A Continuous Hopfield Neural Network based on Dynamic Step for the Traveling Salesman Problem

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***Abstract*—For the traveling salesman problem (TSP) which is also an important aspect for mobile robots, a continuous Hopfield neural network based on dynamic step is applied to solve TSP. For combinatorial optimization problems such as TSP can be mapped to a Continuous Hopfield neural network (CHNN). The dynamic step size is used to replace the fixed step size, which can solve the problem of the mutual restriction between the convergence precision and the convergence speed. The energy function is designed to represent the path length. The energy of network is constantly updated and converged to a minimum value eventually. Meanwhile, the optimal solution is obtained for TSP. Simulation results show that the proposed algorithm can accelerate the convergence rate and obtain high precision optimization results for TSP.**

Keyword—traveling salesman problem; continuous Hopfield neural network; energy function; dynamic step

I. INTRODUCTION

The traveling salesman problem (TSP) [1-3] is a classical combination optimization problem. It describes that one salesman wants to visit a set of cities and needs to find a shortest traveling road that can travel back to the starting point, meanwhile he can't travel those cities repeatedly. TSP problem is everywhere in our daily life such as people arrange a tour between several cities with the shortest distance or how to arrange the machine to drill holes on the circuit board, technically TSP can also be used in the robotics' path planning. All of these problems can be mapped to the TSP and be worked out by using an appropriate method. Many of the researchers have already mentioned various solutions to solve TSP, for example, simulated annealing algorithm, ant colony algorithm, genetic algorithm, hybrid optimization strategy tabu search algorithm, quantum-behaved particle swarm optimization algorithm and Hopfield neural network [4-6] optimization .

In 1982, Professor J.J. Hopfield, a professor of physics at the California Institute of Technology in the USA, proposed a feedback neural network, which is called the Hopfield network. After that, Professor J.J. Hopfield introduced the concept of "energy function" in the feedback neural network, and the stability of neural network operation has a basis for judging because of the concept of energy function. The Hopfield network may be operated in a continuous mode (CHNN) or a discrete mode (DHNN), depending on the model adopted for explaining the neurons.

DHNN [7-9] is a single layer feedback network, it has n neurons, the dynamic mathematical model of differential equation can be used to describe the DHNN. Its main feature is that the output of each neuron in the network is able to restrict each other, all neurons can be updated simultaneously or asynchronously. CHNN [10-12] can be described by ordinary differential equations with constant coefficients. But the description of using electronic circuit is more intuitive, and the neurons are updated synchronously in the CHNN network. In 1985, Hopfield and D.W. Tank used a continuous Hopfield neural network successfully solved the TSP problem which is a typical optimize problem among the academe.

As for a TSP problem, an improved continuous Hopfield neural network based on dynamic step [13] is proposed to solve it efficiently in this paper. The rest parts of this paper are organized as follows: In section II, the continuous Hopfield neural network is introduced and the dynamic step is added to improve the optimization effect. In section III, the simulation results using continuous Hopfield neural

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network based on dynamic step are given. Finally, the conclusion is given in section IV.

II. Continuous Hopfield Neural Network Algorithm

A. Neurons

In the artificial neural network, the neuron is a processing unit which is an abstraction of the information processing process of biological neurons, and it is described by mathematical language. Neuron is an important component of neural network, so before researching the whole network, the neuron should be studied firstly.

1) Mathematical Model of Neuron

The interaction of the neurons can be abstracted with a mathematical expression. Suppose that symbol $x_i(t)$ represents the input signal from neuron i at the moment t. $y_i(t)$ represents the output signal when neuron j receives input signal from neuron i at the moment t. Then the outputs of neuron j can be described as:

$$y_j(t)=f\left\{\left[\sum w_{ij}x_i(t-\tau_{ij})\right]-T_j\right\} \tag{1}$$

The parameters in the formula (1) are interpreted as follows: τ_{ij} is delay between input and output of a neuron; T_j is threshold of neuron j; w_{ij} is connection (weighted) coefficient from neuron i to neuron j; $f()$is neuron transformation function.

B. Continuous Hopfield Neural Network

Continuous Hopfield neural network (CHNN) is different from the discrete Hopfield neural network (DHNN), all the neurons in CHNN work synchronously and they are dealing with information parallel. CHNN is described by using a constant coefficient differential equation. It is more directly and easy to understand that it is described by using analog electronic circuits. Fig.1 shows a network topology map based on CHNN:

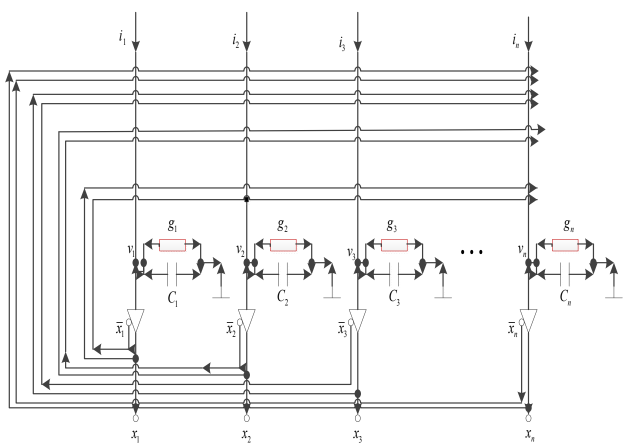

Fig. 1. The Network Topology Map based on CHNN

In Fig.1, C_j and $1/g_j$ are the equivalent input capacitance and resistance of operational amplifiers. They are used to model the output time constant of biological neurons. According to the Kirchhoff's law in the electronic circuit, the following equations can be written as:

$$C_j\frac{du_j}{dt}=\sum_{i=1}^{n}w_{ij}v_i-\frac{u_j}{R_j}+I_j \tag{2}$$

The transfer function in CHNN is S type function, which is expressed as follows:

$$v_j=f(u_j) \tag{3}$$

where, u_j and v_j are the output and input voltage of neuron j respectively. R_j and C_j is the input resistance and capacitance of neuron j, I_j is bias current; $f(u_j)$ is the transfer function of neuron j.

The above differential equation reflects the continuous updating of the state of the network, simulates the characteristics of several biological systems, describes the excitatory and inhibitory connections and the interaction between neurons. The dynamic process of CHNN network can be described based on the equations (2) and (3).

The network's state in neurons has constantly updated and always iterated. With the change of time, the network will eventually converge to a stable state and the output of the network will be stable. The stable state like that can be explained by the energy function.

The energy function of the Hopfield neural network is defined as follows:

$$E=-\frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n}w_{ij}v_iv_j-\sum_{j=1}^{n}v_jI_j+\sum_{j=1}^{n}\frac{1}{R_j}\int_0^{v_j}f^{-1}(v)dv \tag{4}$$

where, f^{-1} is the inverse function of neuron transposition function.

If operational amplifier in Fig.1 is an ideal or nearly an ideal operational amplifier, formula (4) can be simplified as follows:

$$E=-\frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n}w_{ij}v_iv_j-\sum_{j=1}^{n}v_jI_j \tag{5}$$

From the theorem we mentioned before, which is can be seen that the network's energy is always reduced, and finally tends to be stable. When the state of all neurons in the network is no longer changed, the energy will be stable and at the global minimum point, which is corresponding to a steady state in a continuous network. In the optimization problem, the global minimum point will be the optimal solution of the network.

C. Application of CHNN network to solve the TSP problem

1) Traditional CHNN in TSP

The introduction in the previous part has pointed out that the TSP problem is a classic NP problem. For a certain number of cities, a path planning of TSP is not possible with enumeration. Suppose there are n cities, then the total number of possible paths in these n cities is $R_n=n!/2n$. With the increasing of n, the possibilities of paths will show exponential explosive growth. It is hard to find a single route that has the shortest distance among a huge number of the possible paths.

Using continuous Hopfield neural network to solve the TSP problem, one can simplify the difficulty of the work. The idea of this method is to map the TSP problem to the CHNN network, and using the network energy to represent the length of the path. When the network energy is constantly updated, and converge to a minimum value eventually, the optimal solution of the problem will be attained. Because the CHNN network is running in parallel, the speed of calculation will be improved.

The process of solving TSP problem by CHNN shows as follows:

The distance matrix D is initialized;
A transposition matrix ← *Each possible path in TSP problem;*
A neuron array V ← *A transposition matrix;*
Energy function

$$E=E_1+E_2+E_3+E_4 =\frac{1}{2}A\sum_{x=1}^{n}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}v_{xi}v_{xj}+\frac{1}{2}B\sum_{i=1}^{n}\sum_{x=1}^{n-1}\sum_{y=x+1}^{n}v_{xi}v_{yi}+ \frac{1}{2}C(\sum_{x=1}^{n}\sum_{i=1}^{n}v_{xi}-n)^2+\frac{1}{2}D\sum_{x=1}^{n}\sum_{y=1}^{n}\sum_{i=1}^{n}d_{xy}[(v_{xi},v_{y,i+1})+(v_{xi},v_{y,i-1})]$$

While (network is not stable)
Initial the neural network connection weights matrix;
Adjust the weights matrix to make energy function E get the minimum value;
End while
The optimal solution of TSP ← *the steady output of the network;*

In order to solve TSP by using CHNN, we need to find a suitable way to express the travel route first. As for TSP problem with n cities, it is clear and simple by creating a $n\times n$ neurons matrix which is also called the transposition array. The state of a neuron is used to indicate the location of a city in a valid path and the state is expressed as v_{xi}. A city name is expressed by subscript x, x=1, …, n. The location of the city in a valid path is expressed by subscript i, i=1, …, n. For example, v_{xi} = 1 indicates that the city x is accessed in the i place. If v_{xi} = 0, that means in this trip the city x should not be visited in the i place. Because it also has to meet the most basic requirements of the TSP problem, that is, each city can be only visited once. Therefore in the array of each city line only and must have an element “1”, and the remaining elements are “0”. Such as the transposition matrix of 6 cities TSP shows in Fig.2.

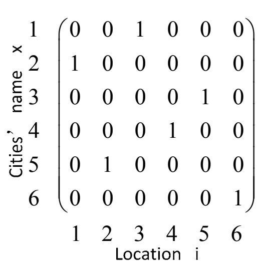

Fig. 1. Transposition Matrix in the TSP of 6-City

The tourist route indicated by the transposition array in Fig.2 is 2→5→1→4→3→6 (the route will eventually return to the beginning city 2). The total length of the path can be obtained according to the order of access. For the above example, the total length of the path is $d_{25}+d_{51}+d_{14}+d_{43}+d_{36}+d_{62}$.

The key of solving a TSP problem with CHNN is to construct a suitable energy function [2]. Here the composition of energy function will be discussed. The energy function of TSP problem is composed of 4 parts, that is:

$$E=E_1+E_2+E_3+E_4 =\frac{1}{2}A\sum_{x=1}^{n}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}v_{xi}v_{xj}+\frac{1}{2}B\sum_{i=1}^{n}\sum_{x=1}^{n-1}\sum_{y=x+1}^{n}v_{xi}v_{yi}+ \frac{1}{2}C(\sum_{x=1}^{n}\sum_{i=1}^{n}v_{xi}-n)^2+\frac{1}{2}D\sum_{x=1}^{n}\sum_{y=1}^{n}\sum_{i=1}^{n}d_{xy}[(v_{xi},v_{y,i+1})+(v_{xi},v_{y,i-1})] \quad (6)$$

Comparing formula (6) with the standard energy function of CHNN written in formula (5), weights between neuron x_i and y_j can be showed as follows:

$$w_{x_i,y_j}=-2A\delta_{xy}(1-\delta_{ij})-2B\delta_{ij}(1-\delta_{xy}) -2C-2Dd_{xy}(\delta_{j,i+1}+\delta_{j,i-1}) \quad (7)$$

where:

$$\delta=\begin{cases}1 & \mathrm{x=y}\\ 0 & \mathrm{x\neq y}\end{cases},\quad \delta_{ij}=\begin{cases}1 & \mathrm{i=j}\\ 0 & \mathrm{i\neq j}\end{cases}$$

And the bias current is

$$I_{xi}=2cn \quad (8)$$

The network needs to be initialized after construction for a stable state corresponding to a travel route. The travel routes of different travel initialization is different, therefore, these routes will be good or the best.

When using a computer to simulate the CHNN network, formula (7) and formula (8) are placed into formula (2) in traditional method and thus it gets:

$$\begin{cases} c_{ij}\dfrac{du_{xi}}{dt}=-2A\sum\limits_{j\neq i}^{n}v_{xi}-2B\sum\limits_{y\neq x}^{n}v_{yi}-2C(\sum\limits_{x=1}^{n}\sum\limits_{j=1}^{n}v_{xj}-n) \\ \qquad -2D\sum\limits_{y\neq x}^{n}d_{xy}(v_{y,i+1}+v_{y,i-1})-\dfrac{u_{xi}}{R_{xi}C_{xi}} \\ v_{xi}=f(u_{xi})=\dfrac{1}{2}[1+\tanh(\dfrac{u_{xi}}{u_0})] \end{cases} \quad (9)$$

where, u_0 is an initial value.

In practical applications, the results of TSP only need to be close to the optimal solution rather than to obtain the optimal solution. CHNN is a very appropriate method used to solve the TSP optimization problem. Using a total length of travel path energy function, not only simplifies the calculation difficulty, but also makes the neural network energy function that tends to be stable. Energy function opens up a new method for solving optimization problems.

2) *The improvement of CHNN algorithm and the solution of TSP*

There is an important parameter index in the CHNN network. Each iteration is related to a step size factor in the process of convergence of the neural network, the step size plays a very important role in the convergence. The operation steps of the traditional Hopfield neural network can be summarized as:

a) Initial network, random setting network initial input $u_i(t_0)$;

b) The output $v_i(t+1)$ of neuron i is computed by:

$$v_i(t+1) = f(u_i(t)) \tag{10}$$

c) The input $u_i(t)$ is calculated by formula (11). "Step" is the iteration step of gradient descent, which is a constant value.

$$u_i(t+1) = u_i(t) + step \cdot \frac{dE}{dt} \tag{11}$$

d) Determine the network to reach a stable state or not. If the given conditions are achieved, then end of the operation. Otherwise go to the second step to continue this process.

In the traditional CHNN algorithm, the step size factor is always a constant term. If the step size is relatively large in the algorithm, the gradient descent and the convergence rate are fast. But the jump of energy function is relatively large, which is easy to skip the optimal solution. If the step size is relatively small, the gradient descent is small. The convergence speed is very slow, which will increase calculation.

Dynamic step is proposed to solve the above problem. It is effective to use dynamic step to replace fixed step for the problem that the mutual restriction between convergence accuracy and speed. In the early stage of energy function convergence of neural network, large step size is used to accelerate the convergence rate. With the convergence process, the step is gradually reduced to obtain high precision optimization results.

The step size in formula (12) is a function that changes over the number of iterations t, which can be calculated by formula (13).

$$u_i(t+1) = u_i(t) + step(t) \cdot \frac{dE}{dt} \tag{12}$$

$$step(t)=step_0(1 - r \cdot \tan sig(t.\frac{a}{L})) \tag{13}$$

In formula (13), "step0" is a constant value, t is iteration number, L is the maximum number of iterations, r and a are the parameters control function change. As it is described in [13], when a becomes 4, step(t) will get better tend with the change of the number of iterations.

III. Simulation Results

A. 10-city simulation

Simulation is carried out under the MATLAB environment. Taking the TSP of 10 cities as an example, 10 neurons are needed in this system. A 10×10 transposition array mapping to the 10 cities is created, with setting the number of iteration to 5000 times. The location of the10 cities is known on the coordinate axes, so it is straightforward to calculate the distance between the 10 cities by distance formula. The distance is shown in the table.1:

Table.1. The Distance Between the10 Cities

	1	2	3	4	5	6	7	8	9	10
1	0	0.3	0.3	0.2	0.5	0.5	0.2	0.1	0.1	0.3
2	0.3	0	0.4	0.4	0.5	0.6	0.5	0.4	0.4	0.4
3	0.3	0.4	0	0.1	0.2	0.2	0.5	0.5	0.4	0.1
4	0.2	0.4	0.1	0	0.4	0.3	0.4	0.4	0.3	0.1
5	0.5	0.5	0.2	0.3	0	0.3	0.7	0.7	0.6	0.3
6	0.5	0.6	0.2	0.3	0.3	0	0.6	0.6	0.5	0.2
7	0.2	0.5	0.5	0.4	0.7	0.6	0	0.1	0.1	0.4
8	0.1	0.4	0.5	0.4	0.7	0.6	0.1	0	0.1	0.5
9	0.1	0.4	0.4	0.3	0.6	0.5	0.1	0.1	0	0.4
10	0.3	0.4	0.1	0.1	0.3	0.2	0.4	0.5	0.4	0

The initial path is random in Fig.3, all of the 10 cities are traveled without a specific order because there are a large number of unknown factors in the real applications that travelers can't control. It shows the total distance of the path is 3.8987.

After optimization based on the constant step, the total distance is expected to become a minimum but it may also fall into a local minimum. Fig.4 shows that it falls into a local minimum situation. Its total length is 2.371. The strategy based on dynamic step can avoid the local minimum. Fig.5 shows the path based on dynamic step, with total length of 2.2316. The path length based on dynamic step of 0.1394 less than in light of constant step

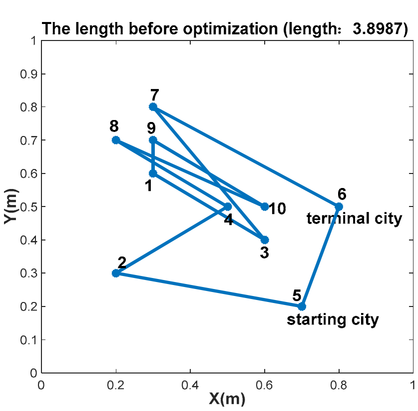

Fig. 2. Path Before Optimization

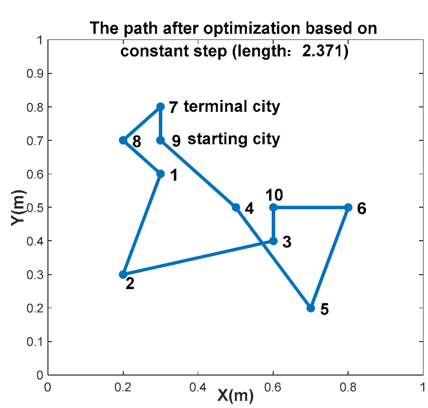

Fig. 3. Path After Optimization Based on Constant Step

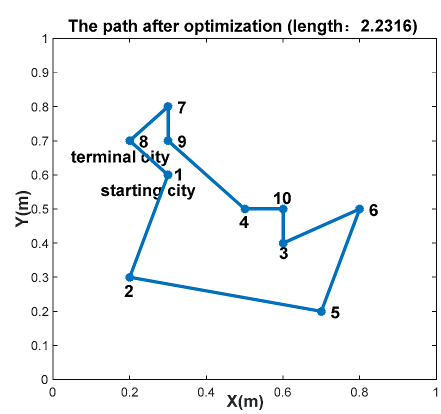

Fig. 4. ath After Optimization Based on Dynamic Step

which means CHNN based on dynamic step contributes in the optimization calculation. Through the program of the network, a system will jump out of the local minimum to attain the global minimum.

TABLE.2. COMPARISON OF TOTAL LENGTH AND ENERGY FUNCTION

	CHNN-Constant step	CHNN-Dynamic step
Total length	2.371	2.2316
Energy function	210.7254	199.2551

After the number of iteration increases to 5000 times, the energy function decreases and becomes stable (shows in Fig.6 and Fig.7), there are no more changes in energy function. The optimal energy of CHNN with constant step is 210.7254. However the network will stop to run based on CHNN with dynamic step while optimal energy is 199.2551. It can be indicate that CHNN with dynamic work is more efficiently than constant step. Using CHNN with dynamic step strategy has generated out the optimal path of the problem of 10-city TSP, so the system gets the best path as always hoped. Table 2 shows the comparison of total length and energy function between CHNN-constant and CHNN-dynamic steps. It can be more intuitive to see that in the case of traveling all cities without repeated access, the total length of CHNN-Dynamic step is shorter and less energy consumption, which means CHNN-Dynamic is more efficiently for solving the TSP problem.

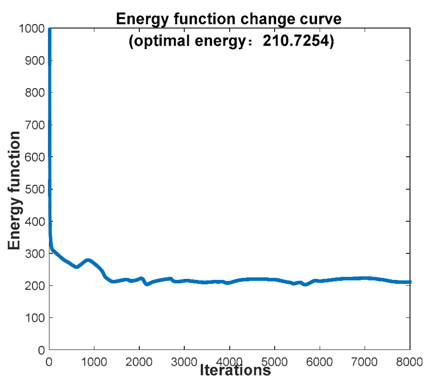

Fig. 5. The Change of Energy Function Based on Constant Step

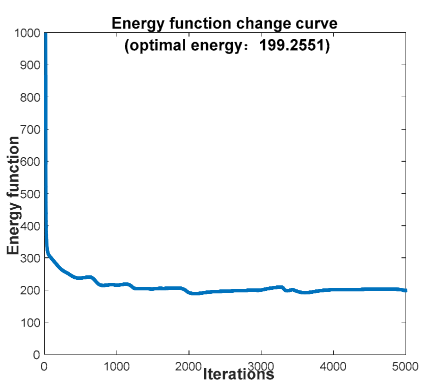

Fig. 6. The Change of Energy Function Based on Dynamic Step

B. *20-city simulation*

In order to improve the dependence and the effectiveness of this continuous Hopfield neural network (CHNN) based on dynamic step in TSP, 20 cities are selected to test in this simulation.

The same as the last simulation, the coordinate of the 20 cities is known, so it is easy to produce the distance between every two cities. The starting city and the end city are selected randomly by the computer. The initial path before optimization can be illustrated in Fig. 8, the dots represent the cities, all of the cities are visited one by one without any repetition.

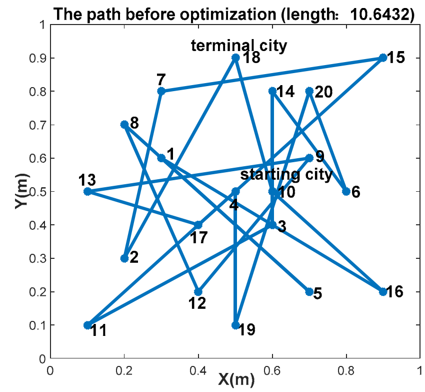

Fig. 7. Path of 20-city Before Optimization

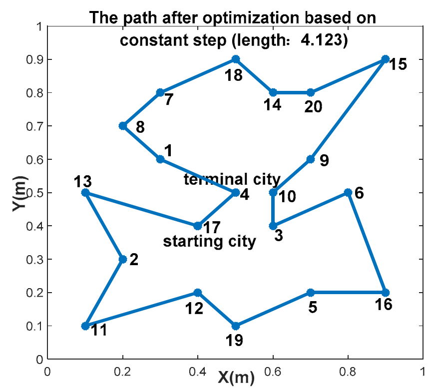

Fig. 8. Path of 20-city after Optimization based on Constant Step

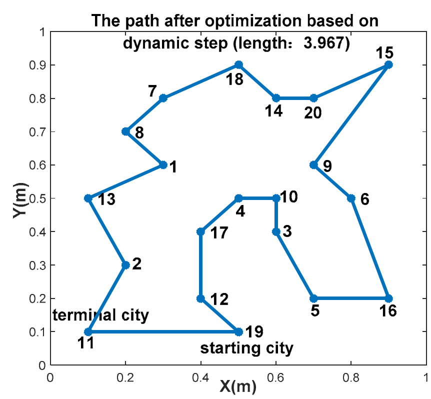

Fig. 9. Path of 20-city after Optimization based on Dynamic Step

TABLE.3. COMPARISON OF TOTAL LENGTH AND ENERGY FUNCTION

	CHNN-Constant step	CHNN-Dynamic step
Total length	4.123	3.967
Energy function	381.9886	376.6066

After optimization calculation based on the constant step, Fig.9 shows it is stuck into a local minimum situation. Its total length is 4.123. The strategy based on dynamic step can avoid the local minimum case. Fig.10 shows the path based on dynamic step, with total length of 3.967. The path length based on dynamic step is 0.156 less than based on constant step which means CHNN based dynamic step performs well in the optimization calculation.

The energy function decrease gradually and becomes stable (shows in Fig.11 and Fig.12), there are no more changes in energy function. The optimal energy of CHNN with constant step is 381.9886. However the network will stop to run based on CHNN with dynamic step while optimal energy is 376.6066. It can be indicated that CHNN with dynamic works more efficiently than that of constant step, which can be depicted in table 3.

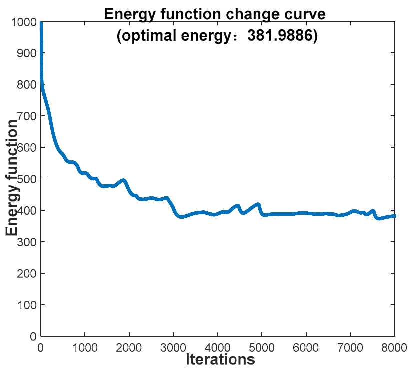

Fig. 10. The Change of Energy Function Based on Constant Step

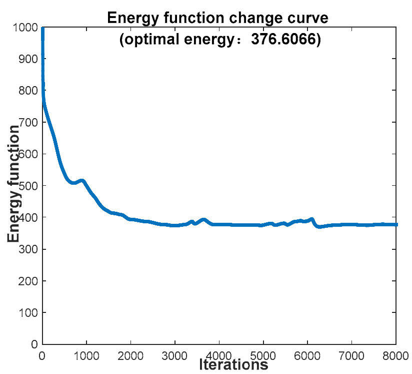

Fig. 11. The Change of Energy Function Based on Dynamic Step

IV. CONCLUSION

In this paper, a continuous Hopfield neural network based on dynamic step is applied to solve the TSP problem. Two experiments are carried out in the paper. The experiments are conducted respectively with 10 cities and 20 cities. From the experimental results, the continuous Hopfield neural network based on dynamic step is effective and feasible for the TSP problem in comparison with CHNN based on constant step.

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