AN AXISYMMETRIC LATTICE BOLTZMANN MODEL
FOR SIMULATION OF TAYLOR–COUETTE FLOWS
BETWEEN TWO CONCENTRIC CYLINDERS

X. D. NIU, C. SHU, and Y. T. CHEW
Department of Mechanical Engineering
National University of Singapore, Singapore 119260

Received 10 October 2002
Revised 4 December 2002

Recently, Halliday et al. presented an idea by inserting the “source” terms into the
two-dimensional (2D) lattice Boltzmann equation (LBE) so that the emergent dynam-
ics of the lattice fluid can be transformed into the cylindrical polar system. This paper
further extends the idea of Halliday et al. to include the effect of azimuthal rotation.
The terms related to the azimuthal effect are considered as inertia forces. By using
our recently developed Taylor-series-expansion and least-square-based lattice Boltzmann
method (TLLBM) for the transformed LBE and a second order explicit finite difference
method for the azimuthal moment equation, Taylor–Couette flows between two concentric
cylinders with the inner cylinder rotating were simulated. To show the performance
of the proposed model, the same problem was also simulated by the three-dimensional
(3D) LBM. Numerical results showed that the present axisymmetric model is much more
efficient than the 3D model for an axisymmetric flow problem.

Keywords: Transformed lattice Boltzmann equation; source term; cylindrical polar co-
dordinates; Taylor–Couette flow.

1. Introduction

Although the lattice Boltzmann method (LBM) has achieved great success in solv-
ing fluid problems, it still faces difficult situation for the three-dimensional (3D)
axisymmetric flows because of the singularity at symmetric axis if a normal O-type
grid is used in the radial–azimuthal plane. Furthermore, if the gradients in the az-
imuthal direction are negligible for flows in cylindrical polar coordinates, the use
of three-dimensional (3D) lattice Boltzmann equation (LBE) to solve this kind of
problems is not economic in computational cost as compared to the two-dimensional
(2D) LBE solver. Traditionally, the 3D Navier–Stokes (N–S) equations can be sim-
plified to a 2D form in the cylindrical polar system for the axisymmetric flow, and
this can greatly reduce the computational effort. Therefore, we believe that there
are some ways to transform the 3D LBE into a 2D form in the cylindrical polar
system for an axisymmetric problem.
Recently, Halliday et al.\textsuperscript{3} suggested that the 3D LBE for the axisymmetric flow can be transformed into a 2D form in the cylindrical polar system by adding variable “source” terms to the standard 2D LBE. In the procedure of their derivation by using the Chapman–Enskog expansion, the source terms are self-consistent of the fluid moment equation. They gave a very accurate result for the forced flow in an infinitely long circular pipe. However, in their derivation, the azimuthal effect was neglected. Note that the use of “source” strategy in the LBE can be tracked back. Higuera et al.\textsuperscript{2} first applied this useful way in simulating flow around a circular cylinder in 1989. After that, flows related to chemical reactions, multiphase, suspensions were studied using this strategy by Ponce et al., Filippova and Hanel, Gunstensen et al., Rothman and Keller, Ladd, and Chen et al.\textsuperscript{9}

Following the idea of Halliday et al.,\textsuperscript{3} in this paper we presented a transformed LBE, in which the “source” terms related to the azimuthal rotation were added in the standard 2D LBE. We take the terms related to the azimuthal rotation as inertia forces. Through the Chapman–Enskog expansion, the transformed 2D LBE in the cylindrical polar system was obtained. By coupling the transformed 2D LBE with azimuthal moment equation in the cylindrical polar system, the emergent dynamics of the lattice fluid was correctly described.

To efficiently apply LBE to solve problems with complex geometry and the use of nonuniform mesh, the Taylor-series-expansion and least-square-based LBM (TLLBM) was recently developed.\textsuperscript{10} In this work, the TLLBM is applied to the transformed 2D LBE and the second order explicit finite difference scheme is applied to solve the azimuthal moment equation. The TLLBM is a meshless method, which is developed from the standard LBM by using the Taylor series expansion and the least squares approach.\textsuperscript{11} The final form of TLLBM is an algebraic formulation, in which the coefficients depend only on the coordinates of mesh points and lattice velocity, and are computed in advance.

Taylor–Couette flow between two concentric cylinders is a typical axisymmetric problem, which is a good test case to validate the new axisymmetric LBM. An understanding of the Taylor–Couette flow between concentric cylinders poses a challenge to both basic research and general applications. Generally, Taylor–Couette flow is the unique outcome of a centrifugal instability resulting from the rotation of an inner cylinder relative to a concentric outer cylinder. At high rotational speed of the inner cylinder, centrifugal forces overcome viscous forces and donut shaped toroidal vortices fill the space between the cylinders. At low rotational speed of the inner cylinder, the flow is steady and the vortices are planar. The critical speed of rotation at which vortices begin to appear depends on the radius ratio of two cylinders. Detailed studies concerning this topic can be found in many literatures using the conventional Navier–Stokes solvers.\textsuperscript{12–17}

To show the performance of the proposed axisymmetric model, the Taylor–Couette flow was also simulated by using the three-dimensional LBM, which is based on the D3Q15 lattice velocity model. Detailed comparison of computational
time and the accuracy of numerical results between the 3D LBM and the present model is given in the paper. Numerical results showed that the present model is very efficient in terms of computational effort.

2. Transformed LBE in Cylindrical Polar System

Considering a general axisymmetric case and making the following replacements,

\[(r, z) \rightarrow (x, y), \quad (u_r, u_z, u_\theta) \rightarrow (u, v, w),\]

the three-dimensional axisymmetric Navier–Stokes equations can be written as

\[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = -\frac{\rho u}{x},\]

\[\rho \frac{D u}{D t} + \frac{\partial \rho}{\partial x} - \rho v \nabla^2 u = \frac{\rho u^2}{x} + \frac{\rho v}{x} \left( \frac{\partial u}{\partial x} - \frac{u}{x} \right),\]

\[\rho \frac{D v}{D t} + \frac{\partial \rho}{\partial y} - \rho v \nabla^2 v = \frac{\rho v \partial v}{x \partial x},\]

\[\rho \frac{D w}{D t} = \rho v \nabla^2 w + \frac{\rho v}{x} \left( \frac{\partial w}{\partial x} - \frac{w}{x} \right) - \frac{\rho uw}{x}.\]

As compared with the forms in the \((x, y)\) plane, the right hand side terms of Eqs. (2)–(4) can be considered as “inertia forces” in the \((r, z)\) plane. Now we follow the idea of Halliday et al.\(^3\) and derive a LBE which recovers Eqs. (2)–(4) from a D2Q9 model.\(^{18,19}\)

The two-dimensional LBE is constructed as

\[f_\alpha(x + e_{ax}\delta_t, y + e_{ay}\delta_t, t + \delta_t) = f_\alpha(x, y, t) - \frac{1}{\tau} (f_\alpha(x, y, t) - f^\text{eq}_\alpha(x, y, t)) + s_\alpha(x, y, t),\]

where \(\tau\) is the single relaxation time; \(f_\alpha\) is the density distribution function along the \(\alpha\) direction; \(e_{ax}, e_{ay}\) is the particle velocity in the \(\alpha\) direction, \(f^\text{eq}_\alpha\) is the corresponding equilibrium state, which can be given as

\[f^\text{eq}_\alpha = w_\alpha \rho \left[ 1 + 3(e_{ax} \cdot \mathbf{U}) + \frac{9}{2} (e_{ax} \cdot \mathbf{U})^2 - \frac{3}{2} \mathbf{U}^2 \right],\]

for the D2Q9 model, where \(w_\alpha\) are weights defined as \(w_\alpha = 1/9\) for \(\alpha\) with \(|e_\alpha| = 1\), \(1/36\) for \(\alpha\) with \(|e_\alpha| = \sqrt{2}\) and \(4/9\) for \(\alpha = 0.1^{18,19}\). Note that the velocity vector \(\mathbf{U}\) in Eq. (7) has two velocity components \(u\) and \(v\). \(s_\alpha(x, y, t)\) is defined as “source” and is a spatial- and velocity-dependent function. Similar to the work of Halliday et al.,\(^3\) \(s_\alpha(x, y, t)\) can be expressed as

\[s_\alpha = \delta_1 s^{(1)}_\alpha + \delta^2 s^{(2)}_\alpha,\]
where \( s^{(1)}_\alpha \) and \( s^{(2)}_\alpha \) respectively represent zeroth order and first order gradients of the macroscopic variables \( \rho, u \). By performing the Chapman–Enskog expansion,\(^3\)\(^{,20}\) we obtain the following equation at \( O(\varepsilon) \)

\[
\partial_t f^{eq}_\alpha + \mathbf{e} \cdot \nabla f^{eq}_\alpha = -\frac{f^{(1)}_\alpha}{\tau \delta t} + s^{(1)}_\alpha ,
\]

and the equation at \( O(\varepsilon^2) \)

\[
\partial_t f^{eq}_\alpha + \left( 1 - \frac{1}{2\tau} \right) \left( \partial_t f^{(1)}_\alpha + \mathbf{e}_\alpha \cdot \nabla f^{(1)}_\alpha \right) = -\frac{f^{(2)}_\alpha}{\tau \delta t} + s^{(2)}_\alpha .
\]

In recovering Eqs. (2)–(4), we obtain the following relations about \( s^{(1)}_\alpha \) and \( s^{(2)}_\alpha \):

\[
\sum_\alpha s^{(1)}_\alpha = -\frac{\rho u}{x},
\]

\[
\sum_\alpha s^{(1)}_\alpha e_{ax} = \frac{\rho u^2}{x} \sum_\alpha s^{(1)}_\alpha e_{ay} = 0 ,
\]

\[
\sum_\alpha s^{(2)}_\alpha = \left( \tau - \frac{1}{2} \right) \left( \partial_t \mathbf{e}_\alpha + \mathbf{e}_\alpha \nabla \right) \sum_\alpha s^{(1)}_\alpha ,
\]

\[
\sum_\alpha s^{(2)}_\alpha e_{ax} = \left( \tau - \frac{1}{2} \right) \delta_t \left( \partial_t \mathbf{e}_\alpha + \mathbf{e}_\alpha \nabla \right) \sum_\alpha s^{(1)}_\alpha e_{ax} ,
\]

\[
\sum_\alpha s^{(2)}_\alpha e_{ay} = \left( \tau - \frac{1}{2} \right) \delta_t \left( \partial_t \mathbf{e}_\alpha + \mathbf{e}_\alpha \nabla \right) \sum_\alpha s^{(1)}_\alpha e_{ay} + \frac{\rho u}{x} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) .
\]

From Eqs. (9) and (10), we obtain

\[
s^{(1)}_\alpha = w_\alpha \left( G e_{ay} + \frac{3 \rho u^2}{x} e_{ax} - \frac{\rho u}{x} \right). \tag{14}
\]

In the above process, we have used the result of \( \sum_\alpha w_\alpha e_\alpha e_\beta = \delta_{\alpha\beta}/3 \) and considered \( G \) as a positive constant body force.

Substituting Eq. (14) into Eqs. (12) and (13) and using the following relation (multiply Eq. (9a) by \( e_\alpha \) and sum on \( \alpha \))

\[
\rho \frac{Du}{Dt_0} + \frac{\partial p}{\partial x} = \frac{\rho u^2}{x},
\]

we can obtain the expression for \( s^{(2)}_\alpha \):

\[
s^{(2)}_\alpha = \frac{v}{x c_s^2} \left( \rho \mathbf{u} \cdot \nabla u + \frac{\partial p}{\partial x} + \rho x \frac{\partial w^2}{\partial x} + \rho (\nabla \times \mathbf{u}) e_{ay} + \frac{\partial w^2}{\partial t} | e_{ax} \right). \tag{15}
\]

By this stage, a new LBE in the cylindrical polar system was developed. Together with Eq. (5), we present an alternative two-dimensional way to solve the three-dimensional axisymmetric flow problems. If the azimuthal rotation is neglected, Eqs. (14) and (15) have exactly the same forms as shown by Halliday \textit{et al.}\(^3\)
As commented by Halliday et al., the explicit inclusion of gradients to the LBE is contrary to the philosophy of the LBM because it undermines the distinction between the solving procedures of the LBE and the N–S equations. Fortunately, these gradients are evaluated at the previous time level. So, the model is still an explicit scheme.

The macroscopic density $\rho$ and momentum density $\rho U$ can be computed by

$$
\rho = \sum_{\alpha=0}^{N} f_{\alpha}, \quad \rho U = \sum_{\alpha=0}^{N} f_{\alpha} e_{\alpha},
$$

where $N$ is the number of discrete particle velocities.

3. The Taylor-Series-Expansion and Least-Squares-Based LBM

The TLLBM developed in Ref. 10 is based on the standard LBE, the well-known Taylor series expansion, the idea of developing Runge–Kutta method, and the least square approach. Due to the restriction of the LBE to regular lattice, at each time step, the particle described by the standard LBE may not stream to the neighboring mesh point if an irregular grid is used in the computation. However, because the particle distribution function is a continuous function in physical space and can be well defined in any mesh system, the Taylor series expansion in space can be used to get the value of the distribution functions at corresponding mesh points. The final algebraic formulation for the transformed LBE (6) can be written as

$$
f_{\alpha}(x_0, y_0, t + \delta t) = \sum_{k=1}^{M+1} a_{1,k} g_{k-1},
$$

where $M$ is the number of neighboring mesh points around the reference mesh point $(x_0, y_0)$, $a_{1,k}$ are the elements of the first row of the following matrix $[A]$

$$
[A] = ([S]^T[S])^{-1}[S]^T,
$$

$[S]$ is a $(M + 1) \times 6$ dimensional matrix, which is given as

$$
[S] = 
\begin{bmatrix}
1 & \Delta x_0 & \Delta y_0 & \frac{(\Delta x_0)^2}{2} & \frac{(\Delta y_0)^2}{2} & \Delta x_0 \Delta y_0 \\
1 & \Delta x_1 & \Delta y_1 & \frac{(\Delta x_1)^2}{2} & \frac{(\Delta y_1)^2}{2} & \Delta x_1 \Delta y_1 \\
& & & & & \\
& & & & & \\
& & & & & \\
1 & \Delta x_M & \Delta y_M & \frac{(\Delta x_M)^2}{2} & \frac{(\Delta y_M)^2}{2} & \Delta x_M \Delta y_M \\
\end{bmatrix}_{(M+1) \times 6}
$$
where
\[ \Delta x_0 = e^{\alpha x_0 \delta t}, \quad \Delta y_0 = e^{\alpha y_0 \delta t}, \quad \Delta x_i = x_i + e^{\alpha x_i \delta t} - x_0, \]
\[ \Delta y_i = y_i + e^{\alpha y_i \delta t} - y_0, \quad i = 1, 2, \ldots, M, \]
\[ g_i = f_\alpha(x_i, y_i, t) + \left[ f_\alpha^{eq}(x_i, y_i, t) - f_\alpha(x_i, y_i, t) \right] \frac{\tau}{\tau} + s_\alpha(x_i, y_i, t), \quad i = 0, 1, \ldots, M. \]

Since the coefficients depend only on the coordinates of mesh points and lattice velocity, and are computed in advance, the new method is essentially a meshless method, and can be applied to any lattice velocity model.

4. Application to Simulate Taylor–Couette Flow

In this part, the laminar Taylor–Couette flow in an annulus with a rotating inner cylinder, a stationary outer cylinder and two end plates was used to validate the axisymmetric LBM. The TLLBM with D2Q9 discrete velocity model\(^{18,19}\) was used to solve Eq. (6) with the source terms (14) and (15). The viscosity was given as
\[ \nu = \left( \frac{(2\nu - 1)}{6} \right) \varepsilon. \]

In our application, we use a structured grid, and take \( M = 8 \) for convenience. As shown in Fig. 1, for an internal mesh point \((i, j)\) (noted as "0" in Eq. (17)), the eight neighboring points are taken as \((i-1, j-1); (i-1, j); (i-1, j+1); (i, j-1); (i, j+1); (i+1, j-1); (i+1, j); (i+1, j+1)\). Therefore, at each mesh point, we only need to store nine coefficients \( a_{1,k}, k = 1, 2, \ldots, 9 \) before Eq. (17) is applied. The azimuthal velocity \( w \) was directly obtained by an explicit evaluation at each evolution step from the azimuthal moment equation (5)
\[ w_{i,j}^{n+1} = w_{i,j}^n + \delta t \left[ \left( \frac{\partial w_{i,j}^n}{\partial x} + v \frac{\partial w_{i,j}^n}{\partial y} \right) + \nu \left( \frac{\partial^2 w_{i,j}^n}{\partial x^2} + \frac{\partial^2 w_{i,j}^n}{\partial y^2} \right) \right. \]
\[ + \frac{v}{x_{i,j}} \left( \frac{\partial w_{i,j}^n}{\partial x} - \frac{w_{i,j}^n}{x_{i,j}} \right) \right], \quad (20) \]

Fig. 1. Schematic plot of neighboring point distribution around the point \((i, j)\).
where the partial derivatives of $w$ were discretized by the second order finite difference scheme.

The radius ratio of two cylinders was set as $\Theta = 0.5$, and the aspect ratio $\Gamma$ was chosen to 3.8. The Reynolds numbers ($Re = WD/\nu$ based on the azimuthal velocity $W$ and the gap of the annulus $D$) were selected as 85, 100 and 150, respectively. All present results were compared with those of Liu\textsuperscript{17} who used the differential quadrature (DQ)\textsuperscript{23} to solve the vorticity-stream function formulation in the cylindrical polar system.

Two kinds of boundary conditions were implemented in the present computation. One is on the outer cylinder wall and two end plates, where a complete bounce back rule is used to describe the nonslip boundary condition. The other is on the inner cylinder wall, where the equilibrium distribution function is used. The sketch of the flow field is shown in Fig. 2. Initially, a constant azimuthal velocity $W = 0.15$ was imposed on the inner cylinder and the flow field was set to be stationary with a constant density $\rho_0$.

\subsection{Convergence criterion and grid sensitivity}

The convergence criterion in this study was set as

\begin{equation}
\sum_{i,j} \frac{\|U(X_{i,j} + e\alpha\delta t, t + \delta t) - U(X_{i,j}, t)\|}{\|U(X_{i,j} + e\alpha\delta t, t + \delta t)\|} \leq 10^{-6}.
\end{equation}

The maximum stream function $\Psi_{\text{max}} (\partial \Psi / \partial y = u)$ in the whole computational domain is chosen as the value to study the grid sensitivity. Table 1 compares the maximum values of the stream function obtained on three different meshes of
Table 1. Comparison of the maximum stream function in $r$–$z$ plane for the Taylor–Couette flow at $Re = 85, 100$ and $150$.

<table>
<thead>
<tr>
<th>Re</th>
<th>TLLBM (axisymmetric model)</th>
<th>Liu\textsuperscript{17}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Psi_{\text{max}}$</td>
<td>$\Psi_{\text{max}}$</td>
</tr>
<tr>
<td>85</td>
<td>$4.95 \times 10^{-2}$</td>
<td>$4.895 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>error 1.87%</td>
<td>0.85%</td>
</tr>
<tr>
<td>100</td>
<td>$5.619 \times 10^{-2}$</td>
<td>$5.580 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>error 1.39%</td>
<td>0.69%</td>
</tr>
<tr>
<td>150</td>
<td>$6.276 \times 10^{-2}$</td>
<td>$6.349 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>error 2.53%</td>
<td>1.4%</td>
</tr>
</tbody>
</table>

Fig. 3. Convergent history of axisymmetric LBM simulation for Taylor–Couette flow at different Reynolds numbers.

$33 \times 57$, $33 \times 65$ and $33 \times 73$. Liu’s results\textsuperscript{17} were also included as the reference data to compute the error. As shown in Table 1, there is little difference among the results obtained on the three mesh sizes, and all results agree very well with those of Liu.\textsuperscript{17} We also measured the relative errors between present results and Liu’s data, and found that the biggest error is below 3%. Therefore, in the following parts, the mesh size of $33 \times 57$ was chosen for all the cases studied. Figure 3 shows the convergent history of present axisymmetric model for simulation of the Taylor–Couette flow at Reynolds numbers of 85, 100 and 150. The mesh size used for these results is $33 \times 57$. It can be seen from this figure that as Reynolds number increases, the convergence rate slightly decreases. Obviously, the present axisymmetric model has a good convergence behavior.

4.2. \textit{Comparison with three-dimensional LBM}

To show the performance of the proposed axisymmetric model, Taylor–Couette flow was also simulated by using three-dimensional (3D) LBM. The 3D simulation
Table 2. Comparison of $\Psi_{\text{max}}$ and computational time between the axisymmetric LBM and 3D LBM for the Taylor–Couette flow at Re = 85 and 100.

<table>
<thead>
<tr>
<th>Re</th>
<th>Present LBE (D2Q9)</th>
<th>3D LBE (D3Q15)</th>
<th>Liu$^{17}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>85</td>
<td>$\Psi_{\text{max}}$</td>
<td>$5.012 \times 10^{-2}$</td>
<td>$4.854 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>CPU (s)</td>
<td>3244</td>
<td>87687</td>
</tr>
<tr>
<td></td>
<td>Iteration steps</td>
<td>17651</td>
<td>14573</td>
</tr>
<tr>
<td>100</td>
<td>$\Psi_{\text{max}}$</td>
<td>$5.794 \times 10^{-2}$</td>
<td>$5.542 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>CPU (s)</td>
<td>2825</td>
<td>86784</td>
</tr>
<tr>
<td></td>
<td>Iteration steps</td>
<td>15243</td>
<td>12435</td>
</tr>
</tbody>
</table>

is based on the D3Q15 lattice velocity model.$^{18}$ The mesh size used for the axisymmetric model is $33 \times 57$, while in the 3D LBM simulation, the mesh size used is $61 \times 33 \times 57$. Note that for the 3D LBM simulation, the sufficient number of mesh points in the circumferential direction is necessary for obtaining converged solution. The 3D LBM also encounters a difficulty for getting converged solution at Re = 150. So, in this comparison, only two Reynolds numbers, that is, 85 and 100, are considered. Table 2 compares the maximum stream function value in the whole computational domain and the CPU time required by the present axisymmetric model with those by the 3D LBM. All the computations were carried out on a personal computer PIII 866. The reference data of Liu$^{17}$ was also included in Table 2 for comparison. It can be observed from Table 2 that the iteration number required by 3D LBM is slightly less than that required by the axisymmetric model. However, the CPU time required by the 3D LBM is almost 30 times of the CPU time required by the present axisymmetric model. Clearly, the proposed model is much more efficient than the 3D model for an axisymmetric flow problem. In addition, as compared with reference data of Liu,$^{17}$ it was found that the axisymmetric LBM results are slightly better than the 3D LBM results. This is probably due to the fact that there is no work done in the circumferential direction in the axisymmetric model, therefore, no numerical error is introduced in this direction. So, overall, the numerical error of the axisymmetric model is slightly less than that of the 3D LBM.

4.3. Effect of increasing Reynolds number on flow pattern

Figures 4–6 show the flow patterns in the $r$–$z$ plane obtained by the present axisymmetric LBM for the Taylor–Couette flow at Re = 85, 100 and 150, respectively. The results of the 3D LBM at Reynolds numbers of 85 and 100 were also shown in Figs. 4 and 5 for comparison. In each figure, the streamlines and vorticity contours were presented to show the effect of Reynolds number. As shown in Figs. 4–6, it was found that a four-cell secondary mode was produced by our LBM simulation for all three Reynolds numbers. As Reynolds number is increased from 85 to 150,
Fig. 4. Flow patterns of Re = 85.

Fig. 5. Flow patterns of Re = 100.
Fig. 6. Flow patterns of Re = 150 obtained by axisymmetric LBE (mesh size: 33 × 57).

the inner two-cell is gradually evolved in size. This can also be found in the vorticity contours. Furthermore, Figs. 4 and 5 show that both the axisymmetric LBM and the 3D LBM give the same flow pattern.

5. Conclusions

An axisymmetric LBE based on the cylindrical polar system was derived in this work by introducing an additional source term to the standard LBE. The new LBE has shown to self-conserve the macrodynamic equations within the usual Chapman–Enskog expansion. Although the new LBE solver involves the gradient operation which is contrary to the philosophy of the LBM, it is explicitly evaluated, and gives a more simple and efficient two-dimensional way to solve the three-dimensional axisymmetric flow. The developed method was successfully applied to simulate the laminar Taylor–Couette flow in an annulus with a rotating inner cylinder, a stationary outer cylinder and two end plates. It was found in this study that the present axisymmetric LBM gives slightly better results than the 3D LBM, but it is much more efficient than the 3D LBM in terms of computational time.

References