Optical nonlinear effects on the performance of IP traffic over GMPLS-based DWDM networks

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Received 2 May 2002; revised 27 January 2003; accepted 27 January 2003

Abstract

With the introduction of Generalized multiprotocol label switching in the common control plane for optical crossconnects and electrical switching devices, the integration of IP and dense wavelength division multiplexing can be efficiently achieved. When IP traffic flows are directly mapped to optical channels without SDH inter-layer, the bursty nature of IP traffic may have significant effect on the performance of the optical channel. On the other hand, the nonlinear effects of the optical channel may also have significant effect on the performance at IP level. This paper focuses on the effect of fiber nonlinearities on the performance of IP traffic. Numerical results including IP packet error probability and high-order distribution functions of IP packet error are presented in terms of IP traffic load, input light signal power and the frequency space between the optical wavelengths. It is demonstrated that when IP traffic load is light in such systems, the effect of either four-wave mixing (FWM) or stimulated Raman scattering (SRS) is much less serious than the worst-case assumption, and the limitation on allowable power of input light is also relaxed. The obtained numerical results demonstrated that FWM and SRS are both sensitive to power level of input light and frequency spacing, but each in a unique way. Effort must be taken in system design to avoid improving the performance of one effect at the expense of deteriorating the other.

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Keywords: Wavelength division multiplexing; IP; Generalized multiprotocol label switching; Four-wave mixing; Stimulated Raman scattering

1. Introduction

Internet Protocol (IP) has been a key factor to propel the dramatic growth of data traffic in the past ten years. With the introduction of QoS mechanisms such as integrated services (IntServ) and Differentiated Services (DiffServ), the Internet is now able to support voice and video applications [1]. Hence IP-based traffic volumes have been continuously growing at a fast speed. Meanwhile, the development of optical communication technologies, especially dense wavelength division multiplexing (DWDM) technology, has provided huge bandwidth up to terabits to meet the bandwidth requirement of the IP traffic. On the other hand, the programmable optical crossconnects (OXC) with wavelength conversion are able to switch multigigabit or even terabit data streams. Therefore, a lot of research and development works in recent years have been concentrating on finding an efficient way to integrate IP and WDM technologies [2–6].

Generalized multiprotocol label switching (GMPLS), also known as multiprotocol lambda switching (MPAS), is the approach that integrates IP and DWDM most efficiently, which has been standardized by IETF [7–10]. GMPLS is based on the idea of multiprotocol label switching (MPLS) but it extends the concept to support devices that perform switching in time, wavelength and space domain, in addition to packet switching devices. Therefore it is especially suitable as the control plane for OXC. Fig. 1 shows a conceptual network diagram of DWDM-based IP backbone using GMPLS, which is the common control plane for the switches in both optical and electrical domains. OXC can be able to exchange state or bandwidth information with other LSRs in the network. On the other hand, LSPs can be established across the entire network through OXCs and electrical switches. The establishment, maintenance and tear down of LSPs in the network are all controlled by GMPLS control plane [8–10].
GMPLS based IP-DWDM system has several advantages: (1) simplified management plane. All switching devices are managed by a single control plane, which results in reduced operational costs. However, this feature requires modifications and extensions to current routing (OSPF) and signaling (RSVP, LDP) protocols, which are standardized by IETF under the umbrella of generalized MPLS. (2) Simplified protocol stack in data plane. IP traffics are directly mapped into optical channels without any intermediate protocol layers, such as ATM or SDH, between IP and DWDM, which results in much more efficient usage of the optical bandwidth. (3) Flexible implementation. Under the framework of GMPLS, different deployment models can be used to meet different service requirements.

However, GMPLS also raises some new issues in the integration of IP and WDM. One of these issues is the coupling of the physical layer characteristics and the IP layer traffic pattern. The removal of ATM or SDH layer implies that the traffic patterns carried in the optical physical layer and IP layer are correlated and affect each other. The performance of optical channels is affected by the bursty nature of IP traffic, while the existence of nonlinearities of optical fiber will also affect the performance in the IP layer. Generally, when only a few wavelengths are used in a strand of fiber, the interference between the light channels is negligible, and optical channels are considered to be independent of each other. However, this is true only when the optical layer parameters including the power of input light and the frequency spacing between wavelengths are within certain ranges so that the optical fiber can be treated as a linear transmission media. When numerous channels are put in the fiber and the power of input light is high enough, optical nonlinearities can cause performance degradation by introducing cross-channel interference or power depletion of the signal light, and pose limitations on the launch power and/or repeater spacing.

Of all the fiber nonlinear effects, four-wave mixing (FWM), simulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS) are the most significant factors in DWDM system design. In Refs. [13,14,22], the FWM cross talk and bit error rate performance in WDM systems are studied. The effects of the stimulated Raman scattering and the optical amplifier noise on DWDM networks are studied in Refs. [11–13,18,20,21] based on the assumption that every channel is carrying continuous optical signal but without considering the traffic pattern carried on the optical channel. However, in GMPLS-based WDM network, the traffic load and the bursty nature of IP traffic may interact with fiber nonlinearities such as FWM and SRS. In order to take full advantage of IP as well as DWDM technology, it is necessary and meaningful to study the effect of IP traffic across DWDM networks. Although a lot of research has been done on fiber nonlinearities and their effect on WDM systems, the effect of IP traffic on these nonlinearities and eventually on the performance of WDM system has not been fully explored yet. This paper concentrates on the effect of FWM and SRS on the performance of IP traffic over GMPLS-based DWDM networks.

This rest of the paper is organized as follows. In Section 2, we discuss the cross-channel interference caused by FWM, and the effect of IP traffic on the interference due to FWM. In Section 3, power depletion caused by SRS and the interaction of SRS, IP traffic load, optical signal power and frequency spacing are addressed in detail. Section 4 gives...
a comparison of the effects of input light power and frequency spacing on FWM and SRS, and the conclusion remarks are presented in Section 5.

2. Effects of four-wave mixing on IP traffic

To utilize the available optical bandwidth, we tend to multiplex numerous channels at different wavelengths on the same fiber. To increase system margins, higher transmitter powers or lower fiber losses are required. But all these attempts to fully utilize the capabilities of silica fibers will ultimately be limited by nonlinear interactions between the information-bearing lightwaves and the transmission medium [19]. These optical nonlinearities can lead to interference, distortion and excess attenuation of the optical signals, resulting in system degradations.

Of all the nonlinear effects, FWM, SRS and SBS are the most significant factors in system design. Each of these effects manifests itself in a unique way. FWM appears at the lowest input power level in a dispersion-shifted fiber system, and causes cross-channel interference [15–17]. In a WDM system with tens of multiplexed channels, the launch power is restricted by FWM [23]. SRS cause power depletion in the lower wavelength channel and is the limiting factor of the launch power in a system with hundreds of channels. Whereas the launch power for individual channels is limited due to SBS.

Since the focus of this paper is on DWDM system with a number of channels, we investigate the FWM and SRS effects in the GMPLS-based IP-DWDM context in the following sections. As the trend of mapping IP traffic directly into light path without other multiplexing scheme is clear in constructing IP backbones based on DWDM, the light paths will most probably be loaded with burst IP traffic such as MPEG video, data transfer applications or multimedia traffic in the network. This section and Section 3 intend to look into the nonlinear effects when loading IP packets directly to WDM channels.

2.1. Cross talk caused by four-wave mixing

FWM originates from the phenomenon that the polarization, induced in the electric dipoles of a medium by an electric field, \( E \), is nonlinear in \( E \). The third-order parametric process results in the lowest order nonlinear effects in optical fibers. Among them, four-wave mixing is the most influential factor for optical FDM network design. In multi-channel systems, a signal channel suffers from FWM and SRS, such as MPEG video, data transfer applications or multimedia traffic in the network. This section and Section 3 intend to look into the nonlinear effects when loading IP traffic in the network. This section and Section 3 intend to look into the nonlinear effects when loading IP traffic in the network.

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Fig. 2. The optical frequency arrangement of four-wave mixing.

\[
P_{\text{FWM}}(f_i, f_j, f_k) = \frac{1024 \pi^6}{n^2 \lambda^6 c^6} \left( \frac{\nu^2}{\lambda^2} \right)^2 \frac{P_i P_j P_k}{A_\text{eff}^4} \left( 1 - e^{-\alpha L} \right)^2 \frac{1}{\alpha^2} \eta
\]

where \( P_i, P_j \) and \( P_k \) represent the input power of the frequencies \( f_i, f_j \) and \( f_k \), respectively; \( P_{\text{FWM}} \) is the power of the lightwave from four-wave mixing at the frequency \( f_{\text{FWM}} \); \( n \) is the fiber refractive index; \( \lambda \) is the wavelength; \( A_\text{eff} \) is the effective mode area of the fiber; \( \alpha \) is the fiber loss coefficients; \( L \) is fiber length; \( d \) is the degeneracy factor (\( d = 3 \) for \( i = j, d = 6 \) for \( i \neq j \)), and \( \chi \) is the third-order nonlinear susceptibility. \( \eta(f_i, f_j, f_k) \) is the mixing efficiency given by [13,14]:

\[
\eta = \frac{\alpha^2}{\alpha^2 + (\Delta \beta)^2} \left\{ 1 + \frac{4e^{-\alpha L} \sin^2(\Delta \beta L/2)}{[1 - e^{-\alpha L}]^2} \right\}
\]

where \( \Delta \beta \) represents the phase mismatch and may be expressed in terms of signal frequency differences [13,14]:

\[
\Delta \beta = \frac{2\pi \lambda^2}{c} |f_i - f_j| f_j
\]

\[
- f_k \left\{ D + \frac{dD}{d\lambda} \left( \frac{\lambda^2}{2c} \right) (|f_i - f_k| + |f_j - f_k|) \right\}
\]

Where \( D \) is the fiber chromatic dispersion. Eqs. (2) and (3) show that the four-wave mixing efficiency decreases with increasing signal frequency difference and chromatic dispersion due to increased phase mismatch between the signals.

In WDM system, spacing between the wavelengths may be uniformly distributed ranging from a few gigahertz to 100 GHz [13]. In such systems, at any particular channel frequency, there will be a number of FWM waves generated from various combinations of interacting signals whose frequencies satisfy: \( f_{\text{FWM}} = f_i + f_j - f_k \).

The total power generated at frequency \( f_m \) may be expressed as a summation:

\[
P_{\text{tot}}(f_m) = \sum_{f_i + f_j - f_k = f_m} \sum_{f_i} \sum_{f_j} \sum_{f_k} P_{\text{FWM}}(f_i, f_j, f_k)
\]

FWM light is detected at the receiver together with the signal light, and induces the interference noise. The FWM
noise power $N_{\text{FWM}}$ is written as [14]:

$$N_{\text{FWM}} = 2b^2 p_s \frac{P_{\text{FWM}}}{8}$$  \hspace{1cm} (5)

where $P_s$ is the signal light power at the receiver. In the case where the input light power to the fiber is $P_0$ and the fiber length is $L$, and fiber loss coefficients is $\alpha$, $P_s = P_0 e^{-\alpha L}$.

The SNR can be expressed as [14]:

$$K = \frac{bP_s}{\sqrt{N_{\text{th}} + N_{\text{sh}} + N_{\text{FWM}}} + \sqrt{N_{\text{th}}}}$$  \hspace{1cm} (6)

Since the thermal noise $N_{\text{th}}$ and shot noise $N_{\text{sh}}$ are very small, $N_{\text{FWM}}$ is the dominant factor of the denominator, so Eq. (6) can be written as:

$$K = \frac{bP_s}{\sqrt{N_{\text{FWM}}}} = \frac{2bP_s}{\sqrt{b^2 P_s P_{\text{FWM}}}} = \frac{2\sqrt{P_s}}{\sqrt{P_{\text{FWM}}}}$$

$$= \frac{2\sqrt{P_s e^{-\alpha L}}}{\sqrt{P_{\text{FWM}}}}$$  \hspace{1cm} (7)

If Gaussian approximation is used to describe the noise caused by FWM interference, the bit error probability $p_e$ for an intensity-modulated on-off-keying signal is written as [14]:

$$p_e = \frac{1}{\sqrt{2\pi}} \int_{K}^{\infty} \exp \left( -\frac{t^2}{2} \right) dt$$  \hspace{1cm} (8)

Therefore, in WDM system, the nonlinear interaction among these channel lights may generate interference light to a signal channel, and cause degradation of signal to increase the bit error probability. Since the power of the interference light is proportional to the input light power and inverse proportional to the frequency spacing, we should note that when the input power is low enough and the frequency spacing is large enough, the effect of FWM is almost negligible. In present WDM systems where the input power is generally much less than 0 dBm, engineers often neglect this factor. But with the rapid development of optical component for all-optical networks, especially for a wide area WDM with the wide use of Erbium Doped Fiber amplifier, the input light power to the fiber can reach 10 dBm or even higher. In such case, the interference of FWM may impose a major limitation to the number of channels or the size of the WDM network.

2.2. Effect of bursty IP traffic on interference caused by FWM

IP traffic can be modeled as an on-off model to describe its bursty nature using three parameters: peak bit rate $R_{pp}$, the average length of a burst $1/\beta$, and the average off-time $1/\varphi$ (as shown in Fig. 3). With different $\varphi$, $\beta$ and $R_{pp}$, the on-off model may be used to model voice, data, images and video.

A source as shown in Fig. 3 alternates between ON and OFF states. In the ON state, the source transmits at its peak rate, and in the OFF state, transmits no data at all. The probability of the source in ON state is

$$p_{on} = \frac{1/\beta}{1/\varphi + 1/\beta} = \frac{\varphi}{\varphi + \beta}$$

The average bit rate of the source is $R_{on} = R_{pp}p_{on}$.

When the light intensity in the wavelength channel is directly modulated by such traffic source, in the ON state, digital information is carried by the light, but in the OFF state, the light is off, or is so small that they can almost be neglected.

Fig. 3(c) shows the digital signal carried by three wavelengths. We know that the FWM interference happens only when the three signal lights $(f_i, f_j, f_k)$ contributing to it at $f_m$ ($f_m = f_i + f_j - f_k$) are all in ON state. When any one of the three lights is OFF, there won’t be FWM light. In Fig. 3(c), FWM interference happens only during $t_1 < t < t_2$. If the on probability of the IP traffic is $p_{on}$, the probability that all the three be in burst mode will be $p_{on}^3$ (if there are only three frequencies in the system). So the actual interference between the channels depends on the on-off state of IP traffic in the light channel [16,17].

When the number of active channels (channels in ON state) is different, the interference caused by FWM varies greatly. When the number of channels increases, the combinations of channel frequencies that contribute to inter-channel interference increases dramatically, thus the light power generated by FWM increases rapidly. Considering the different probability of different numbers of active channels at a time when IP traffic is loaded, the effect of inter-channel interference in a WDM based system can be calculated as follows.

Suppose there are $N$ channels in a WDM system, the average bit error probability caused by FWM interference can be written as:

$$p_e = \sum_{i=1}^{N} p(i)p_{ei}$$  \hspace{1cm} (9)

where $p(i)$ stands for the probability that $i$ channels out of $N$ are in ON state at the same time, $p_{ei}$ is the bit error probability caused by interference noise among these $i$ channels. Suppose the load of all the channels are the same, and the probability of the occurrence of ON state is the same, and is denoted as $p_{on}$, then $p(i)$ can be calculated as:

$$p(i) = \binom{N}{i} p_{on}^i (1 - p_{on})^{N-i}$$  \hspace{1cm} (10)

In order to calculate the worst case interference when $i$ channels out of $N$ are in burst mode, we assume all the $i$ channels are adjacent to each other, and we calculate the FWM interference in the central channel $f_m$, where the frequency combinations are the most that satisfy $f_m = f_j + f_k - f_i$, ($j, k, l = 1, \ldots, i$). Refer to Eq. (8), the bit error
probability on this condition is:

\[ p_{ei} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{r^2}{2}\right) dr \]  

where \( K_i \) is,

\[ K_i = \frac{2\sqrt{P_0}}{\sqrt{P_{\text{FWM}}(f_m)}} \]  

here \( P_{\text{FWM}} \) is the summation of the four-wave mixing light power generated by all the possible frequency combinations when \( i \) channels are active.

\[ P_{\text{FWM}}(f_m) = \sum_{f=j+k} \sum_{j=1}^i P_{\text{FWM}}(f_j, f_k, f_i) \]  

\[ P_{\text{FWM}}(f_j, f_k, f_i) \] can be calculated according to Eq. (1).

### 2.3. Numerical results and discussions

For illustration, the bit error probability caused by FWM interference in a WDM network loaded with IP traffic is calculated using Eqs. (9)–(13), where we do not consider the other noise factors such as SRS, SBS and ASE noise of the EDFAs. The parameters used in the calculation of the following numerical results are listed in Table 1.

Bit error probability versus fiber input light power for different traffic load is shown in Fig. 4. It can be seen that the bit error probability is sensitive to both input light power and traffic load, where traffic load is denoted by

\[ P_{\text{on}} = \frac{1}{\beta} \frac{1/\beta}{1/\beta + 1/\varphi} = \frac{\varphi}{\varphi + \beta}. \]

When light power at the input of optical fiber is low, say less than 4 dBm, in this case, the FWM effect is negligible even for very high traffic load at almost 90% of link capacity, because all corresponding bit error rate is below \( 10^{-15} \). By contrast, when input light power is higher than 4 dBm, for example, the bit error probability increases sharply, even greater than \( 10^{-10} \) or higher for a heavy traffic load ranging from 75 to 90% of the link capacity. However, when traffic load is low, say 30% of the link capacity, the allowable input light power can be up to 8–9 dBm for very reliable transmission at a bit rate below than \( 10^{-20} \). Hence, the traffic load has great influence on the bit error performance. This is because when IP load is heavy, the probability that a WDM channel in burst mode is high, thus more WDM channels are likely to be in burst mode at the same time and more interference noise is generated.

Frequency spacing is another important factor in a dense WDM system. Fig. 5 illustrates the effect of frequency spacing on the system performance in terms of bit error probability. It can be seen that when frequency spacing between the wave lengths is less than 10 GHz, the FWM effect is remarkable and it must be taken into account, however, low traffic load may be able to reduce the effect of FWM to some extents. We note that when frequency spacing between the wavelengths increases, the fiber nonlinear effect is reduced. However, when spacing is large enough, the fiber can be considered as a linear media, and light channels can be deemed as completely independent of each other. And this is often called sparse wavelength division systems. The WDM system used in point-to-point communication at present takes advantage of this.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>16</td>
</tr>
<tr>
<td>( D )</td>
<td>0.3 ps/nm km</td>
</tr>
<tr>
<td>( L )</td>
<td>80 km</td>
</tr>
<tr>
<td>( dD/dA )</td>
<td>0.07 ps/nm²</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>1.55 μm</td>
</tr>
<tr>
<td>( \lambda_{\text{eff}} )</td>
<td>5 × 10⁻⁷ cm²</td>
</tr>
<tr>
<td>( N )</td>
<td>1.46</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.2 dB/km</td>
</tr>
</tbody>
</table>
Fig. 6 illustrates the effects of traffic load on the bit error performance of the wavelength channel for different input light power levels. It can be seen that every 10% increase of the traffic load can cause the increase of bit error rate by almost a magnitude of four. On the other hand, the bit error rate is also sensitive to the input light power level, especially when traffic load is heavy, input light power level must be carefully budgeted in order to ensure that the wavelength channel has the appropriate performance to guarantee packet transmission.

In Fig. 6, the average error probability $p_{pe}$ for a packet of $w$ bits is calculated using $p_{pe} = (1 - (1 - pe)^w)$, where $pe$ is the average bit error rate. The long packet represents that in the on–off model, if the holding time in the ON-state is long, or the peak rate ($R_p$) is high, the resultant packet may be long. In practice, the packet length is determined by user’s operation mode. For example, FTP user may have a large file of hundreds of Megabits to transmit, but for an interactive web user, the message to be transmitted from the user to the server may be short. Obviously, the long packet has higher error probability comparing to the short packet. In order to keep the packet error rate low, one of the tasks for network management or the IP user is to segment these large files to small packets for the actual transmission in the wavelength channel.

Likewise, the bit error probability is illustrated as the function of traffic load for different frequency spacing in Fig. 7. In order to take full advantage of the fiber capacity, we need to place more wavelength channels into the WDM fiber, however, the relative less frequency spacing between the wavelength channel may cause higher error rate. Therefore, the trade-off between the number of wavelength channels in the WDM system and the IP traffic load is one of most important issue to achieve the best performance in terms of error rate and link throughput.
Figs. 8 and 9 illustrate the high order distributions of packet error probability for IP traffic in WDM network. It is important to study the effects of the bursty nature of the IP traffic on the performance of WDM network. Fig. 8 presents the packet error pattern distribution $P(i, N)$ under different traffic loading conditions. The packet error pattern distribution $P(i, N)$ is defined as the probability that a block of $N$ packets transmitted through the WDM link contains exactly $i$ errored packets due to the interference caused by FWM. Fig. 9 shows the packet error free run distribution $P(0'/1)$ which is defined as a conditional probability that given an error packet has occurred, it is followed by $r$ or more consecutive error-free packets. From Figs. 8 and 9, it can be seen that the packet error probability is sensitive to the traffic loading, for example, under heavy traffic load conditions, the corresponding packet error rate is high and the error free run tends to be short. Likewise, the packet error rate is also sensitive to the packet length. The long packets are likely suffering more during the transmission, and the error free runs are correspondingly short.

3. Effect of stimulated Raman scattering on IP traffic

3.1. Power depletion caused by stimulated Raman scattering (SRS)

The utilization of WDM network with hundreds of optical channels might ultimately be limited by stimulated Raman scattering (SRS) [19,23]. Due to SRS effect, if two optical waves separated by up to 15,000 GHz are co-injected into an optical fiber, the lower frequency wave will experience optical gain generated by, and at the expense of, the higher frequency wave. Schematically, the effect of SRS is to produce bit patterns as shown in Fig. 10. Note that SRS happens when there is a mark in both channels involved. If a space (zero light intensity) appears in either channel, no intensity change occurs. Furthermore, the effects of SRS on the two channels are not symmetric. Channel 1 experiences a partial closing of the eye pattern due to the depletion of individual bits, and therefore a degradation in signal-noise ratio. The opening of the eye in channel 2 is, in principle, unaffected because in the worst case some of the bits are amplified while the rest of the bits are unaltered. Although in practice, this can also lead to degradations, especially in receivers with automatic gain control, we will focus on the depletion in channel 1 in this paper.

Traditionally, the effect of SRS has been analyzed by computing the depletion of the shortest wavelength channel in the worst-case assumption that the peak optical power is transmitted in each channel [19,23]. But considering the fact that SRS happens only when there is a mark in both channels involved, it is important to compute the SRS depletion considering the statistics of the modulated signals,
especially when the traffic on the wavelength channel is bursty, which is the case in a DWDM-based IP backbone.

In GMPLS-based WDM networks, because of the bursty nature of IP traffic, the SRS effect in the system varies a lot with different IP traffic load. Accordingly the power penalty, maximum allowable power and performance degradation change under different traffic conditions. Therefore the traditional method of estimating the SRS effect with the worst-case assumption is not adequate in such systems.

In the following sections we study the power penalty and bit error rate caused by SRS power depletion in WDM channels directly modulated by IP traffic, where the effect of IP traffic load is considered. When computing the bit error rate caused by SRS effect, the optical amplifier noise is considered as the major noise source.

### 3.2. SRS power depletion between IP-traffic modulated channels

Suppose there are N equally spaced optical channels with channel separation \(\Delta \nu\) (Hz). The fractional power, \(D\), lost by the shortest wavelength channel (channel 0) is given in

\[
D = \sum_{i=1}^{N-1} D_i = \sum_{i=1}^{N-1} \frac{P_i \gamma_i L_{\text{eff}}}{2A_{\text{eff}}} m_i \tag{14}
\]

where \(P_i\) is the optical power carried in the \(i\)th channel, \(A_{\text{eff}}\) is the effective core area of the fiber, \(L_{\text{eff}}\) is the effective fiber length given by \(L_{\text{eff}} = (1 - \exp(-\alpha L))/\alpha\), where \(\alpha\) is the fiber loss coefficient and \(L\) is the fiber length. \(\gamma_i\) is the Raman gain coefficient coupling the first and the \(i\)th channel which can be expressed using the triangular Raman gain \([13,18]\):

\[
\gamma_i = \begin{cases} 
\frac{i(\Delta \nu)}{1.5 \times 10^{13}} \gamma_p & \text{for } i(\Delta \nu) < 1.5 \times 10^{13} \text{ Hz} \\
0 & \text{otherwise}
\end{cases} \tag{15}
\]

where \(\gamma_p = 6 \times 10^{-12} \text{ cm/W}^2\), is the peak Raman gain coefficient and \(\Delta \nu\) is the channel frequency spacing.

\(m_i\) in Eq. (14) is the modulation factor in the \(i\)th channel where

\[
m_i = \begin{cases} 
1 & \text{When optical lights in both channel 0 and channel } i \text{ are on} \\
0 & \text{otherwise}
\end{cases} \tag{16}
\]

IP traffic can be modeled as an on–off model to describe its bursty nature using three parameters: peak bit rate \(R_p\), average length of burst \(1/\beta\), and average off-time \(1/\varphi\) (Fig. 3). An IP source alternates between ON and OFF states. In the ON state, the source transmits at its peak rate and signal light at the wavelength channel is on. In the OFF state, the source transmits no data and the light is off.

The probability of the source in ON state is

\[
P_{\text{on}} = \frac{1/\beta}{1/\varphi + 1/\beta} = \frac{\varphi}{\varphi + \beta}.
\]

Stimulated Raman scattering happens when both channels involved are in ON state, of which the probability is \(P^2_{\text{on}}\). So the average SRS power depletion in channel 0 due to the \(i\)th channel can be expressed as follows,

\[
E(D_i) = (\lambda/\lambda_0)P_i \gamma_i L_{\text{eff}}/(2A_{\text{eff}}) \times P_{\text{on}} \text{ (Channel 0 is on, Channel } i \text{ is on)}
\]

\[
= (\lambda/\lambda_0)P_i \gamma_i L_{\text{eff}}/(2A_{\text{eff}}) \times P^2_{\text{on}} \tag{17}
\]

Suppose the optical channels are independent of each other, the average of the total power depletion \(D\) is denoted by \(D_t\),

\[
D_t = E(D) = E\sum_{i=1}^{N-1} E(D_i) = \sum_{i=1}^{N-1} \frac{P_i \gamma_i L_{\text{eff}}}{2A_{\text{eff}}} P^2_{\text{on}} \tag{18}
\]

The power penalty \(X\) (dB) due to stimulated Raman scattering is

\[
X = -10\log(1 - D). \tag{19}
\]

When estimating the bit error probability caused by SRS power depletion, we consider the amplified spontaneous emission (ASE) noise as the main noise source. The optical signal power, \(P_s\), and the power of the ASE, at the receiver in the terminating node are given as follows \([13,14]\):

\[
P_s = G \lambda P_i \tag{20}
\]

\[
P_{\text{ASE}} = (G - 1)n_{sp} h \nu L_s (M + 1) B_o \tag{21}
\]

Where \(G\) is the amplifier gain, \(P_i\) is the input power, \(L_s\) is the loss of optical node, \(n_{sp}\) is the population inversion parameter of an optical amplifier, \(h\) is the Planck’s constant, \(\nu\) is the lightwave frequency, and \(B_o\) is the optical bandwidth of the optical amplifier. The power of the noise at the receiver electrical circuit is given as \([13,14]\):

\[
N_{\text{amp}} = 4\hbar^2 P_s P_{\text{ASE}} B_e, \quad b = (\eta e)/(h \nu) \tag{22}
\]

where \(\eta\) is the quantum efficiency and \(e\) is the electron charge, \(B_e\) is the bandwidth of the electrical filter. When Gaussian approximation is adopted to describe the amplifier noise, error probability caused by amplifier noise is computed as \([14]\),

\[
P_e = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp \left(-\frac{t^2}{2}\right)dt \tag{23}
\]

where

\[
Q = \frac{bP_s}{\sqrt{N_{th} + N_{th} + N_{\text{amp}} + \sqrt{N_{th}}}} \tag{24}
\]
Considering the SRS depletion and the fact that $N_{\text{amp}}$ is the dominant noise source, $Q$ can be calculated as:

$$Q = \frac{bP_s(1-D)}{\sqrt{4b^2P_s(1-D)P_{\text{ASE}}B_e}} = \frac{1}{2} \sqrt{\frac{P_s(1-D)}{P_{\text{ASE}}B_e}}$$  \hspace{1cm} (25)

### 3.3. Numerical results and discussions

In this section, we demonstrate some of the numerical results calculated based on the discussions in Section 3.2. Power depletion and power penalty due to stimulated Raman scattering are calculated using Eqs. (18) and (19), where other nonlinear effect such as FWM and SBS are not considered. When calculating the bit error rate caused by stimulated Raman scattering, amplifier noise is considered as the major noise source as in Eqs. (23)–(25). Figs. 11–15 illustrate the numerical results of our calculation. The parameters used in the computing are as in Table 2:

- Fig. 11 shows the power penalty due to SRS under different traffic load conditions. It can be seen that when the power of input light is high (above 5 dBm), the power depletion in the shortest wavelength channel increases rapidly with the traffic load ($P_{\text{in}}$) in the system. This is because when IP traffic load is high, there is more probability for two channels to be in ON state at the same time, thus the SRS power depletion is more serious. Whereas when the power of input light is low, the power penalty caused by SRS is negligible even under a very heavy traffic load. It is also illustrated here that although increasing the traffic load is low, the power penalty caused by SRS is negligible even under a very heavy traffic load.

Fig. 12 depicts the maximum allowable power of input light to keep the power penalty as low as 0.5 dB. Similar influence of IP traffic load is observed. When traffic load is low, the allowable power of the input light is much higher than that when traffic load is high, and this is beneficial in increasing the SNR and the distance between repeaters.

- Fig. 13 shows the relation between SRS power penalty and the frequency spacing. Increasing the frequency spacing increases the SRS depletion until the lowest frequency channel falls out of the effective SRS frequency range. At this point, if we continue to increase the spacing, more and more channels will fall out of the SRS range, and the depletion in the highest frequency channel is reduced. So although increasing the frequency spacing is an effective way to prevent FWM as discussed earlier in this paper, the side effect is the increased SRS depletion. The effect of IP traffic load is very obvious in Fig. 12. According to traditional assumption ($P_{\text{in}} = 1$), 50 GHz spacing may not be acceptable because it causes 3 dB power penalty. However, from Fig. 12 it can be seen that if traffic load is low (e.g. $P_{\text{in}} = 0.3$), the power penalty is not serious in the whole range of frequency spacing.

Fig. 12. Power penalty due to SRS versus frequency spacing ($N = 100$).

Fig. 13. Maximum allowable power of input light ($N = 100$).
Fig. 14 shows the packet error probability at the receiver when SRS power depletion and optical amplifier noise are considered. In Figs. 14 and 15, the average error probability $P_{pe}$ for a packet of $w$ bits is calculated using $P_{pe} = 1 - (1 - p_e)^w$, where $p_e$ is the average bit error rate in Eq. (23). IP packets with average length of 512 bytes are considered in Figs. 14 and 15. The SRS effect under the worst-case assumption is used in the illustration. From the figure it is observed that increasing the power of input light is very effective to overcome the degradation caused by amplifier noise. However, when SRS power depletion is considered, this is true only when the power of the input light is within a certain range. When increasing the power of input light beyond a certain point, it will deteriorate the packet error rate performance rather than improve it.

The effect of IP traffic load is again reflected in Fig. 15. It can be seen that the actual SRS effect in an IP-WDM backbone is much less serious than the worst-case assumption. When IP traffic is as high as 0.7, which is a rather high utilization in practice, the launch power can be 3–4 dB higher than traditional expectation. When traffic is lower, the SRS effect is further reduced and the allowable maximum power is much higher.

### 4. Effects of light power and frequency spacing on FWM and SRS

The numerical results shown in Sections 2 and 3 have demonstrated that the power of input light and the frequency spacing are the two major factors that have significant influences on the nonlinear effects of FWM and SRS. However, the responses of FWM and SRS to these two factors are totally different. In order to demonstrate their different features, Figs. 16 and 17 are used to illustrate the effects of FWM and SRS on the performance of packet error rate under different conditions of light power and frequency spacing, respectively.

Fig. 16 shows the effect of input light power on the FWM and SRS, where the frequency spacing is 10 GHz, the power is ranging from $-2$ to $7$ dBm, and other parameters are the same as that presented in Sections 2 and 3 for the illustration purpose only. From Fig. 16, it can be seen that when the input light power increases, the corresponding packet error rate caused by SRS decreases. By contrast, the corresponding packet error rate caused by FWM increases. This is explained by Eqs. (1), (12), (14) and (25), where the signal-to-noise ratio for FWM decreases when the input light power increases, but the signal-to-noise ratio for SRS

![Fig. 14. Bit error rate caused by amplifier noise (with and without SRS).](image)

![Fig. 15. Bit error rate caused by SRS and optical amplifier noise (with different $P_{on}$).](image)

![Fig. 16. Packet error rate caused by FWM and SRS vs. light power.](image)
From Fig. 17, it can also be seen that the effect of the contradict effect on SRS and FWM, so that it provides range. This fact shows that the power of input light has a increases when the input power grows within a certain range. This fact shows that the power of input light has a contradict effect on SRS and FWM, so that it provides useful information for the system designers to consider. From Fig. 17, it can also be seen that the effect of the frequency spacing has more significant influences on the performance of FWM than on SRS.

In general, when the power of input light increases, the FWM noise becomes more significant; by contrast, the corresponding SRS effect is less significant when the power is within a certain range. On the other hand, when frequency spacing increases, the corresponding FWM effect decreases significantly but the corresponding SRS effect is related less significantly.

5. Conclusion

The effect of optical nonlinearities on the performance of IP traffic over GMPLS-based WDM network is studied. Two of the most significant fiber nonlinear factors of WDM systems—four-wave mixing and stimulated Raman scattering—are addressed. The impact of IP traffic on the nonlinear effects and eventually on the bit error performance of the optical channels is investigated. It can be concluded that when IP traffic load is light in such systems, the effect of either FWM or stimulated Raman scattering is much less serious than the worst-case assumption, and the limitation on allowable power of input light is also relaxed. The obtained numerical results demonstrated that FWM and SRS are both sensitive to power level of input light and frequency spacing, but each in a unique way. Effort must be taken in system design to avoid improving the performance of one effect at the expense of deteriorating the other.

References