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Spectral-Efficient Cellular Communications with Coexistent One- and Two-hop Transmissions

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***Abstract*—The cellular communications scenario involving the coexistence of the one-hop direct transmission and the two-hop relaying is studied in this paper. In contrast to conventional cellular systems featuring orthogonal information flows served by decoupled channel resources via e.g., time-division, we propose a novel protocol in which two information flows are served simultaneously via the shared channel resource, thus constituting a spectral-efficient solution for cellular communications. On the other hand, however, an inevitable issue associated with the proposed protocol is the inter-flow interference which may lead to serious deteriorations on both information flows. To tackle this issue, we develop an overhearing based protocol which utilizes the overheard interference as useful side information in the receiver design. Specifically, depending on the interference levels, an adaptive linear minimum mean squared error (MMSE) and nonlinear MMSE-SIC (successive interference cancellation) receiver exploiting the overheard interference at the direct mobile terminal is developed. To balance between the two information flows, we develop the asymptotically optimum superposition coding at BS in the high power regime. Furthermore, the optimum relay beamforming matrix maximizing the bottleneck of the achievable rates of the two information flows is developed subject to a finite power constraint. Finally, simulations demonstrate a remarkable throughput gain over the conventional cellular systems.**

***Index Terms*—Coexistence, overhearing, interference, one- and two-hop transmission.**

I. INTRODUCTION

COOPERATIVE relay networks have been extensively studied due to the extended cellular coverage, increased channel capacity, and improved diversity performance [1]–[4]. Over the past several decades, significant progress has been made towards developing various relaying schemes, including amplify-and-forward (AF), decode-and-forward (DF), compress-and-forward (CF), and their extensions and unifications [5]–[9]. The two-hop relaying, as a building block of the cooperative relay-based transmissions, has been considered in various modern wireless communications standards, such as WIMAX and LTE.

In mobile cellular networks, the mobile terminals (MTs) may communicate either directly with the base station (BS) through the one-hop transmission when the direct channel is strong, or indirectly via the two-hop relaying when the direct channel is subject to severe path loss or shadowing. This is achieved through the deployment of relay stations (RSs) in cellular networks. In fact, cellular networks are expected to support the two-hop relaying as infrastructured RSs, for example, the Type I and II relays, have already been included in the LTE-advanced [11]. Through the two-hop relaying, the cellular coverage can be substantially extended, thus guaranteeing seamless quality-of-service even for the cell-edge users.

A. Motivation

We consider a typical cellular communication scenario as depicted in Fig. 1, which consists of the two-hop transmission from BS via a RS to MT_1 (hereafter referred to as the two-hop MT) and one-hop direct transmission from BS to MT_2 (hereafter referred to as the direct MT). This may correspond to the case where the direct MT is close to BS and thereby can communicate directly with BS. On the other hand, the two-hop MT might be located at the cell-edge and thus the direct channel between it and BS is too weak to support any data transmission, which motivates the two-hop MT to seek for assistance from RS.

With the co-existence of the one- and two-hop transmissions in cellular networks, an appropriate scheduling protocol coordinating the two information flows is needed. Conventional cellular communications systems typically assign different MTs with *orthogonal* channels via various multiple-access scheduling protocols such as time-division, frequency-division, or code-division [12]. However, these protocols featuring decoupled channel resources for different MTs are rather inefficient in terms of the spectrum efficiency, and thus are highly undesirable given the emergent spectrum sparsity in modern wireless communications. Motivated by this, the main objective of this paper is to develop novel *spectral-efficient* transmission protocol for cellular communications involving the co-existence of one-hop direct transmission and two-hop relayed transmission. To that end, we allocate the shared channel resource to both information flows and allow the

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simultaneous transmission of the two information flows, which automatically guarantees a high spectrum efficiency. Nevertheless, the inter-flow interference between the two information flows becomes an inevitable issue due to the simultaneous transmission. This further motivates us to develop novel receiver structure taking into account the interference. By recognizing the fact that the MTs may overhear the signals intended for other MTs due to the broadcast nature of the wireless medium, we propose to use the overhearing signaling to facilitate the receiver design. The key point is that instead of simply avoiding the interference, we turn the overheard interference into an advantage to facilitate the receiver design.

B. Related Work

It has been demonstrated in various applications that the exploitation of the overhearing signaling can improve the wireless communication performance. In [13] the overheard information was utilized to assist the optimization of the address configuration in *Ad Hoc* networks, which helped to reduce the signaling overhead significantly. The overhearing was also applied in [14] to model the correlation between the proportions of the overheard beacon and the position which resulted in more precise location estimation. In [15], the authors proposed to utilize the overheard feedback signals due to ARQ (Automatic Repeat-reQuest) in the primary system to enable opportunistic spectrum access for the secondary users in cognitive radio networks. In [16], the importance of the overhearing was analyzed for the routing technique with the network coding in wireless *Ad Hoc* networks. Moreover, a distributed way was also proposed for the overhearing in this system. The authors in [17] made use of the overhearing property of wireless communications to design a data caching algorithm in wireless *Ad Hoc* networks.

Very recently, we have designed an overhearing based transmission protocol to deal with two coexistent two-hop information flows [18]. However, the protocol in [18] was developed for the cell-edge users only, where the direct links between BS and MTs are very weak and thus the MTs must be simultaneously served by a shared RS. In this paper, however, we consider a fundamentally different cellular scenario where one of the MTs is directly served by BS while the other is served by RS. Moreover, different from the work of [18] which was limited to *single*-antenna relay terminal, we consider a more general multi-antenna RS which provides additional performance improvement via appropriately designed beamforming.

C. Contribution

The main contributions of this paper are two-fold:

- A novel overhearing-based protocol is developed for cellular communications involving the co-existence of one-hop direct transmission and two-hop relaying, which achieves a significant gain in the spectrum efficiency as compared to the conventional cellular systems.
- The asymptotically optimum superposition coding is developed for BS to balance between the two information flows. In addition, depending on the overheard interference levels, the adaptive minimum mean squared error (MMSE) or MMSE-SIC (successive interference cancellation) receiver was developed for the MT. Finally, the optimum relay beamformer is designed to maximize the minimum of the data rates of the two information flows given the power constraint at RS.

II. System Model and the Proposed Transmission Protocol

Consider a cellular system consisting of one BS, one RS and two MTs as depicted in Fig. 1, where RS is equipped with $M \geq 1$ antennas whereas other terminals have a single antenna. The two-hop MT, denoted by MT_1, cannot receive signals from BS directly due to severe path loss and/or shadowing effect; thus it communicates with BS via the help of the intermediate RS featuring the AF relaying. The direct MT is located close to BS and its wireless connection to BS is more reliable and thus can communicate directly with BS. The considered model represents a typical vehicle-to-vehicle (V2V) communication example in which a vehicle close to BS may communicate directly with the BS, while a vehicle moving outside the coverage region of the BS has to resort to the roadside relay stations. All the nodes work in the half-duplex mode, i.e., each node cannot transmit and receive simultaneously over the same channel. We assume that RS has full channel station information of the whole networks in order to perform the beamforming.

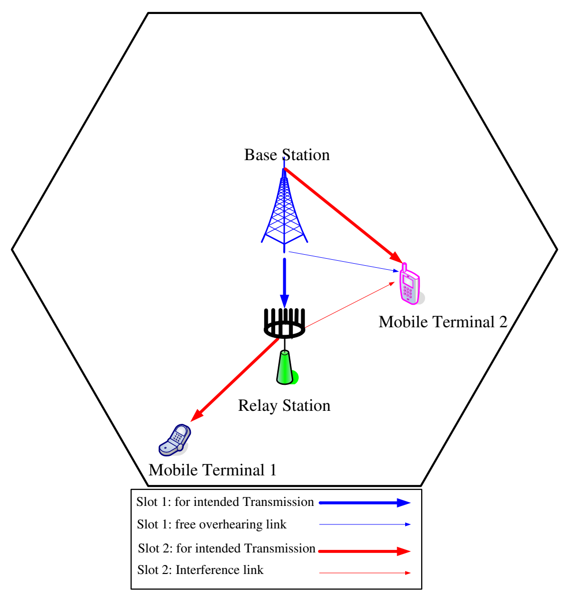

Fig. 1. Proposed transmission protocol for the coexistence of one- and two-hop transmission superposition coding scheme.

Let x_1 and x_2 denote the transmit signals intended for MT_1 and MT_2, respectively, where $\mathbb{E}\{|x_1|^2\} = \mathbb{E}\{|x_2|^2\} = 1$. The superposition coding is performed at the BS to balance between the two information flows. Specifically, in the first slot, the BS transmits $ax_1 + bx_2$, $|a|^2 + |b|^2 = 1$, to the RS with power p_1. The received signal at the RS in the first slot is given by

$$\mathbf{y}_r[1] = \sqrt{p_1}\mathbf{h}_{br}(ax_1 + bx_2) + \mathbf{n}_r[1], \quad (1)$$

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We consider a realistic fading model $\mathbf{h}_{br} = \left(\frac{d_1}{d_0}\right)^{-\frac{\alpha}{2}}\mathbf{h}_{br}^w$ where the small-scale fading coefficient $\mathbf{h}_{br}^w \in \mathbb{C}^{M\times 1}$ as well as large-scale path loss is considered. The fading coefficient is modeled as complex Gaussian $\mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$. The path loss is modeled as $\left(\frac{d_1}{d_0}\right)^{-\frac{\alpha}{2}}$, where d_0 is the reference distance, d_1 is the communication distance between the BS and RS, and α is the path loss exponent that typically ranges from 2 to 6. Also, $\mathbf{n}_r[1]$ is the additive white Gaussian noise (AWGN) in the first slot with the distribution of $\mathbf{n}_r[1] \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$. The signal overheard by the direct user MT_2 in the first slot is written as

$$y_2[1] = \sqrt{p_1}h_{b2}(ax_1 + bx_2) + n_2[1], \tag{2}$$

where $n_2[1] \sim \mathcal{CN}(0,1)$ is the AWGN at MT_2 and $h_{b2} = \left(\frac{d_2}{d_0}\right)^{-\frac{\alpha}{2}}h_{b2}^w$ is the channel from BS to MT_2 in the first slot, where $h_{b2}^w \sim \mathcal{CN}(0,1)$ and d_2 is the direct transmission distance from BS to MT_2.

In the second slot, while BS sends $cx_1 + dx_2$, $|c|^2 + |d|^2 = 1$, to MT_2, RS *simultaneously* amplify-and-forwards its received signal with precoding to MT_1, which automatically ensures a high spectrum efficiency as compared to conventional orthogonal transmissions. Thus, the signals received at MT_1 and MT_2 are expressed as

$$y_1[2] = \mathbf{h}_{r1}^T\mathbf{x}_r + n_1[2], \tag{3}$$

$$y_2[2] = \sqrt{p_2}g_{b2}(cx_1 + dx_2) + \mathbf{h}_{r2}^T\mathbf{x}_r + n_2[2], \tag{4}$$

respectively, where p_2 is the transmit power of BS in the second time slot, $g_{b2} = \left(\frac{d_2}{d_0}\right)^{-\frac{\alpha}{2}}g_{b2}^w$ is the channel coefficient from BS to MT_2 in the second slot, $\mathbf{h}_{r1} = \left(\frac{d_3}{d_0}\right)^{-\frac{\alpha}{2}}\mathbf{h}_{r1}^w$ and $\mathbf{h}_{r2} = \left(\frac{d_4}{d_0}\right)^{-\frac{\alpha}{2}}\mathbf{h}_{r2}^w$ are the channel coefficients from the multi-antenna RS to MT_1 and MT_2. The small-scale fading coefficients are modeled as $g_{b2}^w \sim \mathcal{CN}(0,1)$ and $\mathbf{h}_{rk}^w \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$. The AWGNs, $n_1[2]$ and $n_2[2]$, added at MT_1 and MT_2 are modeled as $\mathcal{CN}(0,1)$, and d_3 and d_4 are the distances from RS to MT_k, $k = 1, 2$, respectively. The transmit signal from RS after the amplification is $\mathbf{x}_r$, which is given by

$$\mathbf{x}_r = \mathbf{W}_r\mathbf{y}_r[1] = \sqrt{p_1}\mathbf{W}_r\mathbf{h}_{br}(ax_1 + bx_2) + \mathbf{W}_r\mathbf{n}_r[1], \tag{5}$$

where $\mathbf{W}_r \in \mathbb{C}^{M\times M}$ is the beamforming/precoding matrix adopted RS. Thus, the two signals received at MT_2 in the two slots are written as

$$\mathbf{y}_2 \triangleq \begin{bmatrix} y_2[1] \\ y_2[2] \end{bmatrix} = \mathbf{\Phi}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_2[1] \\ \tilde{n}_2[2] \end{bmatrix}, \tag{6}$$

where

$$\mathbf{\Phi} \triangleq \begin{bmatrix} \sqrt{p_1}h_{b2}a & \sqrt{p_1}h_{b2}b \\ \sqrt{p_1}\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{h}_{br}a + \sqrt{p_2}g_{b2}c & \sqrt{p_1}\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{h}_{br}b + \sqrt{p_2}g_{b2}d \end{bmatrix} \tag{7}$$

and $\tilde{n}_2[2] \triangleq n_2[2] + \mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{n}_r[1]$. Here, x_2 is the desirable signal for MT_2 yet x_1 is the interference overheard by MT_2 in both time slots. In the next section, depending on the interference levels, we will develop an adaptive receiver exploiting the overheard interference to detect the desirable signal.

In this paper, we consider the general *asymmetric* system setting. Specifically, the transmit powers p_1 and p_2 in the two time slots may be different, which contains the symmetric power setting $p_1 = p_2$ as a special case. Furthermore, the coefficients a and b associated with the superposition coding in the first slot may be different from the coefficients c and d associated with the superposition coding in the second slot. In addition, the channels from BS to MT_2 in the two time slots, h_{b2} and g_{b2}, may be different (corresponding to fast fading) or identical (corresponding to slow fading). Note that the beamforming scheme to be developed in the next section is applicable to the general asymmetric system setting, which includes the symmetric setting as a special case.

III. Optimal Superposition Coding at BS and Optimization of Relay Precoder

In this section, depending on the interference levels, we develop an adaptive linear MMSE and nonlinear MMSE-SIC receiver for the direct MT. Moreover, the asymptotically optimum superposition coding for the BS is developed, which amounts to the pre-cancellation of the interference at BS. The resulting signal-to-noise ratio (SNR) of the two-hop MT, the signal-to-interference-plus-noise ratio (SINR) of the linear MMSE as well as the SNR after the MMSE-SIC for the direct MT are obtained. Finally, the optimization of the precoding matrix $\mathbf{W}_r$ at the multi-antenna RS which maximizes the minimum of the data rates of the two MTs is formulated as a nonconvex problem and solved using the semi-definite relaxation (SDR) method. A standard form of SDR problem can be solved directly by using the convex optimizations, where the rank 1 constraint is first removed. Then, the obtained solution is made to satisfy the rank 1 constraint by applying the randomization method if needed [15].

A. Adaptive MMSE/MMSE-SIC Receiver Structure and S-NR/SINR expressions

From the degree-of-freedom (DoF) point-of-view, the direct MT receives two branches of signals in two slots, which results in the DoF of two at the direct MT. Under the two DoF, both the desirable signal and the interference may be decoded. Thus, depending on the interference levels, an adaptive decoding method may be adopted. Specifically, if the interference is strong, the interference may be decoded first and then cancelled out using the SIC approach. On the other hand, if the interference is relatively weak, it may be treated as noise and a no-SIC decoding method is used to directly decode the desired signal. Using this concept, we now present the receiver structure for direct MT.

For the weak interference scenario, the MMSE receiver is applied at MT_2, where the desired signal x_2 is directly decoded treating the interference x_1 as noise. Following the noise whitening techniques as in [21], [22], the resulting SINR in terms of the desirable signal x_2 is given by in (8) (on the top of next page).

In the alternative case where the interference is stronger, the MMSE-SIC receiver is adopted at MT_2, where the interference signal x_1 is decoded first, resulting in an SINR expression in terms of the interference signal x_1 as in (9) on the top of next page.

$$\mathrm{SINR}_{x_2}|_{\mathrm{MT}_2} = \frac{(1+\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r2}^*)p_1|h_{b2}b|^2 + \left|\sqrt{p_1}\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{h}_{br}b + \sqrt{p_2}g_{b2}d\right|^2 + p_1p_2|h_{b2}g_{b2}|^2(ad-bc)^2}{\left(1+\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r2}^*\right)(1+p_1\left|h_{b2}a\right|^2) + \left|\sqrt{p_1}\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{h}_{br}a + \sqrt{p_2}g_{b2}c\right|^2}. \quad (8)$$

$$\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} = \frac{(1+\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r2}^*)p_1\left|h_{b2}a\right|^2 + \left|\sqrt{p_1}\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{h}_{br}a + \sqrt{p_2}g_{b2}c\right|^2 + p_1p_2|h_{b2}g_{b2}|^2(ad-bc)^2}{\left(1+\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r2}^*\right)\left(1+|\sqrt{p_1}h_{b2}b|^2\right) + \left|\sqrt{p_1}\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{h}_{br}b + \sqrt{p_2}g_{b2}d\right|^2}. \quad (9)$$

Once the interference signal is decoded, it is subtracted from the received signal at MT_2 and then interference-free decoding of x_2 is performed, which results in the SNR (rather than SINR) in terms of x_2 below:

$$\mathrm{SNR}_{x_2}|_{\mathrm{MT}_2} = \frac{\left|\sqrt{p_2}g_{b2}d + \sqrt{p_1}\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{h}_{br}b\right|^2}{1+\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r2}^*} + p_1|h_{b2}b|^2. \quad (10)$$

More rigourously, the adaptive MMSE/MMSE-SIC receiver for the direct MT is summarized as follows:

- If $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} \leq \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}$ (weak interference), the desirable signal x_2 is decoded directly using the linear MMSE receiver;
- If $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} > \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}$ (strong interference), the interference signal x_1 is decoded first using linear MMSE and cancelled by the SIC, followed by the decoding of the desirable signal x_2.

As to the receiver of the two-hop MT, since the equivalent channel model (3) together with (6) for MT_1 corresponds to a single-input single-output (SISO) system, we adopt the simple receiver which directly decodes the desired signal without SIC. Thus, the resulting SINR in terms of the desirable signal x_1 at the two-hop terminal MT_1 is given by

$$\mathrm{SINR}_{x_1}|_{\mathrm{MT}_1} = \frac{p_1|a\mathbf{h}_{r1}^T\mathbf{W}_r\mathbf{h}_{br}|^2}{1+\mathbf{h}_{r1}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r1}^* + p_1|b\mathbf{h}_{r1}^T\mathbf{W}_r\mathbf{h}_{br}|^2}. \quad (11)$$

B. Asymptotically Optimum Superposition Coding for BS

For the coexistent direct and two-hop MTs, the design objective in this paper is to optimize the bottleneck performance of the two end-users. That is, the optimum superposition coding at the BS maximizes the minimum SNR/SINRs of the two end-users as follows (shown in (12) on the top of next page), where $\mathbf{1}(A \leq B)$ is the indicator function that returns one (zero) if the condition $A \leq B$ is true (false), and $\mathbf{1}(A > B) = 1 - \mathbf{1}(A \leq B)$. For the finite power regime (p_1, p_2) with $0 < p_1, p_2 < \infty$, it is difficult, if not impossible, to solve the above optimization problem in general. To tackle this problem, we will investigate the high power regime where the asymptotically optimum solution is derived.

Theorem 1: Consider the high power regime (p_1, p_2) where $p_1 \to \infty$ and the ratio p_2/p_1 is fixed. The superposition coding at the BS which maximizes the minimum SINRs/SNRs of the direct and the two-hop relaying MTs is given by $a^* = 1, b^* = 0, c^* = 0, d^* = 1$.

Proof: See Appendix A. ■

An interesting observation drawn from Theorem 1 is that the asymptotically optimum superposition coding in the high power regime orthogonalizes the codewords and consequently pre-cancels the self-interference at the BS. This indicates that the general superposition coding schemes with non-orthogonal codewords do not yield any performance gain in the high power regime for our considered coexistent one-hop MT and cell-edge two-hop MT communication scenario, where the design objective is to maximize the bottleneck performance of the two MTs.

Applying the asymptotically optimal superposition coding as in Theorem 1, the resulting SINR/SNR expressions in (8)-(11) become as in (13a)-(13d) on the top of next page.

Here, the $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_1}$ reduces to $\mathrm{SNR}_{x_1}|_{\mathrm{MT}_1}$ because there is no interference received at the two-hop MT_1 under the optimal superposition coding with $a = 1, b = 0$.

C. Optimum Relay Beamforming

Based on the SINR/SNR expressions (13a)–(13d), RS beamforming is formulated as an optimization problem which maximizes the minimum of the data rates for the two MTs. Specifically, for the strong interference scenario where the MMSE-SIC receiver is applied at MT_2, the optimization is formulated as

$$\begin{aligned}&\max_{\mathbf{W}_r} \min\left\{\log_2\left(1+\mathrm{SNR}_{x_2}|_{\mathrm{MT}_2}\right), \log_2\left(1+\mathrm{SNR}_{x_1}|_{\mathrm{MT}_1}\right)\right\},\\ &\text{s.t. } \mathrm{Tr}\{\mathbf{W}_r^H\mathbf{W}_r\} \leq p_r, \mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} > \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}.\end{aligned} \quad (14)$$

where $\mathrm{SNR}_{x_2}|_{\mathrm{MT}_2}$ and $\mathrm{SNR}_{x_1}|_{\mathrm{MT}_1}$ are given in (13c) and (13d), respectively. Similarly, for the weak interference case where the linear MMSE receiver is applied at MT_2, the optimization is formulated as

$$\begin{aligned}&\max_{\mathbf{W}_r} \min\left\{\log_2\left(1+\mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}\right), \log_2\left(1+\mathrm{SNR}_{x_1}|_{\mathrm{MT}_1}\right)\right\},\\ &\text{s.t. } \mathrm{Tr}\{\mathbf{W}_r^H\mathbf{W}_r\} \leq p_r, \mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} \leq \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}.\end{aligned} \quad (15)$$

where $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2}$ and $\mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}$ are given in (13a) and (13b), respectively.

Unfortunately, neither of the problems in (14) and (15) is convex with respect to the unknown precoding matrix $\mathbf{W}_r$. To obtain the global optimum solution, the exhaustive numerical search over the $M \times M$ matrix in the complex domain is prohibitively complex to implement. In this paper, we proceed by using the structured solutions based on the bisection method and the feasibility problem similar to that in [19], as explained below.

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$$(a^*, b^*, c^*, d^*) = \arg \max_{\substack{a,b,c,d \\ |a|^2+|b|^2=1 \\ |c|^2+|d|^2=1}} \min \Big\{ \mathrm{SINR}_{x_1}|_{\mathrm{MT}_1}, \quad \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2} \mathbf{1}\big(\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} \le \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}\big) + \mathrm{SNR}_{x_2}|_{\mathrm{MT}_2} \mathbf{1}\big(\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} > \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}\big) \Big\}, \tag{12}$$

$$\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} = \frac{p_1|h_{b2}|^2\big(p_2|g_{b2}|^2 + 1 + \mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r2}^*\big) + p_1|\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{h}_{br}|^2}{p_2|g_{b2}|^2 + 1 + \mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r2}^*}, \tag{13a}$$

$$\mathrm{SINR}_{x_2}|_{\mathrm{MT}_2} = \frac{p_2|g_{b2}|^2\big(1 + p_1|h_{b2}|^2\big)}{(p_1|h_{b2}|^2+1)\big(1 + \mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r2}^*\big) + p_1|\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{h}_{br}|^2}, \tag{13b}$$

$$\mathrm{SNR}_{x_2}|_{\mathrm{MT}_2} = \frac{p_2|g_{b2}|^2}{1 + \mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r2}^*}, \tag{13c}$$

$$\mathrm{SNR}_{x_1}|_{\mathrm{MT}_1} \triangleq \mathrm{SINR}_{x_1}|_{\mathrm{MT}_1} = \frac{p_1|\mathbf{h}_{r1}^T\mathbf{W}_r\mathbf{h}_{br}|^2}{1 + \mathbf{h}_{r1}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r1}^*}. \tag{13d}$$

The feasibility problem for (14) is given by

$$\begin{aligned} &\text{find } \mathbf{W}_r, \\ &\text{s.t.} \log_2\left(1 + \mathrm{SNR}_{x_2}|_{\mathrm{MT}_2}\right) \ge \bar{c}, \mathrm{Tr}\{\mathbf{W}_r^H\mathbf{W}_r\} \le p_r, \\ &\log_2\left(1 + \mathrm{SNR}_{x_1}|_{\mathrm{MT}_1}\right) \ge \bar{c}, \mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} > \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}, \end{aligned} \tag{16}$$

where $\bar{c}$ (bps/Hz) denotes the bound for the feasible region of the cost function. Similarly, the feasibility problem for (15) is given below

$$\begin{aligned} &\text{find } \mathbf{W}_r, \\ &\text{s.t.} \log_2(1 + \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}) \ge \bar{c}, \log_2\left(1 + \mathrm{SNR}_{x_1}|_{\mathrm{MT}_1}\right) \ge \bar{c}, \\ &\mathrm{Tr}\{\mathbf{W}_r^H\mathbf{W}_r\} \le p_r, \mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} \le \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}. \end{aligned} \tag{17}$$

Suppose that the solutions to the two feasibility problems in (16) and (17) are $\mathbf{W}_r^{(1)}$ and $\mathbf{W}_r^{(2)}$, respectively, and the corresponding objective functions in (14) and (15) are $f^{(1)}$ and $f^{(2)}$, respectively. Thus, the optimum RS beamformer is $\mathbf{W}_r^{(j)}$ where $j = \arg\max_{j=1,2} f^{(j)}$, and the maximum data rate achieved by the optimum RS beamforming is given by $\max\left\{f^{(1)}, f^{(2)}\right\}$. To that end, in what follows, we transform the feasibility problems in (16) and (17) into the standard SDR form such that the feasibility problems are readily solved.

1) Equivalent SINR/SNR for standard form of SDR: For the $\mathrm{SNR}_{x_1}|_{\mathrm{MT}_1}$ in (13d), the term in the denominator can be rewritten as

$$\begin{aligned} \mathbf{h}_{r1}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r1}^* &= \mathrm{Tr}\big\{\mathbf{W}_r^H\mathbf{h}_{r1}^*\mathbf{h}_{r1}^T\mathbf{W}_r\big\} \\ &= \mathbf{w}_r^H\left[\mathbf{I}_M \otimes (\mathbf{h}_{r1}^*\mathbf{h}_{r1}^T)\right]\mathbf{w}_r \triangleq \mathbf{w}_r^H\mathbf{Q}_1\mathbf{w}_r, \end{aligned} \tag{18}$$

where $\otimes$ denotes the Kronecker product and $\mathbf{w}_r$ is a vector obtained by stacking the columns of the matrix $\mathbf{W}_r$. The term in numerator of (13d) can be expressed as

$$\begin{aligned} |\mathbf{h}_{r1}^T\mathbf{W}_r\mathbf{h}_{br}|^2 &= \mathbf{w}_r^H\big(\mathbf{h}_{br}^T \otimes \mathbf{h}_{r1}^T\big)^H\big(\mathbf{h}_{br}^T \otimes \mathbf{h}_{r1}^T\big)\mathbf{w}_r \\ &= \mathbf{w}_r^H\left[(\mathbf{h}_{br}^*\mathbf{h}_{br}^T) \otimes (\mathbf{h}_{r1}^*\mathbf{h}_{r1}^T)\right]\mathbf{w}_r \triangleq \mathbf{w}_r^H\mathbf{Q}_2\mathbf{w}_r, \end{aligned} \tag{19}$$

where the first equality holds since the scalar $\mathbf{h}_{r1}^T\mathbf{W}_r\mathbf{h}_{br}$ is invariant with respect to the vectorization operation $\mathrm{vec}\{\cdot\}$, the second equality follows by $\mathrm{vec}\{\mathbf{ABC}\} = (\mathbf{C}^T \otimes \mathbf{A})\mathrm{vec}\{\mathbf{B}\}$, and the third equality holds because $(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = (\mathbf{AC}) \otimes (\mathbf{BD})$ and $(\mathbf{A} \otimes \mathbf{B})^H = \mathbf{A}^H \otimes \mathbf{B}^H$. Substituting (18) and (19) into (13d) yields the equivalent form of SNR at MT_1 as

$$\mathrm{SNR}_{x_1}|_{\mathrm{MT}_1} = \frac{\mathbf{w}_r^H p_1\mathbf{Q}_2\mathbf{w}_r}{1 + \mathbf{w}_r^H\mathbf{Q}_1\mathbf{w}_r}. \tag{20}$$

For the nonlinear MMSE-SIC receiver at MT_2, the SNR in (13c) is equivalent to

$$\mathrm{SNR}_{x_2}|_{\mathrm{MT}_2} = \frac{p_2|g_{b2}|^2}{1 + \mathbf{w}_r^H\left[\mathbf{I}_M \otimes (\mathbf{h}_{r2}^*\mathbf{h}_{r2}^T)\right]\mathbf{w}_r}. \tag{21}$$

For the the case of the linear MMSE at MT_2, the term in the numerator of $\mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}$ in (13b), $\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r2}^*$, can be rewritten as

$$\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r2}^* = \mathbf{w}_r^H\left[\mathbf{I}_M \otimes (\mathbf{h}_{r2}^*\mathbf{h}_{r2}^T)\right]\mathbf{w}_r \triangleq \mathbf{w}_r^H\mathbf{Q}_3\mathbf{w}_r. \tag{22}$$

The term in the denominator of (13b), $|\mathbf{h}_{r2}\mathbf{W}_r\mathbf{h}_{br}|^2$, is equivalent to

$$|\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{h}_{br}|^2 = \mathbf{w}_r^H\left[(\mathbf{h}_{br}^*\mathbf{h}_{br}^T) \otimes (\mathbf{h}_{r2}^*\mathbf{h}_{r2}^T)\right]\mathbf{w}_r \triangleq \mathbf{w}_r^H\mathbf{Q}_4\mathbf{w}_r. \tag{23}$$

By substituting (22) and (23) into (13b), we obtain an equivalent form of $\mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}$ as

$$\mathrm{SINR}_{x_2}|_{\mathrm{MT}_2} = \frac{p_2|g_{b2}|^2\big(1 + p_1|h_{b2}|^2\big)}{p_1|h_{b2}|^2 + 1 + \mathbf{w}_r^H\left[(p_1|h_{b2}|^2+1)\mathbf{Q}_3 + p_1\mathbf{Q}_4\right]\mathbf{w}_r}. \tag{24}$$

Similarly, the $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2}$ at MT_2 in (13a) is equivalent to (25) as given on the top of next page.

The next step is to substitute the obtained equivalent SNR/SINR expressions (20), (21), (24), and (25) into the feasibility problems in (16) and (17). However, the resulting e-quivalent feasibility problems cannot be written in the standard SDR form because the condition $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2}$ >$\mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}$ in (16) and the condition $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} \le \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}$ in (17) leads to the fourth-order constraints in terms of $\mathbf{w}_r$. Thus, it remains to transform the fourth-order constraints into the second-order such that the equivalent problems can be expressed in standard SDR form.

$$\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} = \frac{p_1|h_{b2}|^2\left(p_2|g_{b2}|^2+1\right) + \mathbf{w}_r^H\left(p_1|h_{b2}|^2\mathbf{Q}_3 + p_1\mathbf{Q}_4\right)\mathbf{w}_r}{p_2|g_{b2}|^2 + 1 + \mathbf{w}_r^H\mathbf{Q}_3\mathbf{w}_r}. \tag{25}$$

2) Equivalent second-order constraints on $\mathbf{w}_r$*:* In the following theorem, the fourth-order constraints are transformed to the second-order, for which the SDR method readily applies.

Theorem 2: The condition for the MMSE-SIC at the MT_2, $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} \geq \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}$ is equivalent to

$$p_2|g_{b2}|^2 + 1 \leq \mathbf{w}_r^H\left(p_1|h_{b2}|^2\mathbf{Q}_3 + p_1\mathbf{Q}_4\right)\mathbf{w}_r. \tag{26}$$

Similarly, the condition for the linear MMSE at MT_2, $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} < \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}$, is equivalent to $p_2|g_{b2}|^2+1 > \mathbf{w}_r^H\left(p_1|h_{b2}|^2\mathbf{Q}_3 + p_1\mathbf{Q}_4\right)\mathbf{w}_r$.

Proof: See Appendix B. ■

3) Standard form of SDR: By substituting (20), (21), and (26) into (16), the feasibility problem in (16) for the nonlinear MMSE-SIC receiver at the MT_2 is expressed in the standard form of SDR as

$$\begin{aligned}&\text{find } \mathbf{X},\\ &\text{s.t. } \mathrm{Tr}\left\{\mathbf{X}\left[\mathbf{I}_M \otimes (\mathbf{h}_{r2}^*\mathbf{h}_{r2}^T)\right]\right\} \leq \frac{p_2|g_{b2}|^2}{2^{\bar{c}}-1} - 1, \mathrm{Tr}\{\mathbf{X}\} \leq p_r,\\ &\mathrm{Tr}\left\{\mathbf{X}\left[p_1\mathbf{Q}_2 - (2^{\bar{c}}-1)\mathbf{Q}_1\right]\right\} \geq 2^{\bar{c}} - 1,\\ &\mathrm{Tr}\left\{\mathbf{X}\left(p_1|h_{b2}|^2\mathbf{Q}_3 + p_1\mathbf{Q}_4\right)\right\} \geq p_2|g_{b2}|^2 + 1,\end{aligned} \tag{27}$$

where $\mathbf{X} \triangleq \mathbf{w}_r\mathbf{w}_r^H$. Similarly, the standard SDR form of the problem (17) is given by

$$\begin{aligned}&\text{find } \mathbf{X},\\ &\text{s.t. } \mathrm{Tr}\left\{\mathbf{X}\left[(p_1|h_{b2}|^2+1)\mathbf{Q}_3 + p_1\mathbf{Q}_4\right]\right\} \leq \phi,\\ &\mathrm{Tr}\left\{\mathbf{X}\left[p_1\mathbf{Q}_2 - (2^{\bar{c}}-1)\mathbf{Q}_1\right]\right\} \geq 2^{\bar{c}} - 1, \mathrm{Tr}\{\mathbf{X}\} \leq p_r,\\ &\mathrm{Tr}\left\{\mathbf{X}\left(p_1|h_{b2}|^2\mathbf{Q}_3 + p_1\mathbf{Q}_4\right)\right\} < p_2|g_{b2}|^2 + 1,\end{aligned} \tag{28}$$

where $\phi \triangleq \frac{p_2|g_{b2}|^2\left(1+p_1|h_{b2}|^2\right)}{2^{\bar{c}}-1} - p_1|h_{b2}|^2 - 1$.

The general idea of SDR is to drop the rank one constraint which results in a convex problem that can be directly solved by the convex optimization techniques. If the obtained solution does not satisfy the rank one constraint, the randomization method [15] is further applied to make the solution rank one. The SDR problem is solved by the interior-point algorithm, whose computational complexity is $\mathcal{O}(M^7\log(1/\epsilon))$ for a given solution accuracy $\epsilon > 0$ and the number of antennas M [20].

Remark 1: The design of the relay precoder requires the local CSI associated with the relay $(\mathbf{h}_{br}, \mathbf{h}_{r1})$ and the extra scalar channel coefficients (h_{b2}, g_{b2}) associated with the direct MT. These CSIs can be obtained using standard channel estimation algorithm and CSI feedback method as adopted in numerous relevant references such as [14].

IV. Numerical simulations

Performance in terms of the sum rate is numerically simulated for the optimized precoding scheme at the multi-antenna RS, together with the optimum superposition coding $a=1, b=0, c=0, d=1$ at BS. Monte Carlo simulations with 10^4 trials are performed. As a benchmark, the performance of the nonorthogonal superposition coding scheme employing, for example, $a^2=b^2=c^2=d^2=0.5$, is evaluated to demonstrate the asymptotic optimality of the proposed scheme. In the simulations, we set the path loss exponent as $\alpha = 4$ which corresponds to typical urban environment. Also, the reference distance in the path loss model is set to $d_0 = 1$ km (kilometer) to model the outdoor communications.

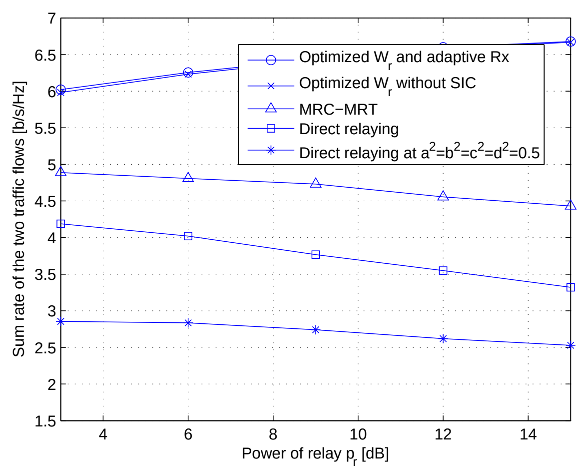

Fig. 2. Data rate comparison at $p_1 = p_2 = 10$dB for the transmission distance setting as $d_1 = d_2 = d_3 = d_4 = 1$ km.

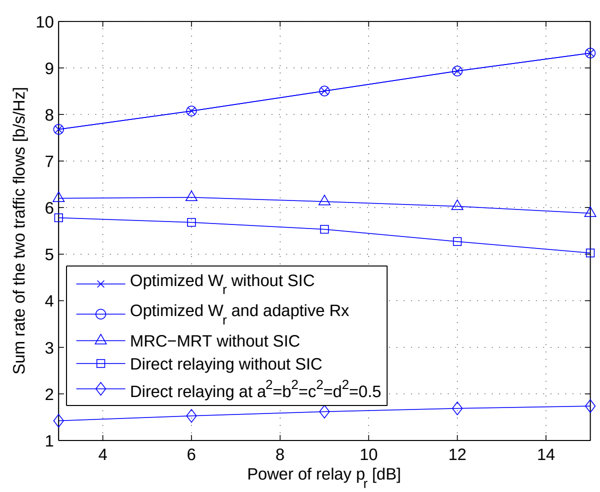

Fig. 3. Data rate comparison at $p_1 = p_2 = 20$dB for the transmission distance setting as $d_1 = 1.5$ km, $d_2 = 1$ km, $d_3 = 2$ km and $d_4 = 1$ km .

First of all, we consider the symmetric setting with $d_1 = d_2 = d_3 = d_4 = 1$ km, i.e., each channel is subject to equal

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path loss. Fig. 2 shows the sum rate of the two information flows versus the RS power p_r at $p_1 = p_2 = 10$ dB. It is seen that the optimum precoding scheme $\mathbf{W}_r$ under the proposed transmission protocol achieves the best sum-rate performance. Moreover, the linear MMSE receiver without SIC employing the optimized $\boldsymbol{W}_r$ is near-optimum. However, the MRC-MRT precoding scheme and the direct relaying scheme at RS result in significant performance loss as compared to the proposed scheme. Here, the existing MRC-MRT [19] denotes the maximum ratio combining of the receive and re-transmit channels $\mathbf{W}_r = \xi\left[\mathbf{h}_{r1}^*, \mathbf{h}_{r2}^*\right]\mathbf{h}_{br}^H$ at the relay for two users. The direct relaying means $\mathbf{W}_r = \eta\mathbf{I}$, where η is chosen to satisfy the relay power constraint. This loss comes from the strong interference received at the direct user from the two-hop user. Yet, all these curves are still much higher than the curve corresponding to the other values of the superposition coding. This is because the SINR of the desirable signal at the direct user becomes smaller at any values of the superposition coding as comparing to the optimal value at $a = 1, b = 0, c = 0, d = 1$.

Secondly, we consider the asymmetric setting with $d_1 = 1.5$ km, $d_2 = 1$ km, $d_3 = 2$ km and $d_4 = 1$ km, i.e., the path losses of different channels may be different. Fig. 3 plots the similar curves as in Fig. 2 to show the gain of the proposed scheme over the other inferior schemes at $p_1 = p_2 = 20$ dB. We observe that the achievable sum rate of our proposed scheme is always higher than any other schemes, such as the nonorthogonal superposition coding, the MRC-MRT scheme, and the direct relaying scheme, which demonstrates the superiority of our proposed scheme.

V. Conclusions

A new transmission protocol has been proposed for the coexistence of the two-hop traffic and the direct traffic flows. The problem comes from the interference received at the direct user from the transmission intended to the dual-hop user. The sufficient condition for the asymptotic optimality of the superposition coding approach at base station is obtained for the high regime of the transmit power. The two degree-of-freedom at the direct terminal is exploited by designing an adaptive MMSE receiver. Using the optimal superposition coding scheme at BS and the adaptive MMSE receiver at MT, the beamforming design is optimized for the multi-antenna RS which is based on the maximization of the minimum of two data rates. The difficulty of this optimization lies in the fourth-order constraint on the unknown beamforming vector, which is successfully transformed to an equivalent form such that the optimization is readily solved by the SDR method. Thus, the coexistent one- and two-hop transmission is enabled by the proposed superposition coding scheme as BS, the optimized precoding scheme at RS, and the adaptive MMSE receiver at MT.

Appendix A: Proof of Theorem 1

First of all, we show that for the high power regime, the bottleneck of the two-user system is the two-hop relaying MT, i.e., $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_1} < \min\left(\mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}, \mathrm{SNR}_{x_2}|_{\mathrm{MT}_2}\right)$ for $p_1 \to \infty$ with fixed ratio p_2/p_1. We proceed by bounding $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_1}$ of (11) as follows

$$\mathrm{SINR}_{x_1}|_{\mathrm{MT}_1} < \frac{p_1|a\mathbf{h}_{r1}^T\mathbf{W}_r\mathbf{h}_{br}|^2}{p_1|b\mathbf{h}_{r1}^T\mathbf{W}_r\mathbf{h}_{br}|^2} \tag{A.1}$$

$$= \left|\frac{a}{b}\right|^2. \tag{A.2}$$

Also, we express $\mathrm{SNR}_{x_2}|_{\mathrm{MT}_2}$ of (10) as $\mathrm{SNR}_{x_2}|_{\mathrm{MT}_2} = p_1\varphi$, where

$$\varphi \triangleq |h_{b2}b|^2 + \frac{\left|\sqrt{\frac{p_2}{p_1}}g_{b2}d + \mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{h}_{br}b\right|^2}{1 + \mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r2}^*} > 0. \tag{A.3}$$

It is thus easy to see that $\frac{\mathrm{SINR}_{x_1}|_{\mathrm{MT}_1}}{\mathrm{SNR}_{x_2}|_{\mathrm{MT}_2}} = \frac{1}{p_1}\frac{1}{\varphi}\left|\frac{a}{b}\right|^2 \to 0$ as $p_1 \to \infty$ under fixed ratio p_2/p_1. Therefore, the relationship $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_1} \ll \mathrm{SNR}_{x_2}|_{\mathrm{MT}_2}$ holds for the high power regime.

Similarly, we rewrite $\mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}$ of (8) as $\mathrm{SINR}_{x_2}|_{\mathrm{MT}_2} = p_1\phi$, where ϕ is given in (A.5) on the top of next page.

In (A.5), the limit of ϕ is taken with respect to p_1 for $p_1 \to \infty$ while the ratio p_2/p_1 is fixed. It is then easy to show that $\frac{\mathrm{SINR}_{x_1}|_{\mathrm{MT}_1}}{\mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}} = \frac{1}{p_1}\frac{1}{\phi}\left|\frac{a}{b}\right|^2 \to 0$ as $p_1 \to \infty$ under fixed ratio p_2/p_1. Therefore, we have $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_1} \ll \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}$ for the high power regime.

So far, we have shown that $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_1} < \min\left(\mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}, \mathrm{SNR}_{x_2}|_{\mathrm{MT}_2}\right)$ holds for the high power regime. Using this relationship, the joint optimization of the superposition coding scheme in (12) for the high power regime is decoupled, equivalently, to two sub-problems as in (A.7)(A.8) on the top of next page.

In order to maximize $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_1}$ for (A.7), the value of a in the numerator of $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_1}$ in (11) should be set as large as possible and the value of b in the denominator should be as small as possible, while the constraint $|a|^2 + |b|^2 = 1$ must be satisfied. Thus, it is easy to see that $a^* = 1, b^* = 0$ is the solution to (A.7).

To tackle the problem of (A.8), we consider two mutually exclusive but collectively exhaustive subcases. For $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} > \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}$, the problem of (A.8) reduces to $(c^*, d^*) = \arg\max_{c,d} \mathrm{SNR}_{x_2}|_{\mathrm{MT}_2}$. From (10), we see that the coefficient d appears in the numerator of $\mathrm{SNR}_{x_2}|_{\mathrm{MT}_2}$ while c is not present. Thus, we should set $c^* = 0, d^* = 1$ to maximize $\mathrm{SNR}_{x_2}|_{\mathrm{MT}_2}$. For the alternative case where $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} \leq \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}$, the problem of (A.8) becomes $(c^*, d^*) = \arg\max_{c,d} \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}$. From (8), we see that d is involved only in the numerator and c is involved in the denominator only. Thus, to maximize $\mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}$, we should set $c^* = 0, d^* = 1$ as well.

Therefore, we conclude that the solution to the problem of (A.8) is $c^* = 0, d^* = 1$, and consequently, the solution to (12) is $a^* = 1, b^* = 0, c^* = 0, d^* = 1$ for the high power regime.

Appendix B: Proof of Theorem 2

For any real positive scalars α, β, γ and $\xi \in \mathbb{R}^+$, it follows that

$$\frac{\alpha+\beta}{\gamma+\xi} - \frac{\gamma+\beta}{\alpha+\xi} = \frac{\alpha(\alpha+\beta+\xi) - \gamma(\gamma+\beta+\xi)}{(\gamma+\xi)(\alpha+\xi)}. \tag{B.1}$$

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$$\phi \triangleq \frac{\frac{1}{p_1}\left\{\left(1+\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r2}^*\right)|h_{b2}b|^2 + \left|\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{h}_{br}b + \sqrt{\frac{p_2}{p_1}}g_{b2}d\right|^2\right\} + \frac{p_2}{p_1}|h_{b2}g_{b2}|^2(ad-bc)^2}{\left|h_{b2}a\right|^2\left(1+\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r2}^*\right) + \frac{1}{p_1}\left(1+\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r2}^*\right) + \left|\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{h}_{br}a + \sqrt{\frac{p_2}{p_1}}g_{b2}c\right|^2} \quad (A.4)$$

$$\to \frac{\frac{p_2}{p_1}|h_{b2}g_{b2}|^2(ad-bc)^2}{\left|h_{b2}a\right|^2\left(1+\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{W}_r^H\mathbf{h}_{r2}^*\right) + \left|\mathbf{h}_{r2}^T\mathbf{W}_r\mathbf{h}_{br}a + \sqrt{\frac{p_2}{p_1}}g_{b2}c\right|^2} \quad (A.5)$$

$$> 0. \quad (A.6)$$

$$(a^*, b^*) = \arg\max_{|a|^2+|b|^2=1} \mathrm{SINR}_{x_1}|_{\mathrm{MT}_1}, \quad (A.7)$$

$$(c^*, d^*) = \arg\max_{|c|^2+|d|^2=1} \Big\{\mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}\mathbf{1}\big(\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} \le \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}\big) + \mathrm{SNR}_{x_2}|_{\mathrm{MT}_2}\mathbf{1}\big(\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} > \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}\big)\Big\}. \quad (A.8)$$

where we have $\alpha(\alpha+\beta+\xi)-\gamma(\gamma+\beta+\xi)\ge 0$ if $\alpha\ge\gamma$ and $\alpha(\alpha+\beta+\xi)-\gamma(\gamma+\beta+\xi)< 0$ if $\alpha<\gamma$. Thus, the following inequalities hold

$$\frac{\alpha+\beta}{\gamma+\xi} \ge \frac{\gamma+\beta}{\alpha+\xi}, \text{ if } \alpha\ge\gamma, \quad \frac{\alpha+\beta}{\gamma+\xi} < \frac{\gamma+\beta}{\alpha+\xi}, \text{ if } \alpha<\gamma. \quad (B.2)$$

We introduce the definitions as follows:

$$\alpha \triangleq p_2|g_{b2}|^2+1,\ \beta \triangleq p_1|h_{b2}|^2(p_2|g_{b2}|^2+1),$$
$$\gamma \triangleq \mathbf{w}_r^H[p_1|h_{b2}|^2\mathbf{Q}_3 + p_1\mathbf{Q}_4]\mathbf{w}_r,\ \xi \triangleq \mathbf{w}_r^H\mathbf{Q}_3\mathbf{w}_r. \quad (B.3)$$

Based on the definitions above, (25) is rewritten as $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} = \frac{\beta+\gamma}{\alpha+\xi}$. Applying the result in (B.2), we have

$$\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} = \frac{\beta+\gamma}{\alpha+\xi} \ge \frac{\alpha+\beta}{\gamma+\xi}, \text{ if } \alpha\le\gamma. \quad (B.4)$$

Substituting (B.3) into $(\alpha+\beta)/(\gamma+\xi)$ yields

$$\frac{\alpha+\beta}{\gamma+\xi} = \frac{p_1|h_{b2}|^2(p_2|g_{b2}|^2+1)+p_2|g_{b2}|^2+1}{\mathbf{w}_r^H[(p_1|h_{b2}|^2+1)\mathbf{Q}_3+p_1\mathbf{Q}_4]\mathbf{w}_r}. \quad (B.5)$$

By defining $\breve{\alpha} \triangleq p_2|g_{b2}|^2+1,\ \breve{\beta} \triangleq p_1|h_{b2}|^2(p_2|g_{b2}|^2+1), \breve{\gamma} \triangleq 0,\ \breve{\xi} \triangleq \mathbf{w}_r^H[(p_1|h_{b2}|^2+1)\mathbf{Q}_3+p_1\mathbf{Q}_4]\mathbf{w}_r$,we have

$$\frac{\alpha+\beta}{\gamma+\xi} = \frac{\breve{\beta}+\breve{\alpha}}{\breve{\gamma}+\breve{\xi}} \ge \frac{\breve{\gamma}+\breve{\beta}}{\breve{\alpha}+\breve{\xi}} = \mathrm{SINR}_{x_2}\big|_{\mathrm{MT}_2}, \quad (B.6)$$

where the inequality always holds due to the fact that $\breve{\alpha} > 0 = \breve{\gamma}$. Thus, combining (B.4) and (B.6), we have

$$\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} \ge \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}, \text{ if } \alpha\le\gamma. \quad (B.7)$$

Following a similar proof, we can show that $\mathrm{SINR}_{x_1}|_{\mathrm{MT}_2} < \mathrm{SINR}_{x_2}|_{\mathrm{MT}_2}$ if $\alpha>\gamma$. This completes the proof.

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