Abstract - In this paper, the problem of $H^\infty$ filtering design is studied for nonlinear sampled-data systems using the Takagi-Sugeno (T-S) fuzzy model approach. Traditionally, the sufficient conditions for the existence of such $H^\infty$ filter are characterized in terms of the solution of a differential Hamilton-Jacobi inequality with jumps, which is equivalent to solving the partial differential inequalities. There is no analytic solution for this nonlinear partial differential inequalities in general. First, in this study, the T-S fuzzy model is proposed to represent a class of nonlinear sampled-data systems. Next, using the T-S fuzzy model, the $H^\infty$ fuzzy filtering design problem for nonlinear sampled-data systems is characterized in terms of a linear matrix inequality problem (LMIP). Hence, the $H^\infty$ fuzzy filter of nonlinear sampled-data systems can be given via solving linear matrix inequalities (LMIs) instead of a differential Hamilton-Jacobi inequality with jumps.

I. INTRODUCTION

The celebrated Kalman filter is the optimal state estimator that minimizes either the average root-mean-square power of the estimation error or the variance of the terminal state estimation error, where the signal generating system is assumed to be driven by a white noise process and the measured output is also assumed to be corrupted by a white noise process, both with known statistical properties, while the purpose of an $H^\infty$ filter is to ensure that the $L^2$-energy gain from the disturbance, which is assumed to be unknown deterministic but of finite energy, to the estimation error is less than a prescribed level. In contrast to the traditional Kalman filter, an $H^\infty$ filter has some practical advantages. First, it does not require knowledge of the statistical properties of the noise, instead, the only requirement for the noise is that its energy is bounded. Consequently, $H^\infty$ filters are less sensitive to the noise uncertainty. Second, it is more robust than Kalman filter to the unmodeled uncertainties of signal systems.

In this paper, the filtering problem for nonlinear systems with sampled measurements in an $H^\infty$ setting is addressed. This problem is to estimate the states of a continuous-time system using only sampled measurements at discrete instants of time. Motivation of studying this problem comes from the fact that in many practical situations the underlying plant is continuous-time while the measurements are usually taken only at discrete-time instants, and virtually all physical systems are nonlinear in nature. Moreover, such systems are important in practice because of the widespread use of digital computers in implementation. Typically, sampled-data filtering is designed either by discretizing an analog design by, e.g., the bilinear transformation, or directly through their discrete-time behavior by the use of the modified Z-transform. However, many performances, such as disturbance and noise attenuation, overshoot, require a deeper insight into the intersampling behavior. The $H^\infty$ performance criterion is defined directly in terms of the continuous-time signals and thus intersampling behavior is taken into account. Our goal is to design a filter that achieves a given bound on the ratio between the $L^2$-energy of a given function of the estimation error and the $L^2$-energy of the disturbances that consist of the continuous-time process noises and the discrete-time measurement noises.

The $H^\infty$ filtering problem was first addressed by Elsayed and Grimble [3] and Grimble [5]. Later, a game theoretic approach has been given by Yaesh and Shaked [13] and a state-space approach has been offered by Nagpal and Khargoneker [8] for the linear $H^\infty$ filtering problem. On the other hand, using a game theoretic approach, the nonlinear $H^\infty$ filtering problems have been extensively studied by Berman and Shaked [1], Fridman and Shaked [4], and Yung et al. [14] for continuous-time systems. Compare with continuous-time systems, the sampled-data systems are more realistic while the information processing are performed by digital hardware. Sun et al. [9] treated the linear sampled-data $H^\infty$ filtering problem using the theory of linear continuous-time systems with jumps. Necessary and sufficient conditions are derived which are expressed in terms of the solutions of a differential Riccati equation with jumps. The filtering problems for nonlinear sampled-data systems have been extensively studied by Li et al. [7]. Using the concepts of dissipativity and differential game, sufficient conditions are derived for the existence of such $H^\infty$ filters. These conditions are expressed in terms of the solutions of a differential Hamilton-Jacobi inequality with jumps. The solutions of a differential Hamilton-Jacobi inequality with jumps are equivalent to solving the partial differential inequalities. To the best of our knowledge, there are no analytic solutions for nonlinear partial differential inequalities in general.

Recently, there have been many applications of fuzzy systems theory in various fields. In most of these applications, the fuzzy systems were thought of as universal approximators for any nonlinear systems. The T-S fuzzy model [10] which has been proved to be a very good representation for a certain class of nonlinear dynamic systems was exten-
sively studied in estimation [11] and control [12] systems. However, the topic of the $H^\infty$ fuzzy filtering problem for nonlinear sampled-data systems has not been addressed at present. In this study, using T-S fuzzy model approach, the $H^\infty$ fuzzy filtering design for a class of nonlinear sampled-data systems can be given via solving linear matrix inequalities (LMIs) instead of Hamilton-Jacobi inequalities. First, a T-S fuzzy model is proposed to approximate a class of nonlinear sampled-data systems. Next, based on the T-S fuzzy model, the $H^\infty$ fuzzy filtering design for nonlinear sampled-data systems is characterized in terms of minimizing the attenuation level subject to some linear matrix inequalities (LMIs), which is also called eigenvalue problem (EVP) [2] and can be efficiently solved by the LMI toolbox in Matlab [6].

II. Preliminaries

Consider a time-invariant nonlinear sampled-data system described by the following dynamic equations
\begin{align*}
\dot{x}(t) &= f(x(t)) + g_1(x(t))w(t), \\
y(kT) &= g_2(x(kT)) + v(kT), \quad k = 0, 1, 2, \ldots, (2)
\end{align*}
where $x$ represents the state defined on a neighborhood of the origin in $\mathbb{R}^n$, and $w \in \mathbb{R}^r$ represents a continuous-time process noise which is assumed to be a member of $L^2[0, \Gamma, \mathbb{R}^r] := \{ w : \|w\|_2 := \int_0^\Gamma \|w(t)\|^2 dt < \infty \text{ for a fixed } \Gamma > 0 \}$. Here $\| \cdot \|$ denotes the Euclidean norm. Eq. (2) defines the discrete-time measured variable $y \in \mathbb{R}^p$ which is available at sampling instant $kT$ with the sampling period $T$, and $v \in \mathbb{R}^q$ represents measurement noise which is assumed to be a member of $l^2[0, \Gamma, \mathbb{R}^q] := \{ v : \|v\|_2 := \sum_{k=0}^{[\Gamma/T]} \|v(kT)\|^2 < \infty \text{ for a fixed } \Gamma > 0 \}$. Here $[e]$ denotes the integer part of $e \in \mathbb{R}$.

We assume that $f(\cdot)$, $g_1(\cdot)$, and $g_2(\cdot)$ are all smooth functions with $f(0) = 0$ and $g_2(0) = 0$. The $H^\infty$ filtering problem considered in this paper is to find a state estimator of the following form
\begin{align*}
\dot{\xi}(t) &= f(\xi(t)), \\
\xi(kT) &= \xi(kT) + \hat{g}(\xi(kT))(y(kT) - g_2(\xi(kT))), \quad t \in (kT, (k+1)T] (3)
\end{align*}
that achieves a given bound $\gamma$ on the ratio between the $L^2$-energy of an estimation error function
\begin{align*}
\varepsilon(t) := E(x - \xi) (4)
\end{align*}
and the energy of the disturbances consisting of the $L^2$-energy of the exogenous input $w$ and the $l^2$-energy of the measurement noise $v$. More precisely, we seek a state estimator (3) such that there exists a neighborhood $\Omega$ of $(x, \xi) = (0, 0)$, and for all $(x(0), \xi(0)) \in \Omega$ and for each input $w(\cdot) \in L^2[0, \Gamma, \mathbb{R}^r]$ and $v(\cdot) \in l^2[0, \Gamma, \mathbb{R}^q]$, the trajectory $(x(t), \xi(t))$ of the whole system remains in $\Omega$ for all $t \in [0, \Gamma]$, and the function (4) satisfies
\begin{align*}
\int_0^\Gamma \|\varepsilon(t)\|^2 dt \leq \gamma^2 \left[ N(x(0), \xi(0)) + \int_0^\Gamma \|w(t)\|^2 dt + \sum_{k=0}^{[\Gamma/T]} \|v(kT)\|^2 \right]
\end{align*}
for some positive function $N(x(0), \xi(0))$.

Remark 1: The equation (3) describes a state estimator under consideration, where $\xi(kT^+) \equiv \lim_{t \to kT} \xi(t)$. This estimator operates on two updating laws. In between the sampling instants (i.e. $kT < t \leq (k+1)T$), the state estimation evolves according to the first equation of (3) with initial state $\xi(kT^+)$, which is built upon the second equation of (3). Therefore, the state $\xi$ is right discontinuous but may be left continuous with possibly finite jumps at $t = kT$. As seen, the resulting filter is implemented with finite jumps at $t = kT$ updated by use of the measurement $y(iT)$.

Remark 2: The solution, quoted from [7], of above $H^\infty$ filtering problem for nonlinear sampled-data systems can be obtained via finding a positive definite function $Q(x, t)$ which is the solution of a differential Hamilton-Jacobi inequality with jumps. The search for function $Q(x, t)$ is equivalent to solving nonlinear partial differential inequalities. There are no analytic solutions for nonlinear partial differential inequalities in general. This motivates the study of solving the nonlinear $H^\infty$ filtering problem using fuzzy approach. Based on the fuzzy approach, the $H^\infty$ filtering problem for nonlinear sampled-data systems can be given via solving linear matrix inequalities (LMIs) instead of nonlinear partial differential inequalities.

A fuzzy dynamic model has been proposed by Takagi and Sugeno [10] to represent locally linear input/output relations for nonlinear systems. This fuzzy dynamic model is described by fuzzy If-Then rules and will be employed here to deal with the filtering design problem for affine nonlinear sampled-data systems. Sampled-data systems operate in continuous time, but some output measurements are sampled at certain time instants (usually periodically), yielding discrete-time signals. Hence, the sampled-data systems are thus hybrid systems, involving both continuous-time and discrete-time signals. Consequently, the fuzzy model contains both continuous-time part and discrete-time part. The $i$th rule of the fuzzy model for a nonlinear sampled-data system is proposed as the following form:

(1) Continuous-time part:

Rule $i$:
\begin{align*}
\text{If} \quad z_1(t) = F_{i1} \cdots \text{and} \quad z_q(t) = F_{iq} \\
\text{Then} \quad \dot{x}(t) = A_ix(t) + B_iw(t), \quad t \neq kT
\end{align*}

(2) Discrete-time part:

Rule $i$:
\begin{align*}
\text{If} \quad z_1(kT) = F_{i1} \cdots \text{and} \quad z_q(kT) = F_{iq} \\
\text{Then} \quad x(kT^+) = x(kT), \\
y(kT) = C_ix(kT) + v(kT),
\end{align*}
for $i = 1, 2, \ldots, L$ where $F_{ij}$ is the fuzzy set; $A_i$ and $B_i$ are known constant matrices with appropriate dimension;
\( L \) is the number of If-Then rules; \( z_1(t), z_2(t), \ldots, z_g(t) \) are the premise variables for the continuous-time part; \( C_i \) is known constant matrices with appropriate dimension; \( z_1(kT), z_2(kT), \ldots, z_g(kT) \) are the premise variables for the discrete-time part; and \( k = 0, 1, \ldots, k_f \) (\( k_f = \lfloor \Gamma/T \rfloor \)).

The final output of the fuzzy system which is the appropriately fuzzy form of the nonlinear sampled-data system (1)-(2) is inferred as follows:

1. Continuous-time part:

\[
\dot{x}(t) = \frac{\sum_{i=1}^{L} \mu_i(z(t)) (A_i x(t) + B_i w(t))}{\sum_{i=1}^{L} \mu_i(z(t))},
\]

\[
= \sum_{i=1}^{L} h_i(z(t)) (A_i x(t) + B_i w(t)), \quad t \neq kT \tag{8}
\]

2. Discrete-time part:

\[
x(kT^+) = x(kT),
\]

\[
y(kT) = \frac{\sum_{i=1}^{L} \mu_i(z(kT)) (C_i x(kT) + v(kT))}{\sum_{i=1}^{L} \mu_i(z(kT))}
\]

\[
= \sum_{i=1}^{L} h_i(z(kT)) (C_i x(kT) + v(kT)), \tag{10}
\]

where

\[
h_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^{L} \mu_i(z(t))},
\]

\[
\mu_i(z(t)) = \prod_{j=1}^{g} F_{ij}(z_j(t)), \tag{11}
\]

\[
z(t) = [z_1(t), z_2(t), \ldots, z_g(t)],
\]

and \( F_{ij}(z_j(t)) \) is the grade of membership of \( z_j(t) \) in \( F_{ij} \).

It is assumed that

\[
\mu_i(z(t)) \geq 0 \text{ and } \mu_i(z(kT)) \geq 0, \quad \forall \ i = 1, 2, \ldots, L
\]

and

\[
\sum_{i=1}^{L} \mu_i(z(t)) > 0, \quad \forall \ t \quad \text{and} \quad \sum_{i=1}^{L} \mu_i(z(kT)) > 0, \quad \forall \ kT
\]

Therefore, we get

\[
h_i(z(t)) \geq 0 \text{ and } h_i(z(kT)) \geq 0, \quad \forall \ i = 1, 2, \ldots, L
\]

and

\[
\sum_{i=1}^{L} h_i(z(t)) = 1 \quad \text{and} \quad \sum_{i=1}^{L} h_i(z(kT)) = 1
\]

### III. Main Results

From aforementioned preliminaries, our goal is to design a fuzzy filter for nonlinear sampled-data system in the \( H^\infty \) performance setting (5). Based on the fuzzy model (6) and (7), we propose the following fuzzy filter for the nonlinear sampled-data system in (6)-(7):

1. Continuous-time state estimation:

\[
\text{Filtering Rule } j : \quad \text{If } \hat{z}_1(t) = F_{j1} \quad \text{and } \hat{z}_g(t) = F_{jg}
\]

\[
\text{Then } \xi(t) = A_j \xi(t) \tag{12}
\]

2. Discrete-time output injection:

\[
\text{Filtering Rule } j : \quad \text{If } \hat{z}_1(t) = F_{j1} \quad \text{and } \hat{z}_g(t) = F_{jg}
\]

\[
\xi(kT) = \xi(kT) + K_j (y(kT) - \hat{y}(kT)) \tag{13}
\]

where \( K_j \) is the fuzzy filter gain for the \( jth \) filtering rule \( j = 1, 2, \ldots, L \), \( \hat{z}(t) = [\hat{z}_1(t), \hat{z}_2(t), \ldots, \hat{z}_g(t)] \) are the premise variables, and \( \hat{y}(kT) = \sum_{r=1}^{L} h_r(\hat{z}(kT)) C_r \xi(kT) \).

The overall fuzzy filter is written as:

\[
\xi(t) = \sum_{j=1}^{L} h_j(\hat{z}(t)) (A_j \xi(t)), \quad t \in (kT, (k+1)T] \tag{14}
\]

and

\[
\xi(kT^+) = \xi(kT) + \sum_{j=1}^{L} h_j(\hat{z}(kT)) K_j (y(kT) - \hat{y}(kT))
\]

\[
= \xi(kT) + \sum_{j=1}^{L} h_j(\hat{z}(kT)) K_j \left[ \sum_{i=1}^{L} h_i(z(kT)) \right.
\]

\[
\left. \times C_i x(kT) + v(kT) - \sum_{r=1}^{L} h_r(\hat{z}(kT)) C_r \xi(kT) \right]
\]

\[
= \xi(kT) + \sum_{i=1}^{L} h_i(z(kT)) \sum_{j=1}^{L} h_j(\hat{z}(kT))
\]

\[
\times \sum_{r=1}^{L} h_r(\hat{z}(kT)) K_j [C_i x(kT) + v(kT) - C_r \xi(kT)] \tag{15}
\]

The sampled-data systems given above is a nonlinear system with finite jumps at discrete instants of time. At the sampling instants, the measurement \( y(kT) \) is used to update the estimated law (13), and to guarantee the whole system having \( L^2 \)-gain \( \leq \gamma \) even though there is no output injection between any two sampling instants. Hence, the \( H^\infty \) filtering problem is to find the fuzzy filter gains \( K_j \ (j = 1, 2, \ldots, L) \), which construct the fuzzy filter, such that the dissipative inequality (5) must be satisfied.
Let $X^e(t) := \text{col}(x(t), \xi(t))$, then the augmented system can be written in the following form:

$$
\dot{X}^e = \begin{bmatrix}
\sum_{i=1}^{L} h_i(z(t))(A_i x(t) + B_i w(t)) \\
\sum_{j=1}^{L} h_j(\ddot{z}(t))(A_j \xi(t)) \\
\end{bmatrix} = \begin{bmatrix}
\sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(\ddot{z}(t))(A_{ij}^e X^e(t) + B_{ij}^e w(t)) \\
\end{bmatrix},
$$

$$
t \in (kT, (k+1)T] \quad \text{and} \quad X^e(kT^+) = \begin{bmatrix}
x(kT) \\
\xi(kT) + \sum_{i=1}^{L} h_i(z(kT)) \sum_{j=1}^{L} h_j(\ddot{z}(kT)) \\
\times \sum_{j=1}^{L} h_j(\ddot{z}(kT)) K_j (C_i x(kT) + v(kT)) \\
- C_r \xi(kT) \\
\end{bmatrix} = \begin{bmatrix}
\sum_{i=1}^{L} h_i(z(kT)) \sum_{j=1}^{L} h_j(\ddot{z}(kT)) \sum_{r=1}^{L} h_r(\ddot{z}(kT)) \\
\times (C_{ijr}^e X^e(kT) + D_{ij}^e v(kT)) \\
\end{bmatrix},
$$

where

$$
A_{ij}^e = \begin{bmatrix} A_i & 0 \\
0 & A_j \end{bmatrix},
$$

$$
B_{ij}^e = \begin{bmatrix} B_i \\
0 \end{bmatrix},
$$

$$
C_{ijr}^e = \begin{bmatrix} I & 0 \\
K_j C_i (I - K_j C_r) \end{bmatrix},
$$

$$
D_{ij}^e = \begin{bmatrix} 0 \\
K_j \end{bmatrix}.
$$

Let $\tilde{E} = \begin{bmatrix} E^T E & -E^T E \\
-E^T E & E^T E \end{bmatrix}$ and

$$
\begin{bmatrix}
M_{11} \\
M_{21} \\
M_{22}
\end{bmatrix}
$$

for concise expression, we obtain the following main result.

**Theorem 1:** For a given $\gamma > 0$ and the augmented system (16)-(17), suppose there exist a positive definite symmetric matrix $Q$ such that the following conditions:

(a) the function

$$
Y_1 = \frac{A_{ij}^e + A_{ij}^r}{2} \hat{Q} + \frac{C_{ijr}^e + C_{ijr}^r}{2} \hat{Q} + \tilde{E} < 0,
$$

(b) the function

$$
Y_{1d} = \begin{bmatrix}
\frac{C_{ijr}^e + C_{ijr}^r}{2} D_{ij}^e \\
0 \end{bmatrix}^T \begin{bmatrix}
\frac{C_{ijr}^e + C_{ijr}^r}{2} D_{ij}^e \\
0 \end{bmatrix} \leq 0
$$

are satisfied for all $i \geq j$, and $i, j, r = 1, 2, \ldots, L$, then the fuzzy filter (12)-(13) renders the system (1)-(2) has $L^2$-gain $\leq \gamma$, i.e., nonlinear sampled-data $H^\infty$ filtering problem can be solved by the fuzzy filter (12)-(13).

**Proof:** We first observe that the problem can be considered as a two players, zero sum, differential game with a value functional

$$
J(K_j, (w, v)) := ||\varepsilon||^2_{T_2} - \gamma^2 \left[ N(x(0), \xi(0)) + \|w\|^2_{L_2} + ||v||^2_{L_2} \right].
$$

The first player is required to choose estimator gains $K_j$ so that the value functional $J$ will be minimized. His opponent is looking for the worst possible disturbance, composed of the continuous part $w$ and the discrete part $v$ and the initial condition $N(x(0), \xi(0))$, in order to maximize $J$. Associated with this differential game setup, we define two Hamiltonian functions $H_1 : \mathbb{R}^{2n} \times \mathbb{R}^r \times \mathbb{R} \rightarrow \mathbb{R},$

$$
H_1(X^c, w, t) := \hat{V}(X^c, t) - \gamma^2 w^T(t)w(t) + \varepsilon^T(t)\varepsilon(t),
$$

for all $t \in (kT, (k+1)T)$, and $H_{1d} : \mathbb{R}^{2n} \times \mathbb{R}^p \rightarrow \mathbb{R},$

$$
H_{1d}(X^c, v) := V(X^c(kT^+), T^+) - V(X^c, T) - \gamma^2 v^T v,
$$

where the function $V(X^c, t)$ is a Lyapunov function for the system (1)-(2). Let us choose

$$
V(t) = X^c^T \hat{Q} X^c,
$$

with the weighting matrix $\hat{Q} = \hat{Q}^T > 0$ and the initial condition

$$
V(0) = \gamma^2 N(x(0), \xi(0)).
$$

Observing the fuzzy filter in (14)-(15), the measurement $y(kT)$ is used to update the estimate at the sampling instants. Hence, the estimated state $\xi(t)$ is left continuous but may be right discontinuous with possibly finite jumps at $t = kT$. That make the function $V(t)$ is piecewise continuous with jumps at $t = kT$. For all $t \in (kT, (k+1)T]$

$$
\hat{V}(t) = \sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(\ddot{z}(t))(X^c A_{ij}^e \hat{Q} X^c + X^c \hat{Q} A_{ij}^e X^c \\
+ w^T(t) \sum_{j=1}^{L} h_j(\ddot{z}(t))(X^c A_{ij}^e \hat{Q} X^c \\
+ X^c \hat{Q} A_{ij}^e X^c \\
+ w^T(t) \sum_{j=1}^{L} h_j(\ddot{z}(t))(X^c A_{ij}^e \hat{Q} X^c + X^c \hat{Q} A_{ij}^e X^c \\
+ \gamma^2 \varepsilon^T(t)\varepsilon(t)).
$$

Substituting above into (20), we obtain

$$
H_1(X^c, w, t) = \hat{V}(X^c, t) - \gamma^2 w^T(t)w(t) + \varepsilon^T(t)\varepsilon(t)
$$

$$
\leq \sum_{i=1}^{L} h_i(z(t)) \sum_{j=1}^{L} h_j(\ddot{z}(t))(X^c A_{ij}^e \hat{Q} X^c \\
+ X^c \hat{Q} A_{ij}^e X^c + \frac{\gamma^2}{2} X^c \hat{Q} B_{ij}^e X^c \\
+ \frac{\gamma^2}{2} X^c \hat{Q} B_{ij}^e X^c \\
+ \frac{\gamma^2}{2} w^T(t)w(t)).
$$

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\[ (x - \xi)^T E^T E(x - \xi) \]
\[ = X^c T \{ \sum_{i=j}^j h_i(z(t)) \sum_{j=1}^L h_j(\hat{z}(t)) Y_1 \]
\[ + 2 \sum_{i>j}^L h_i(z(t)) \sum_{j=1}^L h_j(\hat{z}(t)) Y_1 \} X^c \]

Since \( Y_1 < 0 \) for all \( i \geq j \), and \( i, j = 1, 2, \ldots, L \) by hypothesis (18), we have \( H_1(X^c, w, t) < 0 \) for all \( X^c \neq 0 \).

At the sampling instants, \( t = kT \),
\[ V(X^c(kT^+), T^+) \]
\[ = \sum_{i=1}^L h_i(z(t)) \sum_{j=1}^L h_j(\hat{z}(kT)) \]
\[ \times \left[ \left( \frac{c_{ijr} + c_{irj}}{2} \right) X^c(kT) + D^T_j v(kT) \right] ^T \tilde{Q} \]
\[ \sum_{i=1}^L h_i(z(t)) \sum_{m=1}^L h_m(\hat{z}(kT)) \sum_{n=1}^L h_n(\hat{z}(kT)) \]
\[ \times \left[ \left( \frac{c_{imn} + c_{mni}}{2} \right) X^c(kT) + D^T_m v(kT) \right] \leq \sum_{i=1}^L \sum_{j=1}^L h_i(z(kT)) h_j(\hat{z}(kT)) h_r(\hat{z}(kT)) \]
\[ \times \left[ \begin{bmatrix} X^c & v \end{bmatrix} ^T \left[ \begin{bmatrix} \frac{c_{ijr} + c_{irj}}{2} & D^T_j \end{bmatrix} \right] ^T \tilde{Q} \left[ \begin{bmatrix} \frac{c_{imn} + c_{mni}}{2} & D^T_m \end{bmatrix} \right] \right] \]
\[ = \int_0^\Gamma \left( V(X^c(t), -\gamma^2 w^T(t) w(t) + \epsilon^T(t) \epsilon(t)) \right) dt \]
\[ + \sum_{k=0}^{\Gamma/T} \left( V(X^c(kT^+), T^+) - V(X^c, T) - \gamma^2 v^T v \right) \]
\[ \leq X^c T \{ \sum_{i=j}^j h_i(z(t)) \sum_{j=1}^L h_j(\hat{z}(t)) Y_1 \]
\[ + 2 \sum_{i>j}^L h_i(z(t)) \sum_{j=1}^L h_j(\hat{z}(t)) Y_1 \} X^c \]
\[ + \left[ \begin{bmatrix} X^c & v \end{bmatrix} ^T \left[ \begin{bmatrix} \frac{c_{ijr} + c_{irj}}{2} & D^T_j \end{bmatrix} \right] \right] \left[ \begin{bmatrix} Y_{id} & \sum_{i=j}^j \sum_{j=1}^L h_i(z(kT)) h_j(\hat{z}(kT)) h_r(\hat{z}(kT)) \right] \]
\[ \leq 0. \]

and thus
\[ V(X^c(t), \Gamma) - \gamma^2 N(x(0), \xi(0)) + \int_0^\Gamma \| \epsilon(t) \|^2 dt \]
\[ \gamma^2 \left( \int_0^\Gamma \| w(t) \|^2 dt + \sum_{k=0}^{\Gamma/T} \| v(kT) \|^2 \right) \leq 0. \]

Since \( V(X^c(t), t) \geq 0 \), we obtain
\[ \int_0^\Gamma \| \epsilon(t) \|^2 dt \]
\[ \leq \gamma^2 \left( N(x(0), \xi(0)) + \int_0^\Gamma \| w(t) \|^2 dt + \sum_{k=0}^{\Gamma/T} \| v(kT) \|^2 \right). \]

This completes the proof. \( \Box \)

The fuzzy filter given above is a nonlinear system with finite jumps at discrete instants of time. At the sampling instants, the measure \( y(kT) \) is used to update the estimate with filter gains \( K_j \), and the function \( Y_{id} \) is only available at sampling instant \( kT \). To construct the fuzzy filter (14) and (15), the filter gains \( K_j \) have to be obtained in advance. Theorem 1 shows that the nonlinear \( H^\infty \) sampled-data filtering problem can be solved if there exists a positive definite symmetric matrix \( \tilde{Q} \) satisfying (18) and (19). In general, it is not easy to analytically determine a common solution \( \tilde{Q} \) for (18) and (19). Moreover, this solution may be not unique. For this reason, we can convert the problem in (18)-(19) into a linear matrix inequality problem (LMIP) [2]. For the convenience of design, let
\[ \tilde{Q} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{22} \end{bmatrix} \]
with \( Q_{11} = Q_{11}^T \), and \( Q_{22} = Q_{22}^T \). From (18), we get
\[ Y_1 = \begin{bmatrix} \frac{A^T + A^T}{2} Q_{11} + Q_{11} \frac{A + A}{2} - E^T E \end{bmatrix} \]

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By minimizing the prescribed attenuation level of solving a nonlinear differential inequalities. In general, there are no analytic solutions for nonlinear partial differential inequality. In this paper, the $H^\infty$ filtering problem for nonlinear sampled-data systems is addressed using fuzzy approach. First, a nonlinear sampled-data system is represented by its corresponding T-S fuzzy model. Next, based on the T-S fuzzy model, the $H^\infty$ fuzzy filtering problem is characterized in terms of minimizing the attenuation level subject to some linear matrix inequalities (LMIs), which is also called EVP and can be efficiently solved by the LMI toolbox in Matlab.

REFERENCES


By the Schur complements, (25) is equivalent to the following LMIs form:

\[
\begin{bmatrix}
-1 & 0 & 0 \\
0 & -Q_{11} & 0 \\
0 & 0 & Q_{11}
\end{bmatrix}
\begin{bmatrix}
W_{i}C_{i}+W_{j}C_{j} \\
-W_{i}C_{i}+W_{j}C_{j} \\
W_{i}C_{i}+W_{j}C_{j}
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
-\gamma^{2} I & * & * \\
* & * & * \\
* & * & *
\end{bmatrix}
\leq 0
\]