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Valuing Individual Mortgage Servicing Contracts: A Comparison between Adjustable Rate Mortgages and Fixed Rate Mortgages

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This study constructs a valuation model from which an option-adjusted spread approach is employed to value individual mortgage servicing contracts for both adjustable rate and fixed rate mortgages. The valuation model is comprised of an exogenous OTS prepayment model, a stochastic interest rate process, and other servicing fees and costs, all of which jointly determine the servicing contract's future net cash flows and the rate at which to discount these cash flows. The sensitivity of the price of mortgage servicing rights to the changes in the economic environment is also analyzed. This work is potentially useful for servicers not only servicing mortgages but also servicing other types of loans in order to examine servicing policy-related issues.

Keywords: Mortgage servicing rights; prepayment; mortgage.

1. Introduction

In the operating mechanism of mortgage securitization, servicing plays an important role to service mortgages that underlie the securities. During the course of mortgage securitization, the originators sell off mortgages to either Fannie Mae or Freddie Mac, which in turn securitizes the mortgage loans into the secondary mortgage market. As an industry practice, these

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two government-sponsored entities (GSEs) typically contact outside parties, either the originating institution if the lenders retain the servicing rights or the independent servicer if the lenders do not retain the servicing rights, in order to service the loans.

Mortgage servicing rights (MSR) are contractual rights in exchange for compensation to fulfill a variety of functions. This includes collecting mortgage monthly payments and then remitting the proceeds to mortgage purchasers, managing escrow accounts for collecting and distributing property taxes and hazard insurance, sending notice of delinquency and default, or even initializing foreclosure proceedings on defaulted loans in some cases. The revenue from servicing involves the servicing fee, float income, and other ancillary income. On the other hand, expenses associated with servicing include servicing costs, delinquency, and foreclosure costs. The net cash flows for servicing contracts are determined by the difference between the revenue and the costs.

The value of a MSR is basically the present value of net cash flows obtained from servicing operations. The profile of these cash flows behaves similarly to an IO-like (Interest-Only) instrument. It is essentially due to the fact that a servicing fee, a fixed percentage of the unpaid mortgage balance, is the principal component of servicing income, just as interest payments are.

What is meant by the value of a MSR is the maximum origination cost that the originators are willing to spend so as to retain the servicing for themselves. Or, that is the maximum price that an acquiring firm is willing to pay to receive the servicing. The valuation of a MSR is therefore of crucial importance, no matter if it is being retained or being sold, because it represents a capital expenditure by a servicer.

In this paper, we present a valuation framework for mortgage servicing rights that uses both the Option Adjusted Spread (OAS) model and the Office Thrift Supervision (OTS) dynamic prepayment model.[1] Then we use Monte Carlo simulation to evaluate a MSR given an OAS. While static prepayment models are not particularly good predictors of mortgage prepayment, dynamic prepayment models correlate changes of economic variables with changes in mortgage prepayment rates.[2] With the addition of these

[1]See the OTS net present value model (Page 5A-6), Risk Management Division, Office of the Thrift Supervision.

[2]This is because these static prepayment models ignore the fact that mortgage prepayment rate should be at least partially a function of economic variables such as coupon rates, seasoning, and seasonality. Thus, the static prepayment models are only useful for

economic variables, the OTS dynamic prepayment model is able to predict the future level of mortgage prepayment rates. This is indeed a critical improvement over static prepayment models. Hence, this is the major reason that the OTS prepayment model is adopted in this paper. We then investigate the impact of interest rate variations and the corresponding changes in prepayments on the MSR valuation for both adjustable rate mortgages and fixed rate mortgages. The contribution of this paper is that we incorporate the OTS dynamic prepayment model into the pricing process, which has not been done in previous real estate literature.

The organization of this paper is as follows. Section 2 provides a comprehensive review of related mortgage servicing literature. Section 3 delineates the methodology used in this paper. Section 4 reports the simulation results and compares the results between the adjustable rate mortgages and the fixed rate mortgages. Section 5 presents our conclusions and proposes directions for future research.

2. Literature Review

Relative to the broad academic literature on mortgages, the MSR literature is quite limited. McConnell (1976) develops the first MSR model in which the static discounted cash flow approach is employed. The difficulty with his model is that the author ignores the stochastic property of interest rate term structure and the critical path-dependent nature of the MSR cash flows. Van Drunen and McConnell (1988) develop a two-state, continuous time model. Their model allows for stochastic one-period interest rates and stochastic inflation rates. However, the model only explicitly values the borrower's prepayment option (i.e., it ignores the effect of default on the MSR valuation).

Brown *et al.* (1992) use an Option Adjusted Spread model to price the MSR. The purpose of their paper is to evaluate the prepayment effect on the MSR values. Their results show that in many ways MSRs are similar to Interest-Only (IO) mortgage strips (i.e., they are extremely sensitive to prepayments). Langowski (1999) indicates that the pattern of the MSR cash flows behaves like that of the interest-only strips of a mortgage-backed security, and thus the paper utilizes the IO pricing techniques to price a MSR subject to various shocks. The results show that prepayment is the primary factor on the MSR valuation. The most recent servicing paper by Aldrich,

depicting past prepayment behaviors, whereas the OTS dynamic prepayment model can actually predict usable results for future mortgage prepayment rates.

Greenberg and Payner (2001) indicates that the cash flows of a MSR are generated from various sources such as servicing fees, float incomes and ancillary incomes and late charges. They conclude that around 70–80% of the value of a MSR is attributed to pure servicing fees, and they also recognize that the costs of servicing vary, depending upon the delinquent/early payment patterns of the mortgage borrowers.

Following previous studies, we also regard the MSR as an IO-like instrument and use the OAS model to price the MSR for both adjustable rate mortgages and fixed rate mortgages. However, unlike the work by Brown *et al.* (1992), which uses a static prepayment model, we utilize the Office Thrift Supervision (OTS) prepayment model to capture the dynamic nature of a borrower's prepayment behavior. In addition, the prepayment and the default are treated as exogenous events, which is an effective way of pricing in the capital market.

3. Methodology

The tenet of an OAS approach is to evaluate a financial instrument with an enormous number of random interest rate paths. By feeding an interest rate path into the OTS prepayment model, the prepayment amount can be projected. The corresponding monthly cash flow for a MSR given the interest rate path can then be calculated for both adjustable rate mortgages and fixed rate mortgages. Subtracting servicing costs and foreclosure costs from servicing revenue, the present value of a MSR can be computed by discounting the monthly net cash flows at the rates determined by that single path. The "fair" price of a MSR is then calculated by averaging all the present values generated from all the interest rate paths.

For simplicity, we assume that there are no revenues associated with float income, other ancillary income, and late fees charged. Thus, we treat the servicing fee as the major source of income from servicing. On the other hand, expenses include servicing costs and foreclosures costs but exclude delinquency costs. The stochastic process of the risk-free interest rate is assumed to follow the Cox, Ingersoll and Ross (1985) square-root mean-reverting process[3]:

$$dr = \gamma(\theta - r)\,dt + \sigma_r\sqrt{r}\,dz_r, \tag{3.1}$$

[3]Certainly, an alternative interest-rate process can be used instead of the CIR model. The interest-rate process used in this paper is just one of many proper processes that can be utilized. This specific process chosen is for demonstration purposes and also for consistency with the general real estate literature.

where the interest rate r drifts toward a fixed level θ, governed by the speed of adjustment γ. The increment dz_r is a Wiener process and σ_r is the volatility of interest rates.

The OTS prepayment model and the other formulas are introduced as follows.[4]

$$\text{Prepayment amount in particular month } i = \text{UMB}_{\text{after monthly cash flow}} * \text{CPR}, \tag{3.2}$$

where UMB is the unpaid mortgage balance and CPR is the prepayment rate.

$$\text{CPR} = 1 - (1 - \text{seasoning} * \text{ seasonality} * \text{ refinancing})^{1/12}, \tag{3.3}$$

$$\text{Seasoning} = \min(1.0, 0.0333 * \ i), \tag{3.4}$$

where seasoning is increasing at a rate of 0.0333 until month 30 (then remains at one as a constant thereafter); i denotes month i.

$$\text{Seasonality} = 1 + 0.2 * \sin\{[1.571 * (month + i - 3)/3] - 1\}, \tag{3.5}$$

where *month* is the month the mortgage is issued; and sin is the function of sine.

$$\text{Refinancing} = 0.2913 - 0.1620 * \arctan\{8.3645 * [1.1556 - (c/r + m)]\}, \tag{3.6}$$

where r is the risk-free rate, m is the spread of newly-issued mortgages over the risk-free rate, c is the contract rate and arctan is the function of arctangent.

The OTS prepayment model is comprised of three factors that govern the prepayment behavior of the mortgage pools. These are generally referred to as seasoning, seasonality, and refinancing incentives. Seasoning means that a newly mortgaged pool tends to pre-pay at a lower rate than does an old pool, other things being equal. Seasonality indicates that the level of mortgage payments is tied up with the time of the year, holding all else constant. Refinancing means that prepayments usually increase when mortgage rates drop below the contract rates, all others remaining the same.

[4]The OTS prepayment model is fitted by employing historical data. The benchmark used is based on a conventional 30-year fixed-rate mortgage with moderately-seasoned loans. The estimated coefficients shown on Eqs. (3.4), (3.5), and (3.6) are generated based on long-term prepayment rate forecasts of a number of Wall Street firms. More importantly, the refinancing coefficients shown on Eq. (3.6) are periodically updated. The one that we use are from Q4, 2002.

Equation (3.3) stands for the monthly prepayment rate. Given Eq. (3.3), the monthly prepayment amount can be calculated using Eq. (3.2). Thus, the cash inflow to a MSR at each month can be determined by subtracting the prepayment amount from the unpaid mortgage balance at that month. That is

$$\text{CF in month } i = \text{UMB} - \text{prepayment amount in month } i, \quad (3.7)$$

$$\pi = CF * (sf/12) - (sc/12) - fc, \quad (3.8)$$

where π stands for net cash flows from servicing, sf is the annual servicing fee, sc is the annual servicing cost, and fc is the average monthly foreclosure cost.[5]

$$\text{MSR} = (1/N)\left[\sum_{n=1}^{N}\sum_{t=1}^{T}\pi_{n,t} \Big/ \prod_{t=1}^{T}(1 + r_{n,t} + u)\right], \quad (3.9)$$

where N is the number of interest rate paths and u is the option adjusted spread over the risk-free rate.

Given an assumed spread u, the fair price of the MSR can be calculated according to Eq. (3.9). For each interest rate path, the net cash flows from servicing are determined from Eq. (3.2) through Eq. (3.8), and the present value is achieved. Finally, the "fair" price of a MSR is calculated by averaging the present values from all interest rate paths.

The above pricing procedure is applied to the case of fixed rate mortgages. To make this analysis suitable for adjustable rate mortgages, certain features need to be modified, including the reset interval for the contract rate, index rate, margin, teaser, periodical cap and lifetime cap. First of all, the contract rate is adjusted periodically, and the adjustable rate mortgages behave as a fixed rate mortgages between any two reset dates. Based on the work by Kau *et al.* (1990), the contract rate is reset as follows:

$$c(i) = \max[\min(r(i) + m, c(i-1) + H, c(0) + L), c(i-1) - H], \quad (3.10)$$

where $c(i)$ is the contract rate for any given year except for at the origination.

At the origination, the contract rate is set at $c(0) = r(0) + m - \alpha$. In Eq. (3.10), $c(i-1)$ is the contract rate at the previous adjustment date, m

[5]Foreclosure cost is a one-time cost. Thus, to smooth out this cost, this paper takes the foreclosure cost (fc) divided by the term of the loan (months).

is the margin added to the index to obtain the contract rate for the loan, the initial period discount for the borrowers is denoted by α, the periodical cap is represented by H and the lifetime cap is expressed by L. For interest rate caps, the min function of Eq. (3.10) places a limit on how much the contract rate can change at each reset date, while the max function imposes a ceiling on the contract rate. Equation (3.11) expresses the refinancing function for ARMs.[6]

$$\text{Refinancing} = 0.2006 - 0.0950 * \arctan\{2.401 * \ [1.021 * (c/r + m)]\}. \tag{3.11}$$

With Eq. (3.10), a new contract rate can be determined. This is the new information needed to calculate the new payment and the unpaid mortgage balance. This process is repeated for each rate-adjusted interval. Using Eq. (3.2) through Eq. (3.9), the price of a MSR for adjustable rate servicing contracts can be determined.

To summarize, the major difference between valuing a fixed rate MSR and valuing an adjustable rate MSR is that the contract rate is not constant for the adjustable rate mortgages. Once the contract rate is reset, the new rate has an impact on the mortgage payment, the unpaid mortgage balance, and even the prepayment amount, all of which result in a change in the price of a MSR.

In order to determine the number of iterations that is necessary to achieve the mean price of the MSR under simulated distributions, we examine the relationship between the standard errors and the number of iterations. As shown in Figs. 1 and 2, 5,000 and 9,000 iterations are chosen for valuing a fixed rate MSR and an adjustable rate MSR, respectively, at which numbers the standard errors levering off.

4. Results

Based on the models presented in previous section, the primary work in this section is to analyze the relationships between the MSR value and various mortgage-specific and economic-specific variables. The value of the mortgage-specific and the economic-specific variables are shown in the upper and lower part of Table 1, respectively. Those parameters are commonly used

[6]Please see the OTS net present value model (Page 5G-4), Risk Management Division, Office of the Thrift Supervision. Like the refinancing equation to fixed-rate mortgages, Eq. (3.11) is also subject to periodical revision. The rates used are from Q4, 2002.

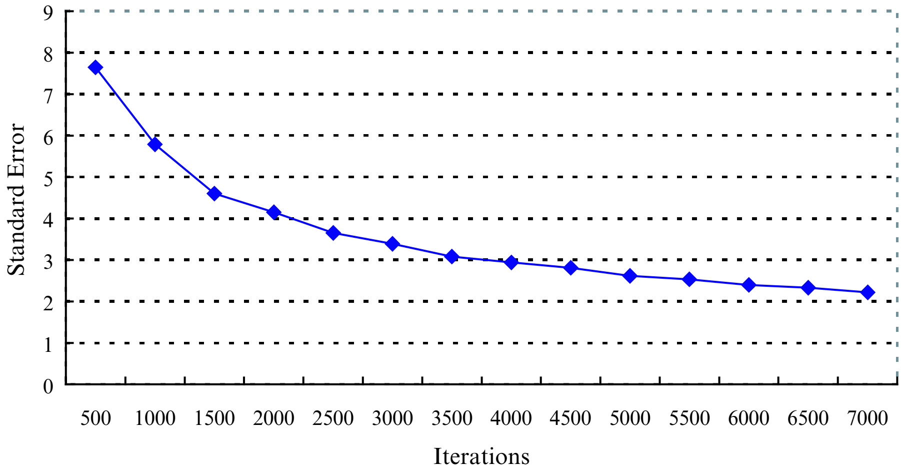

Fig. 1. Fixed rate mortgages: Standard errors by number of iterations.

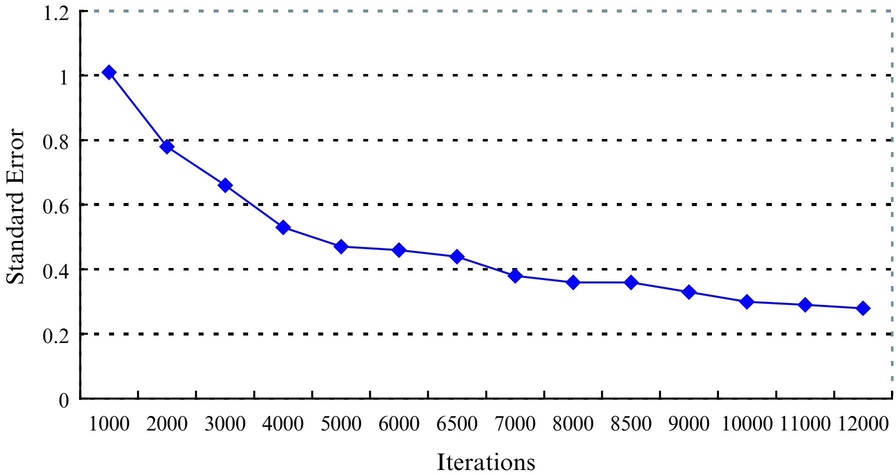

Fig. 2. Adjustable rate mortgages: Standard errors by number of iterations.

in mortgage pricing literature (e.g., Kau *et al.*, 1994 and Ambrose, Buttimer and Capone, 1997).[7]

4.1. *Interest rate volatility*

Changes in the variability of interest rates have a significant effect on MSR valuation for both fixed rate and adjustable rate mortgages. For instance,

[7]The assumed spread u is 2.5%, which is close to a fair OAS spread of 2.0% as shown in the work by Brown *et al.* (1992).

Table 1. Base-case parameters for numerical modeling.

Parameters	Value
The Mortgage Servicing Contract	
Contract rates (C)	9% (FRMs)
Mortgage term	30 years
Loan type	FRMs and ARMs
Margin	2%
Teaser	1.5%
Periodical cap	2%
Lifetime cap	5%
Resetting interval	6 Months
Servicing fee (SF)	0.0025 (Fannie/Freddie Investor)
Servicing cost per loan (SC)	\$44 (Fannie/Freddie Investor)
Foreclosure cost per loan (FC)	1% of the original loan amount
The Economic Parameters	
Steady-state spot rate (θ)	10%
Interest rate volatility (σ_r)	10%
Reversion coefficient (γ)	25%
Spread (u)	2.5%
Original spot rate (r_0)	8%

Data are from Citimortgage, Inc. For Fannie/Freddie Mac investors, the cost of servicing is \$44 per loan annually, and the fee of servicing is 25 basis points per loan annually.

an increase in interest rate volatility σ_r, results in an increase in the probability of mortgage termination due to mortgage refinancing, holding all else constant. The mortgage refinancing truncates the servicing revenues, which is a function of the unpaid mortgage balance, thereby reduces the price of a MSR. In other words, the higher the interest rate variability is, the lower will be the MSR values. Based on our simulations shown in Table 2, an increase in interest rate volatility σ_r, leads to a substantial decrease in price for a fixed-rate MSR relative to that of an adjustable-rate MSR. For instance, when σ_r increases from 5% to 10%, the value of a fixed-rate MSR (given $\gamma = 0.25$) drops from 504.69 to 407.14, and the value of an adjustable-rate MSR decreases from 587.79 to 564.03.

There are two interesting features that deserve particular attention. Firstly, the decrease in the price of a MSR is much smaller for adjustable rate mortgages than that for fixed rate mortgages. This could be explained by the fact that the adjustable rate mortgages allow the rates to vary with the market rates. The incentive of mortgage refinancing is reduced as the new rates are always close to the market rates, thereby causing a much

Table 2. MSR prices under various interest rate volatilities and speed of adjustment factors.

	Fixed Rate Mortgages (FRMs)			Adjustable Rate Mortgages (ARMs)		
	$\sigma_r = 0.05$	$\sigma_r = 0.10$	$\sigma_r = 0.15$	$\sigma_r = 0.05$	$\sigma_r = 0.10$	$\sigma_r = 0.15$
Component						
$\gamma = 0.25$	504.69	407.14	370.92	587.79	564.03	561.77
$\gamma = 0.50$	518.91	427.31	419.76	584.80	559.41	556.87
$\gamma = 0.75$	520.58	458.28	444.12	572.54	558.26	550.65

Simulations are based on the following assumptions. Servicing fee is 0.25% annually, and servicing cost is $44 annually. The original loan amount is $100,000. The foreclosure cost is 1% of the original loan amount per loan per year. The mortgage term is 30 years; contract rate is 9% (FRM); margin is 1.5%; teaser is 1%; initial spot rate is 8%; long-term steady state rate is 10%; periodic cap is 2%; lifetime cap is 5%; and resetting interval is six months.

lower decrease in the price of an adjustable rate MSR. Secondly, the price of a MSR for adjustable rate mortgages is higher than that for fixed rate mortgages. This is, again, because the adjustable rate mortgages are insensitive to changes in interest rates. The chance of the adjustable rate mortgage being refinanced is low as opposed to the fixed rate mortgages. The results also indicate that mortgage servicers, during periods of highly volatile interest rates, should spend much less in retaining the servicing or pay much less in purchasing the servicing rights.

The above results also imply that the impact of high interest-rate volatility on the price of servicing contract is dramatic, especially for a fixed rate MSR. Thus mortgage servicers that originate or purchase new loans should assess the likelihood of mortgage early termination under different scenarios of interest-rate volatilities.

4.2. *Speed of adjustment*

This section investigates the adjustment factor, γ, which governs the speed of interest rate drift on the MSR value. Intuitively, the larger the speed the faster interest rates will revert toward their long-term mean level. Given an upward drifting environment, Table 2 shows that the price of a fixed rate MSR increases, but the price of an adjustable rate MSR drops when the adjustment factor increases, other things being equal.

For a fixed rate MSR, when the speed of the short rate drifting up to the long-term level increases, the possibility of refinancing decreases fast. As shown in Table 2, when γ increases from 0.25 to 0.75 the MSR value rises from 407.14 to 458.28, an increase of 51.14, given a volatility of 0.10. The increase is more apparent, 73.20 (= 444.12 − 370.92), given a larger volatility of 0.15, but less obvious given a smaller volatility.

In the case of an adjustable rate MSR, when the speed of the adjustment factor increases, the short rate may rapidly approach the long-term level before the next coupon reset date arrives, which makes the discount rate higher than the contract rate. Thus, the MSR value drops from 564.03 to 558.26, given a volatility of 0.10.[8] Given a smaller volatility of 0.05 (i.e., a higher certainty about the distribution of the short rate level), the decrease in the MSR value is more apparent, 15.25 (= 587.79 − 572.54), especially when the speed of the adjustment factor triples.

As discussed above, the speed of the adjustment reflects the opposite effect on the price of a MSR between fixed rate and adjustable rate mortgages. The implication is that during periods of an upward drifting environment, servicers will find that servicing fixed rate contracts, instead of adjustable-rate ones, is likely to be accompanied by a rise in the price of the servicing contracts. It also suggests that servicers need to assess the speed of the adjustment factor with great care when acquiring fixed rate or adjustable rate servicing contracts.

4.3. *Interest rate drift*

There are two effects on the MSR valuation (Van Drunen and McConnell, 1988). In an upward drifting environment, $(\theta - r) > 0$, the prepayment effect is a positive effect, which indicates an increase in the price of a MSR due to a diminished probability of prepayment. The discount effect is a negative effect, which indicates a decrease in the price of a MSR due to an increased discount factor. On the contrary, in a downward drifting environment, $(\theta - r) < 0$, the prepayment effect is a negative effect, which indicates a decrease in the price of a MSR due to an increasing probability of prepayment. The discount effect is a positive effect, which indicates an increase in the price of a MSR due to a reduced discount factor.

[8]The present value of a MSR is computed by discounting the net cash flows by a projected interest rate path. In some cases, along a single path where the short rates are below (above) their long-term level, they may move up (down) rapidly before the next coupon reset date arrives, which makes the discount rate higher (lower) than the contract rate. We simulate 9,000 interest rate paths, and the fair value of a MSR is the average of all the expected present values. Given an upward drifting environment, cases where climbing short rates dominate, and thus, the value of an adjustable rate MSR drops as the speed of adjustment increases.

Table 3. MSR prices under different interest rate drift environments.

	Fixed Rate Mortgages (FRMs)	Adjustable Rate Mortgages (ARMs)
	$\sigma_r = 0.10$	$\sigma_r = 0.10$
Component		
$r_0 = 0.08$	407.14	564.03
$r_0 = 0.10$	407.49	579.47
$r_0 = 0.12$	409.54	586.73

Simulations are based on the following assumptions. Servicing fee is 0.25% annually, and servicing cost is $44 annually. The original loan amount is $100,000. The foreclosure cost is 1% of the original loan amount per loan per year. The mortgage term is 30 years; contract rate is 9% (FRM); margin is 1.5%; teaser is 1%; initial spot rate is 8%; long-term steady state rate is 10%; periodic cap is 2%; lifetime cap is 5%; resetting interval is six months, and adjustment factor is 25%.

Table 3 presents the value of a MSR with three scenarios of interest rate drifts for both fixed-rate and adjustable-rate servicing contracts. For the fixed rate mortgages, Table 3 shows that the price of a MSR remains around a tight range regardless of the initial interest rate drift. This indicates that the aforementioned two effects offset virtually each other. However, in the case of adjustable rate mortgages, the discount effect significantly dominates the prepayment effect. For instance, the price of MSR decreases from 579.47 to 564.03 when interest rates drift upward, and increases from 579.47 to 586.73 when interest rates drift downward. This is due to the fact that adjustable rate servicing contracts are less sensitive to mortgage refinancing.

This result indicates that an interest rate drift has a significant effect on the value of an adjustable rate servicing contract. It also means that both originators and servicers should carefully examine the price of a MSR when they acquire or purchase an adjustable rate MSR.

4.4. *Size of loan amount*

This section investigates the effect of the size of the loan amount on the price of a MSR. As reported in Table 4, the price of a MSR increases when the notional loan amount increases, but it increases more than proportionately to the loan amount for both types of mortgage servicing contracts. The servicing revenue rises as the loan amount increases since it is tied to a fixed percentage of the unpaid mortgage balance, while the servicing cost is independent of the size of the mortgage. This implies that servicing a large amount of mortgage is beneficial to servicers, and suggests the servicing industry to go toward consolidation.

Table 4. MSR prices under different loan sizes.

	Fixed Rate Mortgages (FRMs)			Adjustable Rate Mortgages (ARMs)		
	$\sigma_r = 0.05$	$\sigma_r = 0.10$	$\sigma_r = 0.15$	$\sigma_r = 0.05$	$\sigma_r = 0.10$	$\sigma_r = 0.15$
Loan Size						
100,000	504.69	407.14	370.92	587.79	564.03	561.77
200,000	1,395.49	1,123.28	1,116.28	1,532.44	1,476.75	1,427.98
300,000	2,293.93	1,860.74	1,847.72	2,494.17	2,404.21	2,290.52

Simulations are based on the following assumptions. Servicing fee is 0.25% annually, and servicing cost is $44 annually. The foreclosure cost is 1 % of the original loan amount per loan per year. The mortgage term is 30 years; contract rate is 9% (FRM); margin is 1.5%; teaser is 1%; initial spot rate is 8%; long-term steady state rate is 10%; periodic cap is 2%; lifetime cap is 5%; resetting interval is six months, and adjustment factor is 25%.

4.5. *PSA level*

To make this paper more comprehensive, we also illustrate the Public Securities Association (PSA) prepayment model on the MSR valuation.[9] The PSA standard is expressed as a monthly series of annual constant prepayment rates. The reason behind the PSA standard is that the prepayment rates are low for newly-issued loans and speed up when the loans become seasoned. A faster or a slower speed is then referred to as a greater or a smaller percentage of PSA, respectvely. Compared to the OTS prepayment model, the characteristic of the PSA standard is less appropriate, because it only takes the seasoning into account as a prepayment factor.

Table 5 shows that the price of a MSR declines with a faster PSA level in both the fixed-rate and adjustable-rate MSR cases. It is consistent with the intuition that the MSR values are affected by the prepayment speed. Given the same level of a PSA standard, the price of a MSR also drops as the interest rate volatility rises.

An intriguing finding is that when the PSA level is 350% (Table 5: 542.77, given $\gamma = 0.25$ and $\sigma_r = 0.10$), the price of an adjustable rate MSR approximates that of an adjustable rate MSR when the OTS prepayment model is used (Table 2: 564.03, given $\gamma = 0.25$ and $\sigma_r = 0.10$). Furthermore, in order to make the price of a fixed rate MSR calculated by the PSA model closer to that computed by the OTS model, a speed higher than the 350% level is needed. This suggests that the MSR valuation is extremely sensitive to the

[9]The breakeven points chosen for FRM and ARM servicing contracts are 5,000 and 6,000 iterations, respectively.

Table 5. MSR prices under various PSA levels.

Panel A. $\gamma = 0.25$

	FRM — MSR prices			ARM — MSR prices		
$\gamma = 0.25$	$\sigma_r = 0.05$	$\sigma_r = 0.10$	$\sigma_r = 0.15$	$\sigma_r = 0.05$	$\sigma_r = 0.10$	$\sigma_r = 0.15$
PSA level						
PSA = 50	1286.49	1270.08	1121.20	1275.82	1224.88	1094.66
PSA = 100	1079.74	1068.04	964.81	1066.96	1033.99	949.79
PSA = 150	918.65	909.76	849.50	910.51	889.82	837.24
PSA = 200	794.21	789.95	754.42	788.56	773.59	742.92
PSA = 250	696.44	694.35	676.85	691.10	680.72	669.02
PSA = 300	616.42	614.42	613.32	611.55	606.57	604.30
PSA = 350	555.32	551.66	548.79	544.56	542.77	540.50

Panel B. $\gamma = 0.50$

	FRM — MSR prices			ARM — MSR prices		
$\gamma = 0.50$	$\sigma_r = 0.05$	$\sigma_r = 0.10$	$\sigma_r = 0.15$	$\sigma_r = 0.05$	$\sigma_r = 0.10$	$\sigma_r = 0.15$
PSA level						
PSA = 50	1243.22	1240.41	1128.25	1250.10	1217.75	1112.71
PSA = 100	1051.14	1047.57	971.13	1055.39	1033.98	964.44
PSA = 150	904.55	899.58	855.16	908.72	888.49	843.09
PSA = 200	788.14	784.31	754.80	789.83	777.52	747.84
PSA = 250	694.58	691.72	677.04	696.73	686.53	671.87
PSA = 300	618.37	616.76	612.13	619.50	611.53	606.23
PSA = 350	555.40	553.64	551.25	556.12	549.08	547.46

Panel C. $\gamma = 0.75$

	FRM — MSR prices			ARM — MSR prices		
$\gamma = 0.75$	$\sigma_r = 0.05$	$\sigma_r = 0.10$	$\sigma_r = 0.15$	$\sigma_r = 0.05$	$\sigma_r = 0.10$	$\sigma_r = 0.15$
PSA level						
PSA = 50	1226.44	1228.15	1134.38	1241.98	1217.72	1128.74
PSA = 100	1038.85	1041.72	979.87	1049.50	1033.57	972.17
PSA = 150	898.11	896.65	856	903.80	889.63	846.41
PSA = 200	782.60	780.78	757.01	788.60	777.76	752.89
PSA = 250	691.30	689.62	676.44	696.72	686.50	672.34
PSA = 300	617.40	617.11	610.86	621.17	612.85	607.62
PSA = 350	554.72	554.50	554.39	558.33	551.59	550.93

Simulations are based on the following assumptions. Servicing fee is 0.25% annually, and servicing cost is $44 annually. The foreclosure cost is 1% of the original loan amount per loan per year. The mortgage term is 30 years; contract rate is 9% (FRM); margin is 1.5%; teaser is 1%; initial spot rate is 8%; long-term steady state rate is 10%; periodic cap is 2%; lifetime cap is 5%; and resetting interval is six months.

prepayment model. Therefore, one should carefully choose the prepayment model so that it captures the actual prepayment behavior of the mortgage.

5. Conclusions

This paper uses an option-adjusted spread (OAS) approach and incorporates the Office Thrift Supervision (OTS) dynamic prepayment model to evaluate the servicing contracts for both fixed rate and adjustable rate mortgages. The dynamic prepayment model is of crucial importance for servicing contracts in that it generates prepayment rates that adjust with interest rates, thereby changing the unpaid loan balance.

With the OTS prepayment model and the stochastic interest rates model, this study uses Monte Carlo simulation to price a MSR given an OAS. The OAS is the amount above risk-free rate that the market discounts the MSR due to additional prepayment risk embedded in the mortgage servicing contracts. In other words, the expected value obtained by our model is adjusted for risk premium. If the degree of the risk aversion of a representative agent becomes strong (weak), the discount rate will rise (drop) to compensate for risk taking, which lowers (raises) the price of a MSR.

We analyze the effects of the interest rate volatility, the speed of adjustment, the interest rates drift, and the loan size on the price of a MSR. More importantly, this valuation model provides an in-depth insight into the analysis of the risk tolerance for mortgage servicers by examining the price of a MSR due to changes in the economy. We also compare the pricing results using the OTS prepayment model with that using the PSA prepayment standard.

In addition to valuing servicing contracts, the valuation model in this paper could be adopted to evaluate other types of securitized instruments by specifying the appropriate cash flows. These include credit card receivable, auto loan-backed securities, home equity loan securitization and commercial mortgage-backed securities. Furthermore, the parameters used in this paper are obtained from most real restate literature. A calibration of these parameters from empirical data so that the model fits closer to reality is a future challenge in research.

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