated and given the value

\[
\{\mathbf{\tilde{w}}_{q+1}\}_p = -\alpha(\mathbf{\tilde{x}}_q)_p
\]

(33)

all other control coefficients being 0, then the modified vibration response is

\[
\mathbf{z}_{q+1} = (\mathbf{I} - \alpha r_p r_p^T) \mathbf{z}_q
\]

(34)

where \( r_p \) is the \( p \)th column of \( \mathbf{\tilde{R}} \). The analysis thus far has neglected any transient response of the wavelet coefficients that may persist for longer than one cycle. This can be caused by transient rotor dynamics and also the inherent lag in the calculation of wavelet coefficients, which slows the response of the controller. To overcome the lag in reaching steady-state conditions, the relaxation factor \( 0 < \alpha < 1 \) is introduced to slow the convergence rate and thus reduce the transient component of the rotor response to avoid possible instabilities. It follows from Eqs. (32) and (34) and the fact that \( r_p^T r_p = 1 \) that the mean square value of the vibration response will be

\[
\mathbf{z}_{q+1}^T \mathbf{z}_{q+1} = \mathbf{z}_q^T \mathbf{z}_q - \alpha(2 - \alpha)(\mathbf{\tilde{x}}_q)_p^2
\]

(35)

To produce the largest reduction in vibration magnitude the index \( p \) should be chosen to select the largest magnitude element of \( \mathbf{\tilde{x}}_q \).

If more than one coefficient is updated, simply by reiteration of this procedure, the magnitude of the response will be

\[
\mathbf{z}_{q+1}^T \mathbf{z}_{q+1} = \mathbf{z}_q^T \mathbf{z}_q - \alpha(2 - \alpha) \sum_{p \neq \beta} (\mathbf{\tilde{x}}_q)_p^2
\]

(36)

Thus, to minimize vibration while only updating a finite number of control coefficients, the order in which coefficients are calculated is crucial. The update order can be based on calculation of \( \mathbf{\tilde{x}}_q \) from Eq. (32), which can be performed as a background task in a real-time control implementation. In general, nonorthogonal-

![Fig. 3 Schematic of adaptive wavelet-based control scheme for rotor vibration attenuation](image-url)
ity of the columns of $\bar{R}$ implies that $\sum_q z_q^T z_q > \sum_p (x_p)^2$ and so complete cancellation of vibration over one cycle is not possible even with $\alpha = 1$. However, by adapting the wavelet basis used by the algorithm, the controller operates to update the wavelet coefficients in order of decreasing influence on the measured vibration. The adaptive wavelet control scheme is shown schematically in Fig. 3.

4.2 Application to a Flexible Rotor–Magnetic Bearing System. Implementation and evaluation of the vibration control scheme was based on an experimental rotor/magnetic bearing system (Fig. 4). A principal advantage of employing magnetic bearings in rotating machinery is that they enable a wide variety of active vibration control schemes to be implemented. The objectives of any vibration control scheme will depend on the application in hand, but typically rotor displacements relative to the stator, transmitted bearing forces, stator accelerations, magnetic bearing control currents, or weighted combinations thereof, may be usefully minimized. The wavelet control scheme developed in this paper is applied to minimize lateral vibration of the rotor, in the terms of rms values of measured signal wavelet coefficients. Such objectives would be appropriate, for example, to precision milling applications for which cutting tool vibration (relative to the workpiece) should be minimized. However, in magnetic bearing systems, maintaining rotor centering under large disturbances is additionally important to avoid contact between the rotor and backup bearings/bushes, which can result in destabilization of the magnetic bearing control system.

![Schematic of experimental flexible rotor with magnetic bearings](image)

**Fig. 4** Schematics of experimental flexible rotor with magnetic bearings (a) system layout (b) impact mechanism located in plane $G$

![Graph](image)

**Fig. 5** Rotor residual unbalance response: Mean orbit amplitude and phase in (a) sensor planes $A$ and $F$ and (b) sensor planes $C$ and $D$
Fig. 6 Real-time wavelet analysis of rotor vibration measured in plane $F$, showing wavelet coefficient series and corresponding signal components at four scale levels. The controller is activated at zero revolutions. The rotational frequency is 20 Hz.

Fig. 7 Rotor vibration response and control force for controller based on Daubechies D2 wavelet. Rotor displacement and control force are shown for planes $F$ and $D$, respectively. The controller is activated at zero revolutions. The rotational frequency is 16 Hz.
The system employed consists of a 2 m long rotor supported by two radial magnetic bearings. The rotor is made from stainless steel and has a 0.05 m diameter shaft on which are mounted four solid disks of radius 0.15 m. The magnetic bearings are of heteropolar design, having two sets of opposing coil pairs oriented at ±45 deg to the vertical. The bearings coils are driven by eight switching amplifiers operating in a current control mode. Each magnetic bearing has an internally mounted auxiliary bearing with nominal radial clearance of 0.75 mm. The rotor was driven by an alternating current motor through a flexible coupling that additionally prevented axial motion of the rotor.

Lateral displacement of the rotor can be measured in four planes (A, C, D, and F) using orthogonal pairs of eddy current transducers oriented along x- and y-axes at ±45 deg to the vertical, as indicated in Fig. 4(b). The rotor was initially levitated by the magnetic bearings using proportional-integral-derivative (PID) feedback of the rotor displacements measured adjacent to the bearings in planes C and D. Additional forces for vibration control could be applied to the rotor through the magnetic bearings in planes B and E by superposition with the PID control forces.

The rotor was initially levitated using a proportional gain of $K_P=3 \times 10^6$ N/m, derivative gain of $K_D=5 \times 10^4$ N s/m and integral gain of $K_I=2 \times 10^5$ N/ms, which provides moderate levels of stiffness and damping at the magnetic bearings. The rotor was then manually balanced to ensure orbit sizes did not exceed 40% of the radial clearance (0.75 mm) over a running speed range of 0–40 Hz. The synchronous response of the rotor due to residual unbalance can be seen in Fig. 5, which shows the mean orbit amplitude and phase in each of the sensor planes as a function of rotational frequency. There are three main critical speeds evident within this running speed range corresponding to the two rigid body modes (at ~10, 19 Hz) and first flexural mode (at ~29 Hz). However, the modal frequencies are not well separated and so the corresponding modes exhibit a mixture of flexural and rigid body characteristics. Gyroscopic effects are not significant in this speed range.

The wavelet controller inputs comprised four rotor displacement measurements selected from planes A and F or planes C and D, while the controller outputs were the orthogonal pairs of control forces applied at the bearings. Thus the number of control input and output signals were equal. However, there is noncollocation of disturbance force, control force, and measurement planes, which makes PID feedback alone ineffective as a method for vibration attenuation.

The wavelet based control scheme was implemented using digital signal processor (DSP) hardware linked to a personal computer (PC) for purposes of interfacing and data acquisition. Although the main control algorithm was run on a dedicated DSP, basis selection, and calculating the order of coefficient updates was performed on the PC and then communicated to the DSP.

The wavelet coefficients for the four measured signals and four control forces were calculated once per rotor revolution using the pyramid algorithm of Mallat [9], realized as a cascade of discrete time filters. The measured wavelet coefficients used in the control algorithm were restricted to four scale levels. At higher levels (smaller scales) the coefficients became increasingly dominated by noise and therefore were not included in the control calculation. However, this still gave 15 coefficients per signal per rotor revolution, or 60 coefficients in total that were used in the control algorithm.

The basis set for the control force construction was limited to three scale levels in order to avoid amplifier saturation that can occur with high frequency control signals. In practice the magnetic bearings can achieve a maximum control force slew rate of $7.5 \times 10^5$ N/s, which means that discontinuities in the control force signal can only be approximated by the actual force.
resulting error in the control force is only significant for the Haar wavelet, but can be kept small by imposing a maximum scale level for wavelets included in the control force construction. The total number of control force coefficients was 28 per rotor revolution (for all four control forces). However, only the most significant coefficients were updated each control iteration (typically about half) in accordance with Eqs. (29)–(36). The number of measurement coefficients exceeded the number of control force coefficients and so complete cancellation of vibration cannot be expected.

5 Experimental Evaluation

5.1 Method. For evaluation of the vibration control scheme, disturbance forces could be applied at the nondriven end disk (in plane G) using two methods:

1. A sprung hammer mechanism to produce periodic impacts with a wedge of nylon material attached to the circumference of the nondriven end disk (Fig. 4(b)). The impacts occur once per revolution and act on the rotor in plane G. The bending stiffness of the mounting arm of the hammer was sufficiently low that nonlinear stiffness effects arising from contact with the rotor did not significantly change the open-loop dynamics of the rotor system. Thus the rotor was excited by a periodic lateral disturbance force with a fundamental frequency matching the rotational frequency, but containing significant superharmonic components.

2. A small mass was attached to the perimeter of the nondriven end disk using Kevlar cord. The mass could be detached during operation using a solenoid operated blade, thus producing an instantaneous change in rotor unbalance. This type of event provides a useful method for testing controller transient performance.

The action of the wavelet controller was examined by measuring the transient response of the rotor following activation. This was undertaken at a rotational frequency of 20 Hz, close to the critical speed at 19 Hz, using a controller based on the Daubechies D2 wavelet with \( p = 2 \) vanishing moments (Fig. 2(b)).

Figure 6 shows the rotor vibration signal and corresponding real-time wavelet decomposition (as performed by the controller) for a single x-axis sensor in plane F. The wavelet coefficients for the measured displacement are attenuated by the action of the controller. Similar levels of vibration attenuation were measured at all four sensors in the planes A and F, but for brevity these are not shown.
shown. The initial vibration of the rotor, produced by unbalance, is approximately sinusoidal in character and the wavelet coefficients time variation has the same periodicity as the original signal. This means that constant value level 0 coefficients are produced, as only one coefficient is generated per rotor revolution, while at higher levels (smaller scales) oscillating coefficients of diminishing amplitude are generated. Thus the components of the sinusoidal vibration signal are spread across many wavelet levels, although the largest component occupies the level 1 subspace. This is evident in Fig. 6, which shows the corresponding signal components at each scale level. Attenuation of the rotor vibration occurs at all four wavelet levels over approximately 60 revolutions. However, the degree of attenuation weakens as the scale decreases and the component frequencies increase. The residual vibration of the rotor is small in amplitude, but there is a notable high frequency component that cannot be eliminated by the control action. Note that the wavelets basis functions form an orthonormal basis for the measured signals such that the values of the coefficients satisfy Eq. (28), where the time base for signals has been rescaled so that the rotational period is unity. This means that the rms value of the wavelet coefficients for each level will approximate the rms rotor displacement for the corresponding scale component, on a cycle-by-cycle basis.

Figure 7 shows the results of a similar test performed between critical speeds at a rotational frequency of 16 Hz, but focuses on the rotor vibration in plane $F$ and the control force in plane $E$ before and after vibration attenuation has occurred. The results are broadly similar to those obtained at 20 Hz although a faster rate of convergence can be achieved at this speed. Close examination of the signal characteristics reveals interesting features. Initially, vibration of the rotor is sinusoidal in character, consistent with rotor unbalance. When the controller is activated, a steady increase in the amplitude of the control force coefficients occurs until steady conditions are reached and the rotor vibration magnitude approaches minimal levels. The residual vibration of the rotor contains a number of harmonics of the synchronous frequency. For synchronous disturbance cancellation, the final control force signal should approximate a sinusoid, however, the wavelet-based construction produces a signal that is periodic, but irregular in character. Thus the residual high frequency vibration of the rotor is generated by the control force, which is non-smooth due to the irregularity of the $D_2$ wavelet used for its synthesis. It is also remarked that, in the implementation of the controller, the wavelet spacing could not be exactly synchronized with the rotation of the rotor.
rotor due to slight variations in rotor speed. This means that a slight mismatch in the rotational frequency and wavelet spacing can result in nonminimal steady-state vibration.

5.2 Impact Tests. To evaluate controller operation when rotor disturbance forces are periodic, but nonsmooth, impact tests were undertaken at a rotational frequency of 20 Hz. A controller based on the Haar \(D1\) wavelet (Fig. 2(a)) was also included to examine the performance dependence on the wavelet choice. Figure 8 shows the transient response of the rotor in plane \(F\). Initially, the impact mechanism was held back from the rotor and the rotor vibration is well controlled (amplitude \(\sim 50 \mu m\)). After ten revolutions the impact hammer was released and contact with the rotor commenced. There is a lag in the reaction of the control force due primarily to the delay in calculating the best order for updating coefficients, but also due to the intrinsic delay of the wavelet decomposition and reconstructions. During this time the vibration of the rotor builds in amplitude over approximately seven revolutions to a peak amplitude of approximately 500 \(\mu m\), at which time a reaction in the control force is evident. The rotor orbit at this time is highly asymmetric in character and contains multiple harmonics of the synchronous frequency. Attenuation of the rotor vibration then occurs over the next 20 revolutions, while the impact disturbance continues. The final rotor vibration is approximately 100 \(\mu m\) in amplitude. Figure 9 shows the wavelet coefficients produced at four scale levels for this rotor vibration signal during the test. It is clear that the rotor impact produces a vibration response that is spread across all four scale levels. Moreover, the action of the control produces attenuation of the coefficients at every level. It is also apparent that residual vibration of the rotor contains components at all four scale levels.

Figure 10 shows the results of the same test, but for control based on the \(D2\) wavelet. The results are broadly similar to those obtained with the Haar wavelet. However, a significant difference is that a slower convergence rate (smaller order) had to be used. This is because the \(D2\) Daubechies wavelet function has longer duration (length of support three units compared with one for \(D1\)) and therefore the intrinsic lag in the wavelet decomposition and synthesis is greater. The final steady-state control force signals for the \(D1\) and \(D2\) wavelet are both irregular, but very different in character. Despite these differences the final levels of rotor vibration are similar. Figure 11 shows the \(D2\) wavelet coefficients produced at four scale levels. Comparison with Fig. 9 shows the vibration signal has larger components at the lower scale levels and therefore the coefficients at the higher levels (finer scales) are smaller than with the Haar wavelet. Figure 12 shows the equivalent results obtained using the \(D4\) Daubechies wavelet with \(p=4\) vanishing moments, as shown in Fig. 2(c). Comparing results with all three wavelet types, the final levels of rotor vibration are similar in all cases, though the control force signals differ markedly in character, displaying the irregular features of the synthesis wavelet. Also, the rate of convergence that can be achieved increases as the wavelet order and length of support of the mother wavelet increases. However, in these tests, the higher order wavelets could accurately approximate the measured vibration signals using fewer scale levels.

5.3 Mass-Loss Tests. Figure 13 shows the results of a mass loss test undertaken at 20 Hz. The controller employs a wavelet analysis and synthesis based on the \(D1\) wavelet. Initially, the rotor is well balanced and the rotor orbit measured in plane \(F\) is very small. After ten revolutions, a mass of 36 g is detached from the disk in plane \(G\) resulting in a step change in synchronous distur-

Fig. 13 Rotor mass loss test at 20 Hz: Rotor vibration and control force are shown for planes \(F\) and \(D\), respectively. Control force construction is based on a \(D1\) Haar wavelet.
bance force of magnitude 71 N. The rotor orbit measured in plane F grows in amplitude over the next five-six revolutions in response to the increase in disturbance force. The controller adapts the control force coefficients over the next 20 rotor revolutions to attenuate the rotor vibration in a least-squares sense. Although the initial response of the rotor and orbits are highly symmetric, the resultant control force is asymmetric in character and produces noncircular orbits. Interestingly, this occurs despite the Haar wavelet being anti-symmetric about its center of support. Figure 14 shows that significant vibration attenuation occurred for all four wavelet levels monitored and regulated by the controller.

Figure 15 shows results of the same mass-loss test undertaken with a controller based on the D2 wavelet. As for the impact tests, the rotor vibration converges more slowly with the higher order wavelet. There is also a clear residual oscillation in the rotor response with a frequency three times the synchronous frequency. Although this residual vibration also occurred with the D1 wavelet, it is significantly more pronounced with the D2 wavelet. The mass-loss tests confirm the findings of Secs. 5.1 and 5.2 that for attenuation of steady-state rotor synchronous vibration, the wavelet approximations with irregular low order wavelets are less effective than with smoother high order wavelets. Control of transient vibration is most effective with the compact wavelet (D1) in terms of the achievable rate of convergence, which decreases considerably for longer duration wavelets due to the intrinsic lag in the calculation of the coefficients.

6 Conclusions

This paper has introduced theoretical arguments for the application of discrete wavelet transforms in the control of machine vibration. By devising and implementing a wavelet based scheme for the control of rotor vibration the approach has been demonstrated experimentally. By examining the operation of the controller using a selection of wavelets under various disturbance conditions, the principles and limiting factors in achieving good steady-state and transient disturbance attenuation were determined.

It was found that the rotor vibration signals could generally be approximated with fewer wavelet levels by using longer duration wavelets that have higher regularity. However, to achieve short time lags in the processing of the wavelet coefficients the wavelets employed must conversely have short duration. This is necessary for rapid vibration attenuation and allows the controller to adapt more quickly to small changes in disturbance signals, for example caused by drift in the rotor speed. Thus quasi-steady-state vibra-
The wavelet types focused on in this study were therefore of low order, but with maximum regulation levels can also be improved. The wavelet types with the best compromise are important areas for future research.

**Acknowledgment**

The authors gratefully acknowledge the support of the Engineering and Physical Sciences Research Council of the UK, through the award of Grant No. GR/R45277/01.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{S}$</td>
<td>Sobolev regularity</td>
</tr>
<tr>
<td>$\mathcal{T}$</td>
<td>cyclic period for vibration control</td>
</tr>
<tr>
<td>$\mathcal{U}$</td>
<td>control force vector</td>
</tr>
<tr>
<td>$\mathcal{V}$</td>
<td>scaling function subspace</td>
</tr>
<tr>
<td>$\mathcal{W}$</td>
<td>wavelet subspace</td>
</tr>
<tr>
<td>$\mathcal{W}$</td>
<td>wavelet coefficient vector for magnetic bearing control force</td>
</tr>
<tr>
<td>$\mathbf{x}$</td>
<td>system state vector</td>
</tr>
<tr>
<td>$\mathbf{y}$</td>
<td>finite energy scalar signal</td>
</tr>
<tr>
<td>$\mathbf{z}$</td>
<td>Z-transform variable</td>
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<tr>
<td>$\mathbf{\alpha}$</td>
<td>convergence relaxation factor</td>
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<tr>
<td>$\mathbf{\varepsilon}$</td>
<td>signal approximation error</td>
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<td>$\mathbf{\phi}$</td>
<td>scaling function</td>
</tr>
<tr>
<td>$\mathbf{\Sigma}$</td>
<td>rescaling matrix</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular frequency (rad/s)</td>
</tr>
</tbody>
</table>

**Subscripts**

- $d, u, y$ = disturbance, control, measurement related parameters
- $j, l$ = scale level
- $k, m$ = basis function indices
- $q$ = control update index / rotor vibration cycle index
- $U$ = control input
- $0$ = uncontrolled

**Superscripts**

- $1$ = rotor vibration cycle
- $2$ = rotor disturbance vector
- $3$ = basis function

**References**


