Power Consumption in Spatial Cognitive Scenarios

Maha Odeh, Nizar Zorba
University of Jordan
Amman, Jordan
{m.odeh; n.zorba}@ju.edu.jo

Christos Verikoukis
Telecommunications Technological Centre of Catalonia
Barcelona, Spain
cveri@cttc.es

Abstract—Multiple Input Multiple Output (MIMO) adds a new dimension to be exploited in Cognitive Radio (CR) by simultaneously serving several users. The spatial domain that is added through MIMO is another system resource that has to be optimized, and shared when possible. In the current paper, we present a spatial sharing that is carried out through Zero Forcing beamforming (ZFB). The power consumption in such a scenario is mathematically formulated and compared to single user case, to check the feasibility of employing spatial cognition from the power’s perspective. Interesting conclusions are obtained about the utility of spatial cognition, thanks to the derived closed form expressions for the data rate and consumed power. To provide a comprehensive measure of the expediency of spatial cognitive scenario, a joint power and rate metric is also proposed and analyzed.

I. INTRODUCTION

The explosive growth in energy demands in the last few years presents a huge challenge to be tackled by operators as they dissipate considerable amount of power, and consequently surmount the running cost of their networks [1]. Moreover, the telecommunication industry generates 183 million tones or 0.7% of carbon dioxide CO₂ emissions which helps escalating the existing environmental issues such as global warming [2]. The aforementioned aspects have motivated the researchers to figure out methods that are capable of saving power, without compromising the Quality of Service to be implemented in future telecommunication standards.

Cognitive Radio (CR) has appeared as a future technology that has the aptitude to enhance the spectrum utilization by allowing secondary transmissions, when it is possible, to overcome the problems of the inherited fixed allocation techniques, that have led to the scarcity of spectrum utility [3]. CR is able to flexibly adapt to the spectrum through spectrum sensing, dynamic frequency selection, transmit power control and interference cancellation to avoid any harmful transmissions [4].

Extending the cognitive concept to spatial domain is possible through adding new degrees of freedom at the transmitter and/or receiver side by implementing more than one antenna (i.e., MIMO technology [5]). They will enable the spatial sharing by vanquishing or mitigating the resulted interference from the simultaneous transmissions over the several antennas, which can tremendously elevate the link efficiency without any bandwidth expansion. Such kind of transmissions should be able to protect the users by constituting well defined priority levels: Primary Users (PU) have strict demands to be satisfied while Secondary Users (SU) get access when it is possible (i.e., cognitive users).

Power consumption in multiuser MIMO was not widely investigated in the literature; the only contributions related to power consumption are focused on the design of robust power allocation techniques for different purposes; to maximize the sum rate without ignoring the fairness among the users is proposed in [6]; or to mitigate the uncertainties [5] that can be introduced by wireless channel and/or feedback quantization. Other studies focus on the power allocation technique in the spatial cognition between two operators; for the goal of maximizing the sum rate of the secondary system, while keeping the generated interference on the PUs’ receiver at tolerable levels [7]. On the other hand, statistical and instantaneous study of the power loading is thoroughly discussed in [8].

Therefore, the calculation of the power consumption is missing in the literature, and this paper gets it in the spatial cognition scenario and mathematically derives its closed form formulations. The achieved results are very important to check whether it is worth to employ spatial cognition in the current and future telecommunication systems from the power perspective, specially as the cognition is known to improve the data rate. Under predefined minimum Quality of Service (QoS) demands at the receivers side, it is reasonable to think that the required power to satisfy the QoS for both PU’s and SU’s, will be larger than the needed power in traditional systems to satisfy one single user demands. But it remains to know if such increase in the power consumption compensates the increase in the data rate. Therefore, a joint rate-power metric is also proposed and deeply analyzed to investigate the feasibility of the spatial cognition.

The rest of this paper is organized as follows: in section II, the system model is presented while in section III a revision for ZFB is made where its probability density function is introduced. Section IV discusses the power consumption in spatial cognition, while the viability of spatial cognition is discussed in section V. The simulations and the results are displayed in section VI, followed by the paper conclusions in section VII.

II. SYSTEM MODEL

We consider a single-cell multiple-antenna Downlink scenario, where a single base station (BS) equipped with \( n_t \) transmit antennas supports data traffic to \( K \) user terminals, each one of them is equipped with a single receiving antenna.
We assume a quasi static block fading channel $h_{[1 \times n_t]}$ between the BS antennas and each one of the users, where the channel from a transmit antenna to a user is characterized by independent and identically distributed (i.i.d) complex Gaussian entries $\sim CN(0,1)$. Let $x(t)$ be the $n_t \times 1$ transmitted vector, while denote $y_k(t)$ as the $k^{th}$ user received signal given by

$$y_k(t) = h_k(t)x(t) + z_k(t) \quad (1)$$

where $z_k(t)$ is an additive i.i.d. complex noise component with zero mean and $E\{|z_k|^2\} = \sigma^2$. Any number of antennas $n_t$ can be employed, but the value of $n_t = 2$ seems to be the most practical one for all IEEE 802.11n commercial products, as well as all LTE proposals consider this option [9]. Therefore, it will be employed along all the paper for easiness in the results presentation, where its extension to any number of antennas is feasible.

The transmitter delivers service to $M$ simultaneous users, where $M \leq n_t$, so that a more compact system formulation is obtained by stacking the received signals and the noise components for the set of $M$ selected users as $y(t) = H(t)x(t) + z(t)$, with $H(t) = [h_1(t); \ldots; h_M(t)]$ as the compound channel. Notice that the transmitted signal $x(t)$ encloses the uncorrelated data symbols $s_m(t)$ to each one of the selected users, where $E\{|s_m|^2\} = 1$ is employed. The Channel State Information is available at the transmitter side. In order to simplify the notation, the time index is dropped whenever possible.

### III. Zero Forcing Beamforming

A lot of research work have been presented within transmit beamforming, and one of the simple and feasible techniques is the Zero Forcing Beamforming (ZFB), that has been already implemented in the commercial standards [10]. The ZFB’s ability of nulling the interference among the scheduled users, through extinguishing the downlink channels and transforming them into orthogonal and independent sub-channels, is accomplished by the transmission to M users at the same time, which makes the transmitted signal $x$ to be as

$$x = Bs \quad (2)$$

where $s$ is the sequence of transmitting symbols, and $B$ is the beamforming matrix, that for the ZFB it is obtained through the Penrose Pseudo inverse [6] as

$$B = H^H(HH^H)^{-1} \quad (3)$$

In order to control the amount of transmitted power and to enable fair comparisons, the sum power of the transmitting vectors has to be normalized to unity (i.e., $P_t = 1$), as follows:

$$|B|^2 = Tr(\alpha^2 I) \quad (4)$$

where $Tr(\cdot)$ denotes the matrix trace, while $\alpha$ is the normalization parameter defined as

$$\alpha^2 = \frac{1}{nt(\text{HH}^H)^{-1}_{i,i}} \quad (5)$$

so that the equivalent channel ($HB$) is now a diagonal channel that guarantees no interference among the users as follows

$$HB = D = \text{diag}(\alpha_1, \alpha_2, \ldots, \alpha_M) \quad (6)$$

which delivers a Signal to Noise Ratio (SNR) given as [6]

$$SNR_i = \gamma_i = \frac{\alpha_i^2}{\sigma_i^2} \quad (7)$$

The most tempting quality of ZFB is the whipped out interference among the selected users, which outstands this technique to be exploited in the cognitive scenario, as it avoids any interference from the secondary (cognitive) users towards the primary user. Nonetheless, this strength point can be envetered by the spatial correlation that may exist between the scheduled users, leading to collapse the SNR value for each one of them [11]. Therefore, the employment of ZFB is further enhanced with an algorithm, that selects the most orthogonal users to carry out ZFB on their channels, as a pre-amended step before the spatial processing, as already been proposed by [11].

#### A. Spatial Cognition in Zero Forcing Beamforming Scenarios

The spatial cognition is a very appealing implementation for the wireless operators, since all modern wireless communication systems (e.g., WLAN, WiMAX and LTE) consider several antennas in their deployment. This approach allows simultaneous transmissions through the spatial sharing, which will enhance the data rate, reduce the delay but at the cost of increased power in comparison to single user service. The latter point makes the employment of spatial cognition questioned, whether the increase of data rate can compensate the power consumption or not. To study such a performance, we need the SNR Probability Density Function (PDF) $\nu_\gamma(\gamma)$ and the Cumulative Density Function (CDF) $\Upsilon_\gamma(\gamma)$, which are thoroughly investigated in [12], that state as follows:

$$\nu_\gamma(\gamma) = \frac{2}{\gamma_0} e^{-\frac{\gamma}{\gamma_0}} \quad (8)$$

$$\Upsilon_\gamma(\gamma) = 1 - e^{-\frac{\gamma}{\gamma_0}} \quad (9)$$

where $\gamma_0$ is the channel average power. From the operator’s perspective, two conflictive objectives are desired: to keep the QoS for the PU users, that is defined as a minimum guaranteed SNR for the user [5], while on the other hand, it is desired to service SUs in order to increase the number of serviced customers, and therefore, the operator profits. To fulfill both objectives, a definition of the required minimum QoS (i.e., a certain SNR threshold $\gamma_{th}$ that enables the detection at the receiver side) should be declared. A major issue that rules the cognition philosophy is that the QoS satisfaction for PU is the highest priority to the operator, and the SU will not be scheduled if the PU won’t fulfill such SNR value.

### IV. Power Consumption of the Cognitive Scenario within Zero Forcing Beamforming

The wireless channel fluctuates over the time. Therefore, to achieve a predefined QoS, the consumed power by the base station also alters over the time, as it is displayed in the following relation: $r_t = \log_2(1 + \gamma_t p_t)$, and consequently the consumed power can be interpreted as:
where $r_i$ is the required QoS rate in bps/Hz, $\gamma_i$ is the instantaneous SNR and $p_i$ is the instantaneous consumed power.

To avoid any misunderstanding, $\gamma_{th}$ is the SNR value above which the user can be awarded the service (i.e. in the cognitive scenario $\gamma_{th}$ is the value of the PU’s SNR that permits the cognition), while the $r_i$ is the QoS that should be guaranteed by the operator, taking into the consideration that user’s SNR is above $\gamma_{th}$.

### A. Power consumption in the single user scenario

In order to make a fair judgement about the power consumption in the case of spatial cognitive scenario, it should be discussed the required power in the single user scenario. For the i.d.d. complex Gaussian channels, the SNR PDF for one serviced user has the following distribution [13]:

$$f_{\gamma} (\gamma) = e^{-\frac{\gamma}{\gamma_0}}$$

with its related CDF:

$$F_{\gamma} (\gamma) = 1 - e^{-\frac{\gamma}{\gamma_0}}$$

and using Eqn.(10) and Eqn.(11), the PDF of the required power [14] is derived as follows

$$f_p (p) = \frac{f_{\gamma} (2^{r_i} - 1)}{p^2 \gamma_0} \frac{dp}{d\gamma}$$

which reformulates by using Eqn.(11) as

$$f_p (p) = \frac{2^{r_i} - 1}{p^2 \gamma_0} e^{-\frac{2^{r_i} - 1}{p \gamma_0}}$$

with its corresponding CDF

$$F_{\gamma} (\gamma) = e^{-\frac{2^{r_i} - 1}{p \gamma_0}}$$

In practical systems, users with very bad channel conditions (i.e., below $\gamma_{th}$) are not serviced, so that the maximum allowable power equals to:

$$P_{max} = \frac{2^{r_i} - 1}{\gamma_{th}}$$

And by using Eqn.(14) and Eqn.(16), the average power in the single user case $E[p]_c$ can be expressed as follows

$$E[p]_c = \sum_{i=0}^{2} R_i \text{Prob}[i] = 2 r_i e^{-\frac{\gamma_{th}}{\gamma_0}} + r_i (e^{-\frac{\gamma_{th}}{\gamma_0}} (1 + \frac{\gamma_{th}}{\gamma_0}) - e^{-\frac{3 \gamma_{th}}{\gamma_0}})$$

where $\gamma_{th}$ is the SNR value above $\gamma_{th}$.

### B. Power consumption in the cognitive scenario

The spatial cognition seems to be a tempting option to be deployed in new systems, as it allows more users to be served and consequently more income to the operator is achieved. However, such a multiuser service increases the power consumption in comparison to the single user service, which makes operator’s net profit questionable. To make a precise judgement, a formulation of the average power for cognitive scenario is now derived.

From Eqn.(8) and Eqn.(10), the required power PDF can be reformulated as

$$\phi_p (p) = 2 \frac{2^{r_i} - 1}{p^2 \gamma_0} e^{-\frac{2^{r_i} - 1}{p \gamma_0}}$$

with the corresponding CDF

$$\Phi_p (p) = e^{-\frac{2^{r_i} - 1}{p \gamma_0}}$$

In the cognitive scenario, PU’s satisfaction has the highest priority; when an SU enters in the system, the PU’s SNR may decrease below $\gamma_{th}$, then the SU should be directly switched off. The scheduled SU’s SNR must also be above the threshold. Therefore, the average power consumption of the cognitive system $E[p]_c$ must consider both situations: only one PU serviced user with its allocated power given in (16); and the case of cognition with the allocated power being formulated in (19). The probability of each case will definitely impact the power consumption formulation, which is obtained in Eqn.(22) on the top of the next page.

It can be noted that the power consumption in the spatial cognitive scenario is more than twice that in single user case due to the spatial correlation between the users, as adding an SU degrades the PU’s SNR from $||H||^2$ to $\frac{||H||^2 (1-\delta^2)}{2}$, where $\delta = \frac{||h_{PU}||^2 ||h_{SU}||^2}{||h_{PU}||^2 + ||h_{SU}||^2}$ is the normalized scalar product between $h_{PU}$ and $h_{SU}$, with $0 \leq \delta^2 \leq 1$. The case 0 occurs when the users are totally orthogonal and 1 when they are colinear. As a result, spatial cognition in ZFB increases the total consumed power.

The obtained average data rate using the SNR distribution in Eqn.(9) can be expressed as follows:

$$E[R]_c = \sum_{i=0}^{2} R_i \text{Prob}[i] = 2 r_i e^{-\frac{3 \gamma_{th}}{\gamma_0}} + r_i (e^{-\frac{\gamma_{th}}{\gamma_0}} (1 + \frac{\gamma_{th}}{\gamma_0}) - e^{-\frac{3 \gamma_{th}}{\gamma_0}})$$

where $R_i$ denotes total target rate that can be obtained from the system for the $i$ simultaneous serviced users. One of the most alluring qualities of the cognition is the increased data rate, due to the multiple simultaneous transmissions introduced by its concept.

### V. POWER RATE RELATION IN THE COGNITIVE SCENARIO

In spite of the fact that employing spatial cognition seems to be very vivid (i.e due to the increased rate), the power consumption in such scenarios makes it less optimistic. Therefore, a metric to assess the joint performance is needed to check
\[ E[p]_c = 2^{2\gamma_t - 1} \frac{1}{\gamma_0} \left( e^{-\frac{2\gamma_t}{\gamma_0}} + e^{-\frac{2\gamma_t}{\gamma_0}} \right) + \frac{2^{2\gamma_t - 1} - 1}{\gamma_0} e^{-\frac{2\gamma_t}{\gamma_0}} (1 - e^{-\frac{2\gamma_t}{\gamma_0}}) \] (22)

whether it is valuable to implement spatial cognition in the modern wireless systems.

The metric should be stated as: if the allowed consumed power is the same for both single-user system and the cognitive system, then which system can generate higher data rate with the same amount of power? and will this fact change if the threshold value is changed? This metric \( R_m \) represents the achieved data rate in single user systems by employing the power employed in cognitive systems. If \( R_m \) is larger than \( E[R]_c \) in Eqn.(21) then it advises against cognition, and vice versa. It can be expressed as follows:

\[ R_m = E[\log_2(1 + E[p]_c)] \] (23)

which by some mathematical manipulations and using equations (11) and (23), its final formulation is given by

\[ R_m = \frac{e^{-\gamma_t \ln(1 + E[p]_c)} + e^{\gamma_t \ln(1 + E[p]_c)}}{\ln(2)} \] (24)

VI. SIMULATIONS AND RESULTS

The performance of the cognitive scenario is presented by simulation, where the goal is to display the impact of cognition on the total system performance, which is expressed in terms of consumed power and average rate. As previously explained, PU has strict demands to be satisfied while SU is served as long as PU is satisfied, where we assume the same minimum QoS (i.e. SNR threshold) for both PU and SU. The performance of the proposed scheme is evaluated by Monte-Carlo simulations, where a wireless scenario is considered with \( n_t = 2 \) transmitting antenna in a cell with a variable number of active users.

In this paper, different scenarios have been considered; where the general procedure is as follows: when the selected PU’s SNR can support minimum QoS (i.e. threshold), an SU is added to be served under the condition that PU’s rate is guaranteed. This step is sequentially carried out till the PU’s rate is not guaranteed anymore or the number of scheduled user reaches to \( n_t \).

Fig.(1) portrays the relation between the selected \( \gamma_{th} \) and the consumed power for the case \( r_t = 1 \), it can be noted that increasing the threshold values, the amount of the consumed power decreases due to two reasons previously explained along the paper: the high threshold value means that the selected user’s channel shows good characteristics and consequently low power consumption. On the other hand, raising the threshold, makes it harder for the user to achieve this minimum QoS and consequently sometimes no user is scheduled which means zero power consumption. The figure also presents a comparison between the cognitive scenario and single user; while for low thresholds values, the probability of cognition is high which means that there are two users being served, and this leads to higher power consumption (more than twice), this percentage will change as the threshold increases. The total rate for the previous scenario is displayed in Fig.(2), as anticipated, the data rate for the cognitive scenario is about (1.6-1.8) times at low threshold value in comparison with single user, this percentage changes with increasing the threshold, as known, higher threshold means lower probability of cognition (returning back to single user case).

Fig.(3) differs from Fig.(2) in that the former is obtained under a fixed total transmitted power \( P_t = 1 \), displaying the data rate in the figure for the different threshold values, where the cognitive scenario excels the single user scenario for all threshold values. It can be also seen the perfect match between the simulations and the mathematically obtained results through the closed form expressions obtained along the paper. This is thanks to the fact that the mathematical results avoid approximations and are based on exact manipulations.

An interesting result is presented in Fig.(4) for several
variables in the system and plotting the performance of the $R_m$ proposed parameter, showing how the spatial cognition can offer lower performance than the single user service for some specific scenarios. Under these conditions, the spatial cognition data rate does not compensate for the higher power consumption, therefore, the single user case is more suitable for its employment. The figure indicates that for a small target rate and a small value of the SNR threshold, the single user service outperforms the cognition scenario under the same power conditions. Therefore, to employ spatial cognition and to guarantee that it is worth to, the operator should avoid to work with these values; where as indicated by the figure, the spatial cognition takes over the single user service for the most of threshold and target rate values.

The feasibility of spatial cognition has been studied throughout this paper, while for future work, this analysis can be extended to orthogonality based selection within the concept of spatial cognition. Moreover, power consumption and delay analysis of the spatial cognition can be incorporated within single framework in hour intelligent radio resource management, which is based on the utilizing nature of requested application across the day to optimize the performance of the network.

VII. CONCLUSION

The feasibility of exploiting the space dimension to employ cognition by delivering service to $n_s$ simultaneous users is thoroughly studied throughout this work. In spite of the brightful characteristics that the spatial cognition have (i.e., increasing the data rate and reducing the delay), the power-rate characteristics of spatial cognition is discussed through closed form expressions and simulations.

The power consumption in spatial cognition is intensively studied through the derivation of the PDF and finding the expected value, enabling to compare it to the single user (no cognition) scenario, which its formulations of the average rate for different transmission scenarios are are obtained. Finally, the feasibility of cognition from the power perspective is tested and proved to be suitable by finding a power-rate metric that is capable of making such an assessment. The results showed that the single user service can outperform the cognition case for some specific scenarios.

![Graph showing data rate under fixed power.](image1)

**Fig. 3.** Obtained data rate under fixed power.

![Graph showing rate metric performance.](image2)

**Fig. 4.** Rate metric performance of the cognition and the single-user service.

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