Modification to mass flow rate correlation in oil-water two-phase flow by a V-cone flow meter in consideration of oil-water viscosity ratio

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Abstract. Oil-water two-phase flow widely exists and its measurement is of significance in oil well logging, oil transportations and etc. One of the techniques in common use in oil-water two-phase flowrate measurement is the Differential Pressure (DP) meters combining a theoretic model connecting the mass flowrate of the mixture with the differential pressures generated by the throttling element inside the pipeline. Though a number of publications focus on DP meters in measuring gas-water two-phase flows or wet gas, the existing models are still not very compatible in oil-water two-phase flow. In this work, a series of oil-water two-phase flow experiments were conducted in a horizontal pipe of 50mm diameter and the flowrates were measured by a V-cone meter with a diameter ratio of 0.65. Available correlations of DP meters developed from gas-water flows are studied and compared with the measured data from the V-cone meter. A modified correlation is proposed based on the influence of viscosity of oil upon the differential pressure model and three-dimensional Computerized Fluid Dynamic (CFD) simulations. The results have shown that the proposed method achieves better accuracy in oil-water two-phase flowrate measurement than other DP correlations.

Keywords: oil-water; two-phase flow; V-cone flow meter; differential pressure; CFD simulations

1. Introduction

In petroleum industry, the oil-water two-phase flows are widely encountered in production processes such as explorations and transportations. The accurate measurement to the flowrate of an oil-water two-phase flow is of great value to industrial processes (Trallero et al., 1997). Many methods have been proposed to solve this issue, such as, capacitance probes, ultrasonic techniques, tomographies and single-phase flow meters and etc (Skea and Hall, 1999; Falcone et al., 2001). Among them the differential pressure (DP) meters play a significant role. At present, the application of differential pressure flow meters in multiphase flow measurements are of increasing research interests, like Venturi and Orifice meters, and a number of correlations have also been developed for multiphase flow measurements with DP meters (Steven, 2002; Reimann et al., 1982). As a newly developed DP meter, V-cone meters have attracted wide research interests on their applications in two-phase flow measurement (Stewart et al., 2002).

The measuring model that adopted by the DP meters in flow measurement is the pressure drop model which in two-phase flow measurement is mainly classified into homogeneous model and separated model based on the assumptions on the flow conditions. In oil-water two-phase
flows, the homogeneous model assumes that the oil and water are well mixed to be considered as a single phase fluid flowing in the pipeline with the density derived from the weight of oil and water respectively, while the separated model treats the two phases separately each as a single fluid flowing alone in the pipeline. Unlike gas-water two-phase flows, the individual phase of oil-water two phase flows has similar flow characteristics (Viswanathan, 1998; Fang et al., 2007).

Though a considerable number of reports and academic papers were published on gas-liquid two-phase flows on a wide range of fluid properties, however, whether those results and correlations can be applied in oil-water two-phase flows still remains as a question. Because the flow condition of oil-water flows is quite different from that of gas-liquid two-phase flows. The different flow structure is mainly caused by the large liquid/liquid momentum transfer capacity and small buoyancy effects. And the lower free energy at the interface allows the formation of shorter interfacial waves and smaller dispersed phase droplet size (Trallero et al., 1997). Another reason that the oil-water flows are difficult to measure is that the oil properties can be quite diverse. Take the ratio of oil/water viscosity for example, it varies from more than a million to less than one. Different properties of oil that flows with water present so different in both geometrical and rheological behaviors (Xu, 2007).

In the following part of this paper, several available correlations for DP meters are discussed with the oil-water two-phase flow measurements from a V-cone meter with the diameter ratio of 0.65 implemented on a 50mm diameter horizontal pipe. A modified correlation is suggested based on the form that Chisholm suggested to achieve better results. The related coefficients are analyzed and determined through the 3-Dimensional Computerized Fluid Dynamic (CFD) simulations, in consideration of the affect of oil viscosity. A modified correlation is then proposed for oil-water two-phase flowrate measurement, and the results show that the measurement accuracy is improved with the proposed method.

2. V-cone flow meters and available correlations

The throttling set is one of the primary instruments employed in flowrate measurement by introducing pressure drop to the fluid. When fluid passes the throttling set, the contraction of the cross section area increases the flow velocity according to Bernoulli’s Equation. Consequently the pressure difference between the upstream and downstream of the throttling set results in the differential pressure which contains the information of the flowrate.

For the advantages of high reliability and accuracy, simple structure as well as easy installation, the throttling sets have gained wide applications in single phase flow measurement and in broad investigation on two-phase flow measurement (Baker, 2000). In the middle of 1980s, came out a new kind of DP meter, V-cone flow meter whose structure is shown in Figure 1.

![Figure 1. Structure of a V-cone flow meter](image)

The streamlined structure of V-cone flow meters overcomes the disadvantages of other throttling sets that they can be easily jammed due to the high viscosity of oil flows. The noise signal created by the fluid flowing through the V-cone is much lower than that generated by other throttling sets (Stewart et al., 2002). With a wider measurement scale, V-cone meters are more...
accurate and effective in flow rate measurement, and for which are becoming a popular research subject in two-phase flow measurement.

2.1. Principle of V-cone meters

From the momentum equations, the basic correlation of a fluid flowing through a throttling set is:

\[ \frac{p_1'}{\rho_1} + \frac{u_1'^2}{2} = \frac{p_2'}{\rho_2} + \frac{u_2'^2}{2} \]  

(1)

Along with the continuity equation:

\[ A_1 \dot{u}_1 = A_2 \dot{u}_2' \]  

(2)

where \( p_1', p_2', u_1, u_2', \rho_1, \rho_2, A_1, A_2 \) are respectively the pressures, velocity, density and the area from the upstream and downstream of the V-cone. For an incompressible fluid, \( \rho_1 = \rho_2 = \rho \) and \( A_2 = \mu A_0 \) (\( A_0 \) is the throat area of the V-cone, \( A_0 = \frac{\pi}{4} (D^2 - d^2) \)). Therefore

\[ u_2' = \frac{1}{\sqrt{1 - \beta^4}} \sqrt{\frac{2}{\rho} (p_1' - p_2')} \]  

(3)

(\( \beta \) is the diameter ratio, \( \beta = \frac{\sqrt{D^2 - d^2}}{D} \)). Considering the correction coefficient \( \psi \) to different tapping method, correction coefficient \( \epsilon \) to the energy loss due to fluid viscosity and the contraction coefficient \( \mu \), the volumetric flow rate is determined from:

\[ q_v = u_2' \mu A_0 = \frac{\mu \epsilon \sqrt{\psi}}{\sqrt{1 - \beta^4}} A_0 \sqrt{\frac{2}{\rho} \Delta p} \]  

(3)

Define the discharge coefficient as the combination of the above coefficients as \( C_0 = \mu \epsilon \sqrt{\psi} \), the mass flow rate of the fluid flowing through a V-cone is calculated with:

\[ W = \frac{C_0}{\sqrt{1 - \beta^4}} A \sqrt{2 \Delta p \rho} \]  

(4)

where \( W \) is mass flow rate, \( \Delta p \) is the differential pressure.

As an equation for single phase flow, (4) needs modifications for two-phase flows. Several correlations have been proposed based on (4) and flow governing equations. Some of them are developed for wet-gas, and others are for gas-water two-phase flows with orifice or Venturi meters. Therefore, certain modifications are needed when it comes to a V-cone meter measuring oil-water two-phase flow.

2.2. Homogeneous Model

The homogeneous model treats two-phase flows as if they were single-phase flows by introducing a homogeneous density defined as:

\[ \frac{1}{\rho_n} = \frac{x}{\rho_2} + \frac{1-x}{\rho_1} \]  

(5)

where \( \rho_n \) is density of the two-phase mixture, \( x \) is the mass fraction of the second phase (gas or oil), \( \rho_2 \) and \( \rho_1 \) are the density of the two phases respectively. Assumptions on homogeneous model are that the two phases well mix with each other, and no relative velocity and mass transfer involved.
Combine (4) and (5), the mass flowrate of the homogeneous flow is:

$$W_m = \frac{C_o A \sqrt{\Delta p_w \rho_2}}{\sqrt{1 - \beta^4} \left[ x + (1-x) \frac{\rho_2}{\rho_1} \right]}$$

where $W_m$ is the mass flowrate of the oil-water mixture, $\Delta p_w$ is the differential pressure. The homogeneous model is a theoretical model that can be used particularly for a well mixed flow, such as one phase evenly dispersed into the other.

2.3. Separated Model

The basic idea of the separated model is to treat the two phases separately each as a single fluid flowing alone in the pipeline with its own flow parameters and properties but with identical discharge coefficient and differential pressure with each other. Based on these assumptions, the flowrate of each phase when it flows alone in the pipeline is:

$$W_i = \frac{C_o}{\sqrt{1 - \beta^4}} A_i \sqrt{2 \Delta p_i \rho_i}$$

$$W_o = \frac{C_o}{\sqrt{1 - \beta^4}} A_o \sqrt{2 \Delta p_o \rho_o}$$

where $W_i$ and $W_o$ are the mass flowrate of water and oil respectively, $\Delta p_i$ and $\Delta p_o$ are the pressure difference when each phase flows independently in the pipeline with the same mass flow rate as in the two-phase flows. When the two fluids flow together:

$$W_i = \frac{C_o}{\sqrt{1 - \beta^4}} A_i \sqrt{2 \Delta p_i \rho_i}$$

$$W_o = \frac{C_o}{\sqrt{1 - \beta^4}} A_o \sqrt{2 \Delta p_o \rho_o}$$

where $A_i$ and $A_o$ are the respective cross-section areas that the two individual fluid flows through, they have the relationship as:

$$A = A_i + A_o$$

Combining equation (7)-(11), the pressure relationship is:

$$\sqrt{\frac{\Delta p_w}{\Delta p_o}} = \sqrt{\frac{\Delta p_1}{\Delta p_2}} + 1$$

So the mass flow rate of the two-phase flow is detained:

$$W_m = \frac{C_o A \sqrt{2 \Delta p_w \rho_2}}{\sqrt{1 - \beta^4} \left[ x + (1-x) \frac{\rho_2}{\rho_1} \right]}$$

A theoretical correlation as well, it therefore is hard to represent the practical flows. However, a number of correlations were developed from this model with certain modifications to acquire a better performance. Two-Phase Multipliers are usually adopted in the pressure drop analysis, defined as:
where $\Delta p_1$ represents the pressure drop when the fluid of first phase flows alone in the pipe with the same mass velocity as of the two-phase flow:

$$W_a = \frac{C_0}{\sqrt{1 - \beta^2}} A \sqrt{2 \Delta p_{10} \rho_1}$$

(16)

The relationship between $\varphi_{10}^1$ and $\varphi_1^2$ is:

$$\varphi_1^2 = \varphi_{10}^1 \frac{\Delta p_1}{\Delta p_{10}}$$

(17)

And from the definition of $\Delta p_1$ and $\Delta p_{10}$, their ratio is:

$$\frac{\Delta p_1}{\Delta p_{10}} = (1 - x)^2$$

(18)

The multiplier $\varphi_{10}^i$ of two-phase flow is usually analyzed in the frame of homogeneous mode and separated model. In a homogeneous two-phase flow, $\Delta p_\varphi / \Delta p_{10} = \rho_1 / \rho_a$, with the mixture density equation the following correlation obtained:

$$\varphi_{10}^i = \rho_1 / \rho_a = 1 + \left( \frac{\rho_1}{\rho_a} - 1 \right) x$$

(19)

Considering the effect of viscosity, it becomes (Chisholm, 1983):

$$\varphi_{10}^i = \rho_a / \rho_1 = \left[ 1 + \left( \frac{\rho_1}{\rho_a} - 1 \right) x \right]^{1 + \left( \frac{\mu_1}{\mu_a} - 1 \right) x}$$

(20)

While in the separated model, from the definition as well as the assumption of the model, we have:

$$\sqrt{\frac{\Delta p_\varphi}{\Delta p_{10}}} = \frac{W_1/W_a}{A_1/A} = \frac{1 - x}{1 - \alpha}$$

(21)

Substitute $\alpha = 1 \left( 1 + \left( \frac{1 - x}{x} \right) \rho_1 / \rho_a \right)$:

$$\varphi_{10}^i = \Delta p_\varphi / \Delta p_{10} = \left[ 1 + \left( \frac{\rho_1}{\rho_a} - 1 \right) x \right]^2$$

(22)

where $S$ is the velocity ratio of the two phases. Under ideal conditions $S = 1$, the above equation is simplified:

$$\varphi_{10}^i = \Delta p_\varphi / \Delta p_{10} = \left[ 1 + \left( \frac{\rho_1}{\rho_a} - 1 \right) x \right]^2$$

(23)

A general correlation is obtained from the separated model by combining (7)-(10) and (15):
or

\[ W_m = \frac{C_o}{\sqrt{1 - \beta^w}} \frac{A \sqrt{2\Delta p_w \rho_w}}{x \varphi_2} \]  \hspace{2cm} (25)

And (12) is simplified with the Lockhart-Martinelli (L-M) coefficient as:

\[ \varphi_2 = \chi + 1 \]  \hspace{2cm} (26)

and

\[ \chi = \frac{\Delta p_1}{\Delta p_2} = \left( \frac{W_1}{W_2} \right) \left( \frac{\rho_2}{\rho_1} \right) = \left( \frac{1 - x}{x} \right) \sqrt{\frac{\rho_2}{\rho_1}} \]  \hspace{2cm} (27)

The analyses on the two-phase flow multipliers usually focused on homogeneous model based gas-water two-phase flow, and mainly were in the wet gas measurement. Therefore several ad hoc correlations were developed and are introduced in the following sections.

2.4. **Murdock Correlation**

Murdock has developed a gas-water two-phase flow correlation based on the separated model, with a constant 1.26 to (26), as:

\[ \varphi_2 = 1.26 \chi + 1 \]  \hspace{2cm} (28)

Then the mass flowrate is determined by:

\[ W_m = \frac{C_o A \sqrt{2\Delta p_w \rho_w}}{\sqrt{1 - \beta^w [x + 1.26(1 - x)\sqrt{\rho_2/\rho_1}]} \Delta p_2} \]  \hspace{2cm} (29)

It was experimentally developed from orifice meters with different gases, and the accuracy was proved to be around 0.75%. A further modification on this correlation changed 1.26 into 1.5 for different applications (Murdock, 1962).

2.5. **Lin Correlation**

Lin (Lin, 1982) claims that the constant coefficient 1.26 of Murdock correlation (28) should be replaced by a variable \( \theta \) which varies with the ratio of the density of the two phases:

\[ \varphi_2 = \theta \chi + 1 \]  \hspace{2cm} (30)

Parameter \( \theta \) can be fitted in experiments. So Lin correlation is:

\[ W_m = \frac{C_o A \sqrt{2\Delta p_w \rho_w}}{\sqrt{1 - \beta^w [x + \theta(1 - x)\sqrt{\rho_2/\rho_1}]}} \]  \hspace{2cm} (31)

A higher accuracy was obtained when applying this correlation to Venturi meters. When \( \rho_2/\rho_1 \geq 0.328 \), \( \theta \) approaches to unity, which makes Lin correlation transforms into (13).

2.6. **Smith & Leang Correlation**

Smith & Leang correlation was developed from orifice and Venturi meters by introducing a “Blockage Factor” (BF), a function of \( x \), to denote the local blockage of the water. So the mass flowrate is:
\[ W_m = \frac{A(BF)\sqrt{2\rho_2 \Delta p_2}}{x\sqrt{1 - \beta^4}} \] (32)

### 2.7. Steven Correlation

Another correlation of Venturi meters for wet gas metering is given by Steven at 2002 (Steven, 2002):

\[ W_m = \frac{C_\alpha A \sqrt{2\Delta p_\rho \rho_2}}{x\sqrt{1 - \beta^4 \left(1 + A\chi + BF_{Fr_2}\right)}} (33) \]

where \( A \), \( B \), \( C \), \( D \) are the correction parameters and in the functions of density ratio of oil and water.

### 2.8. Chisholm Correlation

Chisholm (Chisholm, 1977) has taken the shear force at the boundary of the two phases into consideration, and developed a gas-water two-phase flow model for orifice meters. He also correlated the differential pressure of two-phase mixtures and each phase alone with the Lockhart-Martinelli parameter \( \chi \),

\[ \phi_2^2 = 1 + \left[ \left( \frac{\rho_1}{\rho_2} \right)^{0.25} + \left( \frac{\rho_2}{\rho_1} \right)^{0.25} \right] \chi + \chi^2 \] (34)

Therefore the correlation for two-phase mass flowrate is:

\[ W_m = \frac{C_\alpha A \sqrt{2\rho_2 \Delta p_\rho \rho_2}}{x\sqrt{1 - \beta^4 \left[1 + \left( \frac{\rho_1}{\rho_2} \right)^{0.25} + \left( \frac{\rho_2}{\rho_1} \right)^{0.25} \right] \chi + \chi^2}} \] (35)

### 2.9. De Leeuw Correlation

De Leeuw (de Leeuw, 1997) believes that the error induced by the liquid is determined not only by pressure and Lockhart-Martinelli parameter, but also by Froude number:

\[ Fr_2 = \frac{U_{ss}}{\sqrt{gD}} \sqrt{\frac{\rho_2}{\rho_1 - \rho_2}} \] (36)

The de Leeuw correlation is close to Chisholm in formation, but with a parameter \( n \) which is solely subject to Froude Number to replace the constant 0.25 in (34):

\[ \phi_2^2 = 1 + \left( \frac{\rho_1}{\rho_2} \right)^n + \left( \frac{\rho_2}{\rho_1} \right)^n \chi + \chi^2 \] (37)

\[ n = \begin{cases} 0.41 & 0.5 \leq Fr_2 \leq 1.5 \\ 0.606(1 - e^{-0.746 Fr_2}) & Fr_2 \geq 1.5 \end{cases} \] (38)

So the mass flowrate is determined by:
3. Experimental Description

The oil-water two-phase flow experiments were conducted at the three-phase flow test loop of Tianjin University, as Figure 2 illustrated. The horizontal pipeline is manufactured of steel tubing with an internal diameter of 50 mm. The total length of this pipeline between entrance nozzle and the outlet is approximately 16.56 m, consists of two horizontal legs with the length of 7.22 and 7.30 m respectively, connected by a horizontal U-bend with the length of 2.04 m. The oil and water was pumped into the entrance nozzle separately and well mixed at the beginning of the pipeline. At the end point of the operating pipeline, the oil-water mixture was fed into a separation tank for stagnant separation. Then the separated oil and water will be pumped back into the oil and water tank separately.

![Figure 2 Test loop of oil-water two-phase flow experiments.](image)

A V-cone meter with the diameter ratio of 0.65 and acquisition rate of 2000 data/second was implemented 14.56 m downstream from the inlet and 2 m upstream to the outlet to allow a 310D distance for the development of flow regimes. A Perspex tube was fitted in the pipeline right before the V-cone meter to provide visual observations on the flow conditions.

The superficial velocity of oil and water was 0–3.6 m/s and 0.63–1.69 m/s respectively, the range of mass fraction of oil was 0–81.7%. The average temperature and operating static pressure were 15.7 °C and 48.6 kPa respectively. There were 55 experiment conditions with 40000 measurements in each point conducted. Oil and water properties are listed in Table 1. The flow condition was recorded by a camera at the Perspex section. The flow regimes observed in experiments were “Dispersion of oil in water and water (Do/w & w)” flow and “Oil in water (o/w)” flow as defined by Trallero (Trallero et al., 1997). The flow map is illustrated in Figure 3.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Properties</th>
<th>Density</th>
<th>Viscosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>998 kg/m³</td>
<td>1 cP</td>
<td></td>
</tr>
<tr>
<td>Oil</td>
<td>842 kg/m³</td>
<td>14.7 cP</td>
<td></td>
</tr>
</tbody>
</table>
The V-cone meters are not standard flow meters, so they must be calibrated before experiments. The parameter that needs calibration is discharge coefficient $C_C$. Ten groups of water experiments were conducted to calculate this parameter, as shown in Figure 4 that all the data points stabilize around 0.8-0.9. An average value of 0.83 was adopted in the models. Before proceeding with the comparisons between each correlation, the error needs to be defined properly:

$$\varepsilon = \frac{Data_{\text{meas}} - Data_{\text{ref}}}{Data_{\text{ref}}} \times 100\%$$

(40)

Define the average relative error $\varepsilon_{\text{ave}}$ as:

$$\varepsilon_{\text{ave}} = \frac{1}{n} \sum \left| \frac{Data_{\text{meas}} - Data_{\text{inlet}}}{Data_{\text{inlet}}} \right| \times 100\%$$

(41)

where $Data_{\text{meas}}$ and $Data_{\text{ref}}$ are the measured value and reference value in the experiments respectively.

4. Results and Analysis

4.1. Experiment Results

The errors of each model discussed in the previous section are presented in Figure 5. It can be seen that half of the Steven correlation and Smith & Leang correlation test points have error greater than 40%, while they decrease to 40% when the mass fraction of oil is higher than 50% and 60% respectively. The error of de Leeuw correlation stays around 10%. Murdock 1.26 has a relatively better performance than Murdock 1.5, and both of the two correlations have the minus
errors that decrease with the increasing oil mass fraction. Besides, it is obvious that homogeneous 
correlation, Lin correlation ($\theta = 1$) and Chisholm correlation have close results and better than 
others.

Figure 5. Measurement errors of each correlation (the errors of several points of Steven 
Correlation and Smith & Leang Correlation are out of 100% range, and thereby not listed in the 
figure.)

4.2. Data Analysis

As indicated in (1)-(4), the pressure drop occurred from V-cone is determined by flow 
velocity and the density of the fluid. Plot the differential pressure versus water mass fraction 
($1 - x$) and the squared fluid velocity $V_s^2$ in Figure 6.

Figure 6. The relationship of differential pressure against water mass fraction and $V_s^2$

The differential pressures are dotted in black in a three-dimensional coordinate against water 
mass fraction and $V_s^2$ from the experiments, and their projections on each coordinate plane are 
indicated respectively with red points, green points and blue points. It shows that the differential 
presures have a nearly linear relationship with the squared mixture velocity and also shows an 
exponential relationship with water mass fraction. Based on (4), the differential pressure could be 
calculated from the following equation:

$$\Delta p = \left(1 - 2\beta^2 \right) \rho \beta \Delta V_s^2$$  \hspace{1cm} (42)

To a fluid with constant density, the differential pressure is linear to the squared flow velocity. 
Plot $\Delta p$ and $V_s^2$ of the experiments in Figure 7.
A good linear relation could be observed as:

$$\Delta p = 2935V_s^2$$

(43)

Substitute the water density with mixture density under homogeneous model assumption:

$$\rho_w = \alpha \rho_o + (1 - \alpha) \rho_w$$

(44)

Then (42) becomes:

$$\Delta p = 3.28 \rho_w V_s^2$$

(45)

Compare (43) with (45), the slope of $V_s^2$ is quite close to each other which explains why the homogeneous model obtains such small errors. Besides, as the mixture density depends on the mass fraction of oil, therefore plotting the differential pressures $\Delta p$ against water mass fraction $(1-x)$ to investigate their relationship. As Figure 8 indicates, under the same water velocity, the increase of $x$ results in the increase of $\Delta p$ in an exponential manner, additionally the curve moves up in the coordinate when increasing water velocity.

$$\phi_w^2 = 1.027(1 - x)^{2.06}$$

(46)
Figure 9. Data trend of $\phi^2_w$ to water mass fraction $(1-x)$

From (46), $\phi^2_w$ has a good exponential relationship with $(1-x)$, besides, in (17) $\phi^2_w$ depends on $\Delta p_{w0}/\Delta p_w$ and $\phi^2_w$. The trend of $\Delta p_{w0}/\Delta p_w$ against water mass fraction is plotted in Figure 10.

Figure 10. The relationship between $\Delta p_w / \Delta p_{w0}$ and water mass fraction $(1-x)$

An exponential relationship is obtained as:

$$\Delta p_w / \Delta p_{w0} = (1-x)^{0.94}$$ (47)

It is very close to the ideal correlation as (18) suggests. In addition, plot $\phi^2_{w0}$ versus $x$ in Figure 11 along with its ideal form under homogeneous and separated assumptions respectively. The dots are the experimental data points, and the black curve and red curve are from the theoretical correlations under homogeneous model (19) and separated model (22) respectively.

Figure 11. Relation of $\phi^2_{w0}$ to $x$ with its ideal form under homogeneous model and separated model
From Figure 10 drawn the observations that \( \phi_w^{20} \) distributed between 0.85-1.20 in the range of \( x \) from 0 to 0.8, and the points have a slightly ascending trend with \( x \) increases. It shows that the oil-water two-phase flow in this work produces relatively close pressure drop as the water with the identical mass flow rate. Besides, the homogeneous model based correlation (19) seems to have a better prediction on the trend of the data. As \( \phi_w^{20} \) approaching to a constant, the two-phase flow is close to the conditions that the two phases are homogeneously mixed and reach the Critical Point where the two phases have identical density and with no concerns on viscosity. However, from Figure 7, though a good linear relationship exists, the slope is still influenced by the mixture density, which shows that the two-phase flow is not well mixed at some point. So the analysis on the form and parameters of the correlations of measuring oil-water two-phase flows is presented in the following section from two aspects of homogeneous model assumption and separated model assumption respectively.

5. Model Discussion

5.1. Model Feasibility

The homogeneous model was developed on the assumptions that the two fluids are well mixed and therefore have identical velocity (though exists phase slips), thermo dynamics and kinematics features, as so they can be dealt with as a single phase flow. The differences of velocity and temperature within the two-phase flow lead the transfer of momentum, heat and mass. Only when one phase uniformly dispersed into the other could the two-phase be considered at the ideal balance. In gas-liquid two-phase flow, it is very difficult to reach this ideal condition, especially in the horizontal pipes where the gravity plays an important role. However, at the high velocity where the kinetic energy overcomes the potential energy (expressed by the coefficient \( Fr \)), the gas-liquid mixtures could be considered homogeneously mixed. While for an oil-water two-phase fluid that the density, as well as viscosity, of oil is close to that of water, when it flows in a horizontal pipe with the velocity increases and oil mass fraction fixed, one phase is easily dispersed into the other. And so forms the pseudo homogeneous flow with identical velocity and temperature and etc, which fits for the presumptions that homogeneous model established on, and thereby achieves a higher accuracy over most of the separated models (Wallis, 1969).

The separated model only takes the continuity of two-phase flow into consideration, excluding the momentum and energy transfer. In addition, the phase properties such as compressibility, viscosity as well as the surface tension were not involved in this model. It is thereby a parameter-reduced description on the two-phase flows, and can be adopted for both gas-liquid and liquid-liquid two-phase flows measurement (Brauner and Moalem, 1992).

5.2. Model Modification

The separated model based correlations are quite similar and easily transferred to each other by adjusting the parameters, for example, the Lin correlation is the adjustment to Murdock correlation, so as the de Leeuw correlation to Chisholm correlation. They all can be included in the general form of correlation (24), and from (9) and analysis of Lin (Lin, 1982) and Chisholm (Chisholm, 1983), the form of \( f(1/\chi) = \sqrt{\Delta p_w/\Delta p_w} = \phi_w \) is suggested during the analysis. In light of the measurement results of each correlation in Figure 5, the form of \( f(1/\chi) \) determines the accuracy of the model. The form that suggested by Chisholm has a stable measuring precision with the change of oil mass fraction and a flexible structurem, and its coefficient has a direct relationship with the well know L-M coefficient \( \chi \):

\[
\phi_w^2 = f(1/\chi) = 1 + C/\chi + 1/\chi^2
\]

where \( C \) is a coefficient and a replacement to (48) in Chisholm correlation. Besides, in Chisholm correlation, due to the property of oil and water, the ratio of density is constant, which
leads to:

$$\left( \left( \frac{\rho_1}{\rho_2} \right)^{0.25} + \left( \frac{\rho_2}{\rho_1} \right)^{0.25} \right)^{\approx 2} \approx 2 \quad (49)$$

Substituting (48), (49) and (27) into (35), Chisholm correlation reduces to Lin correlation with $\theta = 1$, which is the theoretical correlation of separated mode (24). Therefore, the measuring correlation of oil-water two-phase flows can be improved to a higher accuracy with certain modification in the form that Chisholm suggested (Chisholm, 1983).

Parameter $C$ is always greater than two in gas-liquid two-phase flow as the theoretical separated model suggested. And the form of coefficient $C$ that Chisholm discussed also implies that this value is always greater than two. Besides, the modified separated model such as Murdock correlation (both with constant 1.26 and 1.5) and Lin correlation as well as de Leeuw correlation all have shown that a better result would be obtained when a higher value of coefficient $C$ is adopted. The mass flow rate correlation based on (24) and (48) is:

$$W_m = \frac{C_\psi A \sqrt{2 \rho_s \Delta p_w}}{(1 - \chi)\sqrt{1 - \beta^2 \sqrt{1 + C / \chi^2}}} \quad (50)$$

To investigate (50) and coefficient $C$ in oil-water two-phase flow, plot $\phi_1^2$ against $1/\chi$ in Figure 12, coefficient $C$ is fitted as 1.9 (Tan et al., 2009). Substituting $C$ in (50), the measurement errors of homogeneous model, separated model and the modified correlation with the second groups of data that under the same experimental conditions are listed in Table 2. The results also are in line with the analysis by P. Angeli and G.F. Hewitt (Angeli and Hewitt, 1998) that both the homogeneous model and separated model usually over predict the oil-water flows.

The $C$ coefficient is a function of densities as well as viscosities of the two phases, and in a different form with that of gas-water two-phase flows.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\varepsilon_{ave}$</th>
<th>Oil Bubble</th>
<th>Dispersed</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>homogeneous</td>
<td>1.23%</td>
<td>1.22%</td>
<td>1.22%</td>
<td></td>
</tr>
<tr>
<td>Separated</td>
<td>1.17%</td>
<td>1.14%</td>
<td>1.16%</td>
<td></td>
</tr>
<tr>
<td>Modified with $C$</td>
<td>0.55%</td>
<td>0.30%</td>
<td>0.47%</td>
<td></td>
</tr>
</tbody>
</table>

Figure 12. Curve fitting of $\phi_1^2$ and $1/\chi$
5.3. Discussion on the form of coefficient C

Combining (17) and (20), selecting Chisholm and neglecting the viscosity between two phases by setting \( n = 0 \), then introduce \( D \) to formulate a new correlation as:

\[
D = (1 - x)^2 + C \left( \frac{\rho_w}{\rho_o} \right)^{1/2} \left( x(1 - x) + \frac{\rho_w}{\rho_o} x^2 - \left[ 1 + \left( \frac{\rho_w}{\rho_o} - 1 \right) x \right] \right) \tag{51}
\]

To maintain \( D = 0 \) within the range of \( x \), \( C \) must be in the following form:

\[
C = \left( \frac{\rho_w}{\rho_o} \right)^{1/2} + \left( \frac{\rho_w}{\rho_o} \right)^{1/2} \tag{52}
\]

For the given oil and water, neglecting the influence of the slight change of temperature upon the density and viscosity, \( C \) is a constant and equal to 2 in this research which is identical to the ideal separated model. While under the assumptions of separated model, by introducing L-M parameter \( \chi \), (23) becomes:

\[
q_{\ast w}^2 = \left[ 1 + \frac{\rho_w}{\rho_o} \right] \chi \]

Substitute the above equation into \( q_{\ast w}^2 \) and with the Chisholm’s form, we have:

\[
1 + 2 \frac{\rho_w}{\rho_o} \sqrt{\chi} + \frac{\rho_w}{\rho_o} \chi^2 = 1 + C \chi + 1 \chi^2 \tag{54}
\]

And then:

\[
C = \frac{\rho_w}{\rho_o} \left[ 2 + \left( \frac{\rho_w}{\rho_o} - 1 \right) \frac{x}{1 - x} \right] \tag{55}
\]

It clearly shows that, only when \( \rho_w = \rho_o \), \( C \) is a constant and \( C = 2 \). These results show that for a given oil-water two-phase flow, the density of oil and water is fixed, and hence \( C \) is constant. According to the definition of \( \varphi_{\ast w}^2 \), any variation on the viscosity of oil phase results in the change of \( \Delta \rho_w \), and consequently the change of \( \varphi_{\ast w}^2 \) (Grassi et al., 2008). In this way, \( C \) is influenced by the viscosity within the range of \( x \) from 0 to 0.8. In the analysis of multiphase flow, the Blasius factor \( n \) is always introduced for the modification to the equations. Under the assumptions of homogeneously mixed and separated model, we have:

\[
\varphi_{\ast w}^2 = \frac{\Delta \rho_w}{\Delta \rho_w} \varphi_{\ast w}^2 = \left[ 1 + \left( \frac{\rho_w}{\rho_o} - 1 \right) x \right] \left[ 1 + \left( \frac{\mu_w}{\mu_o} - 1 \right) x \right]^{-n} \frac{1}{(1 - x)^2} \tag{56}
\]

And with the form suggested by Chisholm as indicated in (48), we have:

\[
D = (1 - x)^2 + C \left( \frac{\rho_w}{\rho_o} \right)^{1/2} \left[ x(1 - x) + \frac{\rho_w}{\rho_o} x^2 - \left[ 1 + \left( \frac{\rho_w}{\rho_o} - 1 \right) x \right] \right] = 0 \tag{57}
\]

where \( x \) is in the range between 0 to 1. When \( x = 0 \), (58) holds; while \( x = 1 \), the pipe is full of oil and replace \( \rho_w \) with \( \rho_o \), (58) holds as well. Under the given oil and water mixture, take \( x = 0.5 \) into (59) and obtain:
\[ C = \frac{2^{n1}\left(\frac{\rho_w}{\rho_o} + 1\right)\left(\frac{\mu_w}{\mu_o} + 1\right)^{-n} - \left(\frac{\rho_w}{\rho_o}\right)^{-1}}\left(\frac{\rho_o}{\rho_o}\right)\left(\frac{\rho_o}{\rho_o}\right)^{1/2} \] (58)

Under separated model, and by introducing viscosity, (57) becomes:

\[ (1-x)^2 + C\left(\frac{\rho_w}{\rho_o}\right)^{1/2}\left(x(1-x)\right) + \frac{\rho_w}{\rho_o} x^2 - \left[1 + \left(\frac{\rho_w}{\rho_o} - 1\right)x\right]^2\left[1 + \left(\frac{\mu_w}{\mu_o} - 1\right)x\right]^{-n} = 0 \] (59)

This equation holds when \( x \) equals to 0 and 1 as well. Introducing \( D \), then we have:

\[ D = (1-x)^2 + C\left(\frac{\rho_w}{\rho_o}\right)^{1/2}\left(x(1-x)\right) + \frac{\rho_w}{\rho_o} x^2 - \left[1 + \left(\frac{\rho_w}{\rho_o} - 1\right)x\right]^2\left[1 + \left(\frac{\mu_w}{\mu_o} - 1\right)x\right]^{-n} = 0 \] (60)

Likewise, when \( x = 0.5 \), the relation of \( C \) and \( n \) is:

\[ C = \frac{2^{n1}\left(\frac{\rho_w}{\rho_o} + 1\right)\left(\frac{\mu_w}{\mu_o} + 1\right)^{-n} - \left(\frac{\rho_w}{\rho_o}\right)^{-1}}\left(\frac{\rho_o}{\rho_o}\right)\left(\frac{\rho_o}{\rho_o}\right)^{1/2} \] (61)

Considering the form of (58) and (61), the form of \( C \) under homogeneous model and separated model assumptions are very close to each other, and jointly determined with the density ratio, viscosity ratio of oil and water, as well as the coefficient \( n \). Its generalized form can be deduced with the analysis on the properties of oil-water mixture and coefficient \( n \) under the CFD simulations.

5.4. Numerical Simulation with Viscosity Change

Some of experimental conditions were numerically simulated in commercial CFD software named FLUENT, version 6.3. And the geometry and mesh were produced by GAMBIT, commercial software for CFD modeling. A 3-Dimensional mesh of the V-cone and the pipe was built as shown in Figure 14.

![Figure 13. 3D mesh of V-cone and the pipe built with GAMBIT](image)

The length of the simulation pipe is 6 m and a V-cone with the identical size with that used in the experiments is located at 5m away from the inlet. Considering the mixing conditions and the mixture properties observed in experiments, the mixture model along with the \( \varepsilon - k \) model was adopted to describe the oil-water two-phase flow (Wendt, 1992). Three sets of simulations were...
conducted with respective oil viscosity as 14.7 cp, 73.7 cp and 147 cp, and with the identical oil density of 841.2 to examine the influence of viscosity on \( C \).

From Figure 14, value of \( C \) is 1.9, 2.35 and 2.83 are fitted corresponding to oil viscosity 14.7 cp, 73.7 cp and 147 cp respectively. The conclusions can be drawn that, the differential pressure is greater with higher oil viscosity.

Figure 14. Curve fitting of \( \varphi_w^2 \) and \( 1/\chi \), oil viscosity is 14.7 cp (the same as that of experiments), 73.7 cp and 147 cp

Combining (58) and (61), as well as considering the change of \( C \) at each oil viscosity in simulations, a simplified form of \( C \) can be written as:

\[
C = \left( \frac{\rho_w}{\rho_o} \right)^{0.5} \left( \frac{\mu_w}{\mu_o} \right)^n
\]  \hspace{1cm} (62)

Plot (58), (61) and (62) in the plane of \( C - n \), in Figure 15. It shows that when \( n \) is in the range of \((-0.25, 0.25)\), curve of (62) lies between that of (58) and (61) when \( n < 0 \) and increases very quickly afterwards, therefore (62) is considered as a substitute to (58) and (61).

The new modified correlation is:

\[
\varphi_w^2 = 1 + \left( \frac{\rho_w}{\rho_o} \right)^{0.5} \left( \frac{\mu_w}{\mu_o} \right)^n \sqrt[5]{\chi + 1/\chi^2}
\]  \hspace{1cm} (63)

Figure 15. Curve trend of \( C \) versus \( n \) under correlation (58), (61) and (62)

And then the proposed modified correlation for oil-water two-phase flow measurement with the V-cone meter is:
\[ W_m = \frac{C_d A \sqrt{2 \rho_l \Delta p_w}}{(1-x) \sqrt{1 - \beta^2} \left[ 1 + \left( \frac{\rho_w}{\rho_g} \right)^{10} \left( \frac{\mu_w}{\mu_g} \right)^{0.5} / \chi + 1 / \chi^2 \right]} \]  

(64)

where \( n = 0.2 \) when \( C = 1.9 \) in correlation (62), correspondingly, as the oil viscosity is 73.7 cp and 147 cp, \( C \) equals to 2.57 and 2.96 respectively, which are in line with the results from simulations. The measurement error \( \varepsilon_{\text{ave}} \) of (64) on oil-water two-phase flow mass flowrate measurement is 0.52%.

6. Conclusions

A series of experiments of oil-water two-phase flow were conducted on a 50mm diameter horizontal pipe in the three-phase flow loop in Tianjin University. The differential pressure from a V-cone flow meter with diameter ratio of 0.65 was measured to investigate its link to the mass flow rate of oil-water two-phase flow. The drawn conclusions are as follows:

1. The gas-liquid based differential pressure correlations were tested with the differential pressure data measured from the V-cone meter in oil-water two-phase flow. The results show that the separated flow model has a better accuracy than homogeneous flow model, and the Lin correlation (\( \theta = 1 \)) and Chisholm correlation have the best performance within the separated models, and the correlation with the form that Chisholm suggested has a stable prediction accuracy.

2. The accuracy of some gas-water based correlations applied on oil-water flows is acceptable within the herein experimental range, and that it will be improved with certain modification based on the flow properties of the oil and water mixture.

3. Based on the analysis of the model feasibility, the accuracy of the Chisholm correlation is further improved with a modified coefficient \( C \). Two major types of the possible form of \( C \) are presented and find that this coefficient is affected not only by density ratio but also by viscosity ratio. To further investigate the possible form of coefficient \( C \), three sets of CFD simulations were conducted with different oil viscosity but identical oil density. The numerical simulations have shown that, the more viscose that oil phase is, the higher value that \( C \) is.

4. A correlation of \( C \) is then proposed based on the simulation results and the average relative errors of the proposed correlation is 0.52%. This DP correlation is extensively applicable in oil-water two-phase flow rate measurement by adapting coefficient \( C \) to the flow properties with parameter \( n \).

However, the experiment in oil-water two-phase flow with different oil viscosity is beyond the current capability of this flow loop, and thereby no available experimental data from different oil viscosity can be used to test the performance of the proposed correlation. As such, further investigation will be presented with the extended experiment conditions on the updated flow facility.

Acknowledgment

Authors are grateful for the support from Chinese Scholarship Council, and would like to express their gratitude towards the help from Dr. Hoi Yeung in Cranfield University, UK, and Prof. Yanfeng Geng in China University of Petroleum.

This work is supported by the National Natural Science Foundation of China (50776063) and the Natural Science Foundation of Tianjin (08JCZDJC17700).

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