# A Novel Algorithm for Wafer Sojourn Time Analysis of Single-Arm Cluster Tools With Wafer Residency Time Constraints and Activity Time Variation 

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#### Abstract

This paper addresses the scheduling problem of single-arm cluster tools with both wafer residency time constraints and activity time variation in semiconductor manufacturing. Based on a Petri net model developed in our previous work, polynomial algorithms are proposed to obtain the exact upper bound of the wafer sojourn time delay for the first time. With the obtained results, one can check the feasibility of a given schedule or find a feasible and optimal one if it exists. Illustrative examples are given to show the applications of the proposed method.


Index Terms-Cluster tools, discrete event system, Petri net (PN), scheduling, semiconductor manufacturing.

## Nomenclature

| $a_{i}$ | The shortest time needed for completing a wafer at <br> Step $i, i \in \mathbf{N}_{n}$. |
| :--- | :--- |
| $B_{i}$ | The upper bound of the wafer sojourn time delay <br> at Step $i$. |
| $b_{i} \quad$The longest time needed for completing a wafer at <br> Step $i, i \in \mathbf{N}_{n}$. |  |
| $I$ | Input function in a Petri net (PN). |
| $K$ | Capacity function in a PN. |
| $M$ | Marking in a PN. |
| $\mathbf{N}=\quad\{0,1,2, \ldots\}$. |  |
| $\mathbf{N}_{\boldsymbol{n}}=$ | $\{1,2, \ldots, n\}$. |

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$O$ Output function in a PN.
PM Process module.
PN Petri net.
$p_{0} \quad$ PN place modeling loadlocks.
$p_{i} \quad$ PN place modeling Step $i, i \in \mathbf{N}_{n}$.
$q_{i 1} \quad$ PN place modeling the robot waiting before loading a wafer into Step $i, i \in \boldsymbol{\Omega}$.
$q_{i 2} \quad$ PN place modeling the scheduled robot waiting before unloading a wafer from Step $i, i \in \boldsymbol{\Omega}$.
$q_{i 3} \quad$ PN place modeling the unscheduled robot waiting before unloading a wafer from Step $i, i \in \boldsymbol{\Omega}$.
RCP Real-time control policy.
$r \quad$ PN place modeling the single-arm robot.
$s_{i 1} \quad$ PN transition modeling the robot task of loading a wafer into Step $i, i \in \mathbf{N}_{\boldsymbol{n}}$.
$s_{01} \quad$ PN transition modeling the robot task of loading a completed wafer into a loadlock.
$s_{i 2} \quad$ PN transition modeling the robot task of unloading a wafer from a PM at $p_{i}$ and moving to $p_{i+1}$, $i \in \mathbf{N}_{\boldsymbol{n} \boldsymbol{1}}$.
$s_{02} \quad$ PN transition modeling the robot task of unloading a wafer from a loadlock at $p_{0}$ and moving to $p_{1}$.
$s_{n} 2 \quad$ PN transition modeling the robot task of unloading a wafer from $p_{n}$ and moving to a loadlock.
$y_{i 1} \quad$ PN transition modeling robot's moving from $p_{i+2}$ to $p_{i}$ without carrying a wafer, $i \in \mathbf{N}_{n-2} \cup\{0\}$.
$y_{(n-1) 1}$ PN transition modeling robot's moving from $p_{0}$ to $p_{n-1}$ without carrying a wafer.
$y_{n 1} \quad$ PN transition modeling robot's moving from $p_{1}$ to $p_{n}$ without carrying a wafer.
$\Lambda_{i} \quad$ Wafer sojourn time at Step $i$ under normal conditions, $i \in \mathbf{N}_{n}$.
$\boldsymbol{\Omega}=\quad\{0\} \cup \mathbf{N}_{\boldsymbol{n}}$.
$\Theta_{i} \quad$ Accumulated robot time delay when the robot arrives at $q_{i 3}$ for unloading a completed wafer.
$\mu_{y_{i 1}} \quad$ Time needed for firing $y_{i 1}, i \in \boldsymbol{\Omega}$ and $\mu_{y_{i 1}} \in[\alpha, \beta]$.
$\theta \quad$ Cycle time of the system.
$\theta_{i} \quad$ Cycle time of Step $i, i \in \mathbf{N}_{\boldsymbol{n}}$.
$\kappa_{i d}^{j} \quad$ Transition firing sequence for robot's going from Steps $i$ to $d$ in the $j$ th robot cycle.
$\lambda_{i 1} \quad$ Time needed for firing $s_{i 1}, i \in \boldsymbol{\Omega}$, and $\lambda_{i 1} \in[c, d]$.
$\lambda_{i 2} \quad$ Time needed for firing $s_{i 2}, i \in \mathbf{N}_{\boldsymbol{n}}$, and $\lambda_{i 2} \in[c+\alpha, d+\beta]$.


Fig. 1. Cluster tools. (a) Single-arm robot. (b) Dual-arm robot.
$\lambda_{02}$ Time needed for firing $s_{02}$ and $\lambda_{02} \in\left[c_{0}+\alpha\right.$, $\left.d_{0}+\beta\right]$.
$\tau_{i} \quad$ Wafer sojourn time in Step $i, i \in \mathbf{N}_{n}$.
$\omega_{i 1} \quad$ Scheduled robot waiting time before loading a wafer to Step $i, i \in \boldsymbol{\Omega}$.
$\omega_{i 2} \quad$ Scheduled robot waiting time before unloading a wafer from Step $i, i \in \boldsymbol{\Omega}$.
$\omega_{i 3} \quad$ Unscheduled robot waiting time before unloading a wafer from Step $i, i \in \boldsymbol{\Omega}$.
$\delta_{i} \quad$ The longest time for which a wafer can stay in a PM at Step $i$ after it is processed, $i \in \mathbf{N}_{n}$.
$\varsigma_{i} \quad$ Time needed for processing a wafer at Step $i$, $i \in \mathbf{N}_{n}$, and $\varsigma_{i} \in\left[a_{i}, b_{i}\right]$.
$\zeta^{j} \quad$ The actual time taken for completing the $j$ th activity.
$\psi \quad$ Robot cycle time.
$\psi_{1} \quad$ Robot cycle time with no robot waiting.
$\psi_{2} \quad$ Robot waiting time in a cycle.
$\|\bullet\| \quad$ The exact upper bound of time delay during the execution of an activity sequence.

## I. Introduction

IN SEMICONDUCTOR manufacturing, more and more manufacturers adopt cluster tools to process wafers by using single-wafer processing technology. A cluster tool consists of several process modules (PMs), an aligner, a wafer handling robot, and two loadlocks for wafer cassette loading/unloading. In general, raw wafers in a cassette have an identical recipe [1], [2]. They are loaded into a cluster tool through its loadlock, and then processed in one or more PMs with a prespecified order. After all operations are completed, they are returned to their loadlocks by the robot [3]. Such a tool can provide a flexible, reconfigurable, and efficient environment for semiconductor manufacturing, resulting in higher yield, shorter cycle time, better utilization of costly space, and lower capital cost [4]-[8]. With one or two robot arms, it is called a single and dual-arm cluster tool as shown in Fig. 1, respectively.

Extensive work has been done about modeling and analysis of cluster tools [5], [7]-[17]. It is found that, under the steady state, they operate in either the process or transport-bound region. For the former, the robot has idle time and the processing time in PMs determines the cycle time. For the latter, the robot is always busy and the cycle time is determined by its activity time. It is also shown that the PM activities follow the robot tasks [18], [19]. Hence, the key is to schedule the robot. Dispatching or priority rules are developed to do so [14], [20]. The robot moving time from one PM to another can be treated as a constant and is much shorter than the wafer processing time [1]. For single-arm cluster tools, a backward scheduling strategy is optimal [21], [22]. This is true only if there is no limit on how long a wafer can stay in PMs after it is done.

Some wafer fabrication processes pose a strict constraint on the wafer sojourn time in a PM called a wafer residency time constraint [1]-[3], [23]-[27]. With such constraints, methods for finding an optimal periodic schedule for dual-arm cluster tools are proposed in [1], [2], and [24]. Their computational efficiency is improved by deriving necessary and sufficient schedulability conditions for both single and dual-arm cluster tools as revealed in [3] and [28]. If schedulable, closed-form algorithms are given to find an optimal one.

Some wafer fabrication processes are repeated processes, or there is wafer revisiting. In [5], [7], [8], [37], and [38], scheduling strategies are presented for such tools dealing with wafer revisiting. Furthermore, an efficient technique is proposed in [39] to schedule a dual-arm cluster tool coping with both wafer revisiting and residency time constraints.

PMs in cluster tools are failure-prone. Thus, effective control policies are proposed to respond to such failures for single-arm cluster tools in [35] and [36].

All the above studies are conducted without considering activity time variation that occurs in practice. Such variation can make a feasible schedule obtained under the assumption of deterministic activity time infeasible. Methods are proposed to deal with abnormal events and activity time fluctuation in [18] and [29]. Kim and Lee [30] studied the schedulability
problem for dual-arm cluster tools with bounded activity time variation. They identify so-called always schedulable and never schedulable cases by using PNs and a branching technique.

Wu and Zhou [25] show that some never schedulable cases identified in [30], in fact, are always schedulable by using their newly proposed real-time controller. By using PNs, for dual-arm cluster tools with wafer residency time constraints and activity time variation, the wafer sojourn time fluctuation is analyzed and closed-form scheduling algorithms are proposed to find an optimal schedule [25], [26], [31]. With wafer residency time constraints and activity time variation, it is much more complex to schedule single-arm cluster tools than dual-arm ones [23], [32]. Thus, by following the idea in [25] and [26], the scheduling problem of single-arm cluster tools is solved in [23], [32], and [34].

Since the time variation of both robot activities and wafer processing affects the wafer sojourn time delay in a PM in a complex way, it is very difficult to calculate the wafer sojourn time delay in a PM. In fact, the upper bound of wafer sojourn time delay in a PM obtained in [32] is not the exact one but overestimated. This makes the schedulability conditions in [23] and [34] sufficient only, but not necessary. This implies that, by the method in [23] and [34], some nonschedulable cases are, in fact, schedulable. Then, can an exact upper bound of the wafer sojourn time delay in a PM be found? If so, the schedulability conditions in [23] and [34] become necessary and sufficient conditions. This motivates us to conduct this investigation.

This work makes the following contributions: 1) the mechanism about how the activity time variation affects the wafer sojourn time is revealed; 2) algorithms are derived to calculate its exact upper bound; and 3) they are shown to be polynomial with respect to the number of parallel PMs and number of operations. Therefore, the results are significant in this research field.

The remainder of this paper is organized as follows. The next section introduces a Petri net (PN) model and real-time control policy (RCP). Then, Section III presents the algorithms for calculating the exact upper bound of wafer sojourn time delay. Illustrative examples are used to show their applications in Section IV. Finally, the conclusion is given in Section V.

## II. PN Modeling and RCP

In this section, we briefly introduce the PN model developed in [32] and [33] such that this paper is self-complete.

## A. PN Model for the Wafer Flow

PNs are widely used in modeling and analysis of discrete event systems [1], [17], [29], [40]-[53]. The PN model in [32] and [33] is a kind of finite capacity PN whose concept is based on [19] and [54]. It is defined as $\mathrm{PN}=(P, T, I, O, M, K)$, where $P$ is a finite set of places; $T$ is a finite set of transitions with $P \cup T \neq \emptyset$ and $P \cap T=\emptyset$; $I: P \times T \rightarrow \mathbf{N}=\{0,1,2, \ldots\}$ is an input function; $O: P \times$ $T \rightarrow \mathbf{N}$ is an output function; $M: P \rightarrow \mathbf{N}$ is a marking representing the number of tokens in places with $M_{0}$ being the initial marking; and $K: P \rightarrow \mathbf{N} \backslash\{0\}$ is a capacity function, where $K(p)$ represents the largest number of tokens that $p$ can hold.


Fig. 2. PN model for a single-arm cluster tool with $n$ steps.
The preset of transition $t$ is the set of all input places to $t$, i.e., ${ }^{\bullet} t=\{p: p \in P$ and $I(p, t)>0\}$. Its postset is the set of all output places from $t$, i.e., $t^{\bullet}=\{p: p \in P$ and $O(p, t)>0\}$. Similarly, $p$ 's preset ${ }^{\bullet} p=\{t \in T: O(p, t)>0\}$ and postset $p^{\bullet}=\{t \in T: I(p, t)>0\}$. The transition enabling and firing rules can be found in [19] and [41].

The wafer flow pattern can be denoted as $\left(m_{1}, m_{2}, \ldots, m_{n}\right)$ [1], [55], where $n$ is the number of steps for processing a wafer and $m_{i}$ is the number of PMs used to process wafers at Step $i, i \in \mathbf{N}_{n}=\{1,2, \ldots, n\}$. Let $\boldsymbol{\Omega}=\{0\} \cup \mathbf{N}_{n}$. Based on the wafer flow pattern, the PN model [32], [33] for a single-arm cluster tool is shown in Fig. 2 with the meaning of places and transitions being presented in Table I.

By $K\left(p_{0}\right)=m_{0}=\infty$, we mean that the loadlocks can handle all raw and finished wafers in a tool. With a backward strategy, $m_{i}$ wafers are being processed at Step $i, i \in \mathbf{N}_{n}$. Thus, without loss of generality, we let $M_{0}\left(p_{i}\right)=m_{i}, i \in \mathbf{N}_{n}$, $M_{0}(r)=1$ to indicate that the robot is idle, and $M_{0}\left(p_{0}\right)=n$ to indicate that there are always wafers to be processed. To avoid deadlock [56]-[58], we give a control policy to make the PN model live [33].
Definition 1 [33]: At marking $M$, transition $y_{i 1}$, $i \in \mathbf{N}_{n-1} \cup\{0\}$ is said to be control-enabled if $M\left(p_{i+1}\right)=$ $m_{i+1}-1$; and $y_{n 1}$ is said to be control-enabled if $M\left(p_{i}\right)=m_{i}, i \in \mathbf{N}_{n}$.

Under the control policy given in Definition 1, the PN is shown to be deadlock-free [33].

## B. Modeling Activity Time

In the developed PN model, time is associated with both places and transitions. Time duration $\left[\zeta_{1}, \zeta_{2}\right]$ is used to denote a robot task's time interval. The wafer processing time is denoted as ( $\left.\left[\zeta_{1}, \zeta_{2}\right], \delta\right)$ which indicates that after the completion of a wafer with $\zeta \in\left[\zeta_{1}, \zeta_{2}\right]$ time units at Step $i$, the longest time delay in its corresponding PM must be no more than $\delta$. For a robot task or wafer processing at a PM, $\zeta \in\left[\zeta_{1}, \zeta_{2}\right]$ is obtained by measuring the real-time

TABLE I
Meaning of Places and Transitions of PN in Fig. 2

| Transition |  |
| :---: | :--- |
| or place |  |
| $p_{0} \in P$ | The loadlocks called Step 0 with $K\left(p_{0}\right)=m_{0}$ |
| $p_{i} \in P$ | The PMs for Step $i$ with $K\left(p_{i}\right)=m_{i}, i \in \mathbf{N}_{n}$ |
| $r \in P$ | The single-arm robot with $K(r)=1$ |
| $q_{i 1} \in P$ | The scheduled robot waiting before loading a wafer to Step $i$, |
| $q_{i 2} \in P$ | $i \in \boldsymbol{\Omega}$ |
| $q_{i 3} \in P$ | The scheduled robot waiting before unloading a wafer from Step $i, i \in \boldsymbol{\Omega}$ |
| $s_{i 1} \in T$ | Loading a wafer into a PM at Step $i$ modeled by $p_{i}, i \in \mathbf{N}_{n}$ |
| $s_{01} \in T$ | Loading a completed wafer into a loadlock modeled by $p_{0}$ |
| $s_{i 2} \in T$ | Unloading a wafer from a PM at $p_{i}$ and moving to $p_{i+1}, i \in \mathbf{N}_{n-1}$ |
| $s_{02} \in T$ | Unloading a wafer from a loadlock at $p_{0}$ and moving to $p_{1}$ |
| $s_{n 2} \in T$ | Unloading a wafer from $p_{n}$ and moving to a loadlock |
| $y_{i 1} \in T$ | The robot moves from $p_{i+2}$ to $p_{i}$ without carrying a wafer, $i \in \mathbf{N}_{n-2} \cup\{0\}$ |
| $y_{(n-1) 1} \in T$ | The robot moves from $p_{0}$ to $p_{n-1}$ without carrying a wafer |
| $y_{n 1} \in T$ | The robot moves from $p_{1}$ to $p_{n}$ without carrying a wafer |

operational time. If $\zeta \in\left[\zeta_{1}, \zeta_{2}\right]$ represents a scheduled robot waiting time, $\zeta$ can be set to be any number in $\left[\zeta_{1}, \zeta_{2}\right]$. However, if $\zeta \in\left[\zeta_{1}, \zeta_{2}\right]$ represents an unscheduled robot waiting time, $\zeta$ is obtained by real-time measurement and is variable. The time durations for different transitions and places are shown in Table II. Note that the time taken for the robot to move from one PM to another is same under normal conditions defined later.

With wafer residency time constraints, we need to define the liveness for the PN shown in Fig. 2. Let $\tau_{i}$ denote the sojourn time of a token in $p_{i}$ and $\varsigma_{i}$ be a sample in $\left[a_{i}, b_{i}\right]$. Then, the liveness condition of the PN for single-arm cluster tools with residency time constraints can be defined.

Definition 2 [33]: A PN for single-arm cluster tools with residency time constraints is said to be live, if at any marking reached and for any wafer in $p_{i}, \forall i \in \mathbf{N}_{n}$, and any $\varsigma_{i}$ sampled in $\left[a_{i}, b_{i}\right]$ such that whenever $s_{i 2}$ is enabled, $\tau_{i}-\varsigma_{i} \leq \delta_{i}$ holds.

## C. $R C P$

To understand the activity time variation, we may view a system as if it operates under normal conditions with random disturbance. A cluster tool is said to be operated under normal conditions if, for any activity with time duration $\left[\zeta_{1}, \zeta_{2}\right]$, it takes $\zeta=\zeta_{1}$ time units only. With this definition, in a real-time, the time needed for an activity can be denoted as $\zeta_{1}+\Delta \zeta$ with $\zeta_{1} \leq \zeta_{1}+\Delta \zeta \leq \zeta_{2}$. In this way, nonzero $\Delta \zeta$ can be seen as a disturbance and $\zeta$ can be any number in $\left[\zeta_{1}, \zeta_{2}\right]$. In this way, to obtain a feasible periodic schedule under normal conditions is to determine $\omega_{i 1}$ and $\omega_{i 2}$ such that $a_{i} \leq \tau_{i} \leq a_{i}+\delta_{i}, \forall i \in \mathbf{N}_{n}$.

Let $\zeta^{j}$ denote the actual time taken by completing the $j$ th activity. The random activity time variation can be seen as random disturbance to normal conditions. Thus, we let $\mu_{y_{i 1}}^{j}=\alpha+\sigma_{y_{i 1}}^{j}$ and $\lambda_{i 1}^{j}=c+\rho_{i 1}^{j}$ for all $j$ and $i \in \boldsymbol{\Omega}$;
$\lambda_{02}^{j}=c_{0}+\alpha+\rho_{02}^{j}$ for all $j ; \lambda_{i 2}^{j}=c+\alpha+\rho_{i 2}^{j}$ for all $j$ and $i \in \mathbf{N}_{\boldsymbol{n}}$. We then dynamically regulate $\omega_{i 2}$ 's and $\omega_{i 1}$ 's so as to adapt to the random disturbance based on the real-time observation. With $\sigma_{y_{i 1}}^{j}, \rho_{i 2}^{j}$, and $\rho_{i 1}^{j}$ observed in real-time, if there exists a nonzero value of $\sigma_{y_{i 1}}^{j}, \rho_{i 2}^{j}$, and $\rho_{i 1}^{j}$, the robot waiting time in $q_{i 2}$ and $q_{i 1}$ can be shortened by adjusting $\omega_{i 2}$ and $\omega_{i 1}$ on-line. We have the following RCP [33].

1) Under the normal conditions, find a periodic schedule by determining $\omega_{i 2}$ and $\omega_{i 1}, i \in \boldsymbol{\Omega}$.
2) Transition $s_{01}$ is fired if the $j$ th token stays in $q_{01}$ for $\omega_{01}^{j}=\max \left\{\left(\omega_{01}-\rho_{n 2}^{j}\right), 0\right\}$ time units, and transition $s_{i 1}$ is fired if the $j$ th token stays in $q_{i 1}$ for $\omega_{i 1}^{j}=$ $\max \left\{\left(\omega_{i 1}-\rho_{(i-1) 2}^{j}\right), 0\right\}, i \in \mathbf{N}_{\boldsymbol{n}}$.
3) Transition $y_{i 2}$ is fired if the $j$ th token stays in $q_{i 2}$ for $\omega_{i 2}^{j}=\max \left\{\left(\omega_{i 2}-\sigma_{y_{i 1}}^{j}\right), 0\right\}, i \in \boldsymbol{\Omega}$.
4) Transitions $s_{i 2}$, and $y_{i 1}$ fire once they are enabled.

By RCP, $s_{i 2}$ can fire when there is a token in $q_{i 3}$ and a wafer (token) in $p_{i}$ is completed. This implies that the token waiting time $\omega_{i 3}^{j}$ in $q_{i 3}$ (or firing $s_{i 2}$ ) depends on whether a wafer in $p_{i}$ is completed or not.

## III. Exact Upper Bound of Sojourn Time Delay

## A. Effect of Activity Time Variation

Under the normal conditions, a single-arm cluster tool with residency time constraints should be scheduled such that $a_{i} \leq \tau_{i} \leq a_{i}+\delta_{i}$. Thus, we need to know how the activity time variation affects the wafer sojourn time delay in a PM. We first summarize the results of wafer sojourn time delay caused by activity time variation as obtained in [32].

Under the normal conditions, $\omega_{i 2}$ and $\omega_{i 1}, i \in \boldsymbol{\Omega}$, are the scheduled waiting time and are constant, while $\omega_{i 3}, i \in \boldsymbol{\Omega}$, should be zero. Hence, $\lambda_{02}^{j}=c_{0}+\alpha$ for all $j ; \lambda_{d 2}^{j}=c+\alpha$ for all $j$ and $d \in \mathbf{N}_{n} ; \mu_{y_{d 1}}^{j}=\alpha, \omega_{d 2}^{j}=\omega_{d 2}, \omega_{d 1}^{j}=\omega_{d 1}$ and $\lambda_{d 1}^{j}=c$ for all $j$ and $d$. Thus, at any steady state marking $M$,

TABLE II
Time Durations Associated With Transitions and Places

| Symbol | Transition <br> or place | Actions | Allowed time <br> duration |
| :---: | :---: | :--- | :---: |
| $\lambda_{i 1}$ | $s_{i 1} \in T$ | Robot loads a wafer into Step $i, i \in \boldsymbol{\Omega}$ | $[c, d]$ |
| $\lambda_{i 2}$ | $s_{i 2} \in T$ | Robot unloads a wafer from Step $i$ and moves to $p_{i+1}, i \in \mathbf{N}_{\mathrm{n}-1}$ | $[c+\alpha, d+\beta]$ |
| $\lambda_{02}$ | $s_{n 2} \in T$ | Robot unloads a wafer from Step $n$ and moves to a loadlock |  |
| $\mu_{y_{i 1}}$ | $y_{i 1} \in T$ | Robot unloads a wafer from a loadlock, aligns it, and moves to $p_{1}$ | $\left[c_{0}+\alpha, d_{0}+\beta\right]$ |
| $\mu_{y_{(n-1) 1}}$ | $y_{(n-1) 11} \in T$ | Robot moves from Steps $i+2$ to $i, i \in \mathbf{N}_{\mathrm{n}-2} \cup\{0\}$ |  |
| $\mu_{y_{n 1}}$ | $y_{n 1} \in T$ | Robot moves from Steps 0 to $n-1$ | $[\alpha, \beta]$ |
| $\tau_{i}$ | $p_{i} \in P$ | A wafer being processed and waiting in $p_{i}, i \in \mathbf{N}_{\mathrm{n}}$ |  |
| $\omega_{i 2}$ | $q_{i 2} \in P$ | Scheduled robot waiting before unloading a wafer from Step $i, i \in \boldsymbol{\Omega}$ | $\left(\left[a_{i}, b_{i}\right], \delta_{i}\right)$ |
| $\omega_{i 1}$ | $q_{i 1} \in P$ | Scheduled robot waiting before loading a wafer to Step $i, i \in \boldsymbol{\Omega}$ |  |
| $q_{i 2} \in P$ | Unscheduled robot waiting before unloading a wafer from Step $i, i \in \boldsymbol{\Omega}$ | $[0, \infty]$ |  |

we have

$$
\begin{align*}
\tau_{1}= & m_{1} \times\left[2(n+1) \alpha+(2 n+1) c+c_{0}+\sum_{d=0}^{n} \omega_{d 2}+\sum_{d=0}^{n} \omega_{d 1}\right] \\
& -\left(3 c+c_{0}+3 \alpha+\omega_{02}+\omega_{11}+\omega_{21}\right) \\
= & m_{1} \times \psi-\left(3 c+c_{0}+3 \alpha+\omega_{02}+\omega_{11}+\omega_{21}\right) \\
\tau_{i}= & m_{i} \times\left[2(n+1) \alpha+(2 n+1) c+c_{0}+\sum_{d=0}^{n} \omega_{d 2}+\sum_{d=0}^{n} \omega_{d 1}\right] \\
& -\left(4 c+3 \alpha+\omega_{(i-1) 2}+\omega_{i 1}+\omega_{(i+1) 1}\right) \\
= & m_{i} \times \psi-\left(4 c+3 \alpha+\omega_{(i-1) 2}+\omega_{(i+1) 1}+\omega_{i 1}\right) \\
i= & 2,3, \ldots, n-1 \\
\tau_{n}= & m_{n} \times\left[2(n+1) \alpha+(2 n+1) c+c_{0}+\sum_{d=0}^{n} \omega_{d 2}+\sum_{d=0}^{n} \omega_{d 1}\right] \\
& -\left(4 c+3 \alpha+\omega_{(n-1) 2}+\omega_{n 1}+\omega_{01}\right) \\
= & m_{n} \times \psi-\left(4 c+3 \alpha+\omega_{(n-1) 2}+\omega_{n 1}+\omega_{01}\right) . \tag{3}
\end{align*}
$$

Also, under normal conditions, the robot cycle time is

$$
\begin{align*}
\psi & =2(n+1) \alpha+(2 n+1) c+c_{0}+\sum_{d=0}^{n} \omega_{d 2}+\sum_{d=0}^{n} \omega_{d 1} \\
& =\psi_{1}+\psi_{2} \tag{4}
\end{align*}
$$

where $\psi_{1}=2(n+1) \alpha+(2 n+1) c+c_{0}$ is a constant and known in advance and $\psi_{2}=\sum_{d=0}^{n} \omega_{d 2}+\sum_{d=0}^{n} \omega_{d 1}$ is to be determined by a schedule. It should be noticed that $\psi$ is independent of the wafer processing time.

Let $\theta_{1}=\left(\tau_{1}+3 c+c_{0}+3 \alpha+\omega_{02}+\omega_{11}+\omega_{21}\right) /\left(m_{1}\right)$, $\theta_{i}=\left(\tau_{i}+4 c+3 \alpha+\omega_{(i-1) 2}+\omega_{i 1}+\omega_{(i+1) 1}\right) /\left(m_{i}\right)$, $i \in \mathbf{N}_{n-\mathbf{1}} \backslash\{1\}$, and $\theta_{n}=\left(\tau_{n}+4 c+3 \alpha+\omega_{(n-1) 2}+\right.$ $\left.\omega_{n 1}+\omega_{01}\right) /\left(m_{n}\right)$ denote the cycle time for Step $i$, $i \in \mathbf{N}_{n}$. Further, let $\theta$ be the production cycle time of the system. Since the process of single-arm cluster tools is a serial one, the production rate is same for all the steps and this production rate is the cycle time for the system. We have the following proposition.

Proposition 1: In the steady state, a single-arm cluster tool with a backward strategy has the same cycle time for all
processing steps, that is

$$
\begin{equation*}
\theta=\theta_{1}=\theta_{2}=\cdots=\theta_{n} . \tag{5}
\end{equation*}
$$

Then, the relationship between the production cycle and robot cycle can be analyzed based on the model shown in Fig. 2. Assume that wafer $W_{k}$ is loaded into Step $i$ at time $\tau_{k}$ and $W_{k+1}$ is loaded into it at $\tau_{k+1}$. Then, [ $\tau_{k}$, $\tau_{k+1}$ ] forms a cycle for Step $i$. During this time, $s_{i 1}$ fires twice, and the robot completes the following activities: firing $s_{i 1} \rightarrow y_{(i-2) 1} \rightarrow$ waiting in $q_{(i-2) 2} \rightarrow s_{(i-2) 2} \rightarrow$ waiting in $q_{(i-1) 1} \rightarrow s_{(i-1) 1} \rightarrow y_{(i-3) 1} \rightarrow \ldots \rightarrow y_{01} \rightarrow$ waiting in $q_{02} \rightarrow s_{02} \rightarrow$ waiting in $q_{11} \rightarrow s_{11} \rightarrow y_{n 1} \rightarrow$ waiting in $q_{n 2} \rightarrow s_{n 2} \rightarrow$ waiting in $q_{01} \rightarrow s_{01} \rightarrow y_{(n-1) 1} \rightarrow$ $\ldots \rightarrow y_{i 1} \rightarrow$ waiting in $q_{i 2} \rightarrow s_{i 2} \rightarrow$ waiting in $q_{(i+1) 1} \rightarrow$ $s_{(i+1) 1} \rightarrow y_{(i-1) 1} \rightarrow$ waiting in $q_{(i-1) 2} \rightarrow s_{(i-1) 2} \rightarrow$ waiting in $q_{i 1} \rightarrow s_{i 1}$ again. Note that, during this time, the robot completes exactly one cycle. Thus, we have the following proposition.
Proposition 2: In the steady state, under the normal conditions, a single-arm cluster tool with a backward strategy has the same cycle time for the robot and each step, that is

$$
\begin{equation*}
\theta=\theta_{1}=\theta_{2}=\cdots=\theta_{n}=\psi \tag{6}
\end{equation*}
$$

According to (4), $\alpha, c$, and $c_{0}$ are all deterministic, while $\omega_{d 2}$ and $\omega_{d 1}, d \in \boldsymbol{\Omega}$, are changeable, i.e., $\psi_{1}$ is deterministic while the robot waiting time in $\psi_{2}$ can be regulated. Thus, to schedule the system under the normal conditions is to appropriately set $\omega_{d 2}$ and $\omega_{d 1}, d \in \boldsymbol{\Omega}$, such that (6) holds and at the same time the wafer residency time constraints are satisfied. With activity time variation considered, there may exist a nonzero value of $\sigma_{y_{d 1}}^{j}, \rho_{d 2}^{j}$, and $\rho_{d 1}^{j}$ obtained by real-time measurement. Thus, RCP reduces the effects of the time variation on the wafer sojourn time delay as much as possible. Let $\eta_{d 2}^{j}$ be the time delay that is caused by $\sigma_{y_{d 1}}^{j}$ and can be offset
by adjusting $\omega_{d 2}$, and $\eta_{01}^{j}$ and $\eta_{i 1}^{j}$ be the time delay that is caused by $\rho_{n 2}^{j}$ and $\rho_{(i-1) 2}^{j}$ and can be offset by adjusting $\omega_{01}$ and $\omega_{i 1}$, respectively. Then, we have $\omega_{i 2}^{j}+\sigma_{y_{i 1}}^{j}=\omega_{i 2}+\eta_{i 2}^{j}$, or $\eta_{i 2}^{j}=\max \left\{\left(\sigma_{y_{i 1}}^{j}-\omega_{i 2}\right), 0\right\}$. In this way, the effect of $\sigma_{y_{d 1}}^{j}$ on $\tau_{i}$ can be made as small as possible. Similarly, we have $\omega_{01}^{j}+\rho_{n 2}^{j}=\omega_{01}+\eta_{01}^{j}$ and $\omega_{i 1}^{j}+\rho_{(i-1) 2}^{j}=\omega_{i 1}+\eta_{i 1}^{j}$, or $\eta_{01}^{j}=$ $\max \left\{\left(\rho_{n 2}^{j}-\omega_{01}\right), 0\right\}$ and $\eta_{i 1}^{j}=\max \left\{\left(\rho_{(i-1) 2}^{j}-\omega_{i 1}\right), 0\right\}$. Then, according to [32], the wafer sojourn time in $p_{i}$ is

$$
=\Lambda_{i}+\Theta_{i}, 1<i<n
$$

$$
\tau_{n}=m_{n} \times\left[2(n+1) \alpha+(2 n+1) c+c_{0}+\sum_{d=0}^{n} \omega_{d 2}+\sum_{d=0}^{n} \omega_{d 1}\right]
$$

$$
+\sum_{d=0}^{n} \sum_{j=k}^{k+m_{n}-1} \eta_{d 2}^{j}+\sum_{d=0}^{n} \sum_{j=k}^{k+m_{n}-1} \eta_{d 1}^{j}+\sum_{d=0}^{n} \sum_{j=k}^{k+m_{n}-1} \rho_{d 1}^{j}
$$

$$
+\sum_{d=0}^{n} \sum_{j=k}^{k+m_{n}-1} \omega_{d 3}^{j}-\left(4 c+3 \alpha+\omega_{(n-1) 2}+\omega_{n 1}+\omega_{01}\right.
$$

$$
+\omega_{(n-1) 2}^{k}+\omega_{n 1}^{k}+\omega_{01}^{k}-\omega_{(n-1) 2}
$$

$$
-\omega_{n 1}-\omega_{01}+\omega_{(n-1) 3}^{k}+\rho_{n 1}^{k}
$$

$$
\left.+\rho_{n 2}^{k}+\rho_{01}^{k}+\rho_{(n-1) 2}^{k}+\sigma_{y_{(n-1) 1}}^{k}\right)
$$

$$
\begin{equation*}
=\Lambda_{n}+\Theta_{n} \tag{9}
\end{equation*}
$$

where $\Lambda_{1}=m_{1} \times \psi-\left(3 c+c_{0}+3 \alpha+\omega_{02}+\omega_{11}+\omega_{21}\right)$, $\Lambda_{i}=m_{i} \times \psi-\left(4 c+3 \alpha+\omega_{(i-1) 2}+\omega_{i 1}+\omega_{(i+1) 1}\right)$,

$$
\begin{aligned}
& \tau_{1}=m_{1} \times\left[2(n+1) \alpha+(2 n+1) c+c_{0}+\sum_{d=0}^{n} \omega_{d 2}+\sum_{d=0}^{n} \omega_{d 1}\right] \\
& +\sum_{d=0}^{n} \sum_{j=k}^{k+m_{1}-1} \eta_{d 2}^{j}+\sum_{d=0}^{n} \sum_{j=k}^{k+m_{1}-1} \eta_{d 1}^{j}+\sum_{d=0}^{n} \sum_{j=k}^{k+m_{1}-1} \rho_{d 1}^{j} \\
& +\sum_{d=0}^{n} \sum_{j=k}^{k+m_{1}-1} \omega_{d 3}^{j}-\left(3 c+c_{0}+3 \alpha+\omega_{02}+\omega_{11}+\omega_{21}\right. \\
& +\omega_{02}^{k}+\omega_{11}^{k}+\omega_{21}^{k}-\omega_{02}-\omega_{11} \\
& -\omega_{21}+\omega_{03}^{k}+\rho_{11}^{k}+\rho_{12}^{k} \\
& \left.+\rho_{21}^{k}+\rho_{02}^{k}+\sigma_{y_{01}}^{k}\right) \\
& =\Lambda_{1}+\Theta_{1} \\
& \tau_{i}=m_{i} \times\left[2(n+1) \alpha+(2 n+1) c+c_{0}+\sum_{d=0}^{n} \omega_{d 2}+\sum_{d=0}^{n} \omega_{d 1}\right] \\
& +\sum_{d=0}^{n} \sum_{j=k}^{k+m_{i}-1} \eta_{d 2}^{j}+\sum_{d=0}^{n} \sum_{j=k}^{k+m_{i}-1} \eta_{d 1}^{j}+\sum_{d=0}^{n} \sum_{j=k}^{k+m_{i}-1} \rho_{d 1}^{j} \\
& +\sum_{d=0}^{n} \sum_{j=k}^{k+m_{i}-1} \omega_{d 3}^{j}-\left(4 c+3 \alpha+\omega_{(i-1) 2}+\omega_{i 1}+\omega_{(i+1) 1}\right. \\
& +\omega_{(i-1) 2}^{k}+\omega_{i 1}^{k}+\omega_{(i+1) 1}^{k}-\omega_{(i-1) 2} \\
& -\omega_{i 1}-\omega_{(i+1) 1}+\omega_{(i-1) 3}^{k}+\rho_{i 1}^{k} \\
& \left.+\rho_{i 2}^{k}+\rho_{(i+1) 1}^{k}+\rho_{(i-1) 2}^{k}+\sigma_{y_{(i-1) 1}}^{k}\right)
\end{aligned}
$$

$i \in \mathbf{N}_{n-1} \backslash\{1\}$, and $\Lambda_{n}=m_{n} \times \psi-\left(4 c+3 \alpha+\omega_{(n-1) 2}+\right.$ $\left.\omega_{n 1}+\omega_{01}\right)$ are the scheduled sojourn time under the normal conditions given by (1)-(3) and are constant when the periodic schedule is determined.

Now, the sojourn time disturbance is given as

$$
\begin{aligned}
& \Theta_{1}=\sum_{d=0}^{n} \sum_{j=k}^{k+m_{1}-1} \eta_{d 2}^{j}+\sum_{d=0}^{n} \sum_{j=k}^{k+m_{1}-1} \eta_{d 1}^{j}+\sum_{d=0}^{n} \sum_{j=k}^{k+m_{1}-1} \rho_{d 1}^{j} \\
& +\sum_{d=0}^{n} \sum_{j=k}^{k+m_{1}-1} \omega_{d 3}^{j}-\left(\omega_{02}^{k}+\omega_{11}^{k}+\omega_{21}^{k}-\omega_{02}-\omega_{11}\right. \\
& -\omega_{21}+\omega_{03}^{k}+\rho_{11}^{k}+\rho_{12}^{k} \\
& \left.+\rho_{21}^{k}+\rho_{02}^{k}+\sigma_{y_{01}}^{k}\right) \\
& \Theta_{i}=\sum_{d=0}^{n} \sum_{j=k}^{k+m_{i}-1} \eta_{d 2}^{j}+\sum_{d=0}^{n} \sum_{j=k}^{k+m_{i}-1} \eta_{d 1}^{j}+\sum_{d=0}^{n} \sum_{j=k}^{k+m_{i}-1} \rho_{d 1}^{j} \\
& +\sum_{d=0}^{n} \sum_{j=k}^{k+m_{i}-1} \omega_{d 3}^{j}-\left(\omega_{(i-1) 2}^{k}+\omega_{i 1}^{k}+\omega_{(i+1) 1}^{k}-\omega_{(i-1) 2}\right. \\
& -\omega_{i 1}-\omega_{(i+1) 1}+\omega_{(i-1) 3}^{k}+\rho_{i 1}^{k} \\
& +\rho_{i 2}^{k}+\rho_{(i+1) 1}^{k}+\rho_{(i-1) 2}^{k} \\
& \left.+\sigma_{y_{(i-1) 1}}^{k}\right), i \in \mathbf{N}_{\boldsymbol{n}-\mathbf{1}} \backslash\{1\} \\
& \Theta_{n}=\sum_{d=0}^{n} \sum_{j=k}^{k+m_{n}-1} \eta_{d 2}^{j}+\sum_{d=0}^{n} \sum_{j=k}^{k+m_{n}-1} \eta_{d 1}^{j}+\sum_{d=0}^{n} \sum_{j=k}^{k+m_{n}-1} \rho_{d 1}^{j} \\
& +\sum_{d=0}^{n} \sum_{j=k}^{k+m_{n}-1} \omega_{d 3}^{j}-\left(\omega_{(n-1) 2}^{k}+\omega_{n 1}^{k}+\omega_{01}^{k}-\omega_{(n-1) 2}\right. \\
& -\omega_{n 1}-\omega_{01}+\omega_{(n-1) 3}^{k}+\rho_{n 1}^{k} \\
& \left.+\rho_{n 2}^{k}+\rho_{01}^{k}+\rho_{(n-1) 2}^{k}+\sigma_{y_{(n-1) 1}}^{k}\right) .
\end{aligned}
$$

Note that $\eta_{d 2}^{j}, \eta_{d 1}^{j}, \rho_{i 1}^{k}, \rho_{i 2}^{k}, \rho_{(i+1) 1}^{k}, \rho_{(i-1) 2}^{k}$, and $\sigma_{y_{i-1}}^{k}$ are obtained via real-time observation, while $\omega_{d 1}$ and $\omega_{d 2}$ are determined by an off-line schedule and thus known in advance. $\omega_{d 3}^{j}$ is uncontrollable and varies with $j$. In fact, $\Theta_{i}$ represents the accumulated robot time delay when the robot arrives at $q_{i 3}$ for unloading a completed wafer there. Under the normal conditions, the necessary and sufficient schedulability conditions are presented in [3]. Thus, if we can find a method to obtain $\Theta_{i}$ for the worst case, with the results in [3], can we find the necessary and sufficient schedulability conditions for single-arm cluster tools with wafer residency time constraints and activity time variation? To answer it, we are required to find the exact upper bound of the wafer sojourn time delay.

## B. Computing Exact Upper Bound

With wafer flow pattern $\left(m_{1}, m_{2}, \ldots, m_{n}\right), m=m_{1}$ $+m_{2}+\cdots+m_{n}$ wafers are being processed concurrently. We number them as $W_{1}-W_{m}$. Let $\Xi_{i}$ denote the set of wafers that are being processed in $p_{i}$. Further, let $E 1=m_{2}+\cdots+m_{n}+1, L 1=m, E i=m_{i+1}+\cdots+m_{n}$ $+1, L i=m_{i}+\cdots+m_{n}, \quad i \in \mathbf{N}_{n-1}, E n=1$, and $L n$ $=m_{n}$, such that $W_{E i}$ and $W_{L i}$ are the earliest and latest wafers released into $p_{i}$, respectively. Let $E i_{-} j=E i+j$,
then, we have $\Xi_{i}=\left\{W_{E i}, W_{E i_{-} 1}, \ldots, W_{L i}\right\}, i \in \mathbf{N}_{n}$. Assume that it takes $v_{i} \in\left[a_{i}, b_{i}\right]$ time units to complete $W_{E i}$, leading to a time delay $\max \left\{\left(v_{i}-\Lambda_{i}\right), 0\right\}$. Let $H_{i}=\max \left\{\left(b_{i}-\Lambda_{i}\right), 0\right\}, i \in \mathbf{N}_{\boldsymbol{n}}$, be the longest time delay caused by processing a wafer at $p_{i}$ and $H_{0}=0$ since there is no processing time delay at Step 0 . Further let $\eta_{11}=$ $\max \left\{\left(d_{0}-c_{0}\right)+(\beta-\alpha)-\omega_{11}, 0\right\}, \eta_{i 1}=\max \{(d-c)+$ $\left.(\beta-\alpha)-\omega_{i 1}, 0\right\}$, and $\eta_{i 2}=\max \left\{(\beta-\alpha)-\omega_{i 2}, 0\right\}$.

The infeasibility of a schedule is caused by delay $\tau_{i}, i \in \mathbf{N}_{n}$. It follows from (7)-(9) that $\tau_{i}=\Lambda_{i}+\Theta_{i}$, where $\Lambda_{i}$ is the robot task time in a cycle under the normal conditions, while $\Theta_{i}$ is the accumulated robot time delay. Under the normal conditions, the system can be scheduled such that $a_{i} \leq \tau_{i}=\Lambda_{i} \leq a_{i}+\delta_{i}$. With activity time variation, it is required that $\tau_{i}=\Lambda_{i}+\Theta_{i} \leq a_{i}+\Delta a_{i}+\delta_{i}$, where $\Delta a_{i}$ $\in\left[0, b_{i}-a_{i}\right]$. To do so, with $\Lambda_{i}$ being known, we have to find $\Theta_{i}$. When $\Theta_{i}$ reaches its largest value, or the upper bound, in the worst case, if $\tau_{i}=\Lambda_{i}+\Theta_{i} \leq a_{i}+\Delta a_{i}+\delta_{i}$ holds, the system operates in a feasible state. Thus, the key is to calculate the upper bound of $\Theta_{i}$. Notice that the worst case occurs when $\Delta a_{i}$ is zero such that the robot does not need to wait at $q_{i 3}$ for unloading a processed wafer from $p_{i}$ since $a_{i} \leq$ $\Lambda_{i}$, or $\omega_{d 3}^{m_{i}}=0$. Thus, to check the feasibility of a schedule, we need to find the exact upper bound of $B_{i}=\Theta_{i}-\omega_{d 3}^{m_{i}}$. To do so, based on the PN model, we analyze the fabrication process as follows.

After loading a wafer $W_{E i}$ into $p_{i}$, the robot goes to Step $i-2$ by firing $y_{(i-2) 1}$ for unloading a processed wafer there. Then, a sequence of tasks is executed. Finally, the robot comes back to Step $i$ for unloading $W_{E i}$. This process undergoes $m_{i}$ robot cycles. This implies that, before the robot comes back to Step $i$ again, it goes through every Step $d \notin\{i, i-1\}$ for $m_{i}$ times. Hence, to calculate the exact time delay during this process, the robot task sequence can be divided into a number of small segments such that the time delay of each segment can be calculated straightforwardly. Then, the time delay can be calculated in a sequential way. With this idea, we analyze how it can be divided into small segments next.

Let $\kappa_{i d}^{j}$ denote the transition firing sequence for the robot to go from Steps $i$ to $d$ in the $j$ th robot cycle. After loading a wafer into $p_{i}$, starting from Step $i$, in the first robot cycle, the robot goes to Step $i-2$ by firing $y_{(i-2) 1}$. Then, through a number of steps, it goes to Step $d$ for any given $d \in \mathbf{N}_{\boldsymbol{n}}$ and waits there for unloading a wafer, or a token goes into $q_{d 3}$ by executing the following transition sequence:
$\kappa_{i d}^{1}=\left\langle\right.$ firing $y_{(i-2) 1} \rightarrow$ waiting in $q_{(i-2) 2} \rightarrow y_{(i-2) 2} \rightarrow$ waiting in $q_{(i-2) 3} \rightarrow s_{(i-2) 2} \rightarrow$ waiting in $q_{(i-1) 1}$ $\rightarrow s_{(i-1) 1} \rightarrow \ldots \rightarrow y_{d 1} \rightarrow$ waiting in $q_{d 2} \rightarrow y_{d 2}$ $\rightarrow$ waiting in $\left.q_{d 3}\right\rangle, d \in \mathbf{N}_{i-2} \cup\{0\}$
$\kappa_{i d}^{1}=\left\langle\kappa_{i 0}^{1} \rightarrow s_{02} \rightarrow\right.$ waiting in $q_{11} \rightarrow s_{11} \rightarrow y_{n 1} \rightarrow$ waiting in $q_{n 2} \rightarrow y_{n 2} \rightarrow$ waiting in $\left.q_{n 3}\right\rangle, d=n$, or
$\kappa_{i d}^{1}=\left\langle\kappa_{i n}^{1} \rightarrow s_{n 2} \rightarrow\right.$ waiting in $q_{01} \rightarrow s_{01} \rightarrow \ldots \rightarrow$ $y_{d 1} \rightarrow$ waiting in $q_{d 2} \rightarrow y_{d 2} \rightarrow$ waiting in $\left.q_{d 3}\right\rangle$, $d \in \mathbf{N}_{\boldsymbol{n - 1}} \backslash \mathbf{N}_{\boldsymbol{i}-\mathbf{2}}$.
Similarly, in the second robot cycle, starting from Step $i$, the robot goes to Step $i-2$, or a token goes into $q_{(i-2) 3}$, by executing the following sequence in the PN in Fig. 2:

$$
\begin{aligned}
\kappa_{i(i-2)}^{2}= & \left\langle\kappa_{i(i-1)}^{1} \rightarrow s_{(i-1) 2} \rightarrow \text { waiting in } q_{i 1} \rightarrow s_{i 1} \rightarrow\right. \\
& y_{(i-2) 1} \rightarrow \text { waiting in } q_{(i-2) 2} \rightarrow y_{(i-2) 2} \rightarrow \text { waiting } \\
& \text { in } \left.q_{(i-2) 3}\right\rangle .
\end{aligned}
$$

After undergoing $(j-1)<m_{i}$ robot cycles, the robot continues its $j$ th cycle. With $j>1$, the robot goes to Step $d$ in the $j$ th robot cycle by executing the following sequence:

$$
\begin{aligned}
& \kappa_{i d}^{j}=\left\langle\kappa_{i(i-1)}^{j-1} \rightarrow s_{(i-1) 2} \rightarrow \text { waiting in } q_{i 1} \rightarrow s_{i 1} \rightarrow\right. \\
& y_{(i-2) 1} \rightarrow \text { waiting in } q_{(i-2) 2} \rightarrow y_{(i-2) 2} \rightarrow \text { wait- } \\
& \text { ing in } q_{(i-2) 3} \rightarrow s_{(i-2) 2} \rightarrow \text { waiting in } q_{(i-1) 1} \rightarrow \\
& s_{(i-1) 1} \rightarrow \ldots \rightarrow y_{d 1} \rightarrow \text { waiting in } q_{d 2} \rightarrow y_{d 2} \rightarrow \\
& \text { waiting in } \left.q_{d 3}\right\rangle, d \in \mathbf{N}_{i-2} \cup\{0\} \text { and } 1<j<m_{i} \\
& \kappa_{i d}^{j}=\left\langle\kappa_{i 0}^{j} \rightarrow s_{02} \rightarrow \text { waiting in } q_{11} \rightarrow s_{11} \rightarrow y_{n 1} \rightarrow\right. \\
& \text { waiting in } \left.q_{n 2} \rightarrow y_{n 2} \rightarrow \text { waiting in } q_{n 3}\right\rangle, d=n \\
& \text { and } 1<j<m_{i} \text {, or } \\
& \kappa_{i d}^{j}=\left\langle\kappa_{i n}^{j} \rightarrow s_{n 2} \rightarrow \text { waiting in } q_{01} \rightarrow s_{01} \rightarrow \ldots \rightarrow y_{d 1}\right. \\
& \left.\rightarrow \text { waiting in } q_{d 2} \rightarrow y_{d 2} \rightarrow \text { waiting in } q_{d 3}\right\rangle, d \in \\
& \mathbf{N}_{n-1} \backslash \mathbf{N}_{i-2} \text { and } 1<j<m_{i} .
\end{aligned}
$$

Similarly,
$\kappa_{i d}^{m_{i}}=\left\langle\kappa_{i(i-1)}^{m_{i-1}} \rightarrow s_{(i-1) 2} \rightarrow\right.$ waiting in $q_{i 1} \rightarrow s_{i 1} \rightarrow$ $y_{(i-2) 1} \rightarrow$ waiting in $q_{(i-2) 2} \rightarrow y_{(i-2) 2} \rightarrow$ waiting in $q_{(i-2) 3} \rightarrow s_{(i-2) 2} \rightarrow$ waiting in $q_{(i-1) 1} \rightarrow$ $s_{(i-1) 1} \rightarrow \ldots \rightarrow y_{d 1} \rightarrow$ waiting in $q_{d 2} \rightarrow y_{d 2} \rightarrow$ waiting in $\left.q_{d 3}\right\rangle, d \in \mathbf{N}_{i-2} \cup\{0\}$,
$\kappa_{i d}^{m_{i}}=\left\langle\kappa_{i 0}^{m_{i}} \rightarrow s_{02} \rightarrow\right.$ waiting in $q_{11} \rightarrow s_{11} \rightarrow$ $y_{n 1} \rightarrow$ waiting in $q_{n 2} \rightarrow y_{n 2} \rightarrow$ waiting in $\left.q_{n 3}\right\rangle, d=n$, or
$\kappa_{i d}^{m_{i}}=\left\langle\kappa_{i n}^{m_{i}} \rightarrow s_{n 2} \rightarrow\right.$ waiting in $q_{01} \rightarrow s_{01} \rightarrow \ldots \rightarrow$ $y_{d 1} \rightarrow$ waiting in $q_{d 2} \rightarrow y_{d 2} \rightarrow$ waiting in $\left.q_{d 3}\right\rangle$, $d \in \mathbf{N}_{n-1} \backslash \mathbf{N}_{i}$.
Then, after performing $\kappa_{i(i+1)}^{m_{i}}$, the robot performs task sequence $\kappa_{1}=\left\langle s_{(i+1) 2} \rightarrow\right.$ waiting in $q_{(i+2) 1} \rightarrow s_{(i+2) 1} \rightarrow$ $y_{i 1} \rightarrow$ waiting in $\left.q_{i 2}\right\rangle$. Let $\|\bullet\|$ denote the exact upper bound of time delay during the execution of an activity sequence and $\Gamma_{i d}^{j}=\left\|\kappa_{i d}^{j}\right\|$. Note that the exact upper bound of time delay for executing $\left\langle\kappa_{i(i+1)}^{m_{i}} \rightarrow \kappa_{1}\right\rangle$ is $B_{i}$. Because $\left\|\kappa_{1}\right\|=\eta_{(i+2) 1}+$ $\rho+\eta_{i 2}$ and $\left\|\kappa_{i(i+1)}^{m_{i}}\right\|=\Gamma_{i(i+1)}^{m_{i}}$, we have $B_{i}=\Gamma_{i(i+1)}^{m_{i}}+$ $\eta_{(i+2) 1}+\rho+\eta_{i 2}, i \in \mathbf{N}_{\boldsymbol{n - 2}}$. Similarly, we have

$$
B_{i}=\left\{\begin{array}{l}
\Gamma_{i(i+1)}^{m_{i}}+\eta_{(i+2) 1}+\rho+\eta_{i 2}, i \in \mathbf{N}_{n-2}  \tag{10}\\
\Gamma_{i n}^{m_{n-1}}+\eta_{01}+\rho+\eta_{(n-1) 2}, i=n-1 \\
\Gamma_{i 0}^{m_{n}}+\eta_{11}+\rho+\eta_{n 2}, i=n
\end{array}\right.
$$

The remaining problem is how to calculate $\Gamma_{i(i+1)}^{m_{i}}$, $i \in \mathbf{N}_{n} \backslash\{n\}$, or $\Gamma_{i 0}^{m_{n}}, i=n$. To do so, we divide it into several cases. First, we consider the case when $m_{i} \leq m_{u}$ for any $i \neq u$. Without loss of generality, for Step $i$, we analyze the longest sojourn time delay of $W_{E i}$. We have $\kappa_{i(i+1)}^{m_{i}}=\left\langle\kappa_{i(i+2)}^{m_{i}} \rightarrow \kappa_{2} \rightarrow\right.$ waiting in $\left.q_{(i+1) 3}\right\rangle$ with $\kappa_{2}=\left\langle s_{(i+2) 2} \rightarrow\right.$ waiting in $q_{(i+3) 1} \rightarrow$ $s_{(i+3) 1} \rightarrow y_{(i+1) 1} \rightarrow$ waiting in $q_{(i+1) 2} \rightarrow$ firing $\left.y_{(i+1) 2}\right\rangle$, or $\kappa_{i(i+1)}^{m_{i}}$ can be divided into $\kappa_{i(i+2)}^{m_{i}}, \kappa_{2}$, and 〈waiting in $\left.q_{(i+1) 3}\right\rangle$. By the RCP and that firing $y_{(i+1) 2}$ takes no time, we have $\left\|\kappa_{2}\right\|=\eta_{(i+3) 1}+\rho+\eta_{(i+1) 2}$. Thus, $\left\|\kappa_{i(i+1)}^{m_{i}}\right\|=\| \kappa_{i(i+2)}^{m_{i}} \rightarrow$ $\kappa_{2} \|=\Gamma_{i(i+2)}^{m_{i}}+\eta_{(i+3) 1}+\rho+\eta_{(i+1) 2}$. Note that, under the normal conditions, the robot can unload the wafer from $\mathrm{PM}_{i+1}$ immediately after performing $\kappa_{2}$. However, with activity time variation, it should go to $q_{(i+1) 3}$ for waiting after performing $\kappa_{2}$, since there may be a disturbance on wafer processing
in $\mathrm{PM}_{i+1} . W_{E i}$ and $W_{E(i+1)}$ are the first wafers loaded into Steps $i$ and $i+1$ in different cycles, respectively, if $m_{i} \neq$ $m_{(i+1)}$. When $m_{i}=m_{i+1}, W_{E i}$ and $W_{E(i+1)}$ are loaded into Steps $i$ and $i+1$ in the same cycle. Then, with a backward strategy, it follows from $m_{i} \leq m_{u}$ for any $i \neq u$ that wafer $W_{E(i+1)}$ is loaded into Step $i+1$ before $W_{E i}$ into Step $i$.

With the above analysis, in order to analyze the exact longest activity time delay on wafer sojourn time at Step $i$, we should calculate the robot's accumulated activity time delay by starting from the time when $W_{E i}$ has just been loaded into Step $i$. Thus, when the robot goes to $q_{(i+1) 3}$ in the $m_{i}$ th cycle, we should check if the robot waiting in $q_{(i+1) 3}$ is necessary. In other words, at this time, the robot should wait in $q_{(i+1) 3}$ for $\max \left\{H_{i+1}-\left(\Gamma_{i(i+2)}^{m_{i}}+\eta_{(i+3) 1}+\rho+\eta_{(i+1) 2}\right), 0\right\}$ time units. After the robot leaves $q_{(i+1) 3}$, its accumulated activity time delay is $\Gamma_{i(i+1)}^{m_{i}}=\max \left\{\Gamma_{i(i+2)}^{m_{i}}+\eta_{(i+3) 1}+\rho+\eta_{(i+1) 2}\right.$, $\left.H_{i+1}\right\}$. For the sake of clarity, it is assumed that $m_{n} \leq m_{u}, u \in$ $\mathbf{N}_{n-1}$, holds. Then, for Step $n$, we analyze the longest sojourn time delay of $W_{E n}$. The loadlocks, or Step 0 can hold all the wafers, and this is equivalent to $m_{0} \geq \max \left\{m_{j}, j \in \mathbf{N}_{n}\right\}$. In order to calculate $B_{n}$, the key is to calculate $\Gamma_{n 0}^{m_{n}}$. Thus, we have $\kappa_{n 0}^{m_{n}}=\left\langle\kappa_{n 1}^{m_{n}} \rightarrow \kappa_{2} \rightarrow\right.$ waiting in $\left.q_{03}\right\rangle$ with $\kappa_{2}=\left\langle s_{12} \rightarrow\right.$ waiting in $q_{21} \rightarrow s_{21} \rightarrow y_{01} \rightarrow$ waiting in $q_{02} \rightarrow$ firing $\left.y_{02}\right\rangle$, or $\kappa_{n 0}^{m_{n}}$ can be divided into $\kappa_{n 1}^{m_{n}}, \kappa_{2}$, and 〈waiting in $\left.q_{03}\right\rangle$.

By RCP, firing $y_{02}$ takes no time. Thus we have $\left\|\kappa_{2}\right\|=\eta_{21}$ $+\rho+\eta_{02}$. Hence, $\left\|\kappa_{n 1}^{m_{n}} \rightarrow \kappa_{2}\right\|=\Gamma_{n 1}^{m_{n}}+\eta_{21}+\rho+\eta_{02}$. It is known that, under the normal conditions, the robot can unload a wafer from Step 0 (loadlocks) immediately after performing $\kappa_{2}$. Then, with activity time variation, we have to check if the robot task delay caused by $\kappa_{n 1}^{m_{n}} \rightarrow \kappa_{2}$ is larger than the processing time delay at Step 0 . After the robot leaves $q_{03}$, its accumulated activity time delay is $\Gamma_{n 0}^{m_{n}}=\max \left\{\Gamma_{n 1}^{m_{n}}+\eta_{21}+\right.$ $\left.\rho+\eta_{02}, H_{0}\right\}$. In fact, $H_{0}=0$ leads to $\Gamma_{n 0}^{m_{n}}=\Gamma_{n 1}^{m_{n}}+\eta_{21}+\rho$ $+\eta_{02}$. This implies that we can obtain $\Gamma_{n 0}^{m_{n}}$ by calculating $\Gamma_{n 1}^{m_{n}}$ first. Similarly, $\Gamma_{n 1}^{m_{n}}=\max \left\{\Gamma_{n 2}^{m_{n}}+\eta_{31}+\rho+\eta_{12}, H_{1}\right\}$ implies that we can obtain $\Gamma_{n 1}^{m_{n}}$ by calculating $\Gamma_{n 2}^{m_{n}}$ first, $\ldots, \Gamma_{n k}^{m_{n}}$ $=\max \left\{\Gamma_{n(k+1)}^{m_{n}}+\eta_{(k+2) 1}+\rho+\eta_{k 2}, H_{k}\right\}, 0 \leq k \leq n-3$, implies that we can obtain $\Gamma_{n k}^{m_{n}}$ by calculating $\Gamma_{n(k+1)}^{m_{n}}$ first.

Note that $\kappa_{n(n-2)}^{m_{n}}$ can be divided into $\kappa_{n(n-1)}^{m_{n}-1}, \kappa_{3}=\left\langle s_{(i-1) 2}\right.$ $\rightarrow$ waiting in $q_{i 1} \rightarrow s_{i 1} \rightarrow y_{(i-2) 1} \rightarrow$ waiting in $q_{(i-2) 2} \rightarrow$ firing $\left.y_{(i-2) 2}\right\rangle$, and $\left\langle\right.$ waiting in $\left.q_{(i-2) 3}\right\rangle$, since the robot performs $\left\langle\kappa_{n(n-1)}^{m_{n}-1} \rightarrow \kappa_{3} \rightarrow\right.$ waiting in $\left.q_{(i-2) 3}\right\rangle$ in the $\left(m_{n}-1\right)$ th cycle. With RCP, $\left\|\kappa_{n(n-1)}^{m_{n}-1} \rightarrow \kappa_{3}\right\|=\Gamma_{n(n-1)}^{m_{n}-1}+\eta_{n 1}+\rho+$ $\eta_{(n-2) 2}$. Thus, $\Gamma_{n(n-2)}^{m_{n}}=\max \left\{\Gamma_{n(n-1)}^{m_{n}-1}+\eta_{n 1}+\rho+\eta_{(n-2) 2}\right.$, $\left.H_{(n-2)}\right\}$. Then, $\Gamma_{n(n-1)}^{m_{n}-1}=\max \left\{\Gamma_{n n}^{m_{n}-1}+\eta_{01}+\rho+\eta_{(n-1) 2}\right.$, $\left.H_{(n-1)}\right\}$ and $\Gamma_{n n}^{m_{n}-1}=\max \left\{\Gamma_{n 0}^{m_{n}-1}+\eta_{11}+\rho+\eta_{n 2}, H_{n}\right\}$. Furthermore, we have $\Gamma_{n k}^{j}=\max \left\{\Gamma_{n(k+1)}^{j}+\eta_{(k+2) 1}+\rho\right.$ $\left.+\eta_{k 2}, H_{k}\right\}, 0 \leq k \leq n-3$ and $2 \leq j \leq m_{n}-1$, and $\Gamma_{n(n-2)}^{j}=\max \left\{\bar{\Gamma}_{n(n-1)}^{j-1}+\eta_{n 1}+\rho+\bar{\eta}_{(n-2) 2}, H_{(n-2)}\right\}$. Ву continuously doing so, we have $\Gamma_{n(n-2)}^{2}=\max \left\{\Gamma_{n(n-1)}^{1}+\eta_{n 1}\right.$ $\left.+\rho+\eta_{(n-2) 2}, H_{(n-2)}\right\}, \Gamma_{n(n-1)}^{1}=\max \left\{\Gamma_{n n}^{1}+\eta_{01}+\rho+\right.$ $\left.\eta_{(n-1) 2}, H_{(n-1)}\right\}$, and $\Gamma_{n n}^{1}=\max \left\{\Gamma_{n 0}^{1}+\eta_{11}+\rho+\eta_{n 2}\right.$, $\left.H_{n}\right\}$. Generally, we have $\Gamma_{n k}^{1}=\max \left\{\Gamma_{n(k+1)}^{1}+\eta_{(k+2) 1}+\rho\right.$ $\left.+\eta_{k 2}, H_{k}\right\}, 0 \leq k \leq n-4$, and $\Gamma_{n(n-3)}^{1(k)}=\max \left\{\Gamma_{n(n-2)}^{1}\right.$ $\left.+\eta_{(n-1) 1}+\rho+\eta_{(n-3) 2}, H_{(n-3)}\right\}$. According to [23], the

```
Algorithm 1: Calculate \(\Gamma_{n 0}^{m_{n}}\) when \(m_{n} \leq m_{u}, u \in \mathbf{N}_{\boldsymbol{n}-\mathbf{1}}\)
    If \(m_{n} \leq m_{u}, u \in \mathbf{N}_{\boldsymbol{n}-\mathbf{1}}\), find \(\Gamma_{n 0}^{m_{n}}\) as follows.
        1) \(\Gamma_{n(n-2)}^{1}=\max \left\{\eta_{(n-2) 2}, H_{n-2}\right\}\);
        2) If \(n>2\)
        3) \(k=n-3\);
        4) Otherwise \(k=n\);
        5) \(j=1\);
        6) While \(j \leq m_{n}\)
        7) While \(k \neq n-2\)
            If \(0 \leq k \leq n-3\)
            \(\Gamma_{n k}^{j}=\max \left\{\left(\Gamma_{n(k+1)}^{j}+\eta_{(k+2) 1}+\rho+\eta_{k 2}\right), H_{k}\right\} ;\)
                If \(j=m_{n}\) and \(k=0\)
                    Go to (24);
            If \(k=n-1\)
        \(\Gamma_{n(n-1)}^{j}=\max \left\{\left(\Gamma_{n n}^{j}+\eta_{01}+\rho+\eta_{(n-1) 2}\right), H_{n-1}\right\}\)
            If \(k=n\)
        \(\Gamma_{n n}^{j}=\max \left\{\left(\Gamma_{n 0}^{j}+\eta_{11}+\rho+\eta_{n 2}\right), H_{n}\right\}\)
        If \(k=0\)
                \(k=n ;\)
                Otherwise \(k=k-1\);
        \(\Gamma_{n(n-2)}^{j+1}=\max \left\{\left(\Gamma_{n(n-1)}^{j}+\eta_{n 1}+\rho+\eta_{(n-2) 2}\right)\right.\),
        \(\left.H_{n-2}\right\}\);
        If \(n>2\)
            \(k=k-1 ;\)
            Otherwise \(k=n\)
        \(j=j+1 ;\)
        Stop;
```

activity time variation before loading wafer $W_{E n}$ has no effect on the wafer sojourn time of $W_{E n}$ at Step $n$. Thus, we have $\Gamma_{n(n-2)}^{1}=\max \left\{\eta_{(n-2) 2}, H_{n-2}\right\}$ where $\eta_{(n-2) 2}$ and $H_{n-2}$ are known in advance. This implies that $\Gamma_{n 0}^{m_{n}}$ can be computed by calculating $\Gamma_{n(n-2)}^{1}, \Gamma_{n(n-3)}^{1}, \ldots, \Gamma_{n 0}^{1}, \Gamma_{n n}^{1}, \Gamma_{n(n-1)}^{1}$, $\Gamma_{n(n-2)}^{2}, \ldots, \Gamma_{n(n-2)}^{j}, \Gamma_{n(n-3)}^{j}, \ldots, \Gamma_{n 0}^{j}, \Gamma_{n n}^{j}, \Gamma_{n(n-1)}^{j}, \Gamma_{n(n-2)}^{j+1}$, $\ldots, \Gamma_{n(n-2)}^{m_{n}}, \Gamma_{n(n-3)}^{m_{n}}, \ldots$, and $\Gamma_{n 0}^{m_{n}}$ in a sequential way. Then, by (10), we can obtain $B_{n}$. Note that, in a cluster tool, there are at least two steps, or $n \geq 2$. Algorithm 1 finds $\Gamma_{n 0}^{m_{n}}$ if $m_{n} \leq m_{u}, u \in \mathbf{N}_{\boldsymbol{n - 1}}$.
By Algorithm 1, we follow the transition firing sequence to calculate the time delay. After firing $s_{n 1}$, wafer $W_{E n}$ is loaded into a PM for processing at Step $n$ modeled by $p_{n}$. Then, the robot goes to Step $n-2$ for unloading a wafer. Hence, $\Gamma_{n(n-2)}^{1}=\max \left\{\eta_{(n-2) 2}, H_{n-2}\right\}$ (Line 1 in Algorithm 1) is the exact longest time delay for this process. Lines 3 and 4 lead to a different activity sequence for different $n$, i.e., $\left\langle s_{(n-2) 2} \rightarrow\right.$ $\left.s_{(n-1) 1} \rightarrow y_{(n-3) 1} \rightarrow y_{(n-3) 2}\right\rangle$ if $n>2$, and $\left\langle s_{(n-2) 2} \rightarrow s_{(n-1) 1}\right.$ $\left.\rightarrow y_{n 1} \rightarrow y_{n 2}\right\rangle$ if $n=2$. Then, via Lines 9, 13, and 15, $\Gamma_{n k}^{1}$, $\Gamma_{n(n-1)}^{1}$, and $\Gamma_{n n}^{1}$ are obtained for different $k$. Based on the above results, $\Gamma_{n(n-2)}^{2}$ is obtained at Line 19. Continue this process, $\Gamma_{n(n-2)}^{j}, \Gamma_{n(n-3)}^{j}, \ldots, \Gamma_{n 0}^{j}, \Gamma_{n n}^{j}, \Gamma_{n(n-1)}^{j}, \Gamma_{n(n-2)}^{j+1}, \ldots$, and $\Gamma_{n(n-2)}^{m_{n}}$ are obtained. Finally, when $\Gamma_{n 0}^{m_{n}}$ is obtained at Line 9, the procedure stops.

If $m_{n} \leq m_{u}, u \in \mathbf{N}_{n-1}, \Gamma_{n 0}^{m_{n}}$ can be obtained by Algorithm 1 such that the exact upper bound of $B_{n}$ can be calculated
by (10). If $m_{i} \leq m_{u}$ and $i<n, u \in \mathbf{N}_{n} \backslash\{i\}$, to calculate $B_{i}$, we have to obtain $\Gamma_{i(i+1)}^{m_{i}}$. To do so, we need to renumber the steps as follows: 1) Step $i$ as $n$; 2) Step $j$ as Step $(j+n-i)$, $0 \leq j<i$; 3) Step $j$ as Step $(j-i-1), i<j<n$; and 4) $m_{j}$, $H_{j}, \eta_{j 1}$, and $\eta_{j 2}$ are numbered in the same way. In this way, $B_{i}$ can be calculated just as $B_{n}$ by using Algorithm 1 and (10).

Theorem 1: Assume that: 1) $m_{n} \leq m_{u}, u \in \mathbf{N}_{n-1}$; and 2) $\Gamma_{n 0}^{m_{n}}$ is obtained by Algorithm 1. Then, $B_{n}$ given in (10) is the exact upper bound of the accumulated robot time delay when the robot arrives at $q_{n 3}$ again in the $m_{n}$ th cycle after loading a wafer into Step $n$.

Proof: It is known that the robot tasks performed before loading a wafer into Step $n$ have no effect on $B_{n}, n \in \mathbf{N}_{n}$. Thus, to calculate $B_{n}$, we need to consider the activity sequence that starts from loading a wafer into Step $n$ only as done in Algorithm 1. Assume that after firing $s_{n 1}$, wafer $W_{1}$ is loaded into Step $n$, and then the robot goes to Step $n-2$ by performing the robot task of $y_{(n-2) 1}$ and waits in $q_{(n-2) 3}$ for the completion of a wafer there. By scheduling, when the robot arrives at $q_{(n-2) 3}$ under the normal conditions, there is a wafer completed with sojourn time $\Lambda_{n-2}$. With activity time variation, $\Gamma_{n(n-2)}^{1}=\max \left\{\eta_{(n-2) 2}, H_{n-2}\right\}$ is the exact upper bound for this process. Then, after robot activity sequence $\kappa_{4}=\left\langle s_{(n-2) 2} \rightarrow\right.$ waiting in $q_{(n-1) 1} \rightarrow s_{(n-1) 1} \rightarrow y_{(n-3) 1} \rightarrow$ waiting in $\left.q_{(n-3) 2} \rightarrow y_{(n-3) 2}\right\rangle$, the robot arrives at $q_{(n-3) 3}$ for unloading a wafer there. With $\left\|\kappa_{4}\right\|=\eta_{(n-1) 1}+\rho+\eta_{(n-3) 2}$, when the robot arrives at $q_{(n-3) 3}$, the longest delay is $\Gamma_{n(n-2)}^{1}+$ $\eta_{(n-1) 1}+\rho+\eta_{(n-3) 2}$. Meanwhile, the longest delay caused by processing a wafer at Step $n-3$ is $H_{n-3}$. Thus, as done in Algorithm 1, $\Gamma_{n(n-3)}^{1}=\max \left\{\Gamma_{n(n-2)}^{1}+\eta_{(n-1) 1}+\rho+\eta_{(n-3) 2}\right.$, $\left.H_{n-3}\right\}$ is the exact upper bound. By Algorithm 1, every $\Gamma_{n k}^{j}$ is calculated sequentially in this way, which guarantees that $\Gamma_{n k}^{j}$ is the exact upper bound. After $\Gamma_{n 0}^{m_{n}}$ is obtained, the robot comes back to $q_{n 3}$ for unloading wafer $W_{1}$. At this time, the longest time delay happens when $W_{1}$ is completed normally. It does not need to consider the time delay caused by its processing. Then, $B_{n}$ is calculated according to (10) and it is the exact upper bound.

In [23] and [32], to obtain the time delay analytically, $H=$ $\max \left\{H_{1}, \ldots, H_{n}\right\}$ is used as the delay in processing a wafer for all steps. Robot activities are similarly handled. Thus, they fail to obtain the exact upper bund. This problem is solved by Algorithm 1 in a sequential way. In Theorem 1, we consider just the situation that $m_{n} \leq m_{u}, u \in \mathbf{N}_{n-\mathbf{1}}$. However, if this condition is not true, Algorithm 1 is not applicable. Thus, we give Algorithm 2 for the case: $\exists f \neq n$ such that $m_{n}>m_{f}$.

To calculate $B_{n}$, consider the activity sequence that starts from loading wafer $W_{2}$ into Step $n$. For this case, if $f \neq$ $n-2$, $f \neq n-1$, and $f \neq n$ hold, a wafer named as $W_{3}$ is loaded into $\operatorname{Step} f$ in the first cycle when transition $s_{f 1}$ fires. At this time, by Algorithm 1, the longest accumulated robot delay time is $\Gamma_{n(f-1)}^{1}+\eta_{f 1}+\rho$. After $m_{f}$ robot cycles, the robot goes to Step $f$ for unloading $W_{3}$, or it arrives at $q_{f 3}$ in the $\left(m_{f}+1\right)$ th robot cycle. By Algorithm 1, we have that $\Gamma_{n f}^{m+1}=$ $\max \left\{\Gamma_{n(f+1)}^{m_{f}+1}+\eta_{(f+2) 1}+\rho+\eta_{f 2}, H_{f}\right\}$. However, note that, before loading $W_{3}$ into Step $f$, the longest robot delay is

```
Algorithm 2: Calculate \(\Gamma_{n 0}^{m_{n}}\) when \(\exists f \neq n\) such that \(m_{n}>m_{f}\)
    If \(\exists f \neq n\) such that \(m_{n}>m_{f}\), calculate \(\Gamma_{n 0}^{m_{n}}\) as follows.
        1) \(\Gamma_{n(n-2)}^{1}=\max \left\{\eta_{(n-2) 2}, H_{n-2}\right\}\);
        2) If \(i>2\)
        3) \(k=n-3\);
        4) Otherwise \(k=n\);
        5) \(j=1\);
        6) While \(j \leq m_{n}\)
        7) While \(k \neq n-2\)
        8) If \(0 \leq k \leq n-3\)
        9) \(\quad \Gamma_{n k}^{j}=\max \left\{\left(\Gamma_{n(k+1)}^{j}+\eta_{(k+2) 1}+\rho+\eta_{k 2}\right), H_{k}\right\}\);
        10) If \(j=m_{f}+g, g \in \mathbf{N}_{n}\), and \(k=f\)
    11) \(\quad \Gamma_{n k}^{j}=\max \left\{\Gamma_{n(k-1)}^{j-m_{f}}+\eta_{k 1}+\rho+H_{k}, \Gamma_{n(k+1)}^{j}\right.\)
        \(\left.+\eta_{(k+2) 1}+\rho+\eta_{k 2}\right\} ;\)
            If \(j=m_{n}\) and \(k=0\)
                Go to Statement (30);
        If \(k=n-1\)
            \(\Gamma_{n(n-1)}^{j}=\max \left\{\left(\Gamma_{n n}^{j}+\eta_{01}+\rho+\eta_{(n-1) 2}\right)\right.\),
        \(\left.H_{n-1}\right\}\)
        If \(j=m_{f}+g, g \in \boldsymbol{\Omega}\), and \(k=f\)
    17) \(\quad \Gamma_{n(n-1)}^{j}=\max \left\{\Gamma_{n(n-2)}^{j-m_{f}+1}+\eta_{(n-1) 1}+\rho+H_{n-1}\right.\),
        \(\left.\Gamma_{n n}^{j}+\eta_{01}+\rho+\eta_{(n-1) 2}\right\}\)
        If \(k=n\)
            \(\Gamma_{n n}^{j}=\max \left\{\left(\Gamma_{i 0}^{j}+\eta_{11}+\rho+\eta_{n 2}\right), H_{n}\right\} ;\)
        If \(k=0\)
            \(k=n\);
            Otherwise \(k=k-1\);
        \(\Gamma_{n(n-2)}^{j+1}=\max \left\{\left(\Gamma_{n(n-1)}^{j}+\eta_{n 1}+\rho+\eta_{(n-2) 2}\right)\right.\),
        \(\left.H_{n-2}\right\}\);
        If \(j+1=m_{f}+g, g \in \mathbf{N}_{n}\), and \(f=n-2\)
        \(\Gamma_{n(n-2)}^{j+1}=\max \left\{\Gamma_{n(n-3)}^{j+1-m_{f}}+\eta_{(n-2) 1}+\rho+H_{(n-2)}\right.\),
        \(\left.\Gamma_{n(n-1)}^{j}+\eta_{n 1}+\rho+\eta_{(n-2) 2}\right\} ;\)
    26) If \(n>2\)
    27) \(\quad k=k-1\);
    28) Otherwise \(k=n\);
    29) \(j=j+1\);
    30) Stop;
```

$\Gamma_{n(f-1)}^{1}+\eta_{f 1}+\rho$. Thus, with the delay of the processing time at Step $f$ considered, the accumulated delay time is $\Gamma_{n(f-1)}^{1}+$ $\eta_{f 1}+\rho+H_{f}$. Therefore, with the delay from both of wafer processing time at $\operatorname{Step} f$ and the robot task delay considered, we have $\Gamma_{n f}^{m_{f}+1}=\max \left\{\Gamma_{n(f-1)}^{1}+\eta_{f 1}+\rho+H_{f}, \Gamma_{n(f+1)}^{m_{f}+1}+\right.$ $\left.\eta_{(f+2) 1}+\rho+\eta_{f 2}\right\}$, when the robot leaves place $q_{f 3}$ in the $\left(m_{f}+1\right)$ th cycle. Hence, Lines 10 and 11 in Algorithm 2 are used to calculate it. If $f=n-1$, the robot goes to place $q_{f 3}$ for unloading the wafer in the $m_{f}$ th cycle which was loaded into $\operatorname{Step} f$ in the first cycle. Thus, with the processing time delay, $\Gamma_{n(n-1)}^{m_{f}}=\max \left\{\Gamma_{n(n-2)}^{1}+\eta_{(n-1) 1}+\rho+H_{(n-1)}, \Gamma_{n n}^{m_{f}}+\right.$ $\left.\eta_{01}+\rho+\eta_{(n-1) 2}\right\}$. Lines 16 and 17 are used to calculate it. If $f=n-2$, the robot goes to place $q_{f 3}$ for unloading the wafer in the $m_{f}$ th cycle that was loaded into $\operatorname{Step} f$ in the first cycle. The next robot activity is performed in the $\left(m_{f}+1\right)$ th cycle. Therefore, with processing time delay, $\Gamma_{n(n-2)}^{m_{f}+1}=$


Fig. 3. Illustration of Algorithm 2 with wafer flow pattern (1,2). (a) Schedule under the normal condition. (b) Schedule for the worst case by considering activity time variation.
$\max \left\{\Gamma_{n(n-3)}^{1}+\eta_{(n-2) 1}+\rho+H_{(n-2)}, \Gamma_{n(n-1)}^{m_{f}}+\eta_{n 1}+\rho+\right.$ $\left.\eta_{(n-2) 2}\right\}, n \geq 3$. Lines 24 and 25 are used to calculate it. The explanation of Algorithm 2 is shown in Fig. 3 via an example.

The example shown in Fig. 3 has two steps. $\mathrm{PM}_{1}$ is used to process wafers at Step 1. $\mathrm{PM}_{2}$ and $\mathrm{PM}_{3}$ together are used to process wafers at Step 2. Thus, $m_{1}=1<m_{2}=$ 2 holds. In this case, we explain how to obtain $B_{2}$. The accumulated robot delay at Step 2 is calculated when the robot starts from loading wafer $W_{4}$ into $\mathrm{PM}_{3}$ as shown by the time point T in Fig. 3. Then, it moves to the loadlocks. At this time, delay $\Gamma_{20}^{1}$ is obtained by Algorithm 2. The next robot task is $\left\langle s_{02} \rightarrow s_{11}\right\rangle$ such that wafer $W_{5}$ is loaded into $\mathrm{PM}_{1}$. At this time, the accumulated time delay is $\Gamma_{20}^{1}+\eta_{11}+\rho$ as shown in Fig. 3(b). Then, $y_{21}$ fires. Let $\kappa_{5}=\left\langle s_{02} \rightarrow\right.$ waiting in $q_{11} \rightarrow s_{11} \rightarrow y_{21} \rightarrow$ waiting in $\left.q_{22} \rightarrow y_{22}\right\rangle$. With the RCP, we have $\left\|_{\kappa_{5}}\right\|=\eta_{11}+\rho$ $+\eta_{22}$. With $\Gamma_{22}^{1}=\max \left\{\Gamma_{20}^{1}+\eta_{11}+\rho+\eta_{22}, H_{2}\right\}=H_{2}$, when the robot arrives at $\mathrm{PM}_{2}, W_{3}$ is not completed yet. Thus, the robot goes to $q_{23}$ for an unscheduled waiting as shown in Fig. 3(b). Then, after $W_{3}$ is completed and
$\left\langle s_{22} \rightarrow s_{01} \rightarrow y_{11}\right\rangle$ is performed, the robot goes to $\mathrm{PM}_{1}$ for unloading $W_{5}$. Note that, $W_{5}$ is loaded into $\mathrm{PM}_{1}$ after $W_{4}$ is loaded into $\mathrm{PM}_{3}$. By Lines 16 and 17 in Algorithm 2, we have $\Gamma_{21}^{1}=\max \left\{\Gamma_{20}^{1}+\eta_{11}+\rho+H_{1}, \Gamma_{22}^{1}+\eta_{01}+\rho+\eta_{12}\right\}$. For this case, $\Gamma_{21}^{1}=\max \left\{\Gamma_{20}^{1}+\eta_{11}+\rho+H_{1}\right.$, $\left.\Gamma_{22}^{1}+\eta_{01}+\rho+\eta_{12}\right\}=\Gamma_{22}^{1}+\eta_{01}+\rho+\eta_{12}$ holds. Then, similar to Algorithm 1, we can obtain $\Gamma_{20}^{2}$. Thus, by (10), $B_{2}=\Gamma_{20}^{2}+\eta_{01}+\rho+\eta_{12}$.

Similarly, with (10) and Algorithm 2, we can calculate $B_{i}, i \neq n$, by renumbering the steps and their corresponding parameters. In this way, $\Gamma_{n k}^{j}$ can be calculated in a sequential way, which guarantees that $\Gamma_{n k}^{j}$ is the exact upper bound. Thus, $B_{n}$ given in (10) must be the exact upper bound of the accumulated robot time delay. Hence, we have the following theorem immediately.
Theorem 2: Assume that 1) $\exists f \neq n$ such that $m_{n}>m_{f}$; and 2) $\Gamma_{n 0}^{m_{n}}$ is obtained via Algorithm 2. Then, $B_{n}$ given in (10) is the exact upper bound of the accumulated robot time delay when the robot arrives at $q_{n 3}$ again in the $m_{n}$ th cycle after loading a wafer into Step $n$.

```
Algorithm 3: Calculate \(\Gamma_{n 0}^{m_{n}}\) when \(\exists f\) and \(h, f \neq h \neq n\) such
that \(m_{n}>m_{f}\) and \(m_{f}>m_{h}\)
    If \(\exists f\) and \(h, f \neq h \neq n\), such that \(m_{n}>m_{f}\) and \(m_{f}>m_{h}\),
    calculate \(\Gamma_{n 0}^{m_{n}}\) as follows.
    1) \(\Gamma_{n(n-2)}^{1}=\max \left\{\eta_{(n-2) 2}, H_{n-2}\right\}\);
    2) If \(n>2\)
        3) \(k=n-3\);
        4) Otherwise \(k=n\);
        5) \(j=1\);
        6) While \(j \leq m_{n}\)
        ) While \(k \neq n-2\)
        8) If \(0 \leq k \leq n-3\)
        9) \(\quad \Gamma_{n k}^{j}=\max \left\{\left(\Gamma_{n(k+1)}^{j}+\eta_{(k+2) 1}+\rho+\eta_{k 2}\right), H_{k}\right\}\);
        If \(j=m_{h}+g, g \in \mathbf{N}_{n}\), and \(k=h\)
            \(\Gamma_{n k}^{j}=\max \left\{\Gamma_{n(k-1)}^{j-m_{h}}+\eta_{k 1}+\rho+H_{k}, \Gamma_{n(k+1)}^{j}+\right.\)
        \(\left.\eta_{(k+2) 1}+\rho+\eta_{k 2}\right\} ;\)
    12) If \(j=m_{f}+g, g \in \mathbf{N}_{n}\), and \(k=f\)
    \(\quad \Gamma_{n k}^{j}=\max \left\{\Gamma_{n(k-1)}^{j-m_{f}}+\eta_{k 1}+\rho+H_{k}, \Gamma_{n(k+1)}^{j}+\right.\)
    \(\left.\eta_{(k+2) 1}+\rho+\eta_{k 2}\right\}\)
        If \(j=m_{n}\) and \(k=0\)
            Go to Statement (34);
        If \(k=n-1\)
        \(\Gamma_{n(n-1)}^{j}=\max \left\{\left(\Gamma_{n n}^{j}+\eta_{01}+\rho+\eta_{k 2}\right), H_{k}\right\} ;\)
        If \(j=m_{h}+g, g \in \boldsymbol{\Omega}\), and \(k=h\)
            \(\Gamma_{n(n-1)}^{j}=\max \left\{\Gamma_{n(n-2)}^{j-m_{h}+1}+\eta_{(n-1) 1}+\rho+H_{n-1}\right.\),
    \(\left.\Gamma_{n n}^{j}+\eta_{01}+\rho+\eta_{(n-1) 2}\right\}\)
        If \(j=m_{f}+g, g \in \boldsymbol{\Omega}\), and \(k=f\)
            \(\Gamma_{n(n-1)}^{j}=\max \left\{\Gamma_{n(n-2)}^{j-m_{f}+1}+\eta_{(n-1) 1}+\rho+H_{n-1}\right.\),
    \(\left.\Gamma_{n n}^{j}+\eta_{01}+\rho+\eta_{(n-1) 2}\right\}\)
        If \(k=n\)
            \(\Gamma_{n n}^{j}=\max \left\{\left(\Gamma_{n 0}^{j}+\eta_{11}+\rho+\eta_{n 2}\right), H_{n}\right\} ;\)
        If \(k=0\)
        \(k=n\);
        Otherwise \(k=k-1\);
        \(\Gamma_{n(n-2)}^{j+1}=\max \left\{\left(\Gamma_{n(n-1)}^{j}+\eta_{n 1}+\rho+\eta_{(n-2) 2}\right)\right.\),
    \(\left.H_{n-2}\right\}\);
    28) If \(j+1=m_{h}+g, g \in \mathbf{N}_{n}\), and \(h=n-2\)
        \(\Gamma_{n(n-2)}^{j+1}=\max \left\{\Gamma_{n(n-3)}^{j+1-m_{h}}+\eta_{(n-2) 1}+\rho+H_{(n-2)}\right.\),
    \(\left.\Gamma_{n(n-1)}^{j}+\eta_{n 1}+\rho+\eta_{(n-2) 2}\right\}\)
    30) If \(j+1=m_{f}+g, g \in \mathbf{N}_{n}\), and \(f=n-2\)
    31) \(\quad \Gamma_{n(n-2)}^{j+1}=\max \left\{\Gamma_{n(n-3)}^{j+1-m_{f}}+\eta_{(n-2) 1}+\rho+H_{(n-2)}\right.\),
    \(\left.\Gamma_{n(n-1)}^{j}+\eta_{n 1}+\rho+\eta_{(n-2) 2}\right\} ;\)
    32) If \(n>2\)
    33) \(k=k-1\);
    34) Otherwise \(k=n\);
    35) \(j=j+1\);
    36) Stop;
```

Based on Algorithms 1 and 2, we have Algorithm 3 for the case: $\exists f$ and $h, f \neq h \neq n$, such that $m_{n}>m_{f}$ and $m_{f}>m_{h}$.

For this case, similar to Algorithm 2, every time the robot goes to $q_{k 3}, k=f$ or $h$, for unloading the wafer that was loaded into Step $k$ in or after the first cycle, $\Gamma_{i k}^{j}$ can be calculated according to Lines 10-13, 18-21, and 28-31 in Algorithm 3. Similar to Algorithms 1 and 2, the calculation of $B_{i}$ can be
done by renumbering the steps and their parameters. Then, by this algorithm, every $\Gamma_{n k}^{j}$ is calculated sequentially such that $\Gamma_{n k}^{j}$ is the exact upper bound. Hence, based on Theorems 1 and 2 , we have the following theorem.

Theorem 3: Assume that: 1) if $\exists f$ and $h, f \neq h \neq n$, such that $m_{n}>m_{f}$ and $m_{f}>m_{h}$; and 2) $\Gamma_{n 0}^{m_{n}}$ is obtained via Algorithm 3. Then, $B_{n}$ given in (10) is the exact upper bound of the accumulated robot time delay when the robot arrives at $q_{n 3}$ again in the $m_{n}$ th cycle after loading a wafer into Step $n$.

Note that, our results can easily be extended to the case when $m_{u} \geq m_{i}>m_{f}>m_{h}>\cdots>m_{e}$, where $u \neq i \neq f \neq h$ $\neq \cdots \neq e$. Thus, up to now, we present a method to calculate the exact upper bound of wafer sojourn time delay caused by activity time variation. By Algorithms 1-3, the number of the iteration times depends on the number of parallel PMs at a step and the number of steps. If there are $m_{n}$ parallel PMs at Step $n$, we need to do the iteration for $m_{n}$ cycles and, for each cycle, with $n$ steps, we need to do it for $n$ times. Thus, the computational complexity of the proposed method is $\mathrm{O}\left(n \times m_{n}\right)$. In a cluster tool, both $n$ and $m_{n}$ are limited. Therefore, it is very efficient.

In [32], the upper bound of the wafer sojourn time delay is calculated by using analytical expressions. However, it is overestimated such that the schedulability conditions proposed in [23] are sufficient, but not necessary. By Algorithms 1-3, the exact upper bound for different cases can be obtained. Then, we can exactly check if a given off-line schedule is feasible. To make it feasible, i.e., making the PN model live, we require that $a_{i} \leq \tau_{i} \leq a_{i}+\delta_{i}, \forall i \in \mathbf{N}_{n}$. Thus, if a schedule is feasible under normal conditions, then, $a_{i} \leq \Lambda_{i} \leq a_{i}+$ $\delta_{i}$ must hold. With the activity time variation considered, $\Lambda_{i}$ $\leq \tau_{i}$ is always true. Thus, $a_{i} \leq \tau_{i}$ holds. Based on the presented results, we have $\tau_{i} \leq \Lambda_{i}+B_{i}$, where $B_{i}$ obtained via Algorithms $1-3$ is the exact upper bound of the wafer sojourn time at Step $i$. Hence, if, at the worst case, $\Lambda_{i}+B_{i} \leq a_{i}+$ $\delta_{i}$ holds, the wafer residency constraints are never violated, or the schedule is feasible even if the activity time varies. With this perspective, by replacing the so-called upper bound of the wafer sojourn time in [23] by $B_{i}$ calculated by Algorithms 1-3, the necessary and sufficient schedulability conditions are obtained. Thus, an optimal and feasible schedule can be found by using the scheduling algorithms presented in [23].

## IV. Illustrative Examples

In this section, examples are used to show the applications and usefulness of the proposed approach.

Example 1: It is from [23] and the flow pattern is $(1,1)$. Under normal conditions, it takes 15 time units for the robot to unload a wafer from a loadlock and moves to Step $1\left(c_{0}=15\right)$, and 10 time units for the robot to load a wafer into a PM or loadlock, or unload a wafer from a PM $(c=10), 2$ time units to move from $p_{i}$ to $p_{j}(\alpha=2)$. It needs 100 time units for a PM at both Steps 1 and 2 to process a wafer $\left(a_{1}=a_{2}=100\right)$, respectively. After being processed, a wafer at Steps 1 and 2 can stay there for no more than 20 time units ( $\delta_{1}=\delta_{2}=20$ ). The activity time is subject to random variation with $d_{0}=20$, $d=12, \beta=3$, and $b_{1}=b_{2}=105$.

By applying the approach [23], it is obtained that $\omega_{11}=0$, $\omega_{21}=0, \omega_{02}=0, \omega_{01}=3, \omega_{12}=1, \omega_{22}=70, B_{1}=8$, and $B_{2}=9$. For this case, $m_{1}=m_{2}$ holds. Thus, by the proposed method in this paper, Algorithm 1 and (10) are applied to obtain $B_{1}=6$ and $B_{2}=9$. It shows that, for $B_{2}$, the exact upper bound of the wafer sojourn time delay is obtained by the methods presented both in this paper and in [32]. However, $B_{1}$ is overestimated by $25 \%$ if the method in [32] is applied. This implies that, by the method proposed in this paper, a significant improvement is made. If a cluster tool is schedulable under normal conditions, we have $a_{i} \leq \Lambda_{i} \leq a_{i}+\delta_{i}$. With activity time variation, to check the feasibility, one needs to check if $\tau_{i}=\Lambda_{i}+B_{i} \leq a_{i}+\delta_{i}$ holds. Therefore, overestimation of $B_{i}$ may result in a feasible schedule being treated as an infeasible one. This situation can be completely avoided by the proposed method, which is further discussed via the next example.

Example 2: It is also from [23] and the flow pattern is $(2,2,1)$. Under the normal conditions, $c_{0}=14, c=10$, $\alpha=2, a_{1}=150, a_{2}=140, a_{3}=48, \delta_{1}=\delta_{2}=25$, and $\delta_{3}=20$. An activity time is subject to random variations, and we have $d_{0}=19, b_{1}=156, b_{2}=146$, and $b_{3}=53$.

Under the normal conditions, we have $\psi_{1}=2(n+1) \alpha+$ $(2 n+1) c+c_{0}=100$. By examining this case with the approach in [23], all the robot waiting times are set to be zero. Then, from (7)-(9), we have $\Lambda_{1}=150, \Lambda_{2}=154$, and $\Lambda_{3}=54$. Thus, with the activity time variation, by using the method in [32], one has $B_{1}=11$, $B_{2}=16$, and $B_{3}=11$. For this case, $m_{1}=m_{2}>m_{3}$ holds. Thus, by Algorithm 2 and (10), we have $B_{1}=B_{2}=11$. By Algorithm 1 and (10), we have $B_{3}=11 . B_{2}$ is overestimated by $31.25 \%$ if the method in [32] is applied.

Next, we compare the schedule feasibility check via two methods. With the results obtained by using the approaches presented in [32], we have $B_{1}+\left(\Lambda_{1}-a_{1}\right)=11+$ $(150-150)=11<\delta_{1}, B_{2}+\left(\Lambda_{2}-a_{2}\right)=$ $16+(154-140)=30>\delta_{2}$, and $B_{3}+\left(\Lambda_{3}-a_{3}\right)=11+$ $(54-48)=17<\delta_{3}$. In other words, for Step 2, the residency time constraints are violated. This implies that the schedule is infeasible. However, by the method presented in this paper, we have $B_{1}+\left(\Lambda_{1}-a_{1}\right)=11+(150-150)=11<\delta_{1}$, $B_{2}+\left(\Lambda_{2}-a_{2}\right)=11+(154-140)=25=\delta_{2}$, and $B_{3}+$ $\left(\Lambda_{3}-a_{3}\right)=11+(54-48)=17<\delta_{3}$. This implies that the schedule is, in fact, feasible.

Example 3: The flow pattern is $(1,1,1,1,1)$. Under normal conditions, $c_{0}=12, c=8, \alpha=3, a_{1}=90, a_{2}=80, a_{3}=95$, $a_{4}=90, a_{5}=90$, and $\delta_{1}=\delta_{2}=\delta_{3}=\delta_{4}=\delta_{5}=25$. An activity time is subject to random variation, and we have $d_{0}=16, b_{1}=95, b_{2}=85, b_{3}=105, b_{4}=95$, and $b_{5}=95$.

Under normal conditions, we have $\psi_{1}=2(n+1) \alpha+$ $(2 n+1) c+c_{0}=136$. By examining this case with the approach in [23], all the robot waiting times are set to be zero. Then, from (7)-(9), we have $\Lambda_{1}=91$, and $\Lambda_{2}=\Lambda_{3}$ $=\Lambda_{4}=\Lambda_{5}=95$. Thus, with the activity time variation, by using the method in [32], one has $B_{1}=10, B_{2}=14, B_{3}=8$, $B_{4}=8$, and $B_{5}=14$. For this case, $m_{1}=m_{2}=m_{3}=m_{4}=$ $m_{5}=1$ holds. Thus, by Algorithm 1 and (10) in this paper, we have $B_{1}=10, B_{2}=10, B_{3}=8, B_{4}=8$, and $B_{5}=14$.

Therefore, $B_{2}$ is overestimated by about $28.6 \%$ if the method in [32] is applied.

Then, we compare the schedule feasibility check via two methods. With the results obtained by using the approach presented in [32], we have $B_{1}+\left(\Lambda_{1}-a_{1}\right)=10+(91-$ 90) $=11<\delta_{1}, B_{2}+\left(\Lambda_{2}-a_{2}\right)=14+(95-80)=29$ $>\delta_{2}, B_{3}+\left(\Lambda_{3}-a_{3}\right)=8+(95-95)=8<\delta_{3}, B_{4}+$ $\left(\Lambda_{4}-a_{4}\right)=8+(95-90)=13<\delta_{4}, B_{5}+\left(\Lambda_{5}-a_{5}\right)=$ $14+(95-90)=19<\delta_{5}$. In other words, for Step 2, the residency time constraints are not satisfied. This implies that the schedule is infeasible. However, by the method presented in this paper, we have $B_{1}+\left(\Lambda_{1}-a_{1}\right)=10+(91-90)=$ $11<\delta_{1}, B_{2}+\left(\Lambda_{2}-a_{2}\right)=10+(95-80)=25=\delta_{2}$, $B_{3}+\left(\Lambda_{3}-a_{3}\right)=8+(95-95)=8<\delta_{3}, B_{4}+\left(\Lambda_{4}-\right.$ $\left.a_{4}\right)=8+(95-90)=13<\delta_{4}, B_{5}+\left(\Lambda_{5}-a_{5}\right)=14+$ $(95-90)=19<\delta_{5}$. This implies that the schedule is, in fact, feasible.

## V. Conclusion

Wafers in PMs in cluster tools face strict wafer residency time constraints. Such constraints greatly complicate the scheduling problem of cluster tools. Moreover, activity time variation may make a feasible schedule obtained under the deterministic activity time assumption infeasible. Thus, it is very challenging to operate a cluster tool with wafer residency time constraints and activity time variation. This problem is studied in [23], [25], [26], [31], and [32] for both single and dual-arm cluster tools. Based on a PN model and RCP, analytical expressions are derived to calculate the upper bound of wafer sojourn time delay in a PM. Then, real-time scheduling algorithms are proposed to find an optimal schedule. Nevertheless, the upper bound of wafer sojourn time delay is not the exact one but overestimated. Such overestimation may fail to identify some schedulable cases. To solve such a problem, this paper presents polynomial algorithms to find the exact upper bound of the wafer sojourn time delay. Based on it, one can check if a given schedule is feasible and find a feasible one if it is schedulable. The proposed method is of polynomial complexity.

For some wafer processing processes, a wafer needs to be processed in some PMs more than once, or there is wafer revisiting, which makes the scheduling problem more challenging. Our future work will deal with such cases.

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