Constructing and applying an improved fuzzy time series model: Taking the tourism industry for example

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Abstract

This study develops an improved fuzzy time series models for forecasting short-term series data. The forecasts were obtained by comparing the proposed improved fuzzy time series, Hwang’s fuzzy time series, and heuristic fuzzy time series. The tourism from Taiwan to the United States was used to build the sample sets which were officially published annual data for the period of 1991–2001. The root mean square error and mean absolute percentage error are two criteria to evaluate the forecasting performance. Empirical results show that the proposed fuzzy time series and Hwang’s fuzzy time series are suitable for short-term predictions.

Keywords: Fuzzy time series; Heuristic; Forecasting

1. Introduction

There is a great variety of methodologies for short-term predicting presented in the literature. Frequently used quantitative techniques include ARIMA and econometric models (Goh & Law, 2002; Lim & McAleer, 2002). However, the econometric models need large samples (minimum 50 sample data), normal distribution, and stationary data trends, limiting their application validity. Recently, numerous scholars have developed new forecasting techniques to overcome the limitations of tradition statistical methods such as neural networks (Law, 2000; Law & Au, 1999). Although neural networks still require large sample data sets for training and to establish a learning procedure, they do not require making as many assumptions as do statistical methods.

Zadeh (1965) successfully applied fuzzy theory to different research fields, including decision-making, control theory, business analysis, and forecasting. The forecasting application of fuzzy theory was first presented by Song and Chissom (hereafter SandC) (1993a, 1993b) who developed the fuzzy time series using the enrollment at the University of Alabama from 1971 to 1992 as the sample set. In addition, there are two highly effective research directions. First, Chen (1996) presented these of arithmetic operations instead of the logic max–min composition, which was used by the S&C model. Huarng (2001) incorporated the heuristic rule with Chen’s model into the judging criteria of future trends, making a heuristic fuzzy time series model. Chen (2002) and Lee, Liu, and Chen (2006) further developed the model originally presented in 1996, to be high-order fuzzy time series models. In contrast to S&C’s and Chen’s fuzzy models, Hwang, Chen, and Lee (1998) incorporated the variation of forecasting into the fuzzy time series model, whereby the variation value plus the actual value of the last period yields the forecast value.

Fuzzy theory methodologies for short-term predicting have well been presented in the literature, including temperature (Chen & Hwang, 2000; Lee, Wang, & Chen, 2007a, Lee, Wang, & Chen, 2007b), finance (Lee, Wang, Chen, & Leu, 2006), and disruption prediction in Tokamak...
reactors (Versaci & Morabito, 2003); however, the study of the optimal density of intervals using first-order fuzzy time series was scarce. The research reported here seeks to address that smallest interval (i.e., largest density of interval) uses sub-intervals, then judging the data of interval is upward or downward by rule. The proposed fuzzy time series model that applies and compares the rationale of these fuzzy time series and to determine which is the best forecast model based on the empirical results.

2. Methodology

When economists forecast the future trend using time series data, they must carefully examine whether the time series were stationary. If a time series were nonstationary, that implies the data had a stochastic trend and would yield an incorrect forecast. However, this paper does not examine the characteristics of the time series data, because this study uses a different methodology from that of economists. The following subsections will introduce a novel fuzzy time series model.

The forecasting of the \( t + 1 \) period is compared to that of the \( t \) period either upward or downward. Therefore, a novel fuzzy time series model states that the forecast should use a logical relationship to judge the upward or downward movement of the forecast curve, and then yield the forecast value. The main difference between the proposed model and other fuzzy models is that the proposed model forecasts the trend of forecast curve by mean of changing length in each interval of the universes of discourse and using the differences of variations. The proposed fuzzy model was established by the following steps:

Step 1: Define the universe of discourse. The universe \( U \) is defined as, \( U = [D_{\text{min}} - D_{1}, D_{\text{max}} + D_{2}] \), where \( D_{\text{min}} \) and \( D_{\text{max}} \) denote the minimum and maximum number of units among the historical data, respectively; and \( D_{1} \) and \( D_{2} \), which divide the \( U \) into intervals of equal length, are two proper positive numbers.

Step 2: Calculate the density of intervals. The number of intervals depends on the amount of data. This paper divides the largest density of interval into three sub-intervals of equal length (namely, the density of interval is 3). Furthermore, we divide the second largest density of interval into two sub-intervals of equal length (namely, the density of interval is 2). Finally, we can find the smallest interval. If the data were not distributed in the interval, we can delete the interval.

Step 3: Define the fuzzy set \( A_{t} \) and fuzzify the historical data using the intervals mentioned in step 2, \( A_{t} = f_{A_{t}}(u_{1})/u_{1} + f_{A_{t}}(u_{2})/u_{2} + \ldots + f_{A_{t}}(u_{i})/u_{i} \), where \( f_{A_{t}}(u_{i}) \) denotes the grade of membership of \( u_{i} \) in \( A_{t} \) and \( f_{A_{t}}(u_{i}) \in [0, 1] \); the symbol “/” separates the membership degrees for each element degrees in the universe of discourse \( U \) and the symbol “+” means “union” rather than the commonly used algebraic symbol of summation.

Step 4: Establish the fuzzy logical relationships given the fuzzifying the data. Fuzzy logical relationships can be found: \( A_{t} \rightarrow A_{t-1} \rightarrow A_{t-2} \ldots \), where \( A_{t} = f(t - 1) \) and \( A_{t} = f(t) \), and so \( f(t) \) is said to be caused by \( f(t - 1) \).

Step 5: Forecast the future value. According to step 2, we divide the redistributed interval into four equal lengths; meanwhile, whether the forecasting curve, will be upward or downward, depends on the one-fourth point and three-fourth point within the interval.

The changing trend of forecasting is conducted using the following rules. Assuming the value of \( t \) period to be \( P_{n} \), then the variation of value is denoted by \( \Delta P_{t} = P_{t} - P_{t-1} \), and the difference of the variation is denoted by \( \Delta P_{t} = \Delta P_{t} - \Delta P_{t-1} \). Let \( Q_{t} = |\Delta P_{t}| \times 2 + t - 1 \) period of value; \( Q_{t} = |\Delta P_{t}|/2 + t - 1 \) period of value; \( Q_{t} = |\Delta P_{t}|/2 \).

We propose the rules to decide the changing trend of the \( t + 1 \) period as follows:

Rule 1:
(1) If \( Q_{t} \) belongs to \( A_{t} \) having the membership degree 1, then we judge the changing direction of the \( t + 1 \) period to be upward.
(2) If \( Q_{t} \) belongs to \( A_{t} \) having the membership degree 1, then we judge the changing direction of the \( t + 1 \) period to be downward.
(3) Excepting rule (1) and rule (2), the forecasts for the \( t + 1 \) period will be the midpoint of \( A_{t} \) having the membership degree 1.

Rule 2: If we can exactly know the variation rather than \( \Delta P_{t} \), then for
(1) \( Q_{t} > |\Delta A_{t}|/2 \), we judge the change of \( t + 1 \) period will be upward.
(2) \( Q_{t} = |\Delta A_{t}|/2 \), we judge the change of \( t + 1 \) period will be constant.
(3) \( Q_{t} > |\Delta A_{t}|/2 \), we judge the change of \( t + 1 \) period will be downward.

Meanwhile, the interval length of \( \Delta A_{t} \) belongs to the fuzzy set \( A_{t} \) with the membership degree 1.

3. Empirical study for tourism demand forecasting

In order to explore the application of the proposed novel fuzzy time series model, the present paper uses the sample of Taiwan tourists to the USA as published by the Taiwan Tourism Bureau. Empirical analysis is conducted to compare the forecasting result of the proposed fuzzy time series with those of the other models.

3.1. The empirical of novel fuzzy time series

The forecasted method is presented as follows:

(1) Table 1 lists the actual visitors from historical data. The minimum value \( D_{\text{min}} \) and maximum value \( D_{\text{max}} \)
is 23935 and 651134, respectively. Every length of interval is 6000. We divide the $U$ into seven intervals, $u_1, u_2, \ldots, u_7$. Meanwhile,

\[
\begin{align*}
  u_1 &= [235000, 295000], \quad u_2 = [295000, 355000], \\
  u_3 &= [355000, 415000], \quad u_4 = [415000, 475000], \\
  u_5 &= [475000, 535000], \quad u_6 = [535000, 595000], \\
  u_7 &= [595000, 655000].
\end{align*}
\]

(2) We distribute the tourism historical data into seven intervals (listed in Table 2); the interval $u_6$ has the five largest historical data. The interval $u_1$ has the second largest three historical data. Meanwhile, the interval $u_6$ is divided into three of sub-intervals equal length, which are $[535000, 555000], [555000, 575000], \text{and} [575000, 595000]$. At the same time, the interval $u_1$ is also divided into two of sub-intervals equal length, which are $[235000, 265000]$ and $[265000, 295000]$. We delete interval $u_2$ because its density is “0”. The other intervals are transformed into new intervals. The nine redistribution intervals and the density being three are denoted (9, 3).

\[
\begin{align*}
  u_{1,1} &= [235000, 265000], \quad u_{1,2} = [265000, 295000], \\
  u_{1,3} &= [355000, 415000], \quad u_{2,1} = [415000, 475000], \\
  u_{2,2} &= [475000, 535000], \quad u_{2,3} = [535000, 595000], \\
  u_{3,1} &= [595000, 655000].
\end{align*}
\]

(3) Define the fuzzy set $A_i$ using the linguistic variable “Tourists to the USA”, let $A_1 =$ (very very few), $A_2 =$ (very few), $A_3 =$ (few), $A_4 =$ (moderate), $A_5 =$ (many), $A_6 =$ (many many), $A_7 =$ (very many), $A_8 =$ (too many), $A_9 =$ (too many many). This paper define fuzzy sets on $U$. All the fuzzy sets $A_i$ ($i = 1, 2, \ldots, 9$) are expressed as follows:

\[
\begin{align*}
  A_1 &= \{1/u_1, 1/u_2, 0.9/u_3, 0.6/u_4, 0.3/u_5, 0.1/u_6, 1/u_7, 0.5/u_8, 0.25/u_9\}, \\
  A_2 &= \{0.5/u_1, 1/u_2, 0.6/u_3, 0.4/u_4, 0.2/u_5, 0.1/u_6, 0.3/u_7, 0.6/u_8, 0.4/u_9\},
\end{align*}
\]

\[
\vdots
\]

\[
\begin{align*}
  A_9 &= \{0.1/u_1, 0.2/u_2, 0.3/u_3, 0.4/u_4, 0.5/u_5, 0.6/u_6, 0.7/u_7, 0.8/u_8, 1/u_9\},
\end{align*}
\]

where $u_i$ ($i = 1, 1.1-7$) is the element and the number above “/” is the membership of $u_i$ to $A_i$ ($i = 1, 2, 9$).

(4) According to the results of the fuzzified historical data, the fuzzy logical relationships are constructed and listed in Table 3.
(5) Forecasting. Some examples are used to illustrate the process as follows:
[1991]: The value for 1991 is the initial value which does not have variations, the tourism forecast, which was decided in the middle of the fuzzy set $A_2$ ($265000, 295000$), was 280000.
[1992]: The fuzzy logical relationship group for $A_2$ is $A_2 \rightarrow A_2$ from 1991 to 1992. This means that we cannot forecast the trend of the curve. Therefore, we forecast the trend using the difference of variation. According to the outcome of judgment function, the outcome 14129.5 is smaller than that of 15000, which is the half of interval $[265000, 295000]$. We conclude that the trend will downward and the forecast will fall in one-quarter of interval $[265000, 295000]$. We can calculate the forecast value to be $272500$ ($265000 + (30000/4) = 272500$).

Table 2

<table>
<thead>
<tr>
<th>Interval</th>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$u_4$</th>
<th>$u_5$</th>
<th>$u_6$</th>
<th>$u_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3

<table>
<thead>
<tr>
<th>Fuzzy logical relationships</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1 \rightarrow A_2$</td>
</tr>
<tr>
<td>$A_2 \rightarrow A_3$</td>
</tr>
<tr>
<td>$A_3 \rightarrow A_4$</td>
</tr>
<tr>
<td>$A_4 \rightarrow A_5$</td>
</tr>
<tr>
<td>$A_5 \rightarrow A_6$</td>
</tr>
<tr>
<td>$A_6 \rightarrow A_7$</td>
</tr>
<tr>
<td>$A_7 \rightarrow A_8$</td>
</tr>
<tr>
<td>$A_8 \rightarrow A_9$</td>
</tr>
<tr>
<td>$A_9 \rightarrow A_1$</td>
</tr>
</tbody>
</table>
We decide the forecast tourism for 1993 as the middle of interval [355000, 415000]. The reason is that the outcome (291404.5) of the judgment function does not fall in the interval [355000, 415000] and the outcome (304720) also does not. After the above calculations, all of the forecasts are listed in Table 8. In order to explore whether or not different interval numbers and density affect the forecast, this paper simultaneously lists the fuzzy historical data in Table 8 under interval number 7 and density 1 using the same calculating steps with above mentioning.

3.2. The empirical of Hwang’s fuzzy time series

To demonstrate how Hwang’s fuzzy time series model was applied to forecast the short-term tourists. The process of Hwang’s model is presented as follows (Hwang et al., 1998):

1. Calculate the universe of discourse $U$. Table 1 lists the variations which are calculated as the number of tourists for this year minus that for the last year. The maximum $(D_{\text{max}})$ and minimum $(D_{\text{min}})$ variations were 87143 and −109370, respectively, the $U$ is denoted, $U = [D_{\text{min}} - D_{\text{max}}, D_{\text{max}} + D_{\text{max}}]$, where $D_{\text{max}} = 37$ and $D_{\text{max}} = 100$, $U = [-108470, 87180]$.

2. Separate $U$ into the seven intervals, whose length is 27950, and so there are seven intervals including $u_1, u_2, \ldots, u_7$, where $u_1 = [-108470, -80520], u_2 = [-80520, -52570], u_3 = [-52570, -24620], u_4 = [-24620, 3330], u_5 = [3330, 31280], u_6 = [31280, 59230], u_7 = [59230, 87180]$.

3. Define fuzzy sets $A_i$. In this work, the linguistic variable is “Taiwan tourists to the USA”. Each fuzzy set $A_i$ is assigned to a linguistic term: $A_1$ (very very few), $A_2$ (very few), $A_3$ (few), $A_4$ (moderate), $A_5$ (many), $A_6$ (many many), $A_7$ (very many). Each $A_i$ defined by the intervals: $u_1, u_2, \ldots, u_7$. Through steps (2) and (3), Table 1 lists the fuzzified variations of historical data.

4. Choose the appropriate window basis $w$ and calculate the operation matrix $O^w(t)$ and criterion matrix $C(t)$. The difficulty is to choose the fact whose number is $w$. But the procedure is best explained with the hope of specific example, which we set the $w = 4$ and forecast 2000 tourists.

\begin{align*}
O^w(2000) &= \begin{bmatrix}
\text{fuzzy variation of 1998} \\
\text{fuzzy variation of 1997} \\
\text{fuzzy variation of 1996} \\
\text{fuzzy variation of 1999}
\end{bmatrix} = \begin{bmatrix}
A_4 \\
A_5 \\
A_6 \\
A_4
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 \\
0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 \\
0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\end{align*}

\begin{align*}
C(2000) &= \begin{bmatrix}
\text{fuzzy variation of 1999} \end{bmatrix} = [A_4] = \begin{bmatrix}
0 & 0 & 0.5 & 1 & 0.5 & 0 & 0
\end{bmatrix}.
\end{align*}

5. Compute the relation matrix $R(t)[i, j] = O^w(t)[i, j] \times C(t)[j], 1 \leq i \leq 3, 1 \leq j \leq 7$;

\begin{align*}
R(2000) &= \begin{bmatrix}
0 & 0 & 0.25 & 0.5 & 0.5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0
\end{bmatrix}.
\end{align*}

Then, based on, we get the fuzzified forecasting variation as $F(2000)$ follows:

\begin{align*}
F(2000) &= \begin{bmatrix}
r_1 & r_2 & r_3 & r_4 & r_5 & r_6 & r_7
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0.25 & 1 & 0.5 & 0 & 0
\end{bmatrix}.
\end{align*}

6. Defuzzify forecasted variations computed from step (5). According to the rationale of fuzzy time series: (a) If the membership of an output has only one maximum $u_i$ then select the midpoint of the interval that corresponds to the maximum forecast value. (b) If the membership of an output has one or more consecutive maxima, then select the midpoint of the corresponding conjunct interval as the forecast. (c) If the membership of an output is zero, then no maximum exists. Thus, the predicted degree of change is zero.

7. Calculate the forecasts. Taking $F(2000)$ for example, there is a maximum value 1 in $F(2000)$; therefore, the forecasted variation of 2000 is the midpoint $m_d(-10645) = u_4$, where the maximum value is located. The forecast value in 2000 is calculated by the actual value in 1999, adding the forecasted variation in 2000. We can obtain $563991 + (-10645) = 553346$. Repeating the above procedure, we can get the forecast value from 1991 to 2002 (listed in Table 8).

3.3. The empirical of heuristic fuzzy time series

In this subsection, the present work uses an application of the fuzzy time series of the heuristic model. We further illustrate the heuristic knowledge step by step as follows (Huarng, 2001):

1. Define $U$ and intervals. This work takes $U = [235000, 655000]$; and the interval length is 60000. Consequently, there are seven equal intervals; namely, $u_1, u_2, \ldots, u_7$ where $u_1 = [235000, 295000], u_2 = [295000, 355000], u_3 = [355000, 415000], u_4 = 415000]$. 

In this work, the linguistic variable is ‘tourism’; and each fuzzy sets \( A_i \) is assigned to a linguistic term: \( A_i (i = 1, 2, \ldots, 7) \). \( A_1 \) (quite many), \( A_2 \) (not quite many), \( A_3 \) (not many), \( A_4 \) (many), \( A_5 \) (many many), \( A_6 \) (very many), \( A_7 \) (very very many). Each is defined by the intervals of \( u_1, u_2, \ldots, u_7 \).

\[
A_1 = \{ 1/u_1, 0.5/u_2, 0/u_3, 0.5/u_4, 0.75/u_5, 1/u_6 \}, \quad A_2 = \{ 0.5/u_1, 0.5/u_2, 0/u_3, 0.5/u_4, 1/u_5, 1/u_6 \}, \quad \ldots
\]
\[
A_6 = \{ 0/u_1, 0/u_2, 0/u_3, 0.5/u_4, 1/u_5, 1/u_6 \}, \quad A_7 = \{ 0/u_1, 0/u_2, 0/u_3, 0.5/u_4, 1/u_5, 1/u_6 \}.
\]

Table 1 lists the number of tourists to the USA from 1990 to 2001 and the corresponding fuzzy number of tourists \( A_i \).

(3) Establish Fuzzy logical relationship and fuzzy logical relationship groups. From \( A_i \) listed in Table 1, the fuzzy logical relationship group is obtained, as shown in Table 4. The fuzzy logical relationship can be rearranged into fuzzy logical relationship groups, as shown in Table 5.

(4) Heuristic fuzzy logical relationship groups. This work introduces the heuristic function, showing the increase or decrease in the number of tourists. By using the heuristic function, this work can establish the heuristic fuzzy logical relationship groups, as listed in Table 6. The following examples are used to illustrate the forecasting process:

<table>
<thead>
<tr>
<th>Year</th>
<th>Window basis</th>
<th>MAPE$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991, 1992</td>
<td>( w = 6 )</td>
<td>9.4%</td>
</tr>
<tr>
<td>1991, 1992</td>
<td>( w = 5 )</td>
<td>6.7%</td>
</tr>
<tr>
<td>1991, 1992</td>
<td>( w = 4 )</td>
<td>6.0%</td>
</tr>
</tbody>
</table>

The forecasting of tourists is based on the heuristic fuzzy logical relationship groups of \( F(t-1) \). The following examples are used to illustrate the forecasting process:

\[
[415000, 475000], \quad u_5 = [475000, 535000], \quad u_6 = [535000, 595000], \quad u_7 = [595000, 655000].
\]

(5) Forecasting. If \( h(\cdot); A_1, A_3 \) is for \( A_1, A_3 \). Therefore, the heuristic fuzzy logical relationship group in 1993 is \( A_1 \rightarrow A_1, A_3 \).

\[
[2002]: \quad \text{The actual number of tourists in 2001 is 542764 (\( A_6 \)). From Table 5, the fuzzy relationship group for \( A_6 \) is for \( A_6 \rightarrow A_2 \). Suppose that the heuristic indicates a decrease for the tourists in 2002. The heuristic function is expressed by \( h(\cdot); A_6, A_7 \) is for \( A_6, A_7 \). Therefore, the heuristic fuzzy logical relationship group in 2002 is for \( A_1 \rightarrow A_1, A_3 \).
\]

Table 4 Heuristic fuzzy logical relationships

<table>
<thead>
<tr>
<th>( A_1 \rightarrow A_1 )</th>
<th>( A_1 \rightarrow A_3 )</th>
<th>( A_1 \rightarrow A_4 )</th>
<th>( A_1 \rightarrow A_5 )</th>
</tr>
</thead>
</table>

Table 5 Heuristic fuzzy logical relationship groups

<table>
<thead>
<tr>
<th>( A_1 \rightarrow A_1, A_3 )</th>
<th>( A_2 \rightarrow \Phi )</th>
<th>( A_3 \rightarrow A_4 )</th>
<th>( A_4 \rightarrow A_5 )</th>
</tr>
</thead>
</table>

Table 6 Heuristic fuzzy logical relationship groups

<table>
<thead>
<tr>
<th>Fuzzy logical relationship groups</th>
<th>Heuristic increasing((\uparrow))/ ( h(\cdot); A_1, A_3 )</th>
<th>Heuristic fuzzy logical relationship groups</th>
<th>Forecasted value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 \rightarrow A_1, A_3 )</td>
<td>( \uparrow ) ( A_1 \rightarrow A_1, A_3 )</td>
<td>( \downarrow ) ( A_1 \rightarrow A_1, A_3 )</td>
<td>( A_1 \rightarrow A_1, A_3 )</td>
</tr>
<tr>
<td>( A_2 \rightarrow A_6 )</td>
<td>( \uparrow ) ( A_1 \rightarrow A_1, A_3 )</td>
<td>( \downarrow ) ( A_1 \rightarrow A_1, A_3 )</td>
<td>( A_1 \rightarrow A_1, A_3 )</td>
</tr>
</tbody>
</table>

$^a$ MAPE is calculated from the sample in various window bases.
Table 8
The forecast result and RMSE for three models in in-sample

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual visitors</th>
<th>Hwang’s model (under window basis ( w = 4 ))</th>
<th>Heuristic model</th>
<th>Novel model under (7,1)</th>
<th>Novel model under (9,3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>267584</td>
<td>325000</td>
<td>265000</td>
<td>265000</td>
<td></td>
</tr>
<tr>
<td>1992</td>
<td>286966</td>
<td>325000</td>
<td>250000</td>
<td>250000</td>
<td></td>
</tr>
<tr>
<td>1993</td>
<td>371750</td>
<td>325000</td>
<td>385000</td>
<td>385000</td>
<td></td>
</tr>
<tr>
<td>1994</td>
<td>453924</td>
<td>445000</td>
<td>445000</td>
<td>445000</td>
<td></td>
</tr>
<tr>
<td>1995</td>
<td>522910</td>
<td>505000</td>
<td>505000</td>
<td>505000</td>
<td></td>
</tr>
<tr>
<td>1996</td>
<td>579488</td>
<td>565000</td>
<td>580000</td>
<td>585000</td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td>588916</td>
<td>625000</td>
<td>550000</td>
<td>582500</td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td>577178</td>
<td>595000</td>
<td>565000</td>
<td>585000</td>
<td></td>
</tr>
<tr>
<td>1999</td>
<td>563991</td>
<td>595000</td>
<td>550000</td>
<td>585000</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>651134</td>
<td>625000</td>
<td>625000</td>
<td>625000</td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>542764</td>
<td>696389</td>
<td>565000</td>
<td>550000</td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td>559824</td>
<td>27793</td>
<td>9663</td>
<td>4785</td>
<td></td>
</tr>
</tbody>
</table>

\( a \) RMSE is Root Mean Square Error.

\( b \) MAPE is Mean Absolute Percentage Error.

to the arithmetic average of the midpoints of \( u_1 \) and \( u_3 \): \((265000 + 385000)/2 = 325000\).

[1993]: Since \( h(\cdot; A_1, A_3) = A_1, A_3 \) for 1993; therefore, the forecast of 1993 is equal to the arithmetic average of the midpoints of \( u_1 \) and \( u_3 \): 325000.

[2002]: Since \( h(\cdot; A_6, A_7) = A_6, A_7 \) for 2002; therefore, the forecast of 2002 is equal to the arithmetic average of the midpoint of \( A_1 \) and \( A_3 \); i.e., 595000. All of the forecasts are listed in Table 8.

4. Evaluating of prediction model

This work uses two methods to examine the accuracy of the various fuzzy time series models. The first one is the root mean square error (RMSE) to compare the forecast of in-sample. RMSE is defined as RMSE = \( \sqrt{\frac{1}{n} \sum_{k=1}^{n} (x(k) - \hat{x}(k))^2} \), where \( x(k) \) is the actual number of visitors, and \( \hat{x}(k) \) is the predicted number of visitors. The absolute mean percentage error (AMPE) is used to examine the precision of the various fuzzy time series models of out-of-sample. AMPE is defined as, AMPE = \( \frac{|x(k) - \hat{x}(k)|}{x(k)} \), where \( x(k) \) is the actual number of visitors, and is the predicted number of visitors.

Table 7 shows Hwang’s model calculated using \( w = 4, 5, 6 \); wherein the smallest AMPE value is 0.06 using \( w = 4 \). The forecasts using Hwang’s model, heuristic model, and novel model are also shown in Table 8. This table shows that from 1991 to 2001; the forecast using Hwang’s model yields the largest error rate (RMSE = 27793) among these models, and the proposed model using interval and density \((7, 1), (9, 3)\) yields the smallest forecast error rate (RMSE = 4785). Therefore, we rank the forecast model in RMSE standard as proposed model, heuristic model, and Hwang’s model.

The literature on fuzzy time series generally focuses on the precision of forecast for in-sample. In order to test whether these models will yield the same forecast results for out-of-sample as that of in-sample, this paper forecasts the tourists in 2002 and shows the results in Table 8. Comparing these simultaneous forecasts from in-sample and out-of-sample, we conclude the proposed novel fuzzy model will provide better overall forecasting results for appropriate short-term time series data. The forecasted number of tourists from 1991 to 2002 and the actual number of tourists are shown in Fig. 1 for comparison.

5. Conclusion

The literature has developed a rich picture of forecasting in terms of time series methods, ARIMA model, and neural model. The general conclusion that can be drawn from these contributions is that each approach has specific strengths and weaknesses. The ultimate purpose of forecasting is to assist in management decision-making. The environmental turbulence is a critical factor influencing the forecasting
results. Traditional forecasting methodologies need a large amount of sample data and long-term historical data. Clearly, a manager cannot expect to make good decisions by applying a vast amount obsolete historical data. Fuzzy time series can overcome these limitations and make appropriate short-term forecasting.

As with all research, this work also has limitations. One of the foremost disadvantages is the shock data of the special events. We did not intend to provide accuracy forecasts under the influence of special evident such as the 911 terrorist actions, or the 921 earthquake in Taiwan, the Olympic Games, etc. This study has also presented the important contribution in fuzzy theory. Given the small amount raw data, this paper compares the most current fuzzy time series models, and evaluates the forecast performance using the RMSE and AMPE. Furthermore, this work develops a novel fuzzy time series model which yields accurate forecasts for all situations. In the in-sample situation, the novel fuzzy model excellently fits the historical data. In the out-of-sample situations, the AMPE of Hwang’s model, heuristic model and novel model are 0.012, 0.109, and 0.053, respectively. We can conclude that the proposed novel fuzzy time series model provides appropriate short-term forecasting.

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References