Dynamic reliability degradation based models and maintenance optimization

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I. INTRODUCTION

MAINTENANCE of industrial equipment is a lever to increase the efficiency and the productivity of industries. An intelligent maintenance is achieved by determining the optimum preventive time to replace an item in order to minimize the costs and to increase the availability. In order to reach the optimum, one has to know the reliability and the mean residual lifetime of an item. The reliability $R(t)$ is the probability that an item will perform a required function without failure under stated conditions for a stated period of time. This information may be obtained by three approaches that depend of the available data (figure 1)[1].

The first approach is the statistical reliability which consists in fitting the best distribution on failure times. Those failure times are obtained by the return of experience on several identical items that work within the same operating conditions. The second approach supposes that the evolution of an indicator of the health state of the system is available. This indicator is recorded during the life of the item and pronostic methods are used to determine the distribution of time for which the expected evolution of that indicator goes beyond a failure threshold. The last approach supposes that the physical degradation process is known a priori, which leads to an analytical expression for the evolution of the degradation with uncertainties taken into account. The goal of this paper is to show the interest of the second and third methods compared to the first one when degradation data is available. We generated degradation data with a semi-Markov model, the degradation paths are collected up to a given threshold that indicates the failure. Then we fit some reliability models on the hitting times of the threshold and propose a maintenance model based on inspections and preventive replacements that minimizes the average maintenance cost.

II. RELIABILITY BACKGROUND

A. Definition

Reliability $R(t)$ is the ability of a system or component to perform its required functions under stated conditions for a specified period of time. Reliability is measured by a probability value that decreases as time goes on. If $T$ is a stochastic failure time of an equipment, the mathematical expression of its reliability is:

$$R(t) = P(T > t) \quad (1)$$

The complementary function of the reliability is the failure function $F(t)$. The failure function represents the repartition of failure times for a set of identical equipments that have been tested. The failure function is thus used to fit a reliability model on failure data.

$$F(t) = 1 - R(t) = 1 - P(T > t) = P(T \leq t) \quad (2)$$

The repartition of failure times may be represented by the density function $f(t)$. The density function is the derivative of the failure function.

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt} \quad (3)$$

From the above equations we define the failure rate $h(t)$ as to be the ratio of the density function by the reliability function. It indicates the risk of failure at time $t$ according that the equipment has already survived until $t$.

$$h(t) = \frac{f(t)}{R(t)} \quad (4)$$

B. Reliability models

From a set of failure times, a reliability model is fitted. Usual models are :

- Exponential

$$R(t) = \exp(-\lambda t) \quad (5)$$

$\lambda$ is the scale parameter. Exponential models have a constant failure rate and are memoryless. They are well suited when the failures occur randomly.
• Lognormal

\[ R(t) = \Phi_{\text{nor}} \left( \frac{\ln(t) - \mu}{\sigma} \right) \] (6)

\( \mu \) is the mean parameter and \( \sigma \) is the standard deviation. Lognormal models allow to get only positive failure times by the logarithmic transformation of time.

• Weibull law

\[ R(t) = \exp \left( - \left( \frac{t - \gamma}{\eta} \right)^{\beta} \right) \] (7)

\( \eta \) is the scale parameter, \( \beta \) is the shape parameter and \( \gamma \) is the location parameter (usually \( \gamma = 0 \)). Weibull models are well suited for most cases as they are quite flexible with both parameters \( \eta \) and \( \beta \). They are mainly used for components that undergo some degradation.

C. Non parametric model fitting

From a collection of \( N \) failure data, a non parametric failure function \( \hat{F}(t_i) \) may be calculated. There are several methods to get this function. The basic idea is to sort the failure times by ascending order and to count the number \( (i) \) of failures occurred after each time inspection. Here are some examples :

• Kaplan-Meier

\[ \hat{F}(t_i) = \frac{i}{N} \] (8)

This basic formulation presents the disadvantage of having a probability equal to 1 for the last observed failure time.

• Product limit (or medium rank)

\[ \hat{F}(t_i) = \frac{i}{N+1} \] (9)

The denominator is increased by one to solve the problem of having a probability = 1 for the last observed failure time.

• Approached Rank adjust (or median rank)

\[ \hat{F}(t_i) = \frac{i - 0.3}{N + 0.4} \] (10)

D. Parametric model fitting

From the non parametric failure function \( \hat{F}(t_i) \) we can adjust a reliability model. The methods to fit these models are :

• Regression methods

It consists in taking the linear form of the failure function to obtain the relation \( Y = a + bX \).

Example for the weibull law :

\[ \ln \left( \ln \left( \frac{1}{1 - \hat{F}(t)} \right) \right) = \beta \ln(t) - \beta \ln(\eta) \] (11)

Identifying the parameters, we obtain

\[ \begin{cases} \beta = a \\ \eta = \exp \left( -\frac{b}{\beta} \right) \end{cases} \] (12)

• Maximum likelihood methods

This method consists of finding the parameters of the law that maximize the likelihood to find back the failure times. The likelihood function has the form [2] :

\[ L(p) = \prod_{i=1}^{n} f(t_i|p) \] (13)

Using the logarithmic transformation, the parameters that maximize the likelihood are found by taking the partial derivative for each parameter.

\[ \frac{\delta \ln L(p_1, ..., p_k)}{\delta p_i} = 0 \quad i = 1, ..., k \] (14)

E. Mean Residual Lifetime (MRL)

The Mean Residual Lifetime is a useful information for the maintainer. It represents the expected average lifetime of an item of equipment given the fact that it has survived until some time \( t \). MRL is obtained by [3] :

\[ \text{MRL}(t) = \frac{\int_{t}^{+\infty} R(y)dy}{R(t)} \] (15)

Several drawbacks may be highlighted for the classical reliability estimation :

1. Obviously there must be enough failures to fit a model;
2. The fitted reliability aims at estimating a population characteristic(s) of a system, subsystem or component instead of specifying the reliability of a "particular" component. For example, if an automobile transmission fails at 40000 miles, the fact that the mean time to failure of the average transmission system happens to be 200000 does not comfort the owner [4];
3. "Failure is not an option" meaning that some systems can not tolerate failures for safety reasons; thus the classical reliability fitting is not possible.

III. DEGRADATION BASED RELIABILITY

The degradation based approach considers that the evolution of a degradation process \( Z(t) \) may be tracked over time or is known a priori. Linking the degradation evolution to the reliability is achieved by considering a critical threshold \( z \) that corresponds to a failure level. The reliability is then the probability for the degradation process \( Z(t) \) to stay under that threshold :

\[ R(t) = P(z > Z(t)) \] (16)
At the time of failure, the threshold \( z \) is reached, the reliability is obtained by:
\[
R(t) = P(Z^{-1}(z) > t)
\]  

IV. SEMI-MARKOV DEGRADATION MODEL

A. Markov theory

The Markov theory is a common approach to consider transition state problem for stochastic process. Among the recent models, we find:
- Markov Chain which has a discrete countable state-space.
  In this model transitions occur at each time step increment. It is called Discrete Time Markov Process (DTMP).
- Continuous Time Markov Process (CTMP) for which the difference is that rather than transitioning to a new state at each time step, the system will remain in the current state for some random amount of time. When the distribution of sojourn time is exponential this model is also called Homogeneous Markov Process e.g. the transition rates are constant over time
- Semi-Markov Process (SMP) or Non Homogeneous Markov Process (NHMP) for which the embedded jump chain is a Markov Chain and where the sojourn time in states are random variables with any distribution, whose distribution function may depend on the two states between which the move is made.
- Hidden Markov Model (HMM) is a statistical Markov model in which the system has unobservable states but the output, that depends on the state, is observable.

Several authors used Markov models to assess the reliability and maintainability of equipment. Sadek & Limnios proposed a nonparametric estimation of reliability and survival function for continuous-time finite Markov processes [5]. Guo & Yang proposed an automatic creation of Markov models for reliability assessment of safety instrumented systems [6]. Zhao & al. developed condition-based inspection/replacement policies for non-monotone deteriorating systems with time homogeneous Markov chain environmental covariates [7]. Bloch-Mercier studied a preventive maintenance policy with sequential checking procedure for a Markov deteriorating system [8].

Implementing a degradation based reliability with a Markovian approach may be achieved by two ways. The first one consists in considering a Markov chain with some functional states and an absorbing state that corresponds to the failure level [9]; the reliability is then the probability over time for the Markov chain to reach that absorbing state. The second one refers to multistate degradation that consists in choosing several wear/failure rates according to the load case of the different states [10]; thus each states contributes to the degradation according to its wear rate. A threshold is then fixed and we simulate the Markov chain to calculate the cumulated damage until it reaches the given threshold.

On the semi-Markov applications, Kharoufeh proposed a semi-Markov model for degradation based reliability [10]; the estimation of sojourn time is achieved by using phase type distributions. Solo presented a resume of this methodology and used it to represent the crack growth of a turbine blade under different load cases [11]. Ouhbi proposed a non-parametric reliability estimation of semi-Markov processes [12]. Lefebvre studied a semi-Markov process with an inverse Gaussian distribution as sojourn time [13].

B. Semi-Markov framework

In this section, we present the methodology to fit a semi-Markov model from data as well as the method to generate degradation data.

B.1 Hypotheses and notations

We consider a piece of equipment that works under various load cases, which are represented by different and distinct states. For each state, a constant degradation rate \( r(i) \) is fixed defining the degradation rate vector \( R = [r(1), r(2), \ldots, r(n)] \); those rates can correspond to a Physics-of-Failure model of the degradation (e.g. a crack growth function of the applied stress) or to a failure rate if we know the mean time to failure of the component for a given load case. Then a degradation threshold is fixed which can correspond to a specific value of the degradation process (e.g. a critical crack length) or to the accumulated damage that reaches unicity according to fatigue law. The state space \( E \) is a finite vector of \( n \) distinct states. The process starts from a given state defined by the user and then visits some state \( i \in E \) and spends a random amount of time before a new transition to another state \( j \neq i \). The transition rates between states are defined in the transition probability matrix \( P \). This matrix can be estimated from field data or may be defined in the component design phase. The time spent in state \( i \) is a stochastic value according to any distribution or recorded time from field data.

B.2 Estimation of the transition matrix from field data

We assume that the load cases are known over some time interval \([0, t]\) and that the item of equipment has not failed within this interval. When a transition occurs, the transition time, the current state and the destination state are recorded. Let \( T(i, j) \) be the true rate of transition from state \( i \) to state \( j \), \( i \neq j \). Let \( X_t(i, j) \) be the number of transitions from \( i \) to \( j \) and \( V_t(i) \) the total time spent in state \( i \). If the time interval is long enough, we can show that:
\[
T(i, j) \approx \hat{T}_t(i, j) = \frac{X_t(i, j)}{V_t(i)}, \quad j \neq i \tag{18}
\]
\[
\hat{T}_t(i, j) = -\sum_{j \neq i} \hat{T}_t(i, j) \tag{19}
\]

If \( \hat{P} \) is the estimated transition matrix, the estimated transition rates \( \hat{p}_{i, j} \) are
\[
\hat{p}_{i, j} = \begin{cases} 
\frac{\hat{T}_t(i, j)}{-\hat{T}_t(i, i)} & \text{if } j \neq i \\
0 & \text{if } j = i \end{cases} \tag{20}
\]

B.3 Simulation of degradation using a semi-Markov model

We consider a degradation process that is ruled by a semi-Markov model. A semi-Markov model is like a classical continuous Markov chain but without the restriction for the sojourn times to be exponential. That means that when the process is in a given state, it will stay in that state for a random amount of time.
A degradation based semi-Markov model has the following assumptions:

- we use \( n \) states to represent \( n \) different load cases for an item of equipment;
- for each state \( i \), a deterministic degradation rate \( r(i) \) is known;
- the equipment switches from one state to a different state with a probability defined in a transition matrix \( P \);
- between each transition, the equipment remains in its state for a random amount of time;
- each state contributes to the cumulative damage in proportion of its degradation rate and duration in that state;
- failure is observed when accumulated damage reaches the threshold.

The figure 2 illustrates how degradation data is simulated with a semi-Markov model.

B.4 Reliability calculation

Reliability calculation with a degradation based semi-Markov model may be achieved by two methods. The first one uses Monte Carlo simulation to generate several degradation paths until they reach the threshold and the classic statistical reliability adjustment is made on the hitting times. The second method uses phase-type distributions to represent the states of the semi-Markov model which allows to calculate the failure time distribution with an analytical expression; more information may be find in [10], [11], [14]. In this paper we used the first method to calculate the reliability with a lognormal distribution fitting (this is justified by the highest value obtained for the determination factor when testing the possible statistical distributions on data).

V. MAINTENANCE MODEL

The proposed maintenance model is the well known maintenance model which minimizes a cost criteria by researching the best preventive replacement time. Moreover we discuss the case when there is a penalty cost for degradation inspection. Obviously one has to inspect the equipment to measure the degradation level. This inspection may be achieved by continuous online monitoring device which doesn’t affect the global cost or by manual ponctual inspections which has a cost.

A. Maintenance cost model

The model takes into account a corrective cost \( C_c \) (i.e. the item has failed) and a preventive cost \( C_p \) (i.e. the item has not yet failed). The goal is to find the optimal time for replacement \( (T_p) \) in order to minimize the expected average cost per unit of time \( (\overline{C_m}) \). This cost can obtained by [2]:

\[
\overline{C_m} = \frac{F(T_p)C_c + (1 - F(T_p))C_p}{\int_0^{T_p} R(t)dt}
\] (21)

B. Inspection cost

In order to decide to postpone or not the initial time of replacement, one has to measure the level of degradation of the item of equipment. We define the inspection cost \( C_i \) as to be the cost per inspection. The mean cost per unit of time becomes

\[
\overline{C_m} = \frac{F(T_p)(C_c + N_{insp}C_i) + (1 - F(T_p))(C_p + N_{insp}C_i)}{\int_0^{T_p} R(t)dt}
\] (22)

The problem consists in determining the number of inspections \( N_{insp} \) that allows the item to run as long as possible but without being too costly due to the need of inspections. We introduce the following inspection cost criteria \( K \):

\[
K(t_j) = \frac{(j - 1)C_i + C_p}{T_p} \frac{t_j}{C_i + C_p}
\] (23)

\( j \) being the \( j^{th} \) inspection ; \( t_j \) the current time of inspection ; \( T_p \) the next forecast preventive replacement time. While \( K > 1 \) the preventive replacement is postponed to the next \( T_p \) calculated. When \( K \leq 1 \), we reached the optimal number of inspections \( N_{insp} \) and the last inspection time is the preventive replacement time. A strong assumption is required however : at each inspection time, the maintenance staff is ready for an immediate preventive replacement if necessary. The figure 3 details the followed steps for the maintenance optimization with inspection cost.

The next section illustrates the simulation framework on a theoretical example.

VI. ILLUSTRATIVE EXAMPLE

A. semi-Markov parameters

We consider a three states semi-Markov degradation process with the following parameters:

- the transition matrix \( P = \begin{pmatrix} 0 & 0.9 & 0.1 \\ 0.8 & 0 & 0.2 \\ 0.7 & 0.3 & 0 \end{pmatrix} \)
Fig. 3. Simulation framework for maintenance optimization

- the distribution of sojourn times for each state

<table>
<thead>
<tr>
<th>State</th>
<th>Distribution</th>
<th>Scale</th>
<th>Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>Weibull</td>
<td>20</td>
<td>1.5</td>
</tr>
<tr>
<td>State 2</td>
<td>Weibull</td>
<td>10</td>
<td>1.5</td>
</tr>
<tr>
<td>State 3</td>
<td>Weibull</td>
<td>1</td>
<td>1.5</td>
</tr>
</tbody>
</table>

- the deterministic degradation rates \( r \) for each state

<table>
<thead>
<tr>
<th>State</th>
<th>( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>0.0001</td>
</tr>
<tr>
<td>State 2</td>
<td>0.001</td>
</tr>
<tr>
<td>State 3</td>
<td>0.05</td>
</tr>
</tbody>
</table>

B. semi-Markov degradation simulation

We proceed on the simulation of the degradation process. The figure 4 shows the iterative procedure for the simulation. We start from time 0 with the simulation of the semi-Markov chain. The hitting times of the failure threshold allow us to fit a non-parametric reliability model (with the approached rank adjust estimator (eq. 10) as well as a parametric reliability model by fitting a lognormal distribution ). This reliability is a “general” one as we don’t yet have any information of a specific degradation path. After that, we suppose that the degradation path is observable. We inspect the equipment after 400, 1000 and 1300 units of time. At each inspection time, we proceed on a new simulation of possible degradation paths given the fact that the degradation process had a value \( Z_i \) at time \( t_i \). The new simulated hitting times of the threshold allow for a more accurate reliability model (specific for the equipment) as shown in figure 6, 7, 8.

Fig. 4. Simulations of degradation paths by a semi-Markov approach (late failure case); 20 simulations done for representation in this graphic and 500 for accurate calculation in the results. This figure represents the evolution of the expected hitting times for a degradation process that is ruled by a semi-Markov model. From the observed degradation path, prognostics are made at time 400, 1000 and 1300 in order to obtain the distribution of failure times. The closer the threshold the more accurate the residual life will be as we have now a specific reliability model for the equipment.

VII. RESULTS

We discuss three cases (figure 5):

1. the case where the specific time of failure is close to the ge-
A. Reliability and mean residual lifetime

The figures 6, 7, 8 represent the evolution of the reliability for the different cases. We suppose that the degradation is continuously monitored with time; for the purpose of illustration we calculated the reliability for three arbitrary inspection times but the idea is that the models should be updated continuously with data accumulation. For the three cases investigated, we see that the predicted reliability based on the observation of the degradation path is more accurate (i.e. has less dispersion) than the "general" reliability obtained without the knowledge of any degradation path. This emphasizes the need for degradation measurements in order to improve the reliability and the mean residual life prediction.

B. Maintenance discussion

We divide the discussion into two separate cases depending if there is an inspection cost. The first discussion considers that inspection is achieved automatically by sensor devices and monitoring softwares for which there is no maintenance cost related (i.e. inspections don’t require the item of equipment to be shut down). The second discussion discusses the case when inspection has to be made manually which has a cost (downtime, manhours, ...).

B.1 No inspection costs

The figures 9, 10, 11 represent the evolution of the expected cost per unit of time for different value of the periodicity of preventive replacement ($T_p$). Those curves are obtained from a set of 500 failure times and degradation-based reliability prediction at different inspection times ($400, 1000, 1300$). The mean residual life is calculated at each inspection time.
time of replacement would have lead to a failure which would have increased the average cost. In the case of the late failure, we state that the initial period for preventive replacement was too anticipated due to the “optimistic” evolution of the specific degradation path monitored.

The study on the late failure case but the results and comments remain available for any case. The figure 12 represents the analytical mean cost obtained after 0, 1 and 2 inspections respectively at time 0, 870 and 1266. The table I shows the results of the predicted \( T_p \) and the criteria \( K \) after each inspection.

B.2 With inspection costs

We now discuss the case when inspection has a cost. We choose an inspection cost \( C_i = 500 \) and the purpose is to study the performance of the framework showed in figure 3. We focus

\[
\begin{array}{|c|c|c|c|}
\hline
j & t_j & T_p & K(t_j) \\
\hline
0 & 0 & 870 & +\infty \\
1 & 870 & 1266 & 1.33 \\
2 & 1266 & 1318 & 0.96 \\
\hline
\end{array}
\]

Fig. 12. Evolution of analytical mean cost with the number of inspections. The item is inspected first a time after 870 units of time, the next forecasted \( T_p \) is 1266 units of time. The equipment is thus inspected at \( t = 1266 \) units of time and the next predicted \( T_p \) is 1318 units of time. However, the cost criteria is now below 1 which indicates that it is not recommended to wait for \( T_p = 1318 \) to do the preventive maintenance. Thus the maintenance is achieved at time 1266 units of time and only 1 inspection has been made.
Finally the figure 13 represents the simulation of three maintenance scenario for 500 consecutive simulations. The results obtained emphasise the interest of the proposed approach with an intelligent preventive maintenance scheduling based on punctual inspections of the degradation.

VIII. CONCLUSIONS

In this paper, we presented a dynamic degradation based-reliability and maintenance optimization framework. The reliability background has been developed and we highlighted the pitfalls of the generic statistical reliability estimation and the need for a specific degradation-based approach. We focused our approach on a theoretical approach based on a semi-Markov model for the degradation simulation. After that we proposed a cost maintenance model that takes into account inspection costs. We also introduced a decision criteria to help the maintener to decide to postpone or not the time for preventive replacement based on the inspection. We illustrated our model on a theoretical example. The results showed that the approach may lead to a cost reduction. This has been validated by an analytical approach and by simulations. Future work will consist in developing a more general model that can take into account any degradation data and to apply it to real degradation data (i.e. cutting tool flank wear degradation).

REFERENCES