

Hop Distances and Flooding in Wireless Multihop Networks with Randomized Beamforming

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ABSTRACT

We show that randomized beamforming is a simple yet efficient communication strategy in wireless multihop networks if no neighbor location information is at hand. Already small antenna arrays reduce the hop distance between nodes and speed up the flooding of messages. This result is obtained in different scenarios, using accurate models for circular antenna arrays and a line-of-sight link model between randomly placed nodes.

Categories and Subject Descriptors

C.2.1 [Computer-Communication Networks]: Network Architecture and Design—*Wireless communication, Network topology*

General Terms

Performance

Keywords

Wireless multihop networks, beamforming, adaptive antennas, antenna arrays, topology, hop distance, flooding

1. INTRODUCTION

The use of beamforming antennas in wireless multihop networks has gained increased interest in the research community [1–6]. Beamforming allows network nodes to focus the radiated power into a preferred direction, as opposed to transmitting in all directions. This directional transmission has the potential to increase the transmission range of the nodes and to improve the capacity of the network. Moreover, it enables nodes to suppress signals coming from certain directions, thus reducing the overall interference.

The application of beamforming antennas faces a tradeoff between these advantages and higher hardware costs, additional signal processing, and typically more complex proto-

cols. Several research results promise significant gains with beamforming in wireless multihop networks, but these gains are often based on the assumption that nodes have location information about neighboring nodes to adjust their antenna beam into the optimal direction (see, e.g. [1, 3, 4]). To obtain such information, significant signal processing efforts—e.g. algorithms for estimation of direction-of-arrival [7, 8] or channel covariance matrices for eigenbeamforming [9]—are required. In addition, protocols for coordination between nodes must be employed. We believe that such efforts are sometimes problematic in decentralized multihop networks and could outweigh the inherent benefits of beamforming.

We are thus interested in a scenario where nodes have no information at all about other nodes. If a node has no information, a naive and straightforward approach is to transmit in a random direction. We call this communication strategy *randomized beamforming*. Like omni-directional transmission, it does not require any direction estimation among nodes. Randomized beamforming is thus a very practical approach and has low complexity with respect to signal processing and protocols. Investigating randomized beamforming is further interesting since it can be used to obtain performance bounds. This is to say that, if no neighbor location information is at hand, nodes may perform randomized beamforming; once they obtain information about their neighbors, they can optimize the beamforming.

Most publications on beamforming in wireless multihop networks focus on the design of medium access protocols. It was shown in [1] that the distributed coordination function of IEEE 802.11 does not perform well if beamforming antennas are deployed. Thus, various enhancements of 802.11 have been suggested; the authors' paper [6] provides a survey on this issue. Additional work on beamforming in multihop networks addresses the design of neighbor discovery [10], routing and multicasting [11–13], and capacity issues [14, 15].

Interestingly, the actual changes in the *network topology*—i.e. the properties of the network graph resulting from beamforming, in comparison to the properties of the graph resulting from omni-directional transmission—have not been studied in depth so far. This lack of research is very surprising since the network topology has significant impact on various performance measures, such as routing optimality, end-to-end delay, reachability, and capacity. A thorough understanding of the changes in topology properties is crucial to assess the benefits of beamforming and to answer the question: Are the gains really so significant that they compensate the higher hardware costs as well as the increased complexity and development efforts?

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In a recent paper, we made a first step toward an analysis of network topology properties, where we compared randomized beamforming with omni-directional transmission using the same transmission power [16]. We found that randomized beamforming significantly improves the level of connectivity among nodes. As a measure for connectivity we used the *path probability*; it is defined as the probability that two randomly selected nodes can communicate via a multihop or singlehop path. We found that already beamforming antennas with a small number of antenna elements increase the path probability significantly.

Although this result gave a first insight on the gains in connectivity that can be achieved with beamforming, several fundamental questions remained unanswered and are addressed in this follow-up paper:

- What is the impact of directional transmission on the *hop distance* between two nodes?
- What is the impact on the *network diameter*, i.e. the maximum hop distance in a network?
- How does the impact on hop distances carry over to *flooding* messages?

An answer to these questions is by far not obvious. For example, one could argue that the longer links appearing when deploying beamforming increase the connectivity, but may lead to “zigzag” routes and thus to increased hop distances and slower flooding. Existing work on hop distances [17–19] does not give an answer to these issues, since it mainly focuses on omni-directional antennas.

The main goal of this paper is to assess the benefits of randomized beamforming with respect to hop distances and message flooding in comparison to omni-directional antennas. Section 2 explains the used network model and simulation aspects. Section 3 analyzes three measures for the hop distance between nodes. Next, Section 4 studies the impact of beamforming on flooding. Finally, Section 5 concludes and gives an outlook on future work. Throughout the text, we use standard terminology from antenna theory [20]. Random variables are written in upper-case letters, specific realizations in lower-case letters.

2. MODELING AND SIMULATION

In the following we provide an overview of the used network models and simulation methodology. The network models are equivalent to those described in detail in [16].

2.1 Antenna and Beamforming Model

We assume that each node is equipped with a *uniform circular array* (UCA) comprising m antenna elements. We model the elements as isotropic radiators transmitting with power p_t/m at frequency f . The spacing between neighboring elements, Δ , is half the wavelength of the carrier frequency and can be expressed as $\Delta = c/(2f)$ with $c = 3 \cdot 10^8$ m/s. As an example for the space required for such an array antenna, consider a carrier frequency of $f = 5$ GHz yielding $\Delta = 3$ cm. For a UCA with four elements, this yields an array diameter of 4.2 cm. The beamforming strategy that we use is a simple phase shift approach, i.e. given a chosen target direction θ_b (*boresight direction*), a phase shift between neighboring antenna elements is introduced,

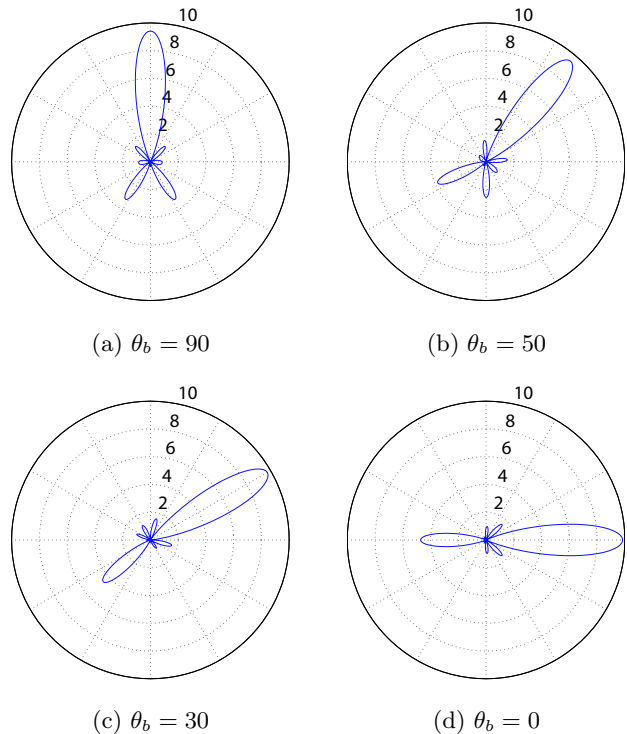


Figure 1: Gain patterns of UCA with 10 elements

such that the maximum gain is achieved towards direction θ_b . Examples in Fig. 1 illustrate that the resulting beam patterns consist of a main lobe and several side lobes with lower gain. The specific shape of the resulting pattern depends on the chosen boresight direction θ_b and the number of antenna elements m . Generally it can be stated that with increasing m the maximum gain of the main lobe increases while its width decreases and the side lobes become smaller. The antenna patterns are effective for both transmission and reception.

2.2 Link Model

This section describes the used link model. It determines whether a link between two specific nodes exists, given the antenna patterns of both nodes, their distance, the transmit power p_t , and the receiver sensitivity $p_{r,0}$. The link model is illustrated in Fig. 2. The received power p_r at the receiving node is obtained by multiplying the transmit power with the overall link gain, which comprises the respective antenna

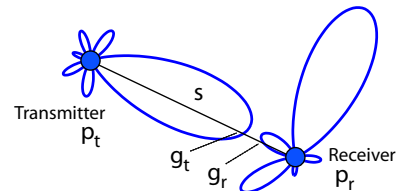


Figure 2: Link model. Both transmission and reception are directional

gains of the transmitter as well as the receiver and the path gain (the reciprocal value of the path loss). Thus, applying a path loss coefficient of α , we can write for the overall link gain

$$\frac{p_r}{p_t} = g_t g_r \left(\frac{s}{1 \text{ m}} \right)^{-\alpha}, \quad (1)$$

where g_t represents the antenna gain of the transmitting node in the direction toward the receiver and g_r is the antenna gain of the receiving node that is effective toward the sending node. It is important to note that, according to our concept of randomized beamforming which will be explained in more detail later, each node uses one fixed beam pattern for both reception and transmission. The pathloss coefficient α must be chosen according to the modeled scenario: $\alpha=2$ applies to a free space scenario, while values of $\alpha=3 \dots 5$ are typical for urban environments. Our model does not include fading and thus rather represents mean values. The reciprocal value of (1) is the attenuation a , which can be expressed in terms of decibel as $a = 10 \log \frac{p_t}{p_r}$ dB. A link between two nodes is established if the received power p_r is above the receiver sensitivity p_{r0} . Assuming that all nodes use the same transmit power p_t and have the same receiver sensitivity p_{r0} , we can state that a link between two nodes exists if the attenuation between them is smaller or equal to the threshold attenuation $a_0 = 10 \log \frac{p_t}{p_{r0}}$ dB. The assumption of homogeneous nodes (same p_t and same p_{r0}) together with the fact that the same pattern is used for transmission and reception gives rise to *bidirectional* (undirected) links. If a node A can receive signals from a node B properly, then node B can consequently receive signals from node A at sufficient power level as well.

2.3 Randomized Beamforming

As stated above, in order to avoid complex adaptive beamforming algorithms and protocol overhead, this paper investigates a strategy that we name *randomized beamforming* or *random direction beamforming*: Each node randomly chooses a boresight direction θ_b uniformly distributed in the interval $[0, 2\pi[$ and applies the resulting beam pattern according to the phase shift beamformer described above. Given this beamforming approach, the shape of the resulting pattern is fully described by the chosen boresight direction relative to the antenna array. The main lobe direction in the system plane is finally given by taking into account the boresight direction and the array orientation. Once chosen, the pattern for each node is fixed and used for transmission as well as for reception.

2.4 Spatial Node Distribution and Scenarios

We consider a square system area of length l . Within this area, n nodes are placed independently and randomly according to a uniform distribution. The physical orientation of each node's antenna array in the plane is randomly chosen from a uniform distribution on the interval $[0, 2\pi[$ and afterward, random direction beamforming is applied in each node to create links.

We are interested in a comparison of omni-directional antennas and UCAs for different parameters n , l , a_0 , and α . To reduce the number of variable parameters, we define a set of three scenarios, which are shown in Table 1. All scenarios have a node density of $n/l^2 = 5000 \text{ km}^{-2}$. With

$a_0 = 40 \text{ dB}$ and $\alpha = 3$, this results in a path probability close to 100 % [16].

Table 1: Scenarios

	Small	Medium	Large
n	50	1250	5000
l	100 m	500 m	1000 m
a_0	40 dB	40 dB	40 dB
α	3	3	3

Example topologies are shown in Fig. 3. For each of the above scenarios, we include topologies for nodes with omni-directional antennas, 4-element UCAs (UCA4), and 10-element UCAs (UCA10). In the topologies with omni-directional antennas, each node has a link to all nodes that are located within a certain distance. In the topologies with beamforming antennas, some of these links are taken away, but in turn some links to nodes further away are added.

While in the illustrations of Fig. 3 all nodes are devices forming a purely self-organized network, we are also interested in topology aspects of multihop access networks to fixed infrastructure. For this investigation, we placed infrastructure nodes (access points or *base stations*) in a regular fashion in the network area. As illustrated in Fig. 4, one, nine and 25 base stations were placed in the small, medium and large network scenario, respectively.

2.5 Simulation

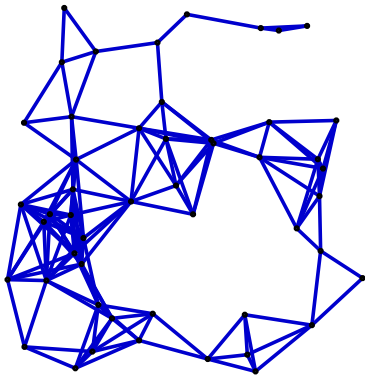
In the sequel, we analyze different hop distance measures as well as the behavior of flooding in the specified scenarios. For this purpose we apply computer simulations using a simulation tool developed in C++ language, incorporating the above models for beamforming and link establishment. All topology plots presented in Fig. 3 are directly taken from the graphical user interface of our simulation tool. All results are estimates generated from a large set of random topologies, with an inherent confidence level of 95% for a confidence interval of 5%. Hereby we assume that the data samples are independent and Gaussian distributed. Section 3.1 explains how the estimates are obtained from analysis of a large number of random topologies. The methodology applied there is used for all further results in this paper as well.

3. ANALYSIS OF HOP DISTANCE

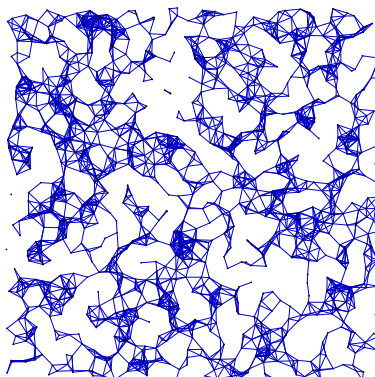
The number of hops that a message traverses between its source and destination has a direct impact on the end-to-end delay of this transmission. Each node in the chain causes various types of delay, such as coding and medium access delay. The purpose of this section is to analyze three different hop distance measures: (a) the number of hops between two random nodes, (b) the network diameter, and (c) the number of hops from a random node to its closest base station in a multihop access network.

3.1 Hop Distance Between Two Nodes

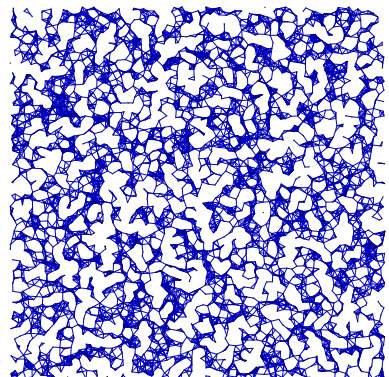
The *hop distance* $h \in \mathbb{N}$ between two given nodes is the minimum number of hops between them. In other words, the two nodes can communicate via at least one h -hop path



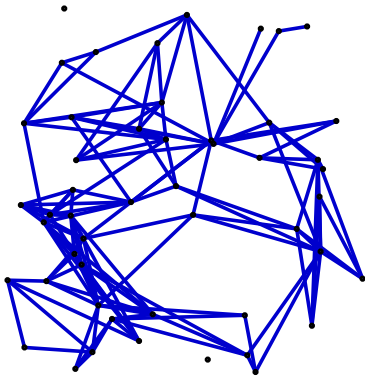
(a) 50 nodes, OMNI



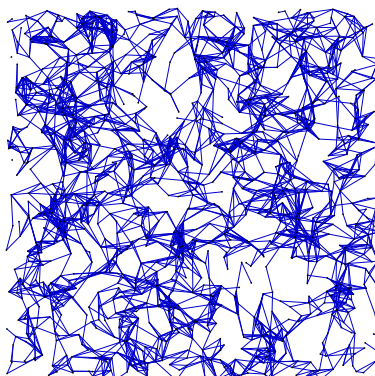
(b) 1250 nodes, OMNI



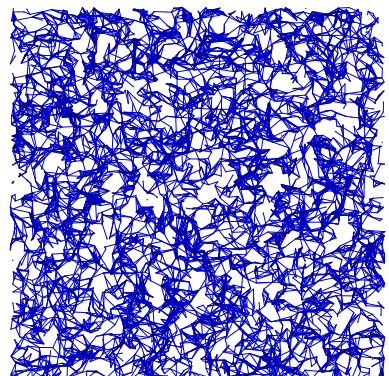
(c) 5000 nodes, OMNI



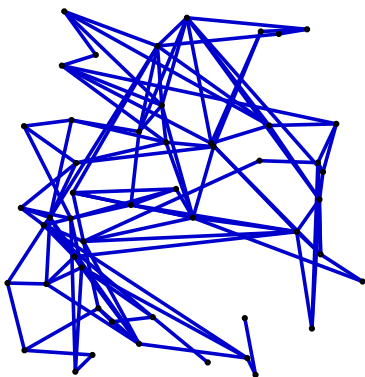
(d) 50 nodes, UCA4



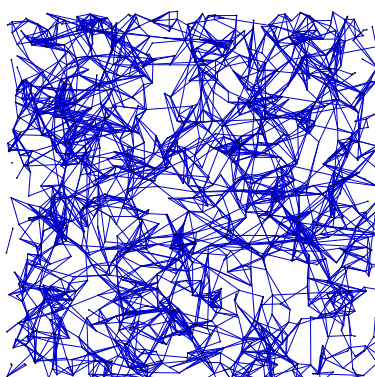
(e) 1250 nodes, UCA4



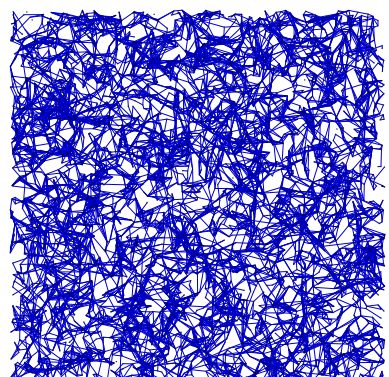
(f) 5000 nodes, UCA4



(g) 50 nodes, UCA10



(h) 1250 nodes, UCA10



(i) 5000 nodes, UCA10

Figure 3: Sample random topologies for the three scenarios small, medium, and large, as defined in Table 1

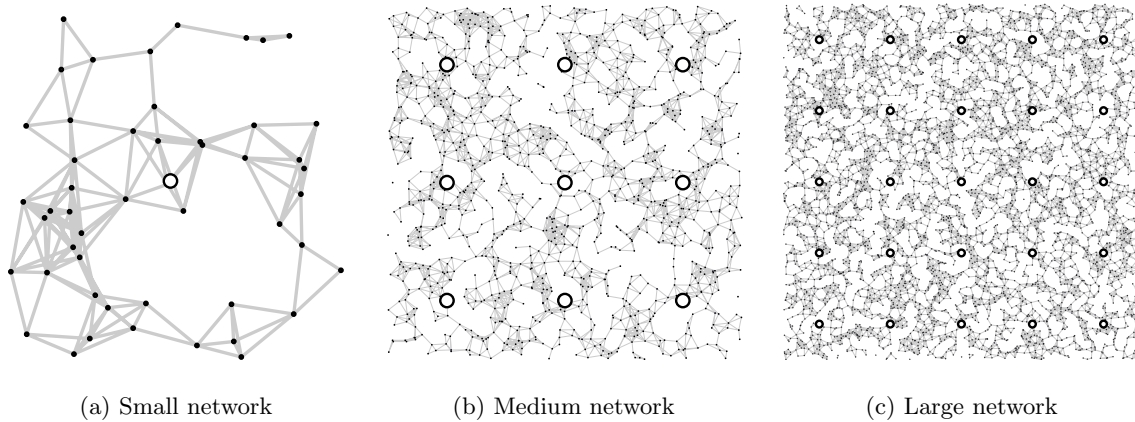


Figure 4: Placement of base stations in multihop access network

(consisting of h links), but there is no path between them with less than h hops. The hop distance between two disconnected nodes is undefined. The hop distance between two randomly chosen nodes in a random network is described by the random variable H . This section analyzes the probability mass function $P(H = h)$ and the expected hop distance $E\{H\}$.

The goal is to obtain a very good estimate of $P(H = h)$. To do so, we generate a random topology \mathcal{T}_1 , measure the hop distances from each node to each other node, and enter these distances into a histogram $p_1(h)$. The histogram represents the hop distance distribution of the given topology \mathcal{T}_1 . Unconnected node pairs are not counted. We repeat this process for many random topologies $\mathcal{T}_1 \dots \mathcal{T}_\Omega$, such that the sample average

$$\frac{1}{\Omega} \sum_{i=1}^{\Omega} p_i(h) \quad \forall h \in \mathbb{N} \quad (2)$$

will converge toward the probability $P(H = h)$. Since the simulation results fulfill the above confidence criteria, we use the notation for the probability, $P(H = h)$, also for its estimate.

Fig. 5 depicts the resulting $P(H = h)$ for the small, medium, and large scenarios, comparing omni-directional, UCA4, and UCA10 antennas. The results show that the use of randomized beamforming lowers the probability of high hop distances. In the small network scenario, hop distances $h \geq 5$ occur less likely with beamforming than with omni-directional antennas. In turn, small hop distances $h = 2, 3$, and 4 occur with a higher probability. These gains become much higher with an increasing size of the network. In the medium scenario, the most likely hop distance with omni-directional antennas is $h = 18$, while nodes with UCA4 antennas can typically communicate via $h = 10$ hops and nodes with UCA10 via only $h = 8$ hops. These gains are emphasized by Table 2, which shows the expected value of H in the given scenarios. Using only four antenna elements reduces $E\{H\}$ by more than 40%.

In summary, beamforming antennas significantly improve the statistics of the hop distance, especially for medium and large networks.

Table 2: Expected Hop Distance $E\{H\}$

	Small	Medium	Large
OMNI	3.87	18.74	35.97
UCA4	3.26	11.18	20.60
	(-16 %)	(-40 %)	(-43 %)
UCA10	3.06	8.75	15.41
	(-21 %)	(-53 %)	(-57 %)

3.2 Network Diameter

The *diameter* $d \in \mathbb{N}$ of a given network topology is the maximum hop distance over all node pairs in this topology. In other words, it is the number of links of the “longest shortest path” between any two nodes in the topology. The random variable representing the diameter of a random topology is denoted by D .

Fig. 6 shows the simulation results for the probability mass function $P(D = d)$. As expected, beamforming reduces the network diameter. For example, the typical diameter of a large network with UCA10 antennas has less than half the diameter of the same network with omni-directional antennas. Since the network diameter determines the maximum occurring delay in the transmission between two nodes, this result nicely illustrates the benefits of randomized beamforming.

3.3 Hop Distance Between a Node and its Closest Base Station

Let us now consider a scenario in which a wireless multihop network is connected to fixed infrastructure (Fig. 7). The variable $k \in \mathbb{N}$ defines the hop distance between a given node and its closest base station, i.e. the base station located at minimum hop distance to this node. The hop distance between a randomly chosen node and its closest base station in a random network is described by the random variable K .

The resulting probability mass functions $P(K = k)$ and the expected distances $E\{K\}$ are shown in Fig. 7 and Table 3. As with H and D , randomized beamforming improves the statistics of K significantly. This means that nodes can

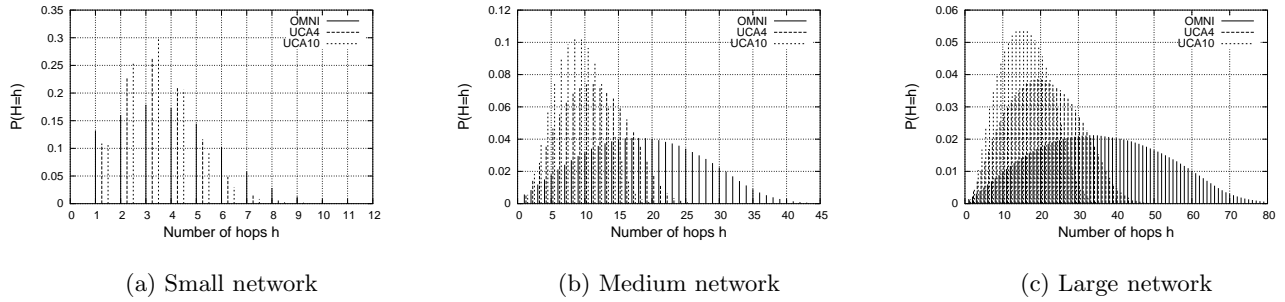


Figure 5: Probability mass function $P(H = h)$ of the hop distance h between two random nodes

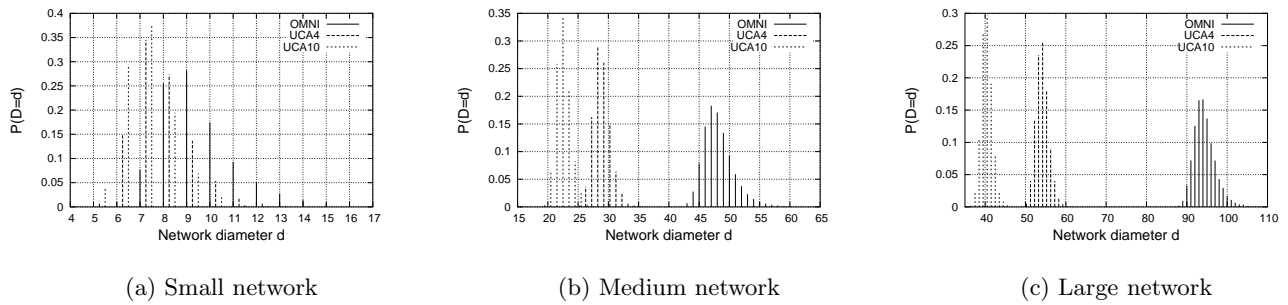


Figure 6: Probability mass function $P(D = d)$ of the network diameter d

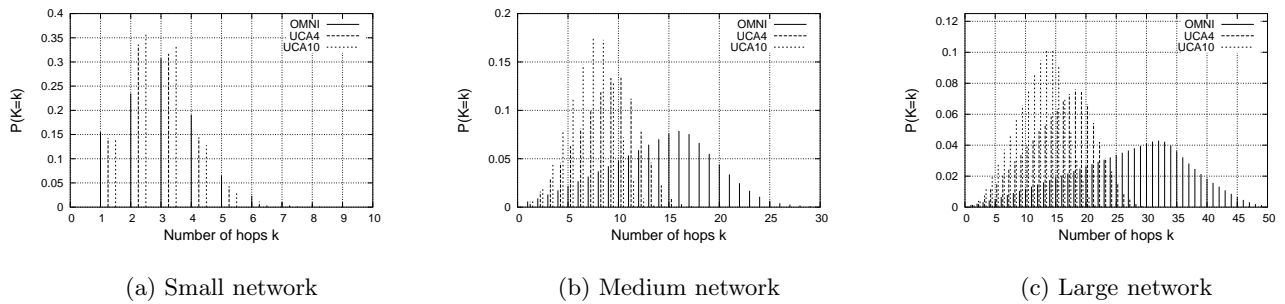


Figure 7: Probability mass function $P(K = k)$ of the hop distance k between a random node and its closest base station

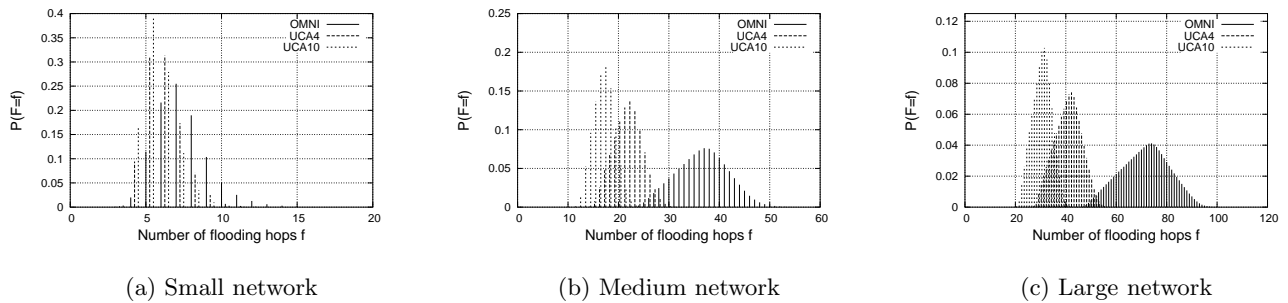


Figure 8: Probability mass function $P(F = f)$ of the number of hops f needed to flood the network

access infrastructure much faster than with omni-directional antennas.

Table 3: Expected Hop Distance $E\{K\}$

	Small	Medium	Large
OMNI	2.96	14.10	26.88
UCA4	2.67	8.59	15.59
	(-10%)	(-39%)	(-42%)
UCA10	2.58	6.90	11.86
	(-13%)	(-51%)	(-56%)

4. ANALYSIS OF FLOODING

Flooding is used for several purposes in wireless multihop networks. It is needed for broadcasting information in a geographical region, route discovery in on-demand routing protocols, and sending data to a destination at unknown location. This section investigates the performance of flooding when randomized beamforming is deployed. We are especially interested in how the impact of randomized beamforming on hop distances carries over to flooding.

A flooding hop denotes the process of transferring a message to all neighbors, if the same message has not been forwarded before. In this sense, flooding is treated as time-discrete process observed stepwise without consideration of MAC-layer issues. Thus, after one flooding step, all one-hop neighbors are assumed to have received the flooded message. Collisions due to multi-point-to-point transmissions are ignored.

As shown in Section 3.2, randomized beamforming reduces the network diameter. It can already be derived from this result that randomized beamforming improves the worst case for flooding, i.e. the case where a node located at the border of the network starts flooding a message. As a generalization of this, we investigate the case where any node of the network may start the flooding process.

The basic measure for evaluation is the *number of hops needed to flood* the network, denoted by $f \in \mathbb{N}$. After f flooding hops, all nodes in the network have received the flooded message. The random variable F , describing the number of hops needed for flooding, is evaluated by a simulative estimation of the probability mass function $P(F = f)$.

Resorting to beamforming for the purpose of flooding may be counterintuitive in the first place. Provided that the network is connected when using omni-directional antennas, one might think that omni-directional transmission is better suited for flooding information. In particular, if flooding is started at a node located in the center of the network, it seems that omni-directional transmission lets the flooded message advance more effectively in all directions than in the beamforming case.

Surprisingly though, as Fig. 8 shows, randomized beamforming evidently *reduces* the number of hops needed to flood a network, in particular in large networks. This reduction in flooding hops corresponds to faster message distribution and reduced signal processing in devices. Figs. 6 and 8 reflect statistically that, for a given topology, f is upper-bounded by d .

It should again be mentioned that in this investigation of flooding — like in all previous sections of this paper — nodes

do not adjust their beamforming pattern when forwarding a flooded message. The pattern remains unchanged for all packets, and remains unchanged when switching between transmission and reception.

5. CONCLUSIONS

The application of beamforming antennas has beneficial impact on the topology properties of wireless multihop networks. Our previous work showed that — even if nodes have no information about their neighbors — beamforming into a random direction increases the network connectivity in terms of path availability [16]. This paper showed that randomized beamforming is further beneficial in reducing hop distances and speeding up the flooding of messages. Our study uses accurate antenna models with a line-of-sight channel model between randomly placed nodes. The mentioned benefits lead to an increased hardware cost for small antenna arrays, but do not require any location estimation or coordination among nodes.

The assumption that nodes do not know anything about their neighbors means that randomized beamforming represents, in some way, a worst case analysis. For example, randomized beamforming can be used in the wake-up phase of a node for initial communication with other nodes. Once a node has obtained information about its neighbors, however, it might optimize its transmission by additional signal processing (e.g. eigenbeamforming or direction-of-arrival estimation). If these techniques are employed, the gains in path probability, hop distance, and flooding are expected to be even higher.

We strongly believe that the use of beamforming has great potential in various types of wireless multihop networks, e.g. networks between mobile robots or sensor buoys on sea. In current work, we perform an analysis of on-demand routing in a beamforming scenario, where we consider different medium access control schemes. Also a thorough analytical study of topology properties with randomized beamforming remains to be done.

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