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Relief effects for passive microwave remote sensing

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Relief effects for passive microwave remote sensing

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Abstract
The signal of a microwave radiometer observing a land surface from space is composed of surface and atmospheric contributions, both of which depend on the relief. For proper interpretation of the data these effects should be quantified and, if necessary, taken into account. Relief effects are twofold: First, the path through the atmosphere between the surface and the sensor depends on the altitude of the emitting surface, thus leading to a height-dependent atmospheric influence. The effect can be taken into account by standard atmospheric radiative transfer models if the elevation of the surface and the atmospheric state are known. Second, and more relevant for the present discussion, is the variable topography of land surfaces, consisting of slopes, ridges and valleys, sometimes with characteristic alignments, and surfaces surrounded by elevated terrain. These surfaces can interact radiatively, not only with the atmosphere, but also with each other, leading to the tendency to enhance the effective emission. Under such circumstances, deviations occur from the standard hemispheric emission of a horizontal surface. The interactions not only depend on topography and emissivity, but also on the bistatic scattering behavior. Special attention will be paid to the radiation enhancement in a landscape of lambertian surfaces with elevated horizons. As an example, simulated data for southern Norway are shown.

Introduction
To assess potential error sources in algorithms based on passive microwave data, the quantification of all factors contributing to the measured microwave radiation is necessary. This paper deals with the effects of a terrain with variable height and with tilted surfaces on the measurement of microwave radiation by means of satellite-based radiometers. Although these effects can be quite significant for land remote sensing, so far they have not received proper attention. An example of relief effects are shown in an early image from Schaeer and Schanda (1974) at 3 mm wavelength where part of the radiation from the mountain is reflected by the lake of Thun (Figure 1). Furthermore the emission itself clearly deviates from a flat horizon. Also shown is a visible version of the same scene. Relief effects are twofold: First, the path between the radiation source at the surface and the sensor depends on the surface altitude, thus leading to a relief-dependent atmospheric contribution. This effect is the topic of Section 2, where the radiation from a horizontal surface with a flat horizon at a given altitude is described. Section 3 is devoted to a variable topography, consisting of valleys and ridges; characteristic effects on the passive microwave signal will be described and illustrated in Section 4.

Flat horizon
The classical geometry of remote sensing of the terrestrial surface is a pair of half spaces separated by a horizontal surface, leading to a flat horizon. In this situation there is no shadow of any kind. The relief effects are determined by the dependence of the emitted radiation on
the altitude $h$ of the surface. Blackbody radiation with a brightness temperature equal to the physical temperature $T_0$ is produced in the lower half space, part of which is transmitted, and thus emitted ($T_e$) into the upper half space where it is sensed by a radiometer (Figure 2). Radiation from the upper to the lower half space ($T_{sky}$) is much smaller than $T_0$. The $p$-polarized emitted brightness temperature $T_e$ above a flat surface with reflectivity $r_p$ is given by

$$T_e = e_p T_0 = (1 - r_p) T_0$$

(1)

The upwelling radiation $T_{up}$ just above the surface is the sum of the radiation emitted by the lower half space and the radiation $T_{sky}$ incident from the upper half space and being reflected at the surface towards the sensor:

$$T_{up} = (1 - r_p) T_0 + r_p T_{sky}$$

(2)

The brightness temperature $T_b$ at satellite level is the attenuated upwelling radiation plus the radiation emitted by the atmosphere in the direction toward the radiometer (atmospheric scattering and atmospheric temperature inhomogeneity being neglected for simplicity):

$$T_b(\theta) = T_{up} t + T_a (1-t)$$

(3)

where $t$ is the atmospheric transmissivity in the observation direction and $T_a$ is the temperature of the atmosphere. The above equation applies to the polarization and direction corresponding to $T_{up}$. In case of an inhomogeneous atmosphere $T_a$ has to be considered as an effective temperature $T_{eff,up}$ for upwelling radiation. For a plane-parallel atmosphere $t$ is given by

$$t(h,\theta) = \exp(-\tau(h,\cos\theta))$$

(4)

where the zenith opacity $\tau(h)$ is the vertical path integral of the absorption coefficient $\alpha(z)$ through the atmosphere, starting at the surface. The reflectivity $r_p$ at polarization $p = h, v$ in (2) may be composed of a specular, polarized component $r_{s,p}$ and of a diffuse, unpolarized component $r_d$:

$$r_p = r_{s,p} + r_d$$

(5)

The following discussion is concentrated on the derivation of expressions for $T_{up}$ depending on properties of the relief. We will assume that we can approximate the total reflectivity $r_p$ by (5), where $r_{s,p}$ is a perfectly specular component and $r_d$ is a lambertian component, i.e. due to a perfectly rough surface. In Ulaby et al. (1981, Sec. 4-16.2), $r_{s,p}(\theta)$ and $r_d$ are expressed by $\Gamma(\theta, p)$ and $0.25\sigma_0^2$, respectively, where $\theta$ is the observation angle with respect to the surface normal, and $p$ is the state of polarization ($p = v, h$). According to Ulaby et al. (1981), $T_{up}$ can be written as

$$T_{up} (\theta, h) = e_p (\theta) T_0 + r_{s,p} (\theta, p) T_{sky} (\theta, h) + \frac{\sigma_0}{4\pi} \int T_{sky} (\theta, \Phi, h) \cos \theta \sin \Phi \; d\Phi ,$$

(6)

where $h$ is the height of the surface above sea level. The integral term in (6) is the diffusely scattered sky radiation, here to be called $T_d$; as a result of Lambert scattering $T_d$ depends only on $h$. We assume that the incident sky radiation is unpolarized. For a plane-parallel atmosphere $T_{sky}$ depends on $\theta_s$ and $h$, thus the integral in (6) can be solved for $\Phi_s$ using $d\Phi_s = \sin \theta_s d\theta_s$:

$$T_d(h) = \frac{\sigma_0}{2} \int_0^{\pi/2} T_{sky} (\theta_s, h) \cos \theta_s \sin \theta_s \; d\theta_s$$

(7)

In case of an isothermal atmosphere at temperature $T_a$ with a cosmic background $T_c$ we have

$$T_{sky} (\theta_s, h) = T_c e^{-\tau_s/\cos \theta_s} + (1 - e^{-\tau_s/\cos \theta_s}) T_a$$

(8)
Equation (8) can be used even for a non-isothermal atmosphere; then \( T_a \) is an effective air temperature \( T_{a,\text{down}} \) for downwelling radiation (e.g. Mätzler, 1992; Ingold et al., 1998). We obtain

\[
T_d - T_c = \frac{\sigma_0}{2} (T_a - T_c) \int_0^{\pi/2} (1 - e^{-\tau_h / \cos \theta_s}) \cos \theta_s \sin \theta_s d\theta_s ,
\]

(9)

where \( \sigma_0 \) is a constant related to the dielectric properties of the scattering surface. For an optically thin atmosphere, the exponential in (9) can be replaced by the first two terms of the Taylor series expansion \( e^{-\tau_h / \cos \theta_s} \approx 1 - \tau_h / \cos \theta_s \), leading to

\[
T_d - T_c = \frac{\sigma_0}{2} (T_a - T_c) \tau_h \int_0^{\pi/2} \sin \theta_s d\theta_s = r_d \cdot 2 \tau_h (T_a - T_c) ,
\]

(10)

where \( \sigma_0 \) has been replaced by \( 4r_d \). As pointed out by Mätzler (1987), the brightness temperature \( T_d \) can be observed from the surface looking up at the incidence angle \( \theta = 60^\circ \), i.e. for 2 air masses. Proper integration of (9) leads to

\[
T_d - T_c = r_d (T_a - T_c) [1 - 2E_3(\tau_h)]
\]

(11)

where \( E_3 \) is the exponential integral of order \( n=3 \) (Abramowitz & Stegun, 1974). Eventually, we get

\[
T_{up} = T_0 e_p(\theta) + T_a r_p - (T_a - T_c) \left\{ r_{sp} \exp(-\tau_h / \cos \theta) + r_d 2E_3(\tau_h) \right\},
\]

(12)

with \( e_p + r_p = 1 \) and \( r_p = r_{sp} + r_d \). Another representation of \( T_{up} \) is obtained if we write \( 2E_3(\tau_h) \) as an effective transmissivity

\[
t_d = 2E_3(\tau_h) = \exp(-\tau_h / \cos \theta_d)
\]

(13)

thus defining \( \theta_d \) as an effective incidence angle for diffuse radiation. For an optically thin atmosphere \( (\tau_h < 0.2) \) we have \( \theta = 60^\circ \) thus \( \cos \theta_d \approx 0.5 \) (Mätzler, 1987). Following this representation, we get

\[
T_{up} = T_0 e_p(\theta) + T_a r_p(\theta) - (T_a - T_c) \left\{ r_{sp}(\theta) t(\theta) + r_d t_d \right\},
\]

(14)

A simplification occurs if \( \theta = \theta_d \); then \( t = t_d \) and

\[
T_{up} = T_0 e_p(\theta_d) + T_a r_p(\theta_d) - (T_a - T_c) r_p(\theta_d) t_d
\]

(15)

Now, since \( \theta_d \) is often between \( 50^\circ \) and \( 60^\circ \) and since this is also the case for the nadir angle \( \theta \) of conical scanning sensors on satellites (SMMR, SSM/I, AMSR), this simplified formula has been used frequently. The difference between (14) and (15) is negligible if at least one of the following conditions is valid (plane-parallel atmosphere):

- the atmosphere is sufficiently transparent \( (t_d \approx 1) \),
- \( \theta \approx \theta_d \),
- \( r_d \approx 0 \).

After having found the most applicable expression for \( T_{up} \) we can determine its value from surface and atmospheric properties. Inserting \( T_{up} \) in Equation (3) leads to the relief-dependent radiation at satellite level, i.e. the dependence of \( T_b \) on \( h \). Note that both \( T_{up} \) and \( t \) depend on \( h \). It is to be expected that this altitude dependence can be a dominant relief effect, if the frequency is above 20 GHz, and if the height variation is significant.
**Terrain with tilted surfaces**

In addition to the altitude effects on atmospheric radiation, there are effects due to tilted surfaces. On the one hand, the local incidence angle of a tilted surface depends on the orientation of the surface with respect to the view direction of the sensor, and on the other hand, the tilted surfaces imply a variable and elevated horizon depending on azimuth, shadowing parts of the sky. In these directions the incident sky radiation is replaced by the radiation of the elevated landscape.

Because the scale of the relief is assumed to be large with respect to the sensing wavelength, both effects (at the large scale) can be described by geometrical optics. A facet model is indicated, see e.g. Schanda (1986, Section 4.3); such a model was used to describe the microwave emission of the rough sea surface at mm wavelengths by Prigent and Abba (1990) who assumed each facet to be a specularly reflecting surface element. In contrast to their model we allow, in accordance to Section 2, that the surface elements have a partly specular and a partly lambertian component.

### 3.1 Emission from large-scale rough surfaces

Reflection on and emission from a local surface facet can be treated as in Section 2 with the exception that the surface normal \( \mathbf{n} \) used to define the plane of incidence deviates from the vertical \( \mathbf{z} \) direction by a tilt angle \( \alpha \), oriented by an azimuth angle \( \phi \) with respect to the global plane of incidence (Figure 3). The transformation from the global to the local plane of incidence affects both the scattering geometry and the polarization. The local angle of incidence \( \theta_L \) is given by

\[
\cos \theta_L = \sin \theta \sin \alpha \cos \phi + \cos \theta \cos \alpha
\]

Furthermore the linear polarization is rotated by an angle \( \varphi \), given by

\[
\sin \varphi = \sin \phi \sin \alpha / \sin \theta_L
\]

After the reflectivities \( r_s(\theta) \) and \( r_d(\theta) \) have been determined in the local reference frame, they can be represented in the global (satellite-earth surface) frame, taking into account the polarization rotation:

\[
r_s(\theta) = r_s(\theta) \cos^2 \varphi + r_d(\theta) \sin^2 \varphi
\]

\[
r_d(\theta) = r_s(\theta) \sin^2 \varphi + r_d(\theta) \cos^2 \varphi
\]

In our choice of the reflectivity, this transformation only acts on the specular components \( r_s, r_p \), since \( r_d \) is independent of incidence angle and polarization.

### 3.2 Shadowing effects

When we speak about shadows we have some active, directed illumination in mind for a given incident ray. Therefore it may not be quite clear what we mean in the present case of passive microwave radiation. First, the concept can be generalized to include diffuse radiation by adding the contributions to the radiance in the observation direction scattered from all incident rays. The incident radiation is distributed over the half space above the local facet.

Second, the concept can be applied to passive radiation by Kirchhoff’s law where emission corresponds to absorption in a reciprocal ray, i.e. one that is transmitted by the radiometer. By doing so, we find the following effects:

- The surface facet appears enhanced or reduced in size (i.e. view solid angle), or is even hidden from a given view direction, depending on incidence angle, slope and orientation of the facet.

- Shadowing effects include radiation reflected by the surface facet, i.e. radiation emitted by some elevated part of the relief which is incident on the facet. This shadow radiation replaces the radiation form the hidden sky. It means also that the incident radiation is composed of a sky term and of a terrestrial term.
Both effects are to be described here, starting with the former, see Figure 3. For a visible facet, the contribution to the received radiation depends on the solid angle \( \Omega \) under which the facet appears; it is given by the relationship \( A \cos \theta / R^2 \), where \( A \) is the true surface area of the facet and \( R \) is the distance to the radiometer antenna. A surface area is usually represented on a map by its projection \( A_h \) on a horizontal plane. If we use this projected area, then we get

\[
\Omega = \frac{A_h \cos \theta_f}{R^2 \cos \alpha}
\]

Now, the total signal at a given polarization collected by the radiometer antenna is a beam-weighted sum over the radiation from all facets (numbered from \( j=1 \) to \( n \)) within the antenna footprint:

\[
T_{b,\text{total}} = \frac{1}{\Omega_{\text{total}}} \sum_{j=1}^{n} T_b(A_j) \cdot \Omega_j
\]

The above summation is limited to the nearest facet per line of sight, taking the closest one to the radiometer.

The remainder of this section will be devoted to the estimation of the former effect, assuming that the surface is a Lambert scatterer (\( r_l = r_h = r_d \)). Let us consider a horizontal profile in some direction \( x \) through a landscape as shown in Figure 4. The profile elements are straight lines whose ends have \( x \) coordinates \( x_1, x_2, x_3, \) etc. The landscape is simplified to table mountains in a horizontal plain. The quantity of interest is the zenith angle \( \theta_H \) of the horizon to the right in Figure 4 at position \( x (x_0 \leq x \leq x_4) \). The slope tangent \( b \) of the dashed line, i.e. \( b(x) = \cot \theta_H \) is given by

\[
b(x) = \begin{cases} \frac{(h_2 - h_1)(x - x_0)}{(x_1 - x_0)(x_3 - x)} & x_0 \leq x \leq x_1 \\ \frac{h_2 - h_1}{x_3 - x} & x_1 \leq x \leq x_2 \\ \frac{h_2 - h_1}{x_3 - x_2} & x_2 \leq x \leq x_3 \\ 0 & x_3 \leq x \leq x_4 \end{cases}
\]

The sky radiation is limited to incidence angles \( \theta \) smaller than \( \theta_H \). For larger angles, enhanced incident radiation at brightness temperature \( T_h \) appears from the elevated landscape. Let us define the total upwelling brightness temperature by \( T_{up} \) (total) as the sum of the radiation from a flat horizon, \( T_{up} \) (Section 2), plus the increase \( \Delta T_{up} \), i.e.

\[
T_{up}\text{(total)} = T_{up}\text{(Section 2)} + \Delta T_{up}.
\]

The increase describes the enhancement, resulting from \( T_h \) being larger than \( T_{sky} \):

\[
\Delta T_{up} = \frac{r_d}{\pi} \int_{0}^{2\pi} d\theta \left( T_h(x, \theta) \cos \sin \theta d\theta \right)
\]

The angle \( \beta \) is the azimuth angle of the surface-profile direction. The upper limit in the first \( \theta \) integral is denoted by \( \theta_{max} \). This value is \( \pi / 2 \) for a horizontal facet, but for a tilted facet, \( \theta_{max} \) may be larger or smaller depending on the orientation of the facet (i.e. the integration has to include all incident directions of the local facet). Note that values of \( \theta_H \) and \( \theta_{max} \) depend on azimuth \( \beta \).

Let us add a comment for the case of a partly specular facet: The above expression for \( \Delta T_{up} \) is still valid as long the specularly reflected component emanates from the sky. This situation can easily be checked at least for horizontal facets. In other situations where specularly
reflected radiation emanates from the surface elements, a ray-tracing technique has to be applied to include all multiple reflections and emissions. In a statistical sense the overall appearance of a very rough surface is one that behaves more and more like a lambertian one. Now we return to Equation (24) to estimate the increase $\Delta T_{up}$ for a lambertian facet. In the simplified case of a horizontal facet at the lower altitude $h_t$ where $\theta_{max} = \pi/2$ we have

$$\Delta T_{up} = \frac{r_d}{\pi} \int_0^{\pi/2} d\theta \int_0^{T_h(x,\theta) - T_{sky}(\theta)} \cos \theta \sin \theta d\theta$$

(25)

In order to estimate an upper limit $\Delta T_{up,max}$ of $\Delta T_{up}$, we assume that the elevated surface is a black body at constant temperature ($T_h = T_0$). This situation is approximated, e.g. by a forest-covered hill. These simplifications lead to

$$\Delta T_{up,max} = \frac{r_d}{2\pi} \int_0^{2\pi} d\theta \cos \theta H = r_d(T_0 - T_{sky}) \cos^2 \theta_H$$

(26)

where the horizontal bar in the last expression means averaging over azimuth $\beta$. With this expression it is rather simple and straightforward to compute $\Delta T_{up,max}$, using a Digital Elevation Model (DEM). The key quantity is $\cos^2 \theta_H$; it can be computed from $b(x)$:

$$\cos^2 \theta_H = \frac{1}{1 + b^2}$$

(27)

Let us assume a hilly relief with an effective angle $\theta_H$ of 70°, thus $\cos^2 \theta_H \approx 0.12$. In order to produce a noticeable increase $\Delta T_{up}$, the diffuse reflectivity $r_d$ has to be large enough. For $r_d = 0.2$ and for $T_0 - T_{sky} = 260 K$, the correction term $\Delta T_{up,max}$ is 6K. The effect gets stronger as the ruggedness of the terrain increases. In an area dominated by steep slopes the azimuthal average of $\cos^2 \theta_H$ within deep valleys may reach 0.5. Since such regions have limited lateral extent, their overall contribution to the radiometer pixel value may still not be very important. In a further situation we assume that besides a constant surface temperature $T_0$, the surface reflectivity $r_p$ is everywhere the same and given by $r_d$. Then, if the zenith opacity is still negligible, $\Delta T_{up}$ becomes

$$\Delta T_{up} = r_d (1 - r_d)(T_0 - T_{sky}) \cos^2 \theta_H$$

(28)

The difference with respect to (26) is the additional factor $(1-r_d)$. If the reflecting facet is a tilted surface we can still use the above expressions for $\Delta T_{up}$, however, with $\theta_H$ being the local angle of incidence of the horizon.

At low elevation angles, atmospheric emission is often not negligible. Increasing atmospheric emission increases $T_{sky}$ and thus reduces $\Delta T_{up}$ to values closer and closer to zero. Simulations of $\Delta T_{up}$ using DEM and realistic atmospheric data will quantify the actual behavior.

An example

As an illustration of pure relief effects (i.e. of the geometrical factors), let us consider the rugged terrain of southern Norway. The relief is shown by the DEM in Figure 5, and Figure 6 shows the computed values of $\cos^2 \theta_H$ whose range extends from 0 to about 0.3 with extremes up to 0.4. Roughly speaking, Figure 6 looks like a negative of Figure 5. However, on a closer look, there are significant differences. The main valleys and Fiords clearly seen in Figure 5 almost disappear in Figure 6 where the brightest areas are found in small and narrow valleys, e.g. in the upper left part of the image. The extended mountain area in the southern part are practically unaffected. The other parameters of interest are the local incidence angle $\theta$, and the polarization rotation angle $\phi$. Instead of $\theta$, its deviation from $\theta$ is shown in Figure 7 for southern Norway, and Figure 8 shows $\phi$ of the same region. These data are of interest if the reflectivity depends on incidence angle and polarization, i.e. for the specular
component \( r_{s,p} \), but not for lambertian surfaces. The radiometer aspect direction was taken from an average descending SSM/I orbit. Figures 6 to 8 were created from Figure 5 with a software package of Frew and Dozier (1986).

Conclusions
We considered relief effects on the upwelling brightness temperature \( T_{up} \) at the surface and on the brightness temperature \( T_b \) to be observed above the atmosphere. At frequencies of relevant atmospheric attenuation, major effects result from the variable atmospheric contributions due to their dependence on the altitude \( h \) of the emitting surface. Additional effects occur due to the shadowing of sky radiation by an elevated horizon. For lambertian surfaces this contribution can be estimated from \( \cos^2 \theta_H \). The formulas were derived for surfaces whose reflectivity can be described by a partly specular and a partly lambertian scattering behavior. The shadowing effects of a hilly terrain with an elevated horizon were computed for lambertian surfaces, and as an example the situation of southern Norway was shown. The effect can be expressed by an increase \( \Delta T_{up} \) of \( T_{up} \). The formulas for \( \Delta T_{up} \) are also valid if a specular component exists, as long as the specularly reflected rays do not lead to an additional increase of \( T_{up} \). A ray-tracing method is required to model multiple specular reflections. In addition to shadowing effects, tilted surfaces lead to changes in incidence angle and to a rotation of the plane of linear polarization.

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References
Figures to:
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Figure 1: (a) Microwave 94GHz and (b) visible relief effects at Lake of Thun with Mountain, Niesen (from Schaefer and Schanda, 1974)
Figure 2: Emitted brightness temperature $T_e$ in direction towards a radiometer above a horizontal surface of emissivity $e_p = 1 - r_p$ ($p = \text{polarization} \ h \text{ or } v$) at physical temperature $T_0$, illuminated from sky by $T_{\text{sky}}$.

Figure 3: Local $\theta_l$ and global $\theta$ incidence angles on a surface tilted by angle $\alpha$.

Figure 4: Profile through a simple landscape to illustrate the horizon at position $x$. 
Figure 5: Digital elevation model of southern Norway, resolution 1km (horizontal), 100m (vertical)

Figure 6: Values of ($\cos \theta_H$)$^2$ in southern Norway.

Figure 7: Change $\theta_l - \theta$ of the local incidence angle for SSM/I by the relief in southern Norway.

Figure 8: Rotation angle $\varphi$ of linear polarization for SSM/I by the relief in southern Norway.