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CENTRALIZING LEFT GENERALIZED DERIVATIONS ON SEMIPRIME RINGS

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ABSTRACT

In this paper we proved that if d is a nonzero derivation of a prime ring R and f be a left generalized derivation, then f is a strong commutativity preserving. Using this we proved that $R$is commutative.

Keywords: *Prime ring, Derivation, Generalized derivation, Left generalized derivation, Homomorphism, Centralizing.*

INTRODUCTION

Bell and Martindale [2] studied centralizing mappings of semi prime rings and proved that if d is a nonzero derivation of prime ring$R$ such that $[d(x), x] = 0$ for all x in a nonzero left ideals of R, then R is commutative. Bell and Daif [3] investigated commutativity in prime and semiprime rings admitting a derivation or an endomorphism which is strong commutativity preserving on a nonzero right ideal. Ali and Shah [1] extended some results of Bell and Martindale [2] or generalized derivations. Throughout this paper, R will denote a semiprime ring and Z its center.Recall that prime if $aRb = (0)$ implise that $a = 0$ or$b = 0$and semi prime if $aRa = (0)$ implies that $a = 0$. As usual $[x, y]$ will denote the commutator $xy - yx$. An additive mapping$d: R \rightarrow R$ is called a derivation if $d(xy) = d(x)y + xd(y)$, holds for all $x, y \in R$.An additive mapping $F: R \rightarrow R$ is called a generalized derivation if there exists a derivation $d: R \rightarrow R$ such that $F(xy) = F(x)y + xd(y)$ for all $x, y \in R$. An additive mapping $F: R \rightarrow R$ is called a left generalized derivation if there exists a derivation $d: R \rightarrow R$ Such that $F(xy) = d(x)y + xF(y)$ for all $x, y \in R$.A mapping f is commuting on a right ideal U of R if $[f(x), x] = 0$, for all $x \in U$ and f is centralizing if$[f(x), x] \in Z$, for all $x \in U$. A mapping $f: R \rightarrow R$ is called strong commutativity preserving if $[f(x), f(y)] = [x, y]$, for all $x, y \in R$.

Rmark1: For a nonzero elements $a \in Z$, if $ab \in Z$, then $b \in Z$.

To prove main result we require the following lemmas:

Lemma 1: If f is an additive mapping from R to R such that f is centralizing on a right ideal U of R, then $f(x) \in Z$, for all $x \in U \cap Z$.

Proof: Since f is centralizing on U, we have $[f(x + y), x + y] \in Z$
$[f(x) + f(y), x + y] \in Z$
$[f(x), x] + [f(x), y] + [f(y), x] + [f(y), y] \in Z$
$[f(x), y] + [f(y), x] \in Z$

Now if $x \in Z$, then from above equation we have
$\Rightarrow [f(x), y] \in Z$we replaced y by $f(x)y$, then
$\Rightarrow f(x)[f(x), y] \in Z$

If $[f(x), y] = 0$, then $f(x) \in C_R^{(U)}$, the centerالizer of U in R and by [1] belongs to Z. But on the other hand, if $[f(x), y] \neq 0$, it again follows from the remark 1 that $f(x) \in Z$

Lemma 2: Let R be a semiprime ring and U a nonzero ideal of R. If Z in R centralizes the set $[U, U]$, then Z centralizes U.

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Now we prove the following result:

Theorem 1: Let $d: R \to R$ be a non zero derivation of prime ring R and f be a left generalized derivation on a nonzero right ideal $U$of R. If f acts as a homomorphism on U, then$f$is strong commutativity preserving on U.

Proof: We assume that $f$acts as homomorphism on U and f be a left generalized derivation on U. Then
$f(xy) = f(x)f(y) = d(x)y + xF(y)$for all x, y in U. (1)

We replace$y$ by zy, $z \in U$, the second equality of (1) we have
$f(xzy) = f(x)f(zy) = d(x)zy + xf(zy) = d(x)zy + xf(z)f(y)$. (2)

Since f is a homomorphism. On the other hand we have
$f(xzy) = f(xz)f(y) = \big(d(x)z + xf(z)\big)f(y)$
$f(xzy) = d(x)zf(y) + xf(z)f(y)$. (3)

From equation (2) & (3), we get
$d(x)zy + xf(z)f(y) = d(x)zf(y) + xf(z)f(y)$
$d(x)z(f(y) - y) = 0$. (4)

We replace y by $[x, y]$ in equation (4), then
$d(x)z(f[x, y] - [x, y]) = 0$

By replacing z by zr, $r \in R$ in the above equation then
$d(x)z\,R(f[x, y] - [x, y]) = 0$

By the prime ness of R, we have either $d(x)z = 0$ or $f[x, y] - [x, y]$=0

Since $d \neq 0$, then $f[x, y] - [x, y] = 0$.
$f[x, y] = [x, y]$
$[f(x), f(y)] = [x, y]$.

Hence f is strong commutativity preserving on U

Theorem 2: Let U be right ideal of a semiprime R such that $U \cap Z \neq 0$. Let d be a non zero derivation and f be a left generalized derivation on R such that f is centralizing on U. Then R is commutative.

Proof: We assume that $Z \neq 0$ because f is commuting on U and there nothing to prove.

Since f is centeralizing on U, we have
$[f(x), x] \in Z$for all $x, y \in U$

Linearizing the above equation we have
$[f(x + y), x + y] \in Z$for all $x, y \in U$
$[f(x), x] + [f(x), y] + [f(y), x] + [f(y), y] \in Z$
$[f(x), y] + [f(y), x] \in Z$ for all $x, y \in U$. (5)

We replaced $x\ by\ yz$ in equation (5), we get
$[f(yz), y] + [f(y), yz] \in Z$
$\big[\big(d(y)z + yf(z)\big), y\big] + [f(y), y]z + y[f(y), z] \in Z$
$[d(y), y]z + d(y)[z, y] + y[f(z), y] + [f(y), y]z + y[f(y), z] \in Z$, then
$[d(y), y]z + y[f(z), y] + [f(y), y]z \in Z$.

Now by lemma 1,$f(z) \in Z$ and there fore $[d(y), y]z + [f(y), y]z \in Z$

But f is centralizing on U. We have $[f(y), y]z \in Z$ and consequently$[d(y), y]z \in Z$.

Since z is non zero, it follows from remark1 that $[d(y), y] \in Z$. This implies that$d$is centralizing on U and hence we conclude that R is commutative.

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