Transition from transient response to steady state for a layered medium

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The transition from transient response to steady-state for a layered medium subjected to antiplane loadings is studied. The steady-state formula for a layered medium is derived and the solutions for a layered half-space are then expressed explicitly in the form of wave number integrals. The transient response solutions for a layered half-space are obtained by the convolution of time harmonic loading function with transient response formula derived analytically from an effective matrix method. Two layered half-spaces with different ratios of wave velocities in the layer and half-space are considered and investigated by means of extensive numerical results to show their quite different transition behavior. The numerical results indicate that transient responses will approach steady state after certain characteristic times when the transient effects die away. The transition phenomena and characteristic times are investigated in detail through the responses from near field to far field as well as from low frequency to high frequency. © 2002 Acoustical Society of America. [DOI: 10.1121/1.1494806]

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I. INTRODUCTION

This work is a continuation of the study of dynamic responses of a layered medium subjected to antiplane loadings given by Ma et al. (2001) in which the attention was focused on the investigation of the transition from transient response to static value in a layered half-space. Not only the characteristic times of the transition could be defined, but also some unusual phenomena were found during the transition period. In this study, a further investigation is performed to understand the transition behavior from transient response to steady state. This study is equivalent to the acoustic problem of sound wave propagating in layered fluid.

The elastic wave propagation in a layered medium has been the interest of considerable studies in recent years, mainly because of its significance in various technological and geophysical problems. Transient responses or steady-state solutions of layered media subjected to time-dependent loadings have been treated by so many authors that it is impossible to give an exhaustive list here. In principle, the steady-state analysis of wave propagation is made equivalent mathematically to the transient-pulse problem by means of the Fourier integral theorem. However, the study related to the transition from transient response to steady state is very limited so far. The steady state can be obtained after some characteristic time when the transient effect dies away.

A tremendous amount of work was devoted in the past to the problem of transient waves excited by an impulsive force, as summarized by Achenbach (1980) and Miklowitz (1978). During the past, several solution methods for this problem have been published. Most of the transient solutions are derived in terms of Laplace or Fourier transform. The evaluation of the inverse transformation integrals that are usually very complicated needs a great deal of computational effort to achieve the desired accuracy. Therefore, various methods or quadrature techniques have been developed to obtain either exact or approximate solutions. It is not too surprising that all these methods are intimately connected, even though they are different in spirit.

The method of obtaining complete synthetic seismograms, based on the generalized ray theory, was implemented for the structure of one homogeneous layer over a homogeneous half-space for an SH-torque pulse by Pekeris et al. (1963). Based on the generalized ray theory and Cagniard’s method (Cagniard, 1939), Pao et al. (1979) investigated the transient waves generated by various loadings in an infinite plate. The generalized ray expansion for a structure containing any number of layers welded on top of a half-space subjected to an $SH$ pulse was presented by Abramovici (1984), along with a rigorous mathematical proof based on the corresponding initial-boundary value problem. The theory of generalized rays is an exact method. An important characteristic of the generalized ray theory is its strong connection with the physical picture of rays propagating through the considered medium. However, it becomes extremely cumbersome for larger receiver–source separation and longer time.
A modified propagator matrix method was used by Franssens (1983) to calculate elastodynamic Green’s functions in layered media. The major advantages of this approach are the elimination of the numerical loss of precision problems associated with the Thomson–Haskell formulation (Thomson, 1950; Haskell, 1953). Kundu and Mal (1985) applied a modified wave number integral approach to the calculation of the motion produced in a multilayered solid by dynamic sources. The well-known numerical difficulties associated with the calculation of the integrands are avoided through the use of delta matrices. Special numerical integration schemes are then used to compute the integrals accurately at smaller distances and higher frequencies. When source and receiver depths are close, it is difficult to compute Green’s functions of the layered medium, because the integrands, whose variable of integration is the horizontal wave number, oscillate with only slowly decreasing amplitude. To remedy this problem, Hisada (1994) derived the asymptotic solutions to obtain accurate Green’s functions.

Recently, the responses of a layered medium subjected to dynamic antiplane impact loading were investigated by Ma and Huang (1995) using the Laplace transform and Cagniard–de Hoop method. The transient responses for both stresses and displacement in each layer are expressed in a closed form. Based on the similar approach, the closed-form solutions for the case of in-plane impact loadings were also given by Ma and Huang (1996). An effective analytical method for determining transient solutions in a strip was developed by Ma and Lee (1999). The analytical solution in the Laplace-transformed domain is expressed in matrix form. It is then decomposed into infinite wave groups. Each group of reflected waves has the same reflection coefficient. Each multireflected wave can be verified by the theory of generalized ray. The inverse transform is performed by the Cagniard method. Both numerical calculations and experimental results were presented. Later, an effective matrix method expanding the matrix solution into a power series of the phase contained in the elastodynamic field. The transient solutions are obtained by the convolution of the time harmonic loading function with transient solution formula given by Ma et al. (2001). Two layered half-spaces with different wave velocity ratios between layer and half-space are considered to show their quite different transition phenomena. The numerical results of displacements and stresses from near field to far field are presented to show the transition from transient response to steady state. The effects caused by different frequencies are also considered.

FIG. 1. Configuration and coordinate system of an n-layered medium.

II. FORMULATION OF STEADY STATE

Consider a multilayered medium consisting of n layers as shown in Fig. 1. Each layer is assumed to be isotropic, homogeneous, and elastic, and can therefore be characterized by its density \( \rho \) and shear wave velocity \( c_T \) for the antiplane problem. A time harmonic loading with magnitude \( \sigma_0 \) is applied at the point \((x_s, y_s)\) in the \( p \)th layer. Let the loading, displacement \( w \), and stresses \( (\sigma_{xz}, \sigma_{yz}) \) have time harmonic behavior of the form \( e^{-i\omega t} \), where \( \omega \) is the circular frequency. For convenience, the time-dependence factor \( e^{-i\omega t} \) will be dropped in the following equations. The equation of motion in the layer without body forces is given by

\[
\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + k_T^2 w = 0,
\]

where \( k_T = \omega/c_T \) is the shear wave number. Similarly, the equation of motion in the source layer can be written as

\[
\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + k_T^2 w = -\frac{\sigma_0}{\mu} \delta(x-x_s) \delta(y-y_s),
\]

where \( \mu \) and \( \delta \) denote the shear modulus and Dirac delta function, respectively. The shear stresses are given by the constitutive relations

\[
\sigma_{xz} = \mu \frac{\partial w}{\partial x}, \quad \sigma_{yz} = \mu \frac{\partial w}{\partial y}.
\]

This problem is solved by the classical wave number integral representation and delta matrix formulation of the elastodynamic field. Let \( \tilde{w}(y;k) \) denote the Fourier transform of \( w(x,y) \) so that

\[
w(x,y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{w}(y;k) e^{ikx} \, dk,
\]

where \( k \) stands for the wave number along the x axis. Substitution of Eq. (4) in the differential equations (1) and (2) yields the following first-order, ordinary differential equations:
where $\gamma_T = \sqrt{k_T^2 - k^2}$. The solution of Eq. (5) can be easily obtained and the corresponding solution of Eq. (1) can be expressed in terms of the wave number integral

$$w(x,y) = \int_{-\infty}^{+\infty} \left[ w_-(k) e^{-i\gamma_T(y-y_s)} + w_+(k)e^{i\gamma_T(y-y_s)} \right] e^{ik(x-x_s)} dk,$$

(7)

where $w_-$ and $w_+$ are unknown functions of $k$, $(x,y)$ are the coordinates of the receiver, and $(x_s,y_s)$ are the coordinates of a reference point. Both $w_-$ and $w_+$ will be determined by the boundary and continuity conditions. It should be noted that $k$ in Eq. (7) is real. In a perfectly elastic material, for $k<k_T$, the terms $e^{i\gamma_T}p$ represent upgoing waves in the layer, whereas the terms $e^{-i\gamma_T}p$ represent downgoing waves. For $k > k_T$, the waves become inhomogeneous with exponential increase or decrease with $y$.

For the solution in the source layer, the following additional term is needed to yield discontinuity in the stress $\sigma_{yz}$ and continuity in the displacement $w$ across the plane $y = y_s$,

$$w_s = \int_{-\infty}^{+\infty} a_p(k) e^{i\gamma_T(y-y_s)} e^{ik(x-x_s)} dk,$$

(8)

where $a_p(k) = i\sigma_0/(4\pi\mu_p \gamma_T)$. This term represents the wave field produced by the force in an infinite homogeneous medium whose properties are identical to those of the source layer.

Let $\{C\}$ denote the coefficient vector $[w_-,w_+]^T$ and $\{B(y)\}$ the displacement-stress vector $[W,\Sigma]^T$ in each layer such that

$$[w,\sigma_{yz}]^T = \int_{-\infty}^{+\infty} [W,\Sigma]^T e^{ik(x-x_s)} dk.$$

(9)

From Eqs. (3), (7), and (8), it can be shown that $\{B(y)\}$ in the $i$th layer may be represented by means of the matrix product

$$\{B(y)\} = \{C_i\}[E(y-y_{i-1})]\{C_i\} + \{B(y)^{\infty}\},$$

(10)

where

$$\{C_i\} = \begin{bmatrix} 1 & 1 \\ -i\mu_i \gamma_T & i\mu_i \gamma_T \end{bmatrix},$$

$$E(y-y_{i-1}) = \begin{bmatrix} e^{-i\gamma_T(y-y_{i-1})} & 0 \\ 0 & e^{i\gamma_T(y-y_{i-1})} \end{bmatrix},$$

$$\{B(y)^{\infty}\} = \begin{cases} 1 \\ \pm i\mu_i \gamma_T \end{cases} a_p(k) e^{i\gamma_T(y-y_s)}, \quad i = p$$

$$= \{0\}, \quad i \neq p. $$

Then, by successively applying the interface continuity conditions, the displacement-stress vector at the $i$th layer can be related to those at the top and bottom surfaces. This is the well-known Thomson–Haskell problem (Thomson, 1950; Haskell, 1953). For the multilayered plate, upon enforcing the traction-free boundary conditions at the top surface and the appropriate boundary conditions at the bottom surface, the formal solution for $\{B(y)\}$ can be obtained through linear operation of matrices $\{C\}$, $\{E\}$ of each layer, and $\{B^{\infty}\}$ of the source layer. A similar procedure can be used for a multilayered half-space where the radiation condition is used in the bottom half-space. Note that the stress component, $\sigma_{xz}$, is not included in $\{B(y)\}$. However, it can be derived directly from Eqs. (3) and (9). Thus, the wave number integral representation of the complete displacement-stress vector is obtained. The steady-state solutions can then be easily obtained from the displacement-stress vector multiplied by the factor $e^{-i\omega t}$.

The magnitude of the elements in the matrix products in the kernels of Eq. (9) should decay as $e^{-k|y-y_s|}$ when the wave number $k$ becomes large. However, the direct evaluation of the matrix product becomes erroneous for large $k$ due to a loss of precision problem. The root cause of this phenomenon is the improper cancellation of the leading terms in the product matrices when they are numerically calculated for large $k$. It should be mentioned that the values of the integrals for large $k$ are often required for the evaluations of the wave number integrals. Here, the delta matrix technique is successively employed to reformulate the problem in avoiding the loss of precision. The details of the delta matrix reformulation are omitted here, and the description of the delta matrix technique can be found in a number of papers (e.g., Kundu and Mal, 1985). Then, the numerical integration is achieved through an effective quadrature scheme (Xu and Mal, 1987) in which the kernels of the wave number integrals are represented by means of polynomials in finite and semi-infinite integrals, and the resulting oscillatory integrals are evaluated analytically. The accuracy of the results is controlled by using an adaptive procedure whereby the number of panels is increased successively until the desired accuracy is reached, with no previous function evaluations wasted.

### III. STEADY-STATE SOLUTIONS FOR A LAYERED HALF-SPACE

In order to understand in detail the transition phenomena from transient response to steady state, the numerical examples of a layered half-space are considered. Therefore, the explicit expressions of the steady-state solutions for a layered half-space are given in this section. The thickness of the layer is $h$. A time harmonic force of magnitude $\sigma_0$ is applied at the point $(x_s,y_s)$ within the layer. The complete steady-state solutions can be obtained from Eqs. (3), (9), and (10) by considering appropriate boundary and continuity conditions. By applying the boundary condition of traction free at the top surface, radiation condition in the half-space, and continuity conditions of displacement and stress at the interface, the expressions of the steady-state solutions in the layer are given by
The solutions in the half-space are expressed as follows:

\[
\sigma_{yz}^{(1)}(x,y,\omega) = i\mu_1 \int_{-\infty}^{\infty} \gamma_{\tau_1}(-w_1^{(1)} e^{-iy\tau_1 y} + w_1^{(1)} e^{iy\tau_1 y}) e^{ik(x-x_s)} dk,
\]

\[
\sigma_{yz}^{(2)}(x,y,\omega) = -i\mu_2 \int_{-\infty}^{\infty} \gamma_{\tau_2} w_2^{(2)} e^{-iy\tau_2 y} e^{ik(x-x_s)} dk.
\]

The simplified integrals can be evaluated more efficiently.

IV. TRANSIENT SOLUTIONS FOR A LAYERED HALF-SPACE

In this section, the transient solutions of a layered half-space subjected to a time harmonic loading \( f(t) = \sigma_0 \sin(\omega t)H(t) \) applied at \((x,y) = (0,0)\) in the layer will be given. It can be expected that these transient solutions will approach the steady-state solutions as time increases. The transient solutions here are obtained by using the similar formulations and techniques presented by Ma et al. (2001). Thus, the details of the formulations are omitted here. Only the solutions related to a layered half-space are given. The main difference between Ma et al. (2001) and this paper is caused by the loading functions. The loading function used in this study is \( f(t) = \sigma_0 \sin(\omega t)H(t) \), while \( f(t) = \sigma_0 H(t) \) is used in Ma et al. (2001). Therefore, the transient solutions are derived directly from the results presented in Ma et al. (2001). For any given loading function \( f(t) \) applied at a point in the layer, the general transient solutions of the displacement and stresses can be obtained by the following convolution techniques:

\[
w(x,y,t) = f(t) * w_1^{(1)}(x,y,t),
\]

\[
\sigma_{yz}^{(1)}(x,y,t) = L^{-1}[\hat{f}(p)] \sigma_0 \sigma_{yz}^{(1)}(x,y,t),
\]

\[
\sigma_{yz}^{(2)}(x,y,t) = L^{-1}[\hat{f}(p)] \sigma_0 \sigma_{yz}^{(2)}(x,y,t),
\]

where \( \hat{f}(p) \) is the Laplace transform of \( f(t) \) with transform parameter \( p \), \( L^{-1} \) denotes the inverse Laplace transform, and \( w_1^{(1)}(x,y,t), \sigma_{yz}^{(1)}(x,y,t), \) and \( \sigma_{yz}^{(2)}(x,y,t) \) can be obtained from Eqs. (43a), (43b), and (43c) in Ma et al. (2001), respectively.

For \( f(t) = \sigma_0 \sin(\omega t)H(t) \) considered here, the transient solutions in the layer can be obtained from the convolutions as follows:

\[
w(x,y,t) = \int_0^t \sin[\omega(t-\tau)]w_1^{(1)}(x,y,\tau)d\tau,
\]

\[
\sigma_{yz}^{(1)}(x,y,t) = \omega \int_0^t \cos[\omega(t-\tau)]\sigma_{yz}^{(1)}(x,y,\tau)d\tau,
\]

\[
\sigma_{yz}^{(2)}(x,y,t) = \omega \int_0^t \cos[\omega(t-\tau)]\sigma_{yz}^{(2)}(x,y,\tau)d\tau.
\]
The Gaussian quadrature is then used to evaluate the integrals. For a high-frequency, long-time, and far-field response, however, direct computations of the convolutions presented in Eqs. (26), (27), and (28) could be very time consuming. Here, the convolutions can be rearranged by taking \( \sin(\omega t) \) and \( \cos(\omega t) \) out of the integrals because the time \( t \), which is the upper bound of the integrals, is not function of \( \tau \). Thus Eqs. (26)–(28) can be rewritten as

\[
w = \sin(\omega t) \int_0^t \cos(\omega \tau) w^{(1)}(\tau) d\tau - \cos(\omega t)
\]

\[
\times \int_0^t \sin(\omega \tau) w^{(1)}(\tau) d\tau,
\]

\[
\sigma_{yz} = \omega \cos(\omega t) \int_0^t \cos(\omega \tau) \sigma_{yz}^{(1)}(\tau) d\tau + \omega \sin(\omega t)
\]

\[
\times \int_0^t \sin(\omega \tau) \sigma_{yz}^{(1)}(\tau) d\tau,
\]

\[
\sigma_{xz} = \omega \cos(\omega t) \int_0^t \cos(\omega \tau) \sigma_{xz}^{(1)}(\tau) d\tau + \omega \sin(\omega t)
\]

\[
\times \int_0^t \sin(\omega \tau) \sigma_{xz}^{(1)}(\tau) d\tau.
\]

Now the entire time histories of transient responses from \( \tau = 0 \) to \( \tau = t \) are obtained by computing the integration once only. It is not necessary to compute the integration starting from \( \tau = 0 \) for each moment \( t \), because the previous integration value can be used for the next moment \( t \). This saves a lot of computational time particularly for the evaluation of transient responses at higher frequencies. In addition, the values of integrals in Eqs. (29)–(31) will oscillate faster for higher frequency. Therefore, an adaptive quadrature procedure depending on the frequency is adopted to achieve the desired numerical accuracy.

V. NUMERICAL RESULTS AND DISCUSSIONS

The transition phenomena from transient response to steady state for a layered half-space are investigated in detail in this section by means of extensive numerical results. However, only the representative ones are shown in this paper.

The layered half-space is composed of a layer with thickness equal to \( h \) and a half-space. All of the wave velocities shown in the numerical results are normalized to the velocity in the layer. The loading is applied at the point \((x, y) = (0, -h/2)\) within the layer. Three receivers located at \((5h, -h/2)\), \((50h, -h/2)\), and \((200h, -h/2)\) are used to study near-field, intermediate-field, and far-field responses, respectively. Numerical results are presented for the displacement \( (w) \) and stress \( (\sigma_{yz}) \). The stress \( (\sigma_{xz}) \) is not presented because of the similar characteristics to \( \sigma_{yz} \). In order to study the effects caused by different frequencies, the low \( (\omega = 1) \), intermediate \( (\omega = 6) \), and high \( (\omega = 12) \) frequencies are taken into account in this paper. Note that the wavelength at \( \omega = 6 \) is of the order of the thickness \( (h) \) of the layer. In the following numerical results, two layered half-spaces with different wave velocity ratios \( (c_{T(2)}/c_{T(1)}) = 0.771 \) and 1.297) are considered to show the completely different transition behavior. The amplitude spectra of stresses located at near field, intermediate field, and far field are shown in Figs. 2 and 3 for the layered half-spaces with

![FIG. 2. Amplitude spectra of stresses in a layered half-space with faster wave velocity in the layer.](image1)

![FIG. 3. Amplitude spectra of stresses in a layered half-space with slower wave velocity in the layer.](image2)

![FIG. 4. Transient response and steady state of the stress \((\sigma_{yz})\) at \((x, y) = (5h, -h/2)\) in the layer with faster wave velocity, circular frequency \( \omega = 1 \).](image3)
wave velocity ratios $c_{T(2)}/c_{T(1)} = 0.771$ and 1.297, respectively. It is obvious from Fig. 2 that the amplitudes of stresses decrease as the distances between source and receiver increase for the case with faster wave velocity in the layer. However, this phenomenon is not observed in the case with slower wave velocity in the layer, as shown in Fig. 3. Furthermore, the amplitudes begin to oscillate at frequency $\omega = 5.5$. Note that the wavelength at this frequency is close to the thickness ($h$) of the layer. The oscillations are more serious for far-field responses.

A. Layer with faster wave velocity

The numerical results for the layered half-space with wave velocity ratio $c_{T(2)}/c_{T(1)} = 0.771$ are presented and discussed in this subsection. The stresses at low frequency ($\omega = 1$) are shown in Figs. 4, 5, and 6 for near-field, intermediate-field, and far-field responses, respectively. The corresponding displacement for the intermediate field is shown in Fig. 7. The transient responses at both intermediate field and far field as shown in Figs. 5 and 6 are quite different from that of near field as shown in Fig. 4. It is found from Figs. 4–7 that the amplitudes for both displacement and stress responses decrease as the distance between source and receiver increases. The transient response of the displacement shown in Fig. 7 reaches a maximum value at around $t/s_{T(1)} = x/c_{T(2)}/c_{T(1)}h$ (i.e., $t = x/c_{T(2)}$), and then takes a very long time to approach to steady state. It is observed from the numerical results that it takes a much longer time for far-field displacement to transfer from transient response to steady state. However, the transient responses of stresses shown in Figs. 4, 5, and 6, go to steady state right after $t = x/c_{T(2)}$. Note that the transient stresses at both intermediate field and far field as shown in the small windows in Figs. 5 and 6 vary tremendously at the very beginning, when a large number of reflected waves arrive in a very short time right after the initial wave. A common feature is observed in the transient responses at low frequency for both displacements and stresses. Although a time-harmonic function is applied in the layer, the transient responses vary smoothly most of the time before $t = x/c_{T(2)}$ without oscillations. The interesting phenomenon found in Fig. 10 presented in Ma et al. (~2001~) is also observed in Fig. 6 here. During the tran-

![FIG. 5. Transient response and steady state of the stress ($\sigma_{xy}$) at (x, y) = (50h, –h/2) in the layer with faster wave velocity, circular frequency $\omega = 1$.](image1)

![FIG. 6. Transient response and steady state of the stress ($\sigma_{xy}$) at (x, y) = (200h, –h/2) in the layer with faster wave velocity, circular frequency $\omega = 1$.](image2)

![FIG. 7. Transient response and steady state of the displacement at (x, y) = (50h, –h/2) in the layer with faster wave velocity, circular frequency $\omega = 1$.](image3)

![FIG. 8. Transient response and steady state of the stress ($\sigma_{xy}$) at (x, y) = (50h, –h/2) in the layer with faster wave velocity, circular frequency $\omega = 6$.](image4)
sition from transient response to steady state, there is a long period \( t/s_{T(1)}h \approx 202–230 \) where the responses are very small and no variation is observed.

The effects caused by different frequencies are considered next. Here, only the intermediate-field stress responses are presented for illustration. The responses at intermediate frequency \( (\omega = 6) \) and high frequency \( (\omega = 12) \) are shown in Figs. 8 and 9, respectively. Compared with transient stress responses at low frequency \( (\omega = 1) \) as shown in Fig. 5, the responses at higher frequencies \( (\omega = 6 \text{ and } 12) \) as shown in Figs. 8 and 9 can transfer to steady state much faster. The transient displacement responses also have a similar tendency. The maximum values observed at low-frequency responses are not so evident at high-frequency responses. It is noticed that the transient stress as shown in Fig. 9, in which the transient effect is not so obvious, becomes steady state much earlier before \( t = x/c_{T(2)} \). From the numerical results studied above, it can be concluded that during the transition stage the behavior of low-frequency responses is strongly related to the Heaviside step function \( H(t) \) as studied in Ma et al. (2001), whereas high-frequency responses are dominated by the harmonic function \( \sin(\omega t) \).

**B. Layer with slower wave velocity**

The numerical results for a layered half-space with slower wave velocity in the layer are considered here. Figures 10, 11, and 12 show the stress responses at low frequency \( (\omega = 1) \) for near field, intermediate field, and far field, respectively. Due to the existence of head waves, the transition phenomena from transient response to steady state are quite different from the last case with faster wave velocity in the layer. The arrival and end times of head waves can be obtained from the formula given by Ma et al. (2001). These times and arrival times of body waves play important roles to the transition phenomena.

Unlike the previous case, the amplitudes of stress responses as shown in Figs. 10–12 apparently do not decrease from near field to far field because more energy can be kept inside the layer. Displacement responses, which are not shown here, have similar behavior. From the numerical results of the intermediate-field \( (x=50h) \) responses at higher

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**FIG. 9.** Transient response and steady state of the stress \( (\sigma_{yz}) \) at \( (x, y) = (50h, -h/2) \) in the layer with faster wave velocity, circular frequency \( \omega = 12 \).

**FIG. 10.** Transient response and steady state of the stress \( (\sigma_{yz}) \) at \( (x, y) = (5h, -h/2) \) in the layer with slower wave velocity, circular frequency \( \omega = 1 \).

**FIG. 11.** Transient response and steady state of the stress \( (\sigma_{yz}) \) at \( (x, y) = (50h, -h/2) \) in the layer with slower wave velocity, circular frequency \( \omega = 1 \).

**FIG. 12.** Transient response and steady state of the stress \( (\sigma_{yz}) \) at \( (x, y) = (200h, -h/2) \) in the layer with slower wave velocity, circular frequency \( \omega = 1 \).
frequencies (not shown here), it is found that no matter lowor high-frequency responses, the characteristics of Heaviside step function are not observed in transient displacements. Compared with body waves, the contribution of head waves is smaller for higher frequency responses. From the numerical results obtained in this study, the transient responses can transfer to steady state between the arrival of first body wave \((t = x/c_{T(1)})\) and the end of last head wave \((t = x c_{T(2)}/c_{T(1)}^2)\).

VI. CONCLUSIONS

The transition from transient response to steady state for a layered half-space subjected to an antiplane harmonic loading has been investigated in detail by extensive numerical results. The transient responses are obtained by the convolution of time harmonic loading function with transient solutions derived analytically from an effective matrix method given by Ma et al. (2001). The steady-state problem is solved by the wave number integral representation. The numerical integration is achieved through an effective quadrature scheme. For the layered half-space with faster wave velocity in the layer, the transition behavior of low-frequency responses is strongly related to the Heaviside step function while high frequency responses are dominated by the harmonic function. Usually high-frequency responses approach steady state faster than low-frequency responses, which can go to steady state after \(t = x/c_{T(2)}\). For the layered half-space with slower wave velocity in the layer, the transient responses transfer to steady state between the arrival of first body wave \((t = x/c_{T(1)})\) and the end of first head wave \((t = x c_{T(2)}/c_{T(1)}^2)\) no matter low- or high-frequency responses.

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