Generalized frequency dependence of output Stokes parameters in an optical fiber system with PMD and PDL/PDG

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Abstract: Dependence of output optical power, Stokes vector and degree of polarization on optical frequency is presented for an optical fiber system with both polarization mode dispersion and polarization-dependent loss or gain. The newly formulated equations are generalized for input light with arbitrary degree of polarization. The spectral resolved measurements of polarization mode dispersion using partially polarized light agree well with our theory.

OCIS codes: (060.2310) Fiber optics; (060.2300) Fiber measurement; (060.2400) Fiber properties; (260.5430) Polarization.

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1. Introduction

In an optical fiber system with both polarization mode dispersion (PMD) and polarization-dependent loss or gain (PDL/PDG), a PMD equation, $d\vec{S}/d\omega = \vec{\Omega}\times\vec{S} - (\vec{\Lambda}\times\vec{S})\times\vec{S}$, was proposed to show that the frequency dependence of the normalized 3D output Stokes vector $\vec{S}$ is governed by the principal states of polarization (PSP) vector $\vec{W} = \vec{\Omega} + i\vec{\Lambda}$ [1]. This equation is considered to be the fundamental tool to study PMD effect in an ultra-high speed

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optical fiber communication system. However this equation only considers completely polarized input light. In general, the input light can have an arbitrary degree of polarization (DOP) from 0 to 1. Furthermore, this equation doesn't give any information about the frequency dependence of output optical power. A rigorous study of frequency dependence of the complete four output Stokes parameters by considering arbitrarily polarized input light is necessary for some optical fiber communication systems and sensor systems. In addition, for an optical fiber link with PDL/PDG, PMD measurement using the generalized Poincaré sphere method proposed in [2] can also be problematic. This is due to the fact that the method assumes the input light is completely polarized. However, this assumption is not always correct. For example, the input light amplified using an EDFA is not 100% in DOP. The purpose of this work is to explore the frequency dependence of the output optical power, normalized 3D Stokes vector and DOP in an optical fiber system with PMD and PDL/PDG while the input light has arbitrary DOP. We also confirm in experiment that, DOP of input light does have influence to the measurement of PMD, and consideration of our model is necessary to minimize error induced by conventional measurement method.

2. Frequency dependence of Stokes parameters and DOP

We concern light transmitting through an optical system formed by conventional single-mode fiber, where index contrast between core and cladding is small so that the weakly-guiding approximation is valid. Such optical fiber system normally has PMD and PDL/PDG, which transforms a completely polarized light into a completely polarized one, so it belongs to the nondepolarizing optical system [3]. Such system can be described by a nondepolarizing Mueller matrix (also called Mueller-Jones matrix) [4]. In Stokes space, the input Stokes parameters $\boldsymbol{S}_{in} = \left(s_{in0}\ s_{in1}\ s_{in2}\ s_{in3}\right)^T$ and the output Stokes parameters $\boldsymbol{S} = \left(s_0\ s_1\ s_2\ s_3\right)^T = \left(s_0\ \vec{s}_F\right)^T$ are linearly related by

$$\boldsymbol{S} = \boldsymbol{M}\boldsymbol{S}_{in} \tag{1}$$

Here the superscript T denotes a transposition, $\boldsymbol{M} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix}$.

It has been found that the nondepolarizing Mueller matrices satisfy the Lorentz transformation (Lorentz Group)[5]. In such optical fiber system, we have [6]

$$\boldsymbol{M}^T\boldsymbol{G}\boldsymbol{M} = \sqrt{\det \boldsymbol{M}}\,\boldsymbol{G} \tag{2}$$

Here $\boldsymbol{G} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ is called Minkowski metric. Based on Eq.(2), we may obtain the inverse matrix of $\boldsymbol{M}$ as follows under the condition that $\det \boldsymbol{M} \neq 0$

$$\boldsymbol{M}^{-1} = \frac{1}{\sqrt{\det \boldsymbol{M}}}\boldsymbol{G}\boldsymbol{M}^T\boldsymbol{G} = \begin{pmatrix} m_{11} & -m_{21} & -m_{31} & -m_{41} \\ -m_{12} & m_{22} & m_{32} & m_{42} \\ -m_{13} & m_{23} & m_{33} & m_{43} \\ -m_{14} & m_{24} & m_{34} & m_{44} \end{pmatrix} / \sqrt{\det \boldsymbol{M}} \tag{3}$$

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Then we have ten independent bilinear equations from $\boldsymbol{M}\boldsymbol{M}^{-1}=\boldsymbol{I}$ ($\boldsymbol{I}$ is 4×4 identity matrix) as

$$\begin{cases} m_{11}{}^2 - m_{12}{}^2 - m_{13}{}^2 - m_{14}{}^2 = \sqrt{\det \boldsymbol{M}} \\ m_{21}{}^2 - m_{22}{}^2 - m_{23}{}^2 - m_{24}{}^2 = -\sqrt{\det \boldsymbol{M}} \\ m_{31}{}^2 - m_{32}{}^2 - m_{33}{}^2 - m_{34}{}^2 = -\sqrt{\det \boldsymbol{M}} \\ m_{41}{}^2 - m_{42}{}^2 - m_{43}{}^2 - m_{44}{}^2 = -\sqrt{\det \boldsymbol{M}} \\ m_{11}m_{21} - m_{12}m_{22} - m_{13}m_{23} - m_{14}m_{24} = 0 \\ m_{11}m_{31} - m_{12}m_{32} - m_{13}m_{33} - m_{14}m_{34} = 0 \\ m_{11}m_{41} - m_{12}m_{42} - m_{13}m_{43} - m_{14}m_{44} = 0 \\ m_{21}m_{31} - m_{22}m_{32} - m_{23}m_{33} - m_{24}m_{34} = 0 \\ m_{21}m_{41} - m_{22}m_{42} - m_{23}m_{43} - m_{24}m_{44} = 0 \\ m_{31}m_{41} - m_{32}m_{42} - m_{33}m_{43} - m_{34}m_{44} = 0 \end{cases} \tag{4}$$

The prerequisite for the above equations is that there is no polarizer in the system, which means $\det \boldsymbol{M} > 0$ to the optical fiber system.

When the input Stokes parameters are fixed, *i.e.*, independent to optical frequency and time, the frequency dependence of Stokes parameters should meet

$$\frac{d\boldsymbol{S}}{d\omega} = \frac{d\boldsymbol{M}}{d\omega}\boldsymbol{M}^{-1}\boldsymbol{S} \tag{5}$$

Based on the 10 bilinear equations in Eq.(4) and the inverse matrix in Eq.(3), we easily obtain

$$\frac{d\boldsymbol{M}}{d\omega}\boldsymbol{M}^{-1} = \begin{pmatrix} \eta_\omega & \Lambda_1 & \Lambda_2 & \Lambda_3 \\ \Lambda_1 & \eta_\omega & -\Omega_3 & \Omega_2 \\ \Lambda_2 & \Omega_3 & \eta_\omega & -\Omega_1 \\ \Lambda_3 & -\Omega_2 & \Omega_1 & \eta_\omega \end{pmatrix} \tag{6}$$

where

$$\eta_\omega = \frac{d\eta}{d\omega} = \frac{d \ln \sqrt[4]{\det \boldsymbol{M}}}{d\omega} \tag{7}$$

$$\begin{cases} \Lambda_{q-1} = \left(-m'_{11}m_{q1} + m'_{12}m_{q2} + m'_{13}m_{q3} + m'_{14}m_{q4}\right)/\sqrt{\det \boldsymbol{M}} \quad q = 2,3,4 \\ \Omega_1 = \left(-m'_{41}m_{31} + m'_{42}m_{32} + m'_{43}m_{33} + m'_{44}m_{34}\right)/\sqrt{\det \boldsymbol{M}} \\ \Omega_2 = \left(-m'_{21}m_{41} + m'_{22}m_{42} + m'_{23}m_{43} + m'_{24}m_{44}\right)/\sqrt{\det \boldsymbol{M}} \\ \Omega_3 = \left(-m'_{31}m_{21} + m'_{32}m_{22} + m'_{33}m_{23} + m'_{34}m_{24}\right)/\sqrt{\det \boldsymbol{M}} \end{cases} \tag{8}$$

Here the prime denotes the derivative with respect to optical frequency. $\vec{\Omega} = (\Omega_1\ \Omega_2\ \Omega_3)^T$, $\vec{\Lambda} = (\Lambda_1\ \Lambda_2\ \Lambda_3)^T$ and their elements are defined in Eq.(8).

Using Eq.(5) and Eq.(6), we have

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$$\begin{cases} \dfrac{ds_0}{d\omega} = \vec{\Lambda}\cdot\vec{s}_F + s_0\eta_\omega \\ \dfrac{d\vec{s}_F}{d\omega} = s_0\vec{\Lambda} + \vec{\Omega}\times\vec{s}_F + \eta_\omega\vec{s}_F \end{cases} \tag{9}$$

These equations show the frequency dependence of four output Stokes parameters. The unknowns are $\vec{\Omega}$, $\vec{\Lambda}$, as well as the determinant of the Mueller matrix.

For a quasi-monochromatic input light with arbitrary DOP, if we use D to denote the DOP of the output light, from Eq.(9), we may further get the following equations

$$\begin{cases} \dfrac{d\vec{s}_N}{d\omega} = \vec{\Omega}\times\vec{s}_N + \vec{\Lambda} - \left(\vec{\Lambda}\cdot\vec{s}_N\right)\cdot\vec{s}_N \\ \dfrac{d\hat{s}}{d\omega} = \vec{\Omega}\times\hat{s} - \dfrac{\left(\vec{\Lambda}\times\hat{s}\right)\times\hat{s}}{D} \\ \dfrac{dD}{d\omega} = \left(1-D^2\right)\vec{\Lambda}\cdot\hat{s} = \dfrac{1-D^2}{D}\vec{\Lambda}\cdot\vec{s}_N \end{cases} \tag{10}$$

Here $\vec{s}_N = \dfrac{\vec{s}_F}{s_0}$, $\hat{s} = \dfrac{\vec{s}_F}{\left|\vec{s}_F\right|}$, $D = \dfrac{\left|\vec{s}_F\right|}{s_0'}$. These equations show the frequency dependence of the 3D Stokes vector $\vec{s}_N$, the normalized 3D Stokes vector $\hat{s}$ and DOP. If $D = 1$, we may find from Eq.(10) that two vectors $\vec{\Omega}$ and $\vec{\Lambda}$ we defined in Eq.(8) are just the two parts of PSP vector presented by Gisin and Huttner [7]. To the best of our knowledge, Eq.(9) and (10) are derived for the first time.

3. DOP effect to the measurement of PMD

If the input light is not completely polarized, from Eq.(10), we may obtain the following equations:

$$\begin{cases} \vec{\Lambda}\cdot\left[\dfrac{\hat{s}_2}{D_1} + \dfrac{\hat{s}_1}{D_2} - \left(\hat{s}_1\cdot\hat{s}_2\right)\left(\dfrac{\hat{s}_1}{D_1} + \dfrac{\hat{s}_2}{D_2}\right)\right] = \hat{s}_1\cdot\dfrac{d\hat{s}_2}{d\omega} + \hat{s}_2\cdot\dfrac{d\hat{s}_1}{d\omega} \\ \vec{\Omega}\cdot\left\{\left[D_2 - D_1\left(\hat{s}_1\cdot\hat{s}_2\right)\right]\hat{s}_1 + \left[D_1 - D_2\left(\hat{s}_1\cdot\hat{s}_2\right)\right]\hat{s}_2\right\} = \left(D_2\dfrac{d\hat{s}_2}{d\omega} - D_1\dfrac{d\hat{s}_1}{d\omega}\right)\cdot\left(\hat{s}_1\times\hat{s}_2\right) \end{cases} \tag{11}$$

Here, the subscripts ‘1’ and ‘2’ denote two outputs under two different inputs. From Eqs.(11), we can employ partially polarized or even unpolarized light to measure PMD in optical fiber system. Also, we notice, if the DOP of input light is not 1, certain amount of error will be induced if we still follow the traditional generalized Poincaré sphere method in Ref.[2] to calculate PMD.

4. Experimental verification

Figure 1 shows the schematics of the experimental setup, which consists of a tunable laser, a fiber depolarizer, a polarization controller (PC) and a polarimeter.

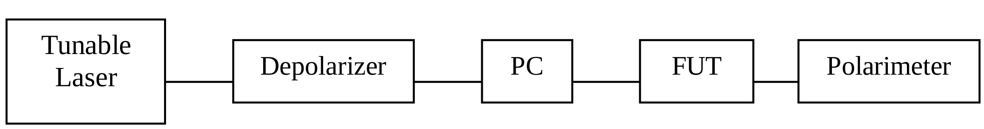

Fig. 1. Experimental setup (PC: polarization controller, FUT: fiber under test)

#8653 - $15.00 USD Received 1 September 2005; revised 20 October 2005; accepted 18 October 2005

Fiber under test (FUT) in our experiment consists of two 3m-long panda-type polarization-maintaining fiber sections whose beat lengths are about 3mm. One of them is coiled to 1.7cm in diameter, so the bending-induced PDL is generated [8]. The scanning wavelength range is from 1549nm to 1551nm with 0.1nm step size. The polarimeter can measure the output optical power, the normalized 3D Stokes vector and DOP simultaneously. The linewidth of our tunable laser is 50MHz so that the DOP of the input light is 35% after passing the depolarizer. Here we purposely use an input light with lower DOP because we want to show the error induced by negligence of DOP more clearly. By tuning PC, we set three different input polarization states. We don't need to know exactly these input polarization states, since actually the three input polarization states can be arbitrary as long as the corresponding output polarization states are not coplanar in Stokes space [2].

Figure 2 shows the wavelength dependence of the output optical power and DOP at one fixed input. Two traces are observed to have similar behavior. To explain this, we can obtain the relationship between optical power and DOP as $\frac{D}{1-D^2}\frac{dD}{d\omega}=\frac{d\ln s_0}{d\omega}-\eta_\omega$ from Eq.(9) and Eq.(10).

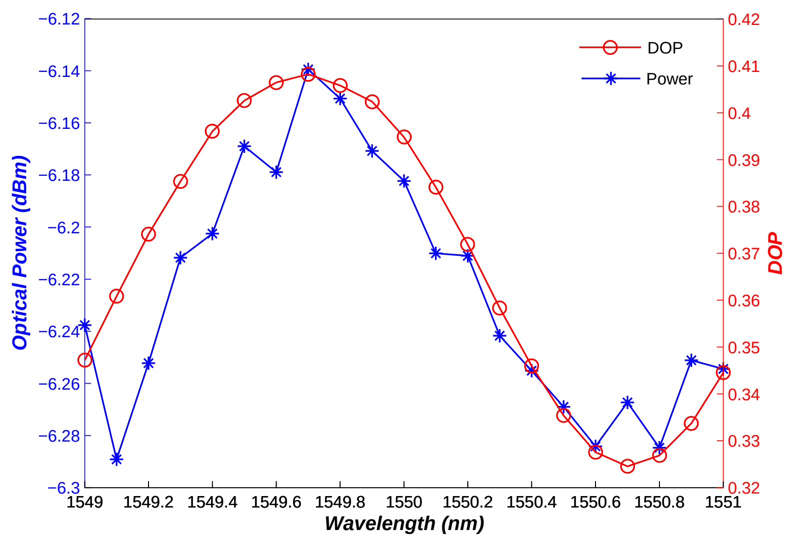

Fig. 2. Wavelength dependence of output optical power and DOP

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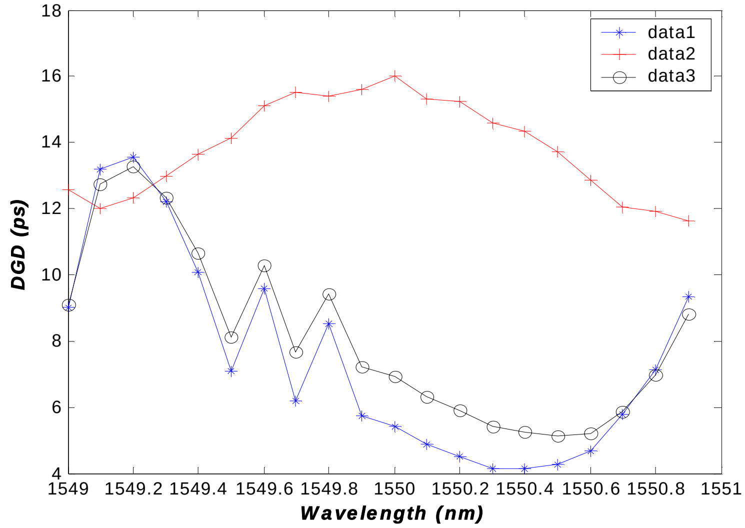
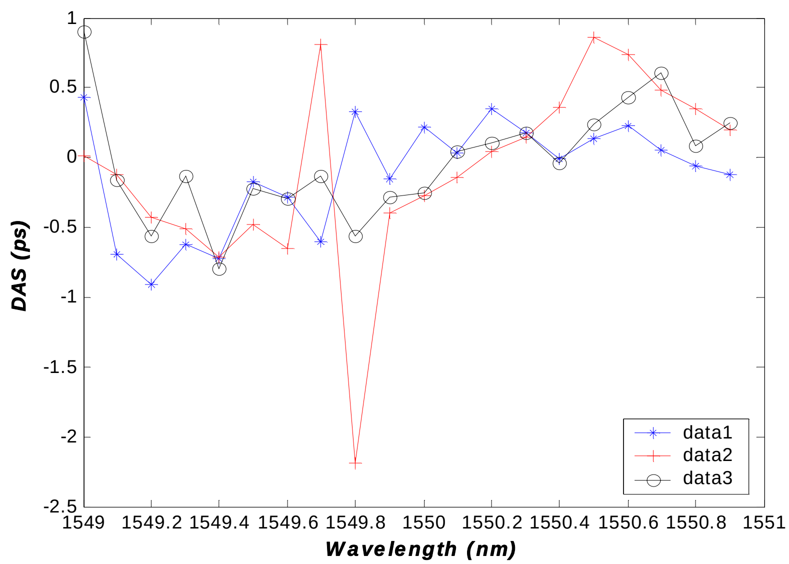


(a)



(b)

Fig. 3. Wavelength dependences of measured (a) DGD and (b) DAS

Figure 3 shows the measured differential group delay (DGD) and differential attenuation slope (DAS) [2], which are calculated from the measured PSP vector [2]. In Fig. 3(a) and (b), the black lines (data3) show the measured DGD and DAS, respectively, by using completely polarized input lights. The generalized Poincaré sphere method is employed. They are the benchmark to verify our theory. The red (data2) and blue (data1) lines show the results measured by using partially polarized input lights. In particular, the red lines are calculated by using the original generalized Poincaré sphere method, namely using normalized 3D Stokes vectors directly without considering DOP of output light; the blues lines are calculated by using Eq.(11), in which the DOP of output light is taken into account. We may see there exist

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obvious discrepancies between the red and the black lines, while the blue and the black lines are observed to have better agreement. This confirms the validity of our theoretical findings.

5. Conclusion

We studied the frequency dependence of all four Stokes parameters in the optical fiber system with PMD and PDL/PDG. Several governing equations are deduced to show that the frequency dependence of the four Stokes parameters is governed by the PSP vector and the determinant of Mueller matrix. Furthermore, we obtained the governing equations on frequency dependence of the 3D Stokes vector, normalized 3D Stokes vector and DOP. These equations are generalized for the input light with arbitrary DOP. In order to verify our theory, experiments are carried out to measure PMD in a PMF link with bending-induced PDL. The results show that we have to consider DOP effect if the input light is partially polarized. The comparison between different algorithms confirms the validity of our theory.

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