A comparison study of polyominoes explorations in a physical and virtual manipulative environment

Y. Yuan, C.-Y. Lee & C.-H. Wang
Graduate School of Education, Chung Yuan Christian University, Chung Li, Taiwan

Abstract
This study develops virtual manipulative, polyominoes kits for junior high school students to explore polyominoes. The current work conducts a non-equivalent group pretest–post-test quasi-experimental design to compare the performance difference between using physical manipulatives and virtual manipulatives in finding the number of polyominoes. Sixty eighth-grade students from two different classes in a junior high school in Taipei County of Taiwan participated in this study. The current research randomly selected one class as the experiment group and the other as the control group. Students in the experiment group used virtual manipulatives to explore polyominoes and those in the control group used physical manipulatives. The results revealed that learning in the experiment group is as effective as that in the control group. This study identifies two obvious strategies (add one and reduce) among students in both groups. New ideas, including using new symbols to record the results and considering the influence of symmetry and rotation on the figures, occurred in the virtual manipulative group. Students in the virtual environment paid much more attention to exploring the polyomino problem.

Keywords
physical manipulatives, polyominoes, problem solving, virtual manipulatives.

Introduction

Problem solving
Problem solving has always been a central focus of mathematical curriculum and considered as a way for students to explore and understand mathematics. Without the ability to solve problems, the usefulness and power of mathematical ideas, knowledge, and skills are severely limited (NCTM 2000). Schroeder and Lester (1989) indicated that problem solving serves as a vehicle for learning new mathematical ideas and skills. Through problem solving, students experience the power and utility of mathematics. A problem-centred approach to teaching mathematics uses interesting and well-selected problems to launch mathematical lessons and engage students (Lee & Chen 2009). In this way, new ideas, techniques and mathematical relationships emerge and become the focus of discussion. Good problems can inspire the exploration of important mathematical ideas, nurture persistence and reinforce the need to understand and use various strategies, mathematical properties, and relationships. Therefore, teaching that provides problems for students to explore is more helpful than a direct teaching approach. However, finding what problems to use to motivate and enhance students’ mathematics learning is the challenge.

Polyominoes have been used in popular puzzles and considered as a fun and challenging game in recreational mathematics (Golomb 1994). A polyomino is a polyform with the square as its base form. It is a connected shape formed as the union of one or more
Learning with representations

The Principles and Standards for School Mathematics (NCTM 2000) emphasized the role of representation in mathematics, reporting that students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare and use various representations. Indeed, representation is an especially important construct not only in problem solving, but also in mathematics learning in general (Janvier 1987; Perkins & Unger 1994; Goldin 2002, 2003; Monk 2003; Smith 2003).

However, evidence shows that students encounter difficulties in using multiple representations and in transforming between representations (Lesh et al. 1987). Learners are faced with complex learning tasks when they are first presented with a novel representation. They must understand how it encodes information and how it relates to the domain it represents. Learners may also need to select an appropriate representation or construct one for themselves, which can provide advantages, but also new cognitive tasks (Ainsworth 2006). Therefore, the ability to use multiple representations and transform between representations is an indicator of whether a student can create a model and understand mathematics concepts.

Both concrete and virtual manipulatives can be used to create visual models through mathematics activities. Students need to determine the purpose of the manipulative in order to complete the task through problem-solving strategies. Virtual manipulatives have been called as ‘dynamic virtual manipulatives’ (Moyer et al. 2002, p. 372). If manipulatives are found to benefit students’ understanding, the effectiveness of virtual manipulatives needs to be determined.

The use of physical and virtual manipulatives

Mathematics education literature has discussed using physical manipulatives as a representation during mathematics instruction for decades. Evidences show that students who use physical manipulatives appropriately to explore mathematical concepts perform better than those who do not (Parham 1983; Raphael & Wahlstrom 1989). Balka (1993) proposed that using manipulatives allows students to make the important linkage between conceptual and procedural knowledge, to recognize relationships among different areas of mathematics, to see mathematics as an integrated whole, to explore problems using physical models and to relate procedures in an equivalent representation. Teachers typically purchase physical manipulatives from stores or make them themselves to help their instruction in mathematics class. However, buying physical manipulatives is costly and making them takes considerable time. Using physical manipulatives in a real classroom also poses some problems, include classroom control, cleaning and keeping problems (Yuan 2006).

Recently, Moyer et al. (2002) reported using virtual manipulatives as a representation for constructing mathematical knowledge. A virtual manipulative is an interactive, Web-based visual representation of a dynamic object that presents opportunities for developing students’ mathematical concepts. Virtual manipulatives tend to be more than just electronic replications of their physical counterparts. They typically include additional features or options that expand on what a physical manipulative offers (Clements & McMillen 1996; Lindroth 2005). Some virtual manipulatives are able to present a representation not easily made or even possible with physical manipulatives, an attribute shared with many types of computer simulations (Steen et al. 2006). For example, a student can break apart and glue together virtual base 10 blocks. These actions often reflect the student’s thought processes more closely than concrete manipulative and are more manageable.

Using virtual manipulatives is still relatively new in the classroom, and research on their impact is limited. Reimer and Moyer (2005) reported a study on third graders learning about fractions with virtual manipulatives that showed statistically significant gains in
students’ conceptual knowledge. The student surveys and interviews indicated that the manipulatives provided immediate and specific feedback, were easier and faster to use than traditional methods, and enhanced students’ enjoyment while learning. Steen et al. (2006) investigated the impact of virtual manipulatives on first-grade students’ geometry learning. Their study randomly assigned 31 first-grade students to either a treatment or a control group. Both groups studied identical objectives, but the treatment group used virtual manipulatives for practice. In addition, both groups of students were given two tests, including grade one and grade two content separately. The treatment group significantly improved on both grade level tests, while the control group only significantly improved on the second-grade testing level.

Suh and Moyer (2007) examined the effect of developing students’ representational fluency using virtual and physical manipulatives. The results showed that although the different manipulative models had different features, both the physical and virtual environments were effective in supporting students’ learning and encouraging relational thinking and algebraic reasoning. These studies showed that virtual manipulatives can offer unique advantages, and can be as or more effective in supporting learning (Wright 1994, Sarama et al. 1996; Clements and Sarama 1998). However, it is important to look beyond pretest and post-test information to examine the characteristics of different learning environments and how these characteristics influence different types of learning experiences. Therefore, this study investigated the impact on problem-solving performance as well as strategies, behaviours and interactions when using the virtual manipulative and the physical manipulative to explore the number of polyominoes.

**Purpose of this study**

The current research focuses on the following questions:

- What differences exist on problem-solving performance between students who use physical manipulatives and those who use virtual manipulatives in exploring the number of polyominoes?
- What are the differences students may have in problem-solving strategies, behaviours and interactions when using different manipulatives to explore the number of polyominoes?
- What are the students’ attitudes towards using virtual manipulatives?

**Methodology**

**Research design and participants**

The study applied the non-equivalent group pretest–post-test quasi-experimental design to compare problem solving performance between the physical manipulative group and the virtual manipulative group in finding the number of polyominoes. A $2 \times 2$ (pretest–post-test by control experiment) mixed model analysis of variance (ANOVA) with $\alpha = 0.05$ was conducted to examine changes from pretest to post-test. The teacher’s class journal, students’ worksheets, and their interviews were collected to analyse the differences of problem-solving strategies, behaviours and interactions in the virtual and physical manipulative environment. Students’ responses of attitudes survey for the virtual manipulative group were also analysed to understand their perceptions toward using the virtual manipulative.

Participants were 60 eighth-grade students, 27 boys and 33 girls, from two different classes in a junior high school in Taipei County of Taiwan. The researchers randomly selected one class as the experiment group and the other one as the control group. Both groups studied identical learning objectives and engaged in the same learning activities. The only difference was that the experiment group used the virtual manipulative, while the control group used the physical manipulative for exploring the number of polyominoes.

**Materials**

Students did not encounter tasks related to the polyomino problem before and did not know the solution methods in the beginning. Therefore, these learning tasks were nonroutine problems for students and provided them with opportunities to integrate their problem-solving skills and apply their pattern finding ability into the context of the polyomino problem. The learning materials were composed of the following three activities.

*Activity 1 (30 min):* Introduce the history of polyominoes and the definition of a polyomino. Then discuss the
examples of monomino, domino and triominoes. Both classes were given a similar instruction except the manipulatives. The teacher used domino to discuss the rule that two polyominoes are considered the same one if they can be rotated and reflected to be congruent. When the teacher introduced triominoes, the ‘add one’ strategy (constructing an existing polyomino with fewer pieces of square and trying to add one more square to formulate a new polyomino) was used in both classes to give the students an idea of finding different triominoes.

Activity 2 (70 min): Students work in groups to find the number of tetrominoes and pentominoes. They are asked to report their problem-solving strategies and thinking processes in their worksheets. The teacher circulates the tables to observe and record students’ strategies to find the expected polyominoes.

Activity 3 (30 min): At the end of explorations, each group was asked to present their answers in the whole class.

Instruments

Pretest and post-test
The pretest measures students’ problem-solving performance in polyominoes explorations before the instructional program. The post-test measures students’ problem-solving performance in polyominoes explorations after the instructional program. The post-test content is the same as the pretest, including only one question and students are asked to find all hexominoes. The test time is 35 min. Students’ test score depends on how many different hexominoes they find. Since the maximum number of hexominoes is 35, the test score ranges from 0 to 35.

Polyominoes kits
The researchers developed the virtual manipulative, polyominoes kits, to explore the number of polyominoes. The Flash tool designed this instructional software. Seven elementary school teachers, six junior high school mathematics teachers and six high school mathematics teachers evaluated the polyominoes kits and modification of this virtual manipulative was based on the comments from these teachers. When using polyominoes kits, students can select a polyomino from the menu and manipulate it. The functions of this virtual manipulative consist of rotating, moving, composing, decomposing and changing polyominoes colours in the working area (See Fig. 1).

Attitudes survey
The attitudes survey aims to understand students’ perceptions toward the virtual manipulative. This survey includes three open questions:

- What difficulties do you encounter while operating the virtual manipulative?
- What are the benefits of using the virtual manipulative in problem solving and what have you learned from this lesson?
- How do you feel about using the virtual manipulative for mathematics learning in the future?
Treatment and data collection procedure

Prior to the instructional program, students in both groups were asked to use six plastic squares to finish the pretest. Students in the virtual manipulative group went to the computer lab and used polyominoes kits to finish Activity 1 and Activity 2. Students in the physical manipulative group also experienced the same activities in their classroom, but used the plastic squares (3 cm × 3 cm) the teachers made in advance as aids for explorations (See Fig. 2). Students in the experiment group used polyominoes kits to complete the post-test and those in the control group used the physical manipulative to finish the post-test. At the end of Activity 3, students in the experiment group completed the attitudes survey to express their perceptions toward the virtual manipulative. During the instruction period, students’ worksheets were also collected to understand their solution processes and problem-solving strategies. The teacher maintained a class journal to record his observations regarding students’ time-on-task, work behaviours, effectiveness of the virtual manipulative and the physical manipulative and overall thoughts on the instructional process.

Results and discussion

Research question 1: What differences exist in problem solving performance between students who use physical manipulatives and those who use virtual manipulatives in exploring the number of polyominoes?

This work conducted a 2 × 2 (pretest–post-test by control experiment) mixed design ANOVA. Because Box’s M-test for homogeneity of covariance matrices was not significant (Box’s $M = 1.591, F = 0.511, P = 0.675$), the observed covariance matrices of the dependent variables are equal across groups. Therefore, the data was appropriate for further parametric analysis. The ANOVA results indicated no significant interaction between pretest to post-test and group membership [$F (1, 58) = 0.874, P = 0.354, \text{partial } \eta^2 = 0.015$], no significant main effect for the between factor [$F (1, 58) = 0.198, P = 0.658, \text{partial } \eta^2 = 0.003$], and a significant main effect for the within factor [$F (1, 58) = 28.075, P < 0.001, \text{partial } \eta^2 = 0.326$]. These results indicate no significant difference of the overall change from pretest to post-test between the experiment group and control group (See Table 1). This is consistent with Suh and Moyer’s (2007) findings suggesting that although the virtual and physical manipulative environments had different features, both the virtual and manipulative representations were effective in supporting students’ learning in different ways. Table 2 shows that both groups made fewer repeating figures on the post-test than they did on the pretest. The reduction of repeating figures might be a manifestation of students’ understanding of polyominoes. However, with the same pretest and post-test, it is possible that the improvement was because of students’ learning from the test. Furthermore, since both groups had relatively high average scores in the pretest, possible ceiling effects may occur in this condition. This indicates that there was no difference of problem solving

<table>
<thead>
<tr>
<th>Manipulative groups</th>
<th>Pretest</th>
<th>Post-test</th>
<th>Total</th>
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<tbody>
<tr>
<td></td>
<td>M</td>
<td>sd</td>
<td>N</td>
</tr>
<tr>
<td>Virtual</td>
<td>27.90</td>
<td>5.040</td>
<td>30</td>
</tr>
<tr>
<td>Physical</td>
<td>26.93</td>
<td>5.558</td>
<td>30</td>
</tr>
<tr>
<td>Total</td>
<td>27.42</td>
<td>5.283</td>
<td>60</td>
</tr>
</tbody>
</table>
performance between the physical group and the virtual group.

**Research question 2:** What differences might students have in problem solving strategies, behaviours and interactions using different manipulatives to explore the number of polyominoes?

From teacher’s observation on students’ discussions in class time and the analysis of students’ solution strategies on their worksheets and post-test, this study identifies two obvious strategies among students in both groups (See Table 3). Students who used the ‘add one’ strategy constructed an existing polyomino with fewer pieces of square, and tried to add one more square to formulate a new polyomino. Students who used the ‘reduce’ strategy arranged the expected squares in a line, and took away one square gradually to formulate a new polyomino. At the beginning of class, the teacher introduced the ‘add one’ strategy to find triominoes. It is not surprising that students in both groups used add one strategy to solve the problem. However, students in both groups used reduce strategy that was not introduced in class. What bring students this new strategy? It is also found that students in the physical manipulative group used reduce strategy more than those in the virtual manipulatives group. Students in the physical manipulative environment might have difficulty rotating and composing squares, and they cannot easily deal with the squares on their small tables. They might use reduce strategy to avoid inconvenient arranging of physical squares.

Unlike students in the physical manipulative group who were afraid of breaking the squares on the tables, students in the virtual manipulative group composed and decomposed the squares easily on the work area of their computer screens. They spent much more time in discussion with their peers in the same group. Therefore, they paid much more attention to exploring the problem and had more new ideas for finding the new polyomino. For example, they used new symbols to record their results and considered the influence of symmetry and rotation on the figures. Figure 3 showed that students in the virtual manipulative group created their own symbols to prevent making repeating figures. The middle section contains one tetromino. The squares around the tetromino are labelled as ‘O’, ‘X’ and ‘Δ’. The squares of the same symbol combined with the tetromino can be constructed to form equivalent pentominoes.

Students in the experiment group understood that solving the symmetry problem increases efficiency for

<table>
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<tr>
<th>Solution strategies</th>
<th>The virtual manipulative group</th>
<th>The physical manipulative group</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>N students (%)</td>
<td>N students (%)</td>
</tr>
<tr>
<td>Add one</td>
<td>16 54</td>
<td>9 30</td>
</tr>
<tr>
<td>Reduce</td>
<td>7 23</td>
<td>14 47</td>
</tr>
<tr>
<td>Not clear</td>
<td>7 23</td>
<td>7 23</td>
</tr>
</tbody>
</table>

Table 2. Summary of the average number of repeating figures per student made on the pretest and the post-test in the experiment group and control group.

<table>
<thead>
<tr>
<th>Items/Groups</th>
<th>Experiment</th>
<th>Control</th>
<th>Total average</th>
</tr>
</thead>
<tbody>
<tr>
<td>The average number of repeating figures per student made on the pretest</td>
<td>8.8</td>
<td>9.5</td>
<td>9.15</td>
</tr>
<tr>
<td>The average number of repeating figures per student made on the post-test</td>
<td>3.97</td>
<td>5.5</td>
<td>4.73</td>
</tr>
</tbody>
</table>

Fig 3 Students’ created symbols.
finding the number of polyominoes. Figure 4 shows that students provided a solution to the symmetry problem, reducing the time for checking the repeating figures for finding the polyominoes. However, students in the physical manipulative group offered neither any solution method to the symmetry problem nor created their own symbols to record ideas. Considering the differences between the physical manipulative environment and the virtual manipulative environment may explain these results (See Table 4). For this reason, the virtual manipulative provided students with flexible representations that allowed them to explore their ideas. The virtual environment enabled students to test their mathematical ideas as they were working with the virtual tool.

Ainsa (1999) indicated that virtual manipulatives make specific mathematics transportation on objects. Sarama et al. (1996) also observed that students perform active thinking and discuss more on the meaning of actions in a computer-based environment. Steffe and Wiegel (1994) proposed that technology provides students with a valuable ‘cognitive play’, so students can develop deep conceptual thinking. These are possible reasons to explain the focus and abundant thoughts in the virtual manipulative group.

**Research question 3**: What are the students’ attitudes toward using the virtual manipulatives?

In this study, qualitative data from the students’ attitudes survey completed by the virtual manipulative group resulted in three important findings regarding students’ experiences with the virtual manipulative:

- From the 30 students’ responses to their attitudes survey in the experiment group, 22 (73%) participants considered it easy to operate the virtual manipulative; only one mentioned difficulty in operating the virtual manipulative, and the others did not mentioned the related control issue of the virtual manipulative. This indicates that the majority of students in the virtual manipulative group had positive attitudes toward the perceived ease of using the virtual manipulative.

- From the 30 students’ responses to their attitudes survey in the experiment group, 21 (70%) participants desired to learn mathematical topics with the virtual manipulatives in the future; only one mentioned that he did not expect to learn using the virtual manipulatives later, and the others did not mentioned this problem. This means that the majority of students in the virtual manipulative group liked to use the virtual manipulative to learn other mathematics concepts in the future.

- The virtual manipulative aroused the students’ motivation and confidence in mathematics learning. The following provides some evidence of students’ responses:

  S1: Trying the virtual manipulative is very fun and interesting.
  S2: When facing a difficult mathematical problem, I recall the problem-solving experiences using the virtual manipulative. Then I persist in finding the solution and never give up.
  S3: When solving a difficult problem in the beginning, I find the solution from the simple case or figure it from multiple perspectives. I do not give up easily.

These findings are in line with Reimer and Moyer’s (2005) study suggesting that the majority of responses...
from students in their experiences with the virtual manipulatives were positive. Overall, students in the virtual manipulative group were excited to use the virtual manipulative and displayed their confidence in solving the problem. They considered the virtual manipulative easy to use and felt that it helped them understand the problem and find the solution.

**Implications and conclusions**

This study develops a virtual manipulative used to emphasize problem solving and compared the performance difference between using the physical manipulative and the virtual manipulative in finding the number of polyominoes. In this initial exploration of comparing the effectiveness of virtual manipulatives versus concrete manipulatives, it was found that using the virtual manipulative was as effective as using the physical manipulative in supporting learning of polyominoes. Physical manipulatives are widely used in mathematics education and their usages in classrooms have long been recommended by educators (NCTM 1989, p.17). The result in present study challenges the general assumption that physical manipulation improves learning. Researchers proposed that using manipulatives does not always guarantee a success of learning (Clements & McMillen 1996) and appropriate instruction with manipulation may be the most important aspect to promote students conceptual understanding (Resnick 1998). Triona and Klahr (2003) conducted a study and found that students reach the science objects equally when taught with either virtual or physical materials. They indicated that whether the materials are virtual or physical will make little difference as long as the method of instruction is preserved. Our study also found both virtual and physical representations enhance students’ problem solving performance. This indicated that simply replacing the physical materials with virtual materials does not affect the amount of learning. It is the instructional design that counts. No matter what instructional aids were used, teachers should pay attention to the instructional design so that manipulatives can be applied appropriately.

Although students in both groups were given a similar instruction to explore the number of polyominoes, different problem-solving behaviours were found between the groups. Based on the teacher’s journal of observations, students reflected that the virtual manipulative was easier to operate for finding the number of polyominoes and it could devote their attention to group discussion. Students consciously thought and talked about the work they were engaged in while using the virtual manipulative in problem solving. On the other hand, students in the physical manipulative had limit space for manipulating the squares on the tables. Teachers apply physical squares to explore the number of polyominoes should consider providing enough space for students explorations. If the physical environment can be modified so that students can easily fix and rotate the squares, students might also come out similar thoughts and behaviours as those who use virtual manipulatives.

Limitations of this study would be the need for a larger sample size and the need to include participant attitudes toward concrete manipulative used in this study. The study also can not explain why students in both groups used reduce strategy that was not introduced by the teacher. Does the complexity of the question bring this result? If students in the physical manipulative group have more space in manipulating objects, will the students in the physical group appear to use more reduce strategy than those in the virtual group? What exactly makes this difference? These issues are an obvious area for further investigation.

<table>
<thead>
<tr>
<th>Table 4. The comparison between the physical manipulative environment and the virtual manipulative environment.</th>
</tr>
</thead>
<tbody>
<tr>
<td>The physical manipulative environment</td>
</tr>
<tr>
<td>1 The table in the classroom can hold limited squares.</td>
</tr>
<tr>
<td>2 The combined squares are easily separated.</td>
</tr>
<tr>
<td>3 The combined squares are not easily rotated.</td>
</tr>
<tr>
<td>4 It is not easy to operate the physical manipulative when students participate in their group discussion.</td>
</tr>
</tbody>
</table>
Acknowledgements

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