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RIGHT IDEALS AND LEFT GENERALIZED DERIVATIONS ON PRIME RINGS

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ABSTRACT

Let R be a prime ring and d be a left derivation on R. If f is a left generalized derivation on R such that f is commuting and centralizing on a right ideal I of R, then R is commutative.

Keyword:

Prime rings, Right ideals, Left derivation, Generalized derivation, Generalized left derivation, Centralizing and commuting..

Preliminaries: Throughout this paper R will denote an associative ring with centre $Z(R)$. A ring R is said to be a prime if $aRb = 0$ implies that either $a = 0$ or $b = 0$. An additive mapping $d: R \to R$ is said to be derivation if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$ and an additive mapping $d: R \to R$ is said to be left derivation if $d(xy) = xd(y) + yd(x)$, for all $x, y \in R$. A mapping f is said to be commutating on a left ideal U of R if $[f(x), x] = 0$, for all $x \in U$ and f is said to be centralizing if $[f(x), x] \in Z(R)$,for all $x \in U$. An additive mapping $f: R \to R$ said to be generalized derivation if there exists a derivation $d: R \to R$ such that $f(xy) = f(x)y + xd(y)$, for all $x, y \in R$. An additive mapping $f: R \to R$ is said to be a left generalized derivation if $f(xy) = xf(y) + yd(x)$, for all $x, y \in R$, where d is a left derivation on R. The commutator $[x, y] = xy - yx$ and anti commutator $[x \circ y] = xy + yx$, for all $x, y \in R$

and commutator identities $[x, yz] = y[x, z] + [x, y]z$, and $[xy, z] = [x, z]y + x[y, z]$, for all $x, y, z \in R$.

Introduction: The study of the commutativity of prime rings with derivations was initiated by E.C.Posner[10]. Recently, M.Bresar [3, 4] define generalized derivation of rings. Hvala [6] studied generalized derivation in prime rings. Golbasi [5] extended some well known results concerning derivations of prime rings to the generalized derivations and a nonzero left ideal of a prime ring which is semi prime as a ring. Jaya Subba Reddy et.al [7, 8] studied centralizing and commutating left generalized derivation on prime ring is commutative. Afrah Mohammad Ibraheem[1] studied right ideal and generalized reverse derivation on prime rings is commutative. In this paper we extended some results right ideals and left generalized derivations on prime ring is commutative.

In order to prove the main results, first we prove the following lemma:

Remark (1) [2]: Let R be a prime ring. For a nonzero element $a \in Z(R)$, if

$ab \in Z(R)$, then $b \in Z(R)$.

Lemma (1): Let R be a prime ring and d be a left derivation on R. For an element $a \in R$,

if $a\, d(r) = 0$, for all $r \in R$, then either $a = 0$ or $d = 0$.

Proof: For $a \in R$, let $a\, d(r) = 0$, for all $r \in R$.
(1)

Replace r by sr in (1), we have $a\, d(sr) = 0$, then

$a\, rd(s) + a\, sd(r) = 0$, for all $s, r \in R$
(2)

By using (1), we get $ard(s) = 0$, for all $r, s \in R$.

if $d(s) \neq 0$, for some $s \in R$, then $a = 0$ by definition of prime ring. Hence proved.

Lemma (2): Let I be a nonzero right ideal of a prime ring R. If R has a zero left derivation

d on I, then d is also zero left derivation on R.

Proof: Let $I \neq \{0\}$ is a right ideal or R.

We assume that $d(I) = 0$
(3)

Since $IR \subseteq R$ we have:

$d(IR) = Rd(I) + Id(R) = 0$ (4)

By using (3), we get $Id(R) = (0)$.

Since $I \neq \{0\}$, then by lemma (1), $d(R) = 0$.

Lemma (3) [9]: Let R be a prime ring and I a non zero right ideal of R. If I is commutave, then R is also commutative.

Theorem (1): Let R be a prime ring and I be a non zero right ideal of R. If d is a non zero left generalized derivation on R, such that d is a centralizing on I, then R is commutative.

Proof: Let d be a centralizing on I

$[a, d(a)] \in Z(R)$, for all $a \in I$.
(5)

Replacing a by a^2 in (5), we get

$[a^2, d(a^2)] \in Z(R)$
(6)

$[a^2, 2ad(a)] \in Z(R)$

$2[a^2, ad(a)] \in Z(R)$

$2a[a^2, d(a)] + 2[a^2, a]d(a) \in Z(R)$

$2a^2[a, d(a)] + 2a^2[a, d(a)] \in Z(R)$

$4a^2[a, d(a)] \in Z(R)$

Thus, $4[a^2[a, d(a)], d(a)] = 0$, for all $a \in I$

$4a^2[\,[a, d(a)], d(a)] + 4[a^2, d(a)][a, d(a)] = 0$

$(a\,[a, d(a)] + [a, d(a)]a)\ [a, d(a)] = 0$

$(a\,[a, d(a)] + a[a, d(a)])\ [a, d(a)] = 0$

and $2a[a, d(a)]\ [a, d(a)] = 0$, for all $a \in I$.
(7)

Also $[a^2, d(a)] = 0$ and $[a, d(a)] = 0$, for all $a \in I$.
(8)

We Replace $d(a)$ by $d(a)b$ in (8) ,we get

$[a^2, d(a)b] = 0$

$d(a)[a^2, b] + [a^2, d(a)]b = 0$

$d(a)[a^2, b] = 0$,for all $a, b \in I$
(9)

Replacing $d(a)$ by $d(a)r$ in (9), we get

$d(a)r[a^2, b] = 0$, for all $a, b \in I$ and $r \in R$.

Since R is a prime, we get either $d(a) = 0$ or $[a^2, b] = 0$

If $d(a) = 0$ for all $a \in I$ then by Lemma (1), $d(R) = 0$ this is a contradiction.

So $[a^2, b] = 0$, for all $a, b \in I$, that's mean I is commutative and hence by Lemma (3), R is commutative.

Theorem (2): Let R be a prime ring and I be a right ideal of R. If f is left generalized derivation on R with a left derivation d on R, such that f is centralizing on I, then for all $a \in I \cup Z(R)$, $f(a) \in Z(R)$.

Proof: Since f is centralizing on I, we have

$[a, f(a)] \in Z(R)$, for all $a \in I$. (10)

By linearizing (10), for all $a, b \in I$, we have

$[a + b, f(a + b)] \in Z(R)$

$[a + b, f(a) + f(b)] \in Z(R)$

$[a, f(a)] + [a, f(b)] + [b, f(a)] + [b, f(b)] \in Z(R)$

$[a, f(b)] + [b, f(a)] \in Z(R)$
(11)

If $a \in Z(R)$, this implies that $[b, f(a)] \in Z(R)$.
(12)

Replacing b by $b\, f(a)$ in (12), we get

$[b, f(a)]f(a) \in Z(R)$, for all $a, b \in I$

If $[b, f(a)] = 0$, then $f(a) \in C_R(I)$, the centralizer of I in R,

and hence $f(a) \in Z(R)$. On the other hand if $[b, f(a)] \neq 0$,

then by remark (1) ,we get $f(a) \in Z(R)$.

Theorem (3): Let I be a non zero right ideal of a prime ring R and f is a left generalized derivation on R with a non zero left derivation d on R. If f is commuting on I, then R is commutative.

Proof: Let f is a commuting on I, then for all $a \in I$,

we have $[a, f(a)] = 0$
(13)

Replacing a by $a + b$ in (13), we get

$[a + b, f(a + b)] = 0$

$[a, f(b)] + [b, f(a)] = 0$ (14)

Substituing $b = ba$ in (14) and using (13), then we get

$[a, f(ba)] + [ba, f(a)] = 0$

$[a, d(b)a + bf(a)] + b[a, f(a)] + [b, f(a)]a = 0$

$[a,d(b)a]+[a,bf(a)]+b[a,f(a)]+[b,f(a)]a=0$

$d(b)[a,a]+[a,d(b)]a+[a,b]f(a)+b[a,f(a)]+b[a,f(a)]+[b,f(a)]a=0$

$[a,d(b)]a+[b,f(a)]a+[a,b]f(a)=0$

$[a,d(b)]a+[b,f(a)]a+[a,b]f(a)=0$, for all $,b\in I$.
(15)

Replacing a by b in (15) and using (13) we get,

$[b,d(b)]b=0$, for all $a,b\in I$
(16)

Replacing $d(b)=rd(b)$ in (16), and using (16) , we get

$[b,rd(b)]b=0$, for all $b\in I$, and $r\in R$

$r[b,d(b)]b+[b,r]d(b)b=0$

$[b,r]d(b)b=0$, for all $b\in I$, and $r\in R$. (17)

Replacing r by sr in (17), and using (17), we get

$[b,sr]d(b)b=0$

$s[b,r]d(b)b+[b,s]rd(b)b=0$

$[b,s]rd(b)b=0$ for all $b\in I$, and $r,s\in R$

Since R is a prime ring, and $d(b)b\neq 0$, then $[b,s]=0$, for all $b\in I$, and $s\in R$.

Therefore $b\in Z(R)$, and so $I\subseteq Z(R)$, which implies that I is commutative

and by Lemma (3), R is commutative.

Theorem (4): Let R be a prime ring and I be a right ideal of R such that $I\cap Z(R)\neq 0$.

Let f be left generalized derivations on R with a non zero left derivation d on R. If f is commuting on I, then R is commutative.

Proof: Let we take $Z(R)\neq 0$, since f is a commuting on I then the proof is complete.

Now , by equation (11) ,we have

$[a,f(b)]+[b,f(a)]\in Z(R)$, for all $a,b\in I$

we replace b by ar ,where $0\neq r\in Z(R)$, we get

$[a,f(ar)]+[ar,f(a)]\in Z(R)$

$[a,d(a)r+af(r)]+a[r,f(a)]+[a,f(a)]r\in Z(R)$

$[a,d(a)r]+[a,af(r)]+a[r,f(a)]+[a,f(a)]r\in Z(R)$

$d(a)[a,r]+[a,d(a)]r+a[a,f(r)]+a[r,f(a)]+[a,f(a)]r\in Z(R)$

$[a,d(a)]r + a[a,f(r)] + [a,f(a)]r \in Z(R)$, for all $\in I$, and $r \in R$
(18)

By using lemma (1) in (18),we get $f(r) \in Z(R)$, and since f is centralizing on I.

We get $r[a,d(a)] \in Z(R)$, for all for all $a \in I$, and $r \in R$
(19)

By using remark (1) in (19), we get

$[a,d(a)] \in Z(R)$, for all $a \in I$ and hence by theorem (1), R is commutative.

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