STATE FEEDBACK CONTROLLER DESIGN OF NETWORKED CONTROL SYSTEMS WITH PARAMETER UNCERTAINTY AND STATE-DELAY

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ABSTRACT

This paper is concerned with the controller design of networked control systems. The continuous time plant with parameter uncertainty and state delay is studied. A new model of the networked control system is provided under consideration of the nonideal network conditions. In terms of the given model, a controller design method is proposed based on a delay dependent approach. The maximum allowable synthetic bounds related with the discarded data packet and network-included delay and the feedback gain of a memoryless controller can be derived by solving a set of linear matrix inequalities for the stabilizability of the networked control system based on Lyapunov functional method. An example is given to show the effectiveness of our method.

KeyWords: Networked control systems, linear matrix inequalities, maximum allowable synthetic bounds, delay system.

I. INTRODUCTION

A networked control system (NCS) involves communication patterns in which both informational and physical control loops are closed through a real-time network. Recently, much attention has been paid to the study of stability analysis and control design of NCS [1–4], since their low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability. In an NCS, one of the important issues to treat is the effect of the network-induced delay on the system performance. Performance of the feedback control in the NCS is directly dependent upon the network-induced delay. Time-varying characteristics of the network-induced delay not only degrade control performance but also introduce distortion of the controller signal [5,6].

In feedback control systems, it is important that sampled data should be transmitted within a sampling period and that stability of control systems should be guaranteed. While a shorter sampling period is preferable in most control systems, for some purposes it can be lengthened up to a certain bound within which stability of the system is guaranteed in spite of the performance degradation. This certain bound is called a maximum allowable delay bound (MADB) [7]. An MADB has been obtained from stability conditions of control systems. There have been some results on the MADB for stability in non-networked control systems [8,9]. In these papers, the MADB is obtained using the Ricatti equation approach, which yields conservative delay bounds. There have been also some results on the MADB for stability in networked control systems, but these were concerned with stability or scheduling of the NCS with an allowable delay [1,4,10]. Walsh et al. introduce the notion of maximum allowable transfer interval (MATI) [11], denoted by τ, which supposes that successive sensor messages are separated by at most τ seconds. Less conservative results on the MADB in networked control systems are reported in [7]. However, these results still remain to be improved. The MADB thus obtained can be extended as a maximum bound of a sampling period in the NCS. That is, the sampling periods determined by the pro-
posed sampling period decision algorithm can be set to values less than the MADB. Furthermore, they can not deal with discarding data packet and error order etc.

There have many studies in the stability of the system in an NCS. Under an assumption that the network-induced delay is less than the sampling period (τ < h), stability of the NCS has been investigated in [4]. Time-invariant delay case was considered in [7,12]. A perturbation method of stability analysis of the NCS was given in [11,13]. However, the control law was used was designed in advance without considering the presence of the network in all above references. The stochastic optimal controller were proposed based on discrete-time model for the cases when the network-induced delay is shorter [14] or larger [15] than the sampling period. But it need a large requirement of controller memory to store a large amount of past information from the initial point. Moreover, to implement the controller, the information of all past delays must be known as a priori. And in these methods, the effect of controller-to-actuator delays was neglected. Moreover, no method was given in the above references how to estimate the noideal network status that guarantees the stabilizability of the NCS.

In this paper, we will be concerned with maximum allowable synthetical bound (MASB) related with the data packet dropout, network-included delay and the controller design of networked control systems. Under consideration of network-induced delay, a model of the NCS is presented based on work of [3,16], where the sensor is clock-driven and the controller and actuator are event-driven and the data is transmitted with a single-packet. In contrast with the controller design method based on continuous linear time-invarying model, our method considers the NCS is a piecewise continuous uncertain parameters system with state-delay. Moreover, the network-induced delay considered in this paper can be slowly or fast time-varying. Then, a controller design method is proposed based on a delay dependent approach. From the derived criteria, the memoryless controller can be designed and the MASB can be determined by solving a set of linear matrix inequalities (LMIs). One example is finally given to show the effectiveness of our method.

**Notation.** \( \mathbb{R}^n \) denotes the \( n \)-dimensional Euclidean space, \( \mathbb{R}^{m \times n} \) is the set of \( n \times m \) real matrices, \( I \) is the identity matrix of appropriate dimensions, \( \| \cdot \| \) stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate. The notation \( X > 0 \) (respectively, \( X \geq 0 \)), for \( X \in \mathbb{R}^{m \times n} \) means that the matrix \( X \) is real symmetric positive definite (respectively, positive semi-definite). \( \lambda_{\text{max}}(P)(\lambda_{\text{min}}(P)) \) denotes the maximum (minimum) of eigenvalue of real symmetric matrix \( P \). For an arbitrarily matrix \( B \) and two symmetric matrices \( A \) and \( C \), \( \begin{bmatrix} A & B \\ * & C \end{bmatrix} \) denotes a symmetric matrix, where * denotes a block matrix entry implied by symmetry.

### 1.1 Modeling of NCS

In this paper we consider the following system with state-delay and parameter uncertainty:

\[
\begin{align*}
\dot{x}(t) &= (A + \Delta A)x(t) + (A_i + \Delta A) x(t - d) + (B + \Delta B) u(t) \\
x(t) &= \varphi(t), \quad t \in [-d, 0]
\end{align*}
\]

(1)

(2)

where \( x(t) \in \mathbb{R}^n \) and \( u(t) \in \mathbb{R}^n \) are the state vector and the control input vector respectively. \( A, A_i, B \) and \( \Delta A \) are constant matrices with appropriate dimensions. \( \Delta A, \Delta A_i, \) and \( \Delta B \) are uncertain matrices which can be time-varying. \( d \) is a scalar representing the delay in the system.

Suppose the parameter uncertainties \( \Delta A, \Delta A_i, \) and \( \Delta B \) of system model are norm-bounded and satisfies

\[
[\Delta A \quad \Delta A_i \quad \Delta B] = DF(t)[E_1 \ E_2 \ E_3]
\]

(3)

where \( F(t) \in \mathbb{R}^{m \times n} \) is an uncertain matrix which satisfies \( F^T(t)F(t) \leq I \). \( D, E_1, E_2, \) and \( E_3 \) are constant matrices of appropriate dimensions.

Suppose the sensor are clock-driven and controller and actuator are event-driven, the data is transmitted with a single-packet and the full state variables are available for measurements, the real input to any controller realize through zero-order hold is piecewise constant function.

Assume that there are no error codes in the transmission and that the state variables are available for state feedback control. Apparently, the data packet transmitted delay \( \tau^{\text{in}}_k \) of sensor to controller do not change the value of \( x(i_k h) \), so \( x(i_k h) = x(i_k h + \tau^{\text{in}}_k) \). Similarly, the data packet transmitted delay \( \tau^{\text{out}}_k \) of controller to actuator do not change the value of \( Kx(i_k h + \tau^{\text{out}}_k) \), so \( u(i_k h + \tau^{\text{out}}_k) = Kx(i_k h + \tau^{\text{out}}_k) \), where \( K \) is the state feedback gain. From the above analysis, we can obtain \( u(i_k h + \tau^{\text{out}}_k) = Kx(i_k h) \) (see the parallel dashed line in Fig. 1) and hence

\[
u(t^+) = Kx(t - \tau^{\text{out}}_k), \quad t \in \{i_k h + \tau^{\text{out}}_k, k = 1, 2, \cdots\}
\]

(4)

where \( u(t^+) = \lim_{t \to t^+} u(t) \), \( h \) be the sampling period, \( i_k (k = 1, 2, 3, \ldots) \) be some integers such that \( \{i_1, i_2, i_3, \ldots\} \subset \{0, 1, 2, 3, \ldots\} \). The network-induced delay \( \tau^{\text{in}}_k \) is the time from the instant \( i_k h \) when sensors sample from the plant to the instant when actuator transmit data to the plant. 

\[
\tau^{\text{in}}_k = \tau^{\text{in}}_k^0 + \tau^{\text{in}}_k^* \quad \tau^{\text{in}}_k^0 \quad \text{is the sensor-to-controller delay}, \quad \tau^{\text{in}}_k^* \quad \text{controller-to-actuator delay and compute and overhead delay is included in } \tau^{\text{in}}_k^*
\]

The time interval between the instant \( i_k h + \tau^{\text{in}}_k \) of a packet arriving at the actuator and the next arrival instant \( i_{k+1} h + \tau^{\text{in}}_{k+1} \) is the effective duration of the hold operation. Obviously, \( \sum_{k} \left(i_k h + \tau^{\text{in}}_k, i_{k+1} h + \tau^{\text{in}}_{k+1} \right) = [t_0, \infty), \quad t_0 \geq 0.\)
According to (4), the input \( u(t) \) realized through a zero-order hold is a piecewise constant function and the effective control system can be modeled as

\[
\dot{x}(t) = (A + \Delta A) x(t) + (A_1 + \Delta A_1) x(t-\tau) + (B + \Delta B) u(t), \quad t \in [i_k h + \tau, i_{k+1} h + \tau_{i_{k+1}})
\]

\[
u(t') = Kx(t - \tau), \quad t' \in [i_k h + \tau, k = 1, 2, \ldots]
\]

Notice that it is not required to have \( i_{k+1} > i_k \). If \( i_{k+1} = i_k \), it means that there is no data packet dropout in the transmission. If \( i_{k+1} > i_k + 1 \), there are some data packet dropout but the data are ordered correctly. If \( i_{k+1} < i_k \), it means unordered data arrival sequence occurs, which includes \( \tau_k = \tau_0 \) and \( \tau_k < h \) as the special cases. These possible nonideal network conditions are taken into account in (5) which are illustrated in Fig. 1.

From Fig. 1, it can be observed that:

- \( h \to 2h \) data packet dropout may occur between sensor and controller or controller and actuator.
- \( 2h \to 3h \) data from sensor to controller are ordered, but data from controller to actuator are unordered.
- \( 4h \to 5h \) data from sensor to controller are unordered and controller to actuator are also unordered.
- \( 6h \to 7h \) data from sensor to controller are unordered, but controller to actuator are ordered.

In (5), when the data arrival sequence is unordered, \( i_{k+1} < i_k \). For example, in Fig. 1 when \( 2h \to 3h \), \( i_k h + \tau = 3h + \tau \), \( i_{k+1} h + \tau_{i_{k+1}} = 2h + \tau_2 \) and \( 6h \to 7h \), \( i_k h + \tau = 7h + \tau_3 \), \( i_{k+1} h + \tau_{i_{k+1}} = 6h + \tau_2 \). The control \( u(t) \) maintains at a constant value of \( u(t = i_k h + \tau) \) by the zero-order hold when \( t \in [i_k h + \tau, i_{k+1} h + \tau_{i_{k+1}}] \) as observed when \( t \in [\tau_0, 3h + \tau_3) \) or \([3h + \tau_3, 2h + \tau_2)\).

In this paper, we assume that \( u(t) = 0 \) before the first control signal reaches the plant. The system (5) can be rewritten as

\[
\dot{x}(t) = (A + \Delta A) x(t) + (A_1 + \Delta A_1) x(t-\tau) + (B + \Delta B) Kx(i_k h), \quad t \in [i_k h + \tau, i_{k+1} h + \tau_{i_{k+1}})
\]

It is easy to see that the solutions of (6) are continuous on \([\tau_0, \infty)\). To facilitate development, we first introduce the following definition.

**Definition 1.** A maximum allowable synthetic bound (MASB) of NCS, denoted by \( \eta \), which satisfies \( (i_{k+1} - i_k)h + \tau_{i_{k+1}} \leq \eta, k = 1, 2, 3, \ldots \)

**Remark 1.** MASB is related with the number of data packet dropout \( |i_{k+1} - i_k| - 1 \), network-induced delay \( \tau_k \) and sampling period \( h \). Therefore, when the MASB is obtained, we can use it as a better scheduling method for NCS.

**II. CONTROLLER DESIGNS OF NCS**

In this section, we assume that the full state variables are available for measurements. We present a MASB calculation method and relevant controller design method for system (6) based on a Lyapunov functional method.

First, let's consider the simple case without the parameter uncertainties \( \Delta A, \Delta A_1, \) and \( \Delta B \). We present a controller design method for system (6) based on an LMI approach.
Theorem 1. For given scalars \( \eta > 0 \) and \( \lambda_i (i = 2, 3, 4) \), if there exist matrices \( \tilde{P}, \tilde{S}, \) and \( \tilde{R} > 0 \), a nonsingular \( \Upsilon \) and matrices \( Y \) and \( \tilde{M}_i (i = 1, 2, 3, 4) \) with appropriate dimensions such that

\[
\begin{bmatrix}
\tilde{P} & \tilde{S} & \tilde{R} \\
\tilde{P}^T & \tilde{S} & \tilde{R} \\
\tilde{P}^T & \tilde{S} & \tilde{R}
\end{bmatrix} < 0
\]

(7)

are satisfied, where

\[
\begin{align*}
\tilde{P}_{11} &= \tilde{S} + AX^T + AX + M_1 + \tilde{M}_1^T, \\
\tilde{P}_{12} &= A X^T + \lambda_2 X A \\
\tilde{P}_{13} &= -\lambda_2 A^T X - \lambda_2 X A^T, \\
\tilde{P}_{14} &= \tilde{P} + \lambda_2 A X A^T - X^T + \tilde{M}_1^2, \\
\tilde{P}_{22} &= -\lambda_2 X^T + \lambda_2 A X^T, \\
\tilde{P}_{23} &= -\lambda_2 X^T + \lambda_2 A X^T, \\
\tilde{P}_{24} &= \lambda_2 B Y + \lambda_4 Y B^T - \tilde{M}_2^T, \\
\tilde{P}_{33} &= -\lambda_3 Y^T + \lambda_4 B^T Y^T - \tilde{M}_3^T, \\
\tilde{P}_{44} &= \eta R - \lambda_4 X^T - \lambda_4 X
\end{align*}
\]

then the system (6) is asymptotically stable with the feedback gain \( K = YX^{-\tau} \).

Proof. We set \( y(t) = \dot{x}(t) \) and construct a Lyapunov functional candidate as

\[
V(t) = V_1(t) + V_2(t) + V_3(t)
\]

where \( V_1(t) = \int_{i-\tau}^{t} \dot{x}^T(s) P \dot{x}(s) ds \), \( V_2(t) = \int_{i-\tau}^{t} \dot{y}(s) R \dot{y}(s) ds \), and \( P > 0 \), \( S > 0 \) and \( R > 0 \). Taking the time derivative of \( V(t) \) for \( t \in [i_h + \tau_i, i_{i+1} h + \tau_{i+1}] \), we have

\[
\begin{align*}
\dot{V}_1(t) &= 2 \dot{x}^T(t) \dot{P} \dot{x}(t) \\
\dot{V}_2(t) &= \dot{y}^T(t) S \dot{x}(t) - \dot{x}^T(t-d) S \dot{x}(t-d) \\
\dot{V}_3(t) &= \eta \dot{y}^T(t) R \dot{y}(t) - \int_{i-\tau}^{t} \dot{y}^T(s) R \dot{y}(s) ds
\end{align*}
\]

(9)

(10)

(11)

Using the Newton-Leibniz formula \( x(t) = x(i_h) - \int_{i_h}^{t} y(s) ds = 0 \) and (6), we can show that, for arbitrary matrices \( N_i \) and \( M_i (i = 1, 2, 3, 4) \) of appropriate dimensions,

\[
[\dot{x}^T(t) M_1 + \dot{x}^T(t-d) M_2 + \dot{x}^T(i_h) M_4 + \dot{y}^T(t) M_4]\]

\[
\begin{bmatrix}
\dot{x}(t) - x(i_h) - \int_{i_h}^{t} y(s) ds
\end{bmatrix} = 0
\]

(12)

and

\[
\begin{align*}
[x^T(t) N_1 + x^T(t-d) N_2 + x^T(i_h) M_1 + y^T(t) M_4]\]

\[
\begin{bmatrix}
x(t) - x(i_h) - \int_{i_h}^{t} y(s) ds
\end{bmatrix} = 0
\]

Then using (12) and (13) and the whole time derivative of \( V(t) \) for \( t \in [i_h + \tau_i, i_{i+1} h + \tau_{i+1}] \) yields:

\[
V(t) = V_i(t) + V_{i+1}(t) + V_{i+2}(t)
\]

\[
\begin{align*}
&+ 2 \int_{i-\tau}^{t} [x^T(t) M_1 + x^T(t-d) M_2 + x^T(i_h) M_4 + y^T(t) M_4]\]
&\begin{bmatrix}
x(t) - x(i_h) - \int_{i_h}^{t} y(s) ds
\end{bmatrix} ds
\end{align*}
\]

(13)

\[
\begin{align*}
&+ 2 \int_{i-\tau}^{t} [x^T(t) N_1 + x^T(t-d) N_2 + x^T(i_h) N_3 + y^T(t) N_4]\]
&\begin{bmatrix}
x(t) - x(i_h) - \int_{i_h}^{t} y(s) ds
\end{bmatrix} ds
\end{align*}
\]

(14)

From (8), we can obtain that, when \( t \in [i_h + \tau_i, i_{i+1} h + \tau_{i+1}] \),

\[
- \int_{i-\tau}^{t} \dot{y}^T(s) R \dot{y}(s) ds \leq - \int_{i_h}^{t} \dot{y}^T(v) R \dot{y}(v) dv
\]

(15)

and

\[
-2 \int_{i-\tau}^{t} [x^T(t) M_1 + x^T(t-d) M_2 + x^T(i_h) M_4 + y^T(t) M_4]\]

\[
\begin{bmatrix}
x(t) - x(i_h) - \int_{i_h}^{t} y(s) ds
\end{bmatrix} \leq \eta \xi^T(t) M^T R^{-1} M \xi(t)
\]

(16)

where \( \xi(t) = [x^T(t) x^T(t-d) x^T(i_h) y^T(t)] \), \( M = [M_1^T M_2^T M_3^T M_4^T] \). Combining (14)-(16), we obtain
\begin{equation}
\dot{V}(t) \leq \xi^T(t) + \eta \xi^T(t) M^T R^{-1} M \xi(t) \tag{17}
\end{equation}

where

\begin{align*}
\Pi_{11} &= S + N_1 A + A^T N_1^T + M_1 + M_1^T \\
\Pi_{12} &= N_1 B K + A^T N_1^T - M_1 + M_1^T \\
\Pi_{13} &= -S + N_1 A + A^T N_1^T \\
\Pi_{14} &= P + A^T N_1^T - N_1 + M_1^T \\
\Pi_{22} &= -S + N_2 A + A^T N_2^T \\
\Pi_{23} &= N_2 B K + A^T N_2^T - M_2 \\
\Pi_{24} &= -N_2 + K^T B^T N_2^T - M_2 \\
\Pi_{33} &= N_3 B K + K^T B^T N_3^T - M_3 - M_3^T \\
\Pi_{34} &= \eta R - N_4 - N_4^T \\
\Pi_{44} &= \eta R - N_4 - N_4^T
\end{align*}

Suppose (7) is satisfied, it is apparent that \( \Pi_{44} = \eta R - \lambda \), \( \Xi^T + \lambda \xi^T + \lambda \xi^T \leq 0 \), According to Theorem 1, \( \eta > 0 \), \( \Xi > 0 \), assume \( \lambda > 0 \), we can obtain \( \Xi^T + \lambda \xi^T + \lambda \xi^T \) is nonsingual. Defining \( N_1 = \lambda_1 N_1, N_2 = \lambda_1 N_1, N_4 = \lambda_1 N_1, \) and \( Y = KX^T, \) defining \( P = X P X^T, \) \( \dot{R} = X R X^T, \) \( \dot{S} = X S X^T, \) and \( M_i = X M_i X^T (i = 1, 2, 3, 4) \) then pre, post-multiplying both sides of (19) with diag \( (XXX) \) and its transpose, we can get (7), therefore, (19) is equivalent to (7).

\begin{equation}
\begin{bmatrix}
\Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \eta M_1 \\
* & \Pi_{22} & \Pi_{23} & \Pi_{24} & \eta M_2 \\
* & * & \Pi_{33} & \Pi_{34} & \eta M_3 \\
* & * & * & \Pi_{44} & \eta M_4 \\
* & * & * & * & -\eta R
\end{bmatrix} < 0 \tag{19}
\end{equation}

According to (19), by Suchr complements, we can obtain that (17) is smaller than zero \( (\dot{V}(t) < 0) \) for \( t \in [i, h + \tau_i, \tau_{i+1} + h + \tau_{i+1}] \). Since \( \bigcup_{i=1}^{\infty} [i, h + \tau_i, \tau_{i+1} + h + \tau_{i+1}] \) \( = [t_0, \infty) \), \( t_0 > 0 \) and \( V(t) \) is continuous in \( t \) [14]. We can deduce \( \dot{V}(t) < 0 \) for \( t \in [t_0, \infty) \). Therefore, by using the Lyapunov-Krasovskii theorem, the closed-loop system (6) is asymptotically stable. This completes the proof.

**Remark 2.** From the proof, we can obtain that \( X \) is nonsingual. So in the proof, we can set \( Y = KX^T \) and obtain \( K = I X X^T \). Otherwise, In this paper, the system is asymptotically stable means that an equilibrium is asymptotically stable. Considering the effect of the parameter uncertainties \( \Delta A, \Delta A_1, \) and \( \Delta B \), we conclude the following result for the uncertain time-delay systems (6).

**Theorem 2.** For given scalars \( \eta \) and \( \lambda (i = 2, 3, 4) \), if there exist scalars \( \varepsilon_i > 0 \) \( (i = 1, 2, 3) \), matrices \( \tilde{P}, \tilde{S}, \) and \( \tilde{R} > 0 \), a nonsingualr \( X \) and matrices \( Y \) and \( M_i (i = 1, 2, 3, 4) \), with appropriate dimension, if

\begin{equation}
\begin{bmatrix}
\Pi_{11} + \Phi & \Pi_{12} & \Pi_{13} & \Pi_{14} & \eta M_1 & \lambda X E_1^T & 0 & 0 \\
* & \Pi_{22} + \lambda_2 \Phi & \Pi_{23} & \Pi_{24} & \eta M_2 & 0 & \lambda X E_2^T & 0 \\
* & * & \Pi_{33} + \lambda_3 \Phi & \Pi_{34} & \eta M_3 & 0 & 0 & \lambda Y E_3^T \\
* & * & * & \Pi_{44} + \lambda_4 \Phi & \eta M_4 & 0 & 0 & 0 \\
* & * & * & * & -\lambda E_4 I & 0 & 0 & 0 \\
* & * & * & * & * & -\lambda E_5 I & 0 & 0 \\
* & * & * & * & * & * & -\lambda E_5 I & 0 \\
* & * & * & * & * & * & * & -\lambda E_5 I
\end{bmatrix} < 0 \tag{20}
\end{equation}
\begin{equation}
(l_{k+1} - l_k) h + \tau_{k+1} \leq \eta
\end{equation}
where \( \Phi = (e_1 + e_2 + e_3) DD^T, \lambda = 1 + \lambda_2 + \lambda_3 + \lambda_4 > 0 \)

and the feedback gain system where (24)

\begin{equation}
1 + \lambda_2 + \lambda_3 + \lambda_4 > 0
\end{equation}

where \( \Phi = (e_1 + e_2 + e_3) DD^T, \lambda = 1 + \lambda_2 + \lambda_3 + \lambda_4 \)
then the system (6) with the feedback gain \( K = YX^T \) is asymptotically stable.

**Proof.** Replace \( A, A_1, \) and \( B \) with \( A + DF(t)E_1, A_1 + DF(t)E_2, \) and \( B + DF(t)E_3 \) in (14). Then, following the similar procedure as in the proof of theorem 1, we can obtain

\[
\begin{bmatrix}
\Theta 
\end{bmatrix} \begin{bmatrix}
\tilde{\Pi}_{12} \\
\tilde{\Pi}_{13} \\
\tilde{\Pi}_{14} \\
\eta M_1 \\
\cdots \\
\cdots \\
\tilde{\Pi}_{14} \\
\eta M_2 \\
\cdots \\
\cdots \\
\tilde{\Pi}_{14} + \lambda_4 \Phi \\n\eta M_4
\end{bmatrix} < 0
\]

where \( \Theta = \tilde{\Pi}_{11} + \Phi + \lambda e_1^T X E_1^T X + X Q X^T, \quad \Xi = \tilde{\Pi}_{22} + \lambda_2 + \lambda e_2^T X E_2^T Y + Y^T R Y, \quad \Phi = \tilde{\Pi}_{13} + \lambda_3 \Phi + \lambda e_3^T Y E_1 Y + Y^T R Y \).

Using Schur complement, we can obtain (20) from (23) and 22 from \( \lambda > 0 \). This completes the proof.

From Theorem 1 and Theorem 2, it is apparent that different \( \lambda \) (\( i = 2, 3, 4 \)) have different value of \( \eta \), so we purpose such search algorithm to find the optimal value of \( \lambda (i = 2, 3, 4) \) to obtain the sub-maximum \( \eta_{\text{max}} \). To obtain the \( \eta_{\text{max}} \), we present following search algorithm:

**Algorithm:**

1. Given the \( \alpha, \beta_i \) as upper and lower bound of \( \lambda, \zeta_i \) (\( i = 2, 3, 4 \)) as step increment. Set \( \eta_0 = 0 \) and \( K = 0 \).
2. According to theorem 1, under the constraint of \( \zeta_i, \alpha_i \), and \( \beta_i \), group different \( \lambda \) to obtain corresponding \( \eta \) and \( K = YX^T \) based on LMI, subject to (25) and (8). If \( \eta > \eta_{\text{max}} \), set \( \eta = \eta_0 \) and \( K = K_0 \).
3. Output \( \eta_0 = \eta_{\text{max}} \) and \( K = K_0 \).

**III. NUMERICAL EXAMPLE**

This section presents an example to obtain the \( \eta_{\text{max}} \) and the feedback gain \( K \). Consider an uncertain state delay system

\begin{equation}
\dot{x}(t) = (A + \Delta A) x(t) + (A_1 + \Delta A_1) x(t - d) + (B + \Delta B) u(t)
\end{equation}

where

\[
A = \begin{bmatrix}
0 & 1 \\
0 & -0.1
\end{bmatrix}, \quad A_1 = \begin{bmatrix}
-0.1 & 0 \\
0 & -0.1
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
0.1
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
0.1 \\
0
\end{bmatrix}, \quad E_1 = [0.2 \ 0], \quad E_2 = [0 \ 0.1], \quad E_3 = [0.4]
\]

The feedback gain of networked controller is designed as \( K = YX^T \), the \( \eta_{\text{max}} \) that guarantees the stabilizability of the system (6), we apply the theorem 2 and the search algorithm to derive the \( \eta_{\text{max}} \) and the corresponding feedback gain \( K \) with \( \lambda_2 = -1.1, \lambda_3 = 0.5, \lambda_4 = 8.4, \) it has been found that the maximum value of \( \eta_{\text{max}} \) is 2.95 and the corresponding feedback gain is \( K = [-0.2275 - 2.1247] \).

In other words, as long as \( \eta \), the (6) with \( K = [-0.2275 - 2.1247] \) is asymptotically stable. It means that, if \( h = 0.2 \) ms and the nonideal transmitted data packet can be neglected in the transmission, the maximum allowable delay \( \tau_k \leq 2.75 \) ms. The designed controller can stabilize the system (5) as long as the upper bound of the network-induced delay is less than 2.75 ms.

A plot of the states of the above uncertain system with different controller feedback gain is shown in Fig. 3. It apparent that the system is asymptotically stable.

According to the \( \eta \), if we have known the MADB of specified NCS, we can obtain the relationship of the sampling period \( h \), number of data packet dropout \( (l_{k+1} - l_k) \), and network-induced delay \( \tau_{k+1} \), it’s shown in Table 2. From the Table 2, if the sampling period is set to 0.4, \( \eta_{\text{max}} \) is 2.95, then it allow data packet dropout and nonordered data arrival sequence happen. For example:

- No data packet dropout \( |i_{k+1} - i_k| = 1 \), no unordered sequence \( i_{k+1} > i_k \), the MADB is 2.55 \( \eta_{\text{max}} \leq \eta - h \).

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- No data packet dropout \( |i_{k+1} - i_k| = 1 \), no unordered sequence \( i_{k+1} > i_k \), the MADB is 2.55 \( \eta_{\text{max}} \leq \eta - h \).
- Have an unordered sequence \( i_{k+1} < i_k \), the MADB is 3.35 \( \tau_{k+1} \leq \eta + h \).

**Fig. 2.** State variable versus sampling time with \( K = [-0.2275 - 2.1247] \).
Have data packet dropout $|i_{k+1}-i_k| > 1$, number of data packet dropout $(|i_{k+1}-i_k| - 1)$ can be 1, 2, 3, ... under the constraint of $(\tau h_k) \leq \eta$. Which including no unordered data arrival sequence $i_{k+1} > i_k$, and unordered data arrival sequence $i_{k+1} < i_k$. When have one data packet dropout $|i_{k+1}-i_k| = 2$, the MADB is $2.35\ (\tau h_k) \leq \eta - 2h$.

MASB provide a better scheduling method to NCS. Such as, to the specified network (e.g. DeviceNet, ControlNet etc.), we can estimate the network-induced delay as a prior knowledge of network scheduling, then adjust the sampling period $(h)$ and the rate of active data packet dropout to get optimal performance index.

**Table 2. The relationship of scheduling parameter.**

<table>
<thead>
<tr>
<th>MAEDB</th>
<th>Sampling Period</th>
<th>Data Dropout Number</th>
<th>Allowable Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.95</td>
<td>0.2</td>
<td>0</td>
<td>2.75</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>2.35</td>
</tr>
<tr>
<td>0.4</td>
<td></td>
<td>0</td>
<td>2.55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2.15</td>
</tr>
</tbody>
</table>

### IV. CONCLUSIONS

Performance of the feedback control in the NCS is directly dependent upon the network-induced delay. Time-varying characteristics of the network-induced delay not only degrade control performance but also introduce distortion of the controller signal. In this paper, the plant with uncertain parameters and state delay is studied. We assure that the full state variables are available for measurements, present a calculational method of a new rule and relevant controller design method for system (5). The maximum allowable synthetical bounds are obtained for the stabilizability of the NCS based on Lyapunov functional method and linear matrix inequalities (LMI) formulation. As future works, the data dropout and the multi-loops network scheduling algorithm will be considered and the plant with performance index such as guaranteed cost and $H_2/H_{\infty}$ are necessary to be studied.

### REFERENCES

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