An adaptive approach to selecting flow partition exponent for multiple flow
direction algorithm

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An adaptive approach to selecting flow partition exponent for multiple flow direction algorithm

Abstract

Most Multiple Flow Direction algorithms (MFD) use a flow partition coefficient (exponent) to determine the fractions draining to all downslope neighbors. The commonly used MFD often employs a fixed exponent over an entire watershed. The fixed coefficient strategy cannot effectively model the impact of local terrain conditions on the dispersion of local flow. This paper addresses this problem based on the idea that dispersion of local flow varies over space due to the spatial variation of local terrain conditions. Thus the flow partition exponent of a MFD should also vary over space. We present an adaptive approach for determining the flow partition exponent based on local topographic attribute which controls local flow partitioning. In our approach, the influence of local terrain on flow partition is modeled by a flow partition function which is based on local maximum downslope gradient (we refer to this approach as MFD based on maximum downslope gradient, MFD-md for short). With this new approach, a steep terrain which conduces a convergent flow condition, can be modeled using a large value for the flow partition exponent. Similarly, a gentle terrain can be modeled using a small value for the flow partition exponent. MFD-md is quantitatively evaluated using four types of mathematical surfaces and their theoretical ‘true’ value of Specific Catchment Area (SCA). The Root Mean Square Error (RMSE) shows that the error of SCA computed by MFD-md is lower than that of SCA computed by the widely used SFD and MFD algorithms. Application of the new approach using a real DEM of a watershed in Northeast China shows that the flow accumulation computed by MFD-md is better adapted to terrain conditions.
1. Introduction

Flow direction is one key factor in many research fields (such as distributed hydrological modeling, watershed feature extraction, geomorphology, soil erosion, etc.) (Beven and Kirkby 1979, Moore et al. 1991, Wilson and Gallant 2000) and thus becomes an important subject of terrain analysis. The computation of flow direction is based on the assumption that the local hydraulic gradient can be estimated with the local slope gradient which is derived from the DEM, especially the gridded DEM (Beven and Kirkby 1979, O’Loughlin 1986, Wilson and Gallant 2000). Therefore, most flow direction algorithms use gridded DEM to determine the flow direction of a point according to the elevations in a 3 by 3 window around it (Wilson and Gallant 2000).

Current flow direction algorithms can be classified into two main types according to the flow direction schemes used: Single Flow Direction (SFD) and Multiple Flow Direction (MFD) algorithms (Wolock and McCabe 1995). The basic idea of SFD algorithms is that all water from a point (or pixel) should flow into one and only one neighboring pixel which has the lowest elevation. This idea of the steepest descent direction is easy to implement, and suited for modeling convergent flow. Many SFD algorithms have been proposed and frequently used in many flow routing applications (O’Callaghan and Mark 1984, Martz and de Jong 1986, Jenson and Domingue 1988, Fairfield and Leymarie 1991, Lea 1992). However, SFD has evident defects, i.e. the inability to model divergence of flow and the production of parallel flow (Freeman 1991, Fairfield and Leymarie 1991, Wilson and Gallant 2000). Detailed discussion of SFD and its shortcomings are beyond the scope of this paper and
interested readers are referred to Bertolo (2000) for more discussion on SFD.

MFD assumes that flow from the current position could drain into more than one downslope neighboring pixel. The flow partitioning proportion among downslope neighboring pixels is determined based on slope gradient (Quinn et al. 1991). Much research has shown that MFD is obviously better than SFD when the spatial pattern of hydrological parameters (such as flow accumulation and topographic index) are computed (Moore et al. 1993a, Wolock and McCabe 1995, Pan et al. 2004). The key issue in MFD is how to partition the flow into multiple downslope pixels. A commonly used approach for modeling divergent flow is the fixed exponent strategy (Freeman 1991, Quinn et al. 1991) although other approaches have been examined (Costa-Cabral and Burges 1994, Tarboton 1997). The fixed exponent strategy is unsuitable when the terrain conditions include both convergence and divergence because the exponent cannot be altered in response to local terrain conditions during the execution of the program. In other words, this approach is not adaptive to local terrain conditions which are important for partitioning local flow. Some efforts have been made to model the flow partition in complex terrain using a varying exponent strategy (e.g. Quinn et al. 1995, Kim and Lee 2004). However, the effect of local terrain conditions on the dispersion of local flow has still not been effectively modeled in these efforts. The shortcomings of current MFDs are further discussed in next section.

This paper presents an adaptive approach for varying a flow partition exponent based on local terrain conditions. Section 2 reviews current MFD algorithms and highlights the key problems associated with the fixed exponent strategy. Section 3 presents the new flow partition approach. Section 4 provides a quantitative evaluation of the new approach using
mathematical surfaces. Section 5 presents an application of the method using a real watershed in Northeast China. Conclusions are made in Section 6.

2. MFD algorithms based on a flow partition exponent

In MFD, the cell with the steeper descent, among all downslope neighboring cells, will receive more flow. Quinn et al. (1991) made the original contributions for MFD and presented the following expression (Equation 1) to model this flow partition:

\[
d_i = \frac{(\tan \beta_i)^p \times L_i}{\sum_{j=1}^{8} (\tan \beta_j)^p \times L_j}
\]

where \( d_i \) is the fraction of flow into the \( i \)th neighboring cell; \( \tan \beta_i \) is the slope gradient of the \( i \)th neighboring cell; the exponent \( p \) is the flow partition exponent \( (p > 0) \) (some authors refer to it as the “variable exponent” (Holmgren 1994) or the “flow partitioning factor” (Kim and Lee 2004)); \( L_i \) is the “effective contour length” of pixel \( i \). The value of \( L_i \) is 0.5 for pixels in cardinal directions and 0.354 for pixels in diagonal directions (Quinn et al., 1991). Recently, Chirico et al. (2005) suggest that the \( L_i \) value should be uniform for both cardinal and diagonal directions when SFD or D\( \infty \) (Tarboton 1997) is applied. For the convenience of comparing the results of the new method with that of Quinn et al. (1991)’s algorithm we decided to follow the effective contour length defined by Quinn et al. (1991).

The selection of \( p \) determines the flow partition scheme of MFD. The larger the value of \( p \), the more similar MFD is to SFD. Actually, SFD can be deemed as a special case of MFD under the extreme condition of \( p \to +\infty \) (Holmgren 1994). The earlier MFD algorithms take a positive constant as the exponent to model divergent flow on hill slopes
(Quinn et al. 1991, Freeman 1991, Holmgren 1994). As a result all terrain conditions, no matter how flat or steep, divergent or convergent, are modeled as the same. The difference of local terrain conditions on flow distribution is ignored.

As an example, Figure 1 shows three areas of 3x3 pixels with a grid resolution of 1 m and the flow dispersion from the central pixel is computed by a representative MFD, Quinn et al. (1991)'s algorithm (which is named as “MFD-Quinn” in this paper). The terrain conditions of these three areas range from steep to smooth (the maximum downslope angles are 35.3º, 8.0º, and 2.4º, respectively). Under the steep terrain condition, most of the flow from the center pixel should drain to the neighboring cell with the steepest downslope, which should be similar to the situation modelled by SFD. The flatter the terrain condition, the more divergent the flow. Thus the flow from the center pixel should be distributed more evenly to the neighboring downslope pixels. Under MFD-Quinn which sets $p = 1$ (Quinn et al. 1991) the flow is partitioned in a uniform manner for all three areas and the partition of flow cannot adapt to different local terrain conditions, no matter how steep or flat (Figure 1). This deficiency is also seen in all other MFD algorithms which set $p$ as a constant (fixed $p$). Although more convergent flow might be modeled using a higher $p$ value (Holmgren 1994), for an MFD with a fixed $p$ value, the $p$ value cannot be altered according to local terrain conditions during the execution of the algorithm. Thus, the particular fixed $p$ value might be good for some locations but not for others. This means that MFDs with a fixed $p$ value cannot adapt to local terrain conditions.

Insert figure 1 about here
Some algorithms try to setup a flow partition scheme to accomplish both MFD for a divergent surface and SFD for a convergent surface. Moore et al. (1993b) set up a FD8/FRho8 method. FD8/FRho8 requires the specification of a critical upslope contributing area for channel initiation. Then the MFD with a fixed $p$ value proposed by Freeman (1991) is applied in upland areas above defined channels and the D8 (O’Callaghan and Mark 1984) or Rho8 (Fairfield and Leymarie 1991) algorithm is applied below the points of channel initiation. But the appropriate value of a critical upslope contributing area might vary depending on the characteristics of catchment (Güntner et al. 2004). The transition from MFD to SFD can cause an irregularity in the frequency distribution of the contributing area (Wilson and Gallant 2000).

Pilesjo et al. (1998) designed a “form-based” MFD. The basic idea is that the topographic form of a sub surface (i.e., “form”) dictates the flow distribution. If the form is concave or flat, SFD is applied. If the form is convex, the MFD-Quinn algorithm is applied. This is a clever idea but the method is also very sensitive to the quality of DEM because small errors in DEM could shift the form from the convex to the concave or vice versa. However, flow on convex forms is still modeled with a fixed flow partition exponent.

Quinn et al. (1995) described a MFD algorithm where the flow partition exponent was changed continuously from 1 (full dispersion) to a large value (SFD) as the contributing area increased, giving a smooth change from MFD to SFD. They used the MFD-Quinn algorithm to compute a spatial distribution of flow accumulation. Then an ‘optimal’ channel initiation threshold (CIT) was identified for computing the flow partition exponent using the following
equation:

\[ p = \left( A / \text{thresh} + 1 \right)^h \]  

(2)

where \( A \) is the pre-computed accumulation of the interested cell; \( \text{thresh} \) is the accumulation threshold of the application region, same as the channel initiation of Moore \textit{et al.} (1993b); \( h \) is a positive constant. The larger the value of \( h \) the faster the method approaches SFD. Some researchers also focused on how to acquire a varying \( p \) value according to the accumulation distribution pre-computed by other SFD or MFD. An example is the SDFAA method which computes a \( p \) value by a genetic algorithm (Kim and Lee 2004). These MFD algorithms are based on an assumption that the pre-computed accumulation distribution can sufficiently reflect the effects of local terrain conditions on flow distribution. But flow accumulation is not actually a ‘local’ topographic attribute. The accumulation of a cell is affected by all cells in its upland areas. Moreover, different flow direction algorithms might produce very different accumulation distributions. So the rationale of this assumption is questionable and this type of MFD is not independent of other flow direction algorithms. Furthermore, the identification of ‘optimum’ CIT is subjective and complicated (Quinn \textit{et al.} 1995, Güntner \textit{et al.} 2004). Despite these deficiencies, the varying flow partition exponent could still get more reasonable results than the fixed exponent (Quinn \textit{et al.} 1995, Kim and Lee 2004).

The above discussion leads us to see both the deficiencies of current MFD algorithms and the need for an adaptive strategy for determining the flow partition exponent according to local terrain conditions. In the next section we present an adaptive approach to varying the exponent \( p \), which is based on local topographic attribute that reflects the effects of local
terrain conditions on flow distribution.

3. An adaptive approach to varying the flow partition exponent for MFD

3.1. Basic idea

We believe that the local terrain conditions control flow partitioning at each cell. It is more reasonable to use local topographic conditions to determine the flow partition exponent than using global topographic attributes (such as flow accumulation). Using local topographic conditions also makes the algorithm more adaptive to local terrain conditions which directly control the local flow partition. This idea results in the development of a new flow partition approach to MFD. There are two steps in building this new approach. The first is the selection of the local topographic attribute \( e \) which can directly describe the effect of local terrain on flow partitioning. The second step is the construction of a function of \( e \) (\( f(e) \)) for computing the flow partition exponent which is adaptive to the local terrain conditions. So the varying flow partition exponent is determined through a flow partition exponent function \( f(e) \) based on local terrain conditions. The two steps are discussed below in detail.

3.2. Selection of local topographic attribute

The candidates of local topographic attribute for our flow partition scheme should be those which not only reflect the local terrain conditions but also control the local drainage. It is natural to use something similar to a local slope gradient because the local hydraulic gradient is estimated with the local slope gradient, which is computed by the center cell and all of its neighboring cells, according to the basic assumption of flow direction algorithms (Beven and Kirkby 1979). Hjerdt et al. (2004) argued that the downslope topography is an
important factor for estimating a hydraulic gradient. Among the topographic attributes related to the local slope gradient, Güntner et al. (2004) also showed that the local downslope gradient computed by the center cell and its downslope neighboring cell can quantify flow of water better than the local slope in explaining the soil water content of catchment. Therefore, we use downslope gradient as the local topographic attribute to model the impact of local terrain on flow partition.

We consider three downslope gradients as candidates for our MFD scheme: the maximum downslope gradient, the minimum downslope gradient, and the average downslope gradient. We determine which candidate of these local topographic attributes is the most appropriate for the new flow partition scheme based on two principles. The first is that the chosen local topographic attribute can express the change in local terrain condition which has evident geomorphological meanings for the change of flow partitioning. As discussed before, the flow partition scheme should be more similar to SFD when the terrain condition consists of a higher downslope gradient in the steepest descent direction. Most directly, the maximum downslope gradient, among the three candidates, can quantify the steepness in the steepest descent direction.

The other principle for choosing the local topographic attribute is related to the sensitivity to subtle variations in DEM. The local topographic attribute should not be too sensitive to subtle variations of DEM because not only is the effect of subtle DEM variations on flow partition often small but also the likelihood of subtle DEM variations being errors is high. Theoretically speaking, the subtle variations of DEM have a much more marked effect on the minimum downslope gradient than the maximum downslope gradient. The average
downslope gradient is also sensitive to the subtle variations in DEM error because these subtle variations can change the number of neighboring downslope cells. The maximum downslope gradient should be less sensitive to the DEM error than the minimum or average downslope gradients.

Figure 2 illustrates the sensitivity of these three candidates and slope gradients to subtle variations in DEM for the watershed described in Section 5. Through visual comparison, all of them show the variation of local terrain conditions, more or less. But there are many disorderly and unsystematic breaks in the image of an average downslope gradient and a minimum downslope gradient. The spatial distribution of the maximum downslope gradient, which is similar to that of a slope gradient, follows the terrain conditions on the contour maps (see also Figure 4 in Section 5) the best. Therefore, the maximum downslope gradient meets the two principles best and was chosen as the local topographic attribute for the new flow partition scheme.

3.3. Determination of flow partition exponent using the maximum downslope gradient

The construction of the function \( f(e) \) for determining the flow partition exponent using the maximum downslope gradient includes two issues: the function form of \( f(e) \) and its upper and lower bounds. The principle used in constructing \( f(e) \) is that \( f(e) \) can properly model the effects of the maximum downslope gradient on flow partitioning.
Furthermore, the function form should be as simple as possible. When the maximum downslope gradient is large, the value of $f(e)$ should be large accordingly to model the convergent flow. If the maximum downslope gradient is small, the corresponding $f(e)$ should also be small to model the divergence of flow. A linear function would meet both conditions. Therefore, we chose a linear function of the maximum downslope gradient as $f(e)$. Equation (3) gives the form of this linear function

$$f(e) = \begin{cases} p_l & (e \leq e_{\min}) \\ \frac{e - e_{\min}}{e_{\max} - e_{\min}} \times (p_u - p_l) + p_l & (e_{\min} < e < e_{\max}) \\ p_u & (e \geq e_{\max}) \end{cases} \tag{3}$$

where $e$ is the tangent value of the maximum downslope gradient ($\tan \beta$ and $\beta$ being the maximum downslope gradient); $f(e)$ is the flow partition function; $p_u$ and $p_l$ are the upper and lower bounds of $f(e)$ and are used as the $p$ values representing completely divergent and convergent flows, respectively (see discussion below); $e_{\min}$ and $e_{\max}$ are the $e$ values which are associated with $p_l$ and $p_u$, respectively. The determination of the bounds (i.e., $p_u$ and $p_l$) for $f(e)$ depends on many researchers’ comparison and discussion about the best value of the flow partition exponent for modeling the complete divergence or convergence of flow. Among integers, $p = 1$ is proposed for the complete divergence of flow (Quinn et al. 1991, Holmgren 1994, Pilesjo and Zhou 1997). Freeman (1991) proposed $p = 1.1$ as the best value after he analyzed the instances of $p$ being 1, 1.1, and 1.25 respectively. So we choose $p = 1.1$ as the lower bound for $f(e)$. Both Holmgren (1994) and Quinn et al. (1995) recommended $p = 10$ for modeling the single directional flow. Their recommendation is taken here as the upper bound of $f(e)$. So the domain of
\( f(e) \) is defined as \([1.1,10]\).

We set \( e_{\min} = 0 \) and \( e_{\max} = 1 \) \((\tan 45^\circ)\) for this experiment. This means that the flow partition exponent is set to \( p_u \) to model the single flow direction when the local maximum downslope gradient is \( 45^\circ \) or above, and to \( p_i \) when the local maximum downslope gradient is 0. It is understandable to choose \( 0^\circ \) for \( e_{\min} \) because we expect flat areas to be typical areas of divergent flow. The determination of 1 \((\tan 45^\circ)\) to be associated with \( p_u \) is made under an assumption that a location with a maximum downslope gradient of \( 45^\circ \) or above will have a complete convergent flow. In addition, the \( 45^\circ \) for \( e_{\max} \) allows us to capture most of the gradient values for flow partition because slope gradient values for most terrain are in the range of \( 0^\circ \) to \( 45^\circ \) and multiple flow directions often occur in areas with low gradient values \((<45^\circ)\). Although the determination of \( e_{\min} = 0 \) and \( e_{\max} = 1 \) are a bit arbitrary, these settings are made to illustrate the advantages of this adaptive approach to varying flow partition exponent for MFD algorithms. A detailed study on the effect of these settings on the algorithm will be reported in a separate paper.

With the above settings the final form of the flow partition exponent function is shown in Equation (4)

\[
f(e) = 8.9 \times \min(e,1) + 1.1
\]

where \( \min(e,1) \) is the minimum between \( e \) and 1.

The new MFD algorithm proposed in this paper then takes the following form:

\[
d_i = \frac{(\tan \beta_i)^{f(e)} \times L_i}{\sum_{j=1}^{8} (\tan \beta_j)^{f(e)} \times L_j},
\]

where \( f(e) \) is computed with Equation (4) and the definition of other symbols is the same
with that in Equation (1). We name the new MFD algorithm as MFD-md (MFD algorithm based on Maximum Downslope Gradient).

Figure 3 shows flow partitioning by MFD-md for the same three areas which were shown in Figure 1. The spatial grid resolution is 1 m. When terrain is steep, MFD-md drains the flow mainly into the neighboring cell in the direction with the steepest downslope, which is similar to the situation modelled by SFD. When the terrain becomes flatter, MFD-md partitions the flow adaptively and more evenly distributes the flow among all downslope neighboring cells, which is similar to the result by MFD-Quinn (Figure 1). Thus, in virtue of the adaptive approach to varying flow partition exponents, MFD-md can more accurately model flow partitioning under different terrain conditions.

4. Quantitative assessment for MFD-md

This method for assessing flow direction algorithms is necessary because different algorithms can produce very different results, even for the same DEM (Wilson and Gallant 2000). The most commonly used method of assessing error is to apply the algorithm to a real DEM. However, this assessment is data-dependent and hard to quantify (Zhou and Liu 2002). A few artificial DEMs were used to show the rationality of the flow direction algorithm. An example is an artificial cone surface used in Freeman (1991). However, the flow accumulation computed by the MFD algorithm of Freeman (1991) was subjectively judged to be
“reasonable” or “unreasonable”. If the ‘true’ value of flow accumulation or Specific Catchment Area (SCA for short, which is defined as “upslope area per unit width of contour” (Wilson and Gallant 2000)) of artificial surfaces can be pre-determined by mathematical inference, the flow direction algorithms can then be quantitatively assessed.

There are two quantitative methods with which artificial DEMs are used to assess the error of grid-based flow routing algorithms. One was developed by Zhou and Liu (2002) and the other by Pan et al. (2004). Both of them create artificial DEMs to simulate typical terrain conditions, such as planar, convergent, divergent, and so on. The Root Mean Square Error (RMSE) can be used to assess the errors between the theoretical flow accumulation (or SCA) and the result computed by flow direction algorithms. Comparatively, the artificial DEMs created by Zhou and Liu (2002) include one more terrain type, i.e. the saddle surface representing the ridge. The ellipsoid surface in Zhou and Liu (2002) is more similar to the practical convex slope than to the cone surface in Pan et al. (2004). We use the method developed by Zhou and Liu (2002) to assess MFD-md.

4.1. Method and Material

According to Zhou and Liu (2002)’s method, four types of artificial surfaces are constructed using mathematical models. They are based on an ellipsoid (representing convex slopes), an inverse ellipsoid (representing concave slopes), a saddle (representing ridges), and a plane (representing straight slopes) respectively. Figure 4 shows an example of the mathematical models, the contour maps of the artificial surfaces, and the ‘true’ SCA inferred from the mathematical models. The artificial DEMs are generated from the mathematical
surfaces at the resolution of 1 m.

Zhou and Liu (2002) take the SCA defined as the upstream length at the point where the contour length tends to zero. The details of computing SCA at any given point on a given mathematical surface are not included in this paper. Interested readers are referred to Zhou and Liu (2002). The error at each cell caused by the flow direction algorithm can be computed by (Zhou and Liu 2002):

\[ E_i = \frac{SCA_i^{true} - SCA_i}{\text{cellsize}} \]  

where \( E_i \) is the error at \( i \) th cell; \( SCA_i^{true} \) and \( SCA_i \) are the ‘true’ value of and computed SCA for the cell, respectively. Thus the Root Mean Square Error (RMSE) can be computed for the assessment and comparison of the flow direction algorithms.

\[ RMSE = \sqrt{\frac{\sum_{i=1}^{n} (E_i)^2}{n}} \]  

where \( n \) is the number of cells for assessment.

We use the artificial DEMs in Figure 4 and the method of Zhou and Liu (2002) to quantitatively assess MFD-md. For comparing MFD-md with classical SFD and MFD, we compute SCAs of artificial DEMs using D8 (a representative SFD) and MFD-Quinn (a representative MFD), and MFD-md, respectively. Here we do not choose current MFD with varying flow partition exponent (e.g. Quinn et al. (1995)’s MFD) as reference MFD. This is
mainly because of their depending on other flow direction algorithm, as we discussed in Section 2.

4.2. Results of quantitative assessment for MFD-md

Table 1 lists the RMSE of errors computed by D8, MFD-Quinn, and MFD-md. Under all tested terrain conditions, MFD-md produces the lowest error among the three tested algorithms. The lowest error of all is produced by the ellipsoid surface as opposed to other terrain conditions. This shows that the flow direction algorithm is more suitable for convex slopes than other typical terrains. All three algorithms get their highest errors for inverse surfaces. This may partly be related to the fact that the surface of an artificial inverse ellipsoid has no outlet for flow. For the plane surface (representing the straight slope), MFD-Quinn gets the highest error because MFD-Quinn does not accurately model the flow on the steeper terrain. In summary, MFD-md can model the four kinds of terrain conditions better than D8 and MFD-Quinn.

5. Application of MFD-md

5.1. Data and the pre-processor for depressions and flat areas in DEM

A typical small catchment in the Nenjiang watershed in Northeast China is applied with MFD-md. The catchment is approximately 1.6 km×1.3 km in size and has relatively low relief as a whole (Figure 5). The DEM is in a regular 10 m grid with more than 20,000 pixels.
A pre-processor for DEM is nearly always necessary for applying the flow routing algorithms. This is because the real DEM often contains depressions or flat areas which interfere with the execution of flow routing algorithms (Jenson and Domingue 1988, Kong and Rui 2003). The usual method for dealing with depressions is to fill them (O’Callaghan and Mark 1984). There are two ways for processing the flat areas. The first way is to keep the elevation unchanged and assign a flow direction for every cell in flat areas according to a set of rules (Jenson and Domingue 1988). This method is aimed at SFD, not MFD. The second way is to mildly amend the elevation of the cells in flat areas according to the terrain conditions. A good example is the DEM-processing method proposed by Martz and Garbrecht (1998). Their method makes every cell in flat areas have reasonable downslope neighboring cell(s) without changing the veracity of the DEM. We use Martz and Garbrecht’s DEM-processing method to fill depressions and amend elevation of cells in flat areas before MFD-md is applied. The details of this method are not included in this paper and interested readers are referred to Martz and Garbrecht (1998) for details.

5.2. Application and discussion of MFD-md, SFD, and MFD

The MFD-md and two comparing algorithms are applied after the DEM is pre-processed. The resulting accumulations are shown in Figure 6. In the flow accumulation computed by D8 (Figure 6a), there are a lot of narrow and parallel lines with higher accumulation values which
demonstrates the shortcoming of D8 for computing the spatial distribution of flow accumulation. This defect of SFD has been discussed by many researchers, such as Fairfield and Leymarie (1991), and Wilson and Gallant (2000). Compared with D8, MFD-Quinn gets a much smoother spatial distribution of flow accumulation in which the state of flow in the low relief areas is much more reasonable (Figure 6b).

\[\text{Insert figure 6 about here}\]

\[\text{Insert figure 7 about here}\]

The spatial distribution of flow accumulation by MFD-md is intergradation between that shown by D8 and MFD-Quinn. MFD-md gets a similar result to MFD-Quinn where the main difference is in the areas of hillslope (Figure 6b, c). Figure 7 shows a map of difference between Figure 6c (MFD-md) and 6b (MFD-Quinn). The negative values in Figure 7 means that the flow accumulation value from MFD-md is smaller than that from MFD-Quinn (i.e. MFD-md minus MFD-Quinn). The positions with large negative difference (i.e. the red region in Figure 7) are mostly areas next to the v-shaped channels where slope gradients are high. This is because the fixed flow partition exponent used in MFD-Quinn treats convergent flow as divergent flow and unnecessarily spreads flow to the neighboring pixels thus producing higher flow accumulation values for these pixels. For the same reason, for the v-shaped valleys, as in this study area, the pixels at valley center receive a smaller flow accumulation based on MFD-Quinn because the flow draining into them has been spread into other
neighboring pixels. This explains the larger positive differences in the valley centers between the two methods. The comparison of MFD-md, D8, and MFD-Quinn when applied to a real DEM shows that the proposed algorithm is advantage in modeling the effect of local terrain on flow partitioning.

6. Conclusion and future work

Following on Quinn et al. (1991)’s research, this paper proposes an adaptive scheme to varying the flow partition exponent for MFD algorithms. Within this scheme, the flow partition exponent is changed adaptively based on local topographic attribute which not only reflect the terrain conditions but also controls the local drainage. Thus, the adaptive scheme can overcome the main problem of all MFD algorithms with a fixed $p$ value.

This paper also presentes the MFD-md algorithm which is an implementation of the proposed adaptive flow partition scheme. MFD-md selects the local maximum downslope gradient as the local topographic attribute in the adaptive scheme. Then MFD-md uses a function of the local maximum downslope gradient to vary the flow partition exponent in response to local terrain conditions. Unlike some current MFDs with varying flow partition exponents (e.g. Quinn et al. 1995), MFD-md does not need a spatial distribution of, or a threshold for, flow accumulation, which must be pre-computed with a flow routing algorithm for determining the flow partition exponent. Therefore, the MFD-md algorithm has lower complexity and is independent of other flow direction algorithms.

MFD-md is evaluated using four artificial surfaces. MFD-md has the lowest error rate in comparison with the D8 algorithm and the MFD-Quinn algorithm. This evaluation suggests
that the MFD-md can adaptively accommodate the effects of terrain conditions on flow distribution. The real world case further suggests that the spatial distribution of computed flow accumulation by MFD-md is more accurate than the result of D8 and is similar to the result of Quinn et al. (1991)’s MFD algorithm. Further analysis of valley regions shows that MFD-md is more adaptive to the terrain conditions than MFD-Quinn.

The adaptive scheme presented in this paper provides a flexible framework for varying flow partition exponent for MFD. During the implementation of the adaptive scheme, the selection of both threshold of parameters and local topographic attributes will deduce different models for local flow partitioning. MFD with a fixed \( p \) value can be considered as a special case of this adaptive scheme. For example, MFD-Quinn is the case when the adaptive scheme takes \( p_l = p_u = 1 \) in Equation (3). In future work, the impact of \( p_u, p_l, e_{\min}, \) and \( e_{\max} \) on MFD-md will be quantitatively studied in order to enhance the usability and robustness of this MFD-md algorithm. On the other hand, although the maximum downslope gradient might be the most important local topographic attribute controlling local drainage, it is not the only one. Other local topographic attributes, such as plan curvature and profile curvature, should be considered together with the maximum downslope gradient to model varying flow partition exponent.
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Table 1. The RMSE for four mathematical surfaces in Figure 4 using D8, MFD-Quinn and MFD-md algorithms

<table>
<thead>
<tr>
<th>Surface</th>
<th>Ellipsoid</th>
<th>Inverse ellipsoid</th>
<th>Saddle</th>
<th>Plane</th>
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<tr>
<td>D8</td>
<td>65.76</td>
<td>1906.44</td>
<td>125.05</td>
<td>96.02</td>
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<tr>
<td>MFD-Quinn</td>
<td>6.60</td>
<td>500.51</td>
<td>109.78</td>
<td>412.45</td>
</tr>
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<td>MFD-md</td>
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<td><strong>431.61</strong></td>
<td><strong>97.01</strong></td>
<td><strong>39.77</strong></td>
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</table>
Figure 1. Examples of the MFD-Quinn algorithm.

Figure 2. Comparison of (a) maximum downslope gradient, (b) minimum downslope gradient, (c) average downslope gradient, and (d) slope gradient.

Figure 3. Examples of the MFD-md algorithm.

Figure 4. The mathematical models, the contour maps of artificial surfaces, and the ‘true’ SCA distributions of four typical mathematical surfaces: (a) ellipsoid; (b) inverse ellipsoid; (c) saddle; and (d) plane. The unit represented is meters.

Figure 5. DEM of application area.

Figure 6. The spatial distribution of flow accumulation (m²) computed by (a) D8, (b) MFD-Quinn, and (c) MFD-md. (Contours with a 2.5 m interval of DEM are overlaid on the flow accumulation distribution)

Figure 7. The difference of flow accumulations (m²) computed by MFD-md and MFD-Quinn. (Contours with a 2.5 m interval of DEM are overlaid on the figure)
<table>
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<th>100.1</th>
<th>100</th>
<th>100.06</th>
<th>100.03</th>
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<td>99.97</td>
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<td>99.9</td>
<td>99.8</td>
<td>100</td>
<td>99.97</td>
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<th>5.7°</th>
<th></th>
<th>1.7°</th>
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</thead>
<tbody>
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<td></td>
<td>26.6°</td>
<td>35.3°</td>
<td>5.7°</td>
<td>8.0°</td>
<td>1.7°</td>
<td>2.4°</td>
<td></td>
</tr>
</tbody>
</table>

| MFD-Quinn    |     |     | 33.3%|       | 33.3% |     |        |        |     |

Figure 1. Examples of the MFD-Quinn algorithm.
Figure 2. Comparison of (a) maximum downslope gradient, (b) minimum downslope gradient,
(c) average downslope gradient, and (d) slope gradient.
Figure 3. Examples of the MFD-md algorithm.
(a) ellipsoid: \[
\frac{(x - 400)^2}{400^2} + \frac{(y - 300)^2}{300^2} + \frac{z^2}{20^2} = 1 \quad (z > 0) ; \quad 0 < x < 800; 0 < y < 600
\]

(b) inverse ellipsoid: \[
\frac{(x - 400)^2}{400^2} + \frac{(y - 300)^2}{300^2} + \frac{(z - 300)^2}{300^2} = 1 \quad (z < 300) ; \quad 0 < x < 800; 0 < y < 600
\]

(c) saddle: \[
\frac{(x - 400)^2}{2^2} - \frac{(y - 400)^2}{1^2} = \frac{z}{0.002} \quad 0 < x, y < 800
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(Contours with a 2.5 m interval of DEM are overlaid on the figure)