Modeling, Analysis, and Visualization of Left Ventricle Shape and Motion by Hierarchical Decomposition

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Abstract—This paper presents an approach to the modeling, analysis, and visualization of left ventricle motion and deformation. Our modeling of left ventricle shape and motion as a hierarchical representation enables us to develop a promising noninvasive technique for monitoring heart dynamics where both image analysis and image synthesis are involved. The proposed hierarchical motion model of left ventricle is constructed by combining several existing simple models and is able to capture major motion and deformation components of the left ventricle. The hierarchical decomposition characterizes the left ventricle motion and deformation in a coarse-to-fine fashion and leads to computationally efficient estimation algorithms. We estimate the global rigid motion of the left ventricle by establishing a time-varying object-centered coordinate system. The global deformations of the left ventricle are obtained by fitting the given data to superquadric modeling primitives. The local deformations are estimated by a tensor-description approach that is based on the locally deformable surface obtained by constructing spherical harmonic local surface from the residues of global shape estimation. We also describe in this paper methods of image synthesis and dynamic animation for visualizing the estimated results of the time-varying left ventricle shape, motion, and deformations. These animation results are consistent with the apparent motion pattern of the left ventricle and therefore show the success of our hierarchical decomposition based approach.

I. INTRODUCTION

LEFT ventricle motion and deformation analysis from image sequences provides an invaluable noninvasive technique for monitoring the dynamic behavior of human heart. To develop a clinically useful system that is able to analyze the given image sequence as well as display the estimated results, we need to apply the rigid and nonrigid object motion analysis algorithms evolved in the field of computer vision as well as the display and animation techniques investigated in the fields of computer graphics and scientific visualization. It is natural that development of such a system requires the integration of image analysis and image synthesis techniques. However, motion estimation of a beating heart from the image sequence is difficult since the heart undergoes a complex dynamic process throughout the cardiac cycle. How to display the estimation results is also a challenging problem because of the time-varying three-dimensional and tensor-based representations of the motion and deformation parameters.

There are four major methods of acquiring image data for extracting 3-D time-varying information of the heart. They are: echocardiography [1], cineangiography [2], [3], computed tomography [4], and nuclear magnetic resonance imaging [5]. Among these attempts of using imaging techniques for monitoring the dynamic behavior of the human heart, it appears that biplane cineangiography is at present the best in terms of both computational requirement and image quality. Several researchers have already been working on angiographic data-based approaches, and the results are rather promising. Kim et al. [6] used biplane coronary artery bifurcation points as natural markers for the kinematic analysis of heart motion. Coppini et al. [7] extended that approach and developed a method for a 3-D kinematic description of the local deformation of the heart surface by the end-diastole and end-systole configurations. Goldgof, Lee, and Huang [8] applied 3-D Gaussian curvature to analyze the heart surface by recovering the stretching factor from the angiographic data. However, all these researchers confined their approaches to the local region of the heart in order to implicitly eliminate the global movement effects, and global motion estimation was never addressed. Recently, Chen and Huang [9] started to apply algorithms developed in computer vision research and presented a new algorithm for heart motion analysis. They suggested that both global motion and local deformation of the beating heart should be considered and proposed a scheme to decouple the global motion from the local deformations.

This paper presents a scheme for left ventricle shape and motion modeling, analysis, and visualization using angiographic data. We are given the 3-D coordinates of the bifurcation points obtained through biplane cineangiography. The correspondences of these bifurcation points are obtained by tracking the coronary artery over a cardiac cycle. The detail of how these points are tracked can be found in recent papers [2], [10]. We propose in this paper a hierarchical motion model that is able to combine various cardiac motion and deformation models into a unified one. We also develop coarse-to-fine estimation algorithms based on the hierarchical decompositions. The hierarchical modeling has been motivated by the intentions to utilize a priori knowledge of the cardiac
motion and reorganize such knowledge into a hierarchy of representations. The modeling of heart motion and deformation plays a central role in this estimation procedure. However, it is the hierarchical decomposition that allows us to convert a complex motion estimation problem into several stages of well-defined and simpler estimation procedures. Furthermore, the idea of hierarchical decomposition of shape and motion can also be applied to a wide variety of nonrigid objects with the necessary modification of motion and deformation classifications to suit the particular objects under consideration.

Model-based estimation of heart motion and shape has been investigated by many researchers. In the earlier study of left ventricle dynamics, simple shape primitives such as spheres and cylinders were assumed and motion descriptions were limited to volume changes of the left ventricle over a cardiac cycle [7], [11]. These simple models have been modified later, and more complex models have been investigated [12]. However, the estimation results based on these models represent only fragmentary characteristics of complex heart motion and deformations since they are based on either crude shape information or local motion information. In this paper, we intend to combine different shape and motion models into a hierarchical representation in an attempt to systematically characterize the complex nature of the heart motion. We have arranged the hierarchy in such a way that the decomposition of motion estimation leads to a series of well-defined estimation procedures.

In Section II, we introduce the motion and shape modeling primitives for our model-based approach. We propose a hierarchical motion model and two types of shape modeling primitives; each characterizes the global deformable surface and the local deformable surface, respectively. In Section III, we present the hierarchical decomposition approach, which converts the original complex estimation problem into a series of well-defined and relatively simple parameter estimation problems. The successful decomposition of the hierarchical representation of the motion and deformations enables us to develop computationally efficient estimation algorithms. The implementation details of this model-based motion and deformation analysis are described in Section IV. This section includes a recursive algorithm developed to produce a good estimate of global motion and deformation even when the given data points are distributed with bias, and an algorithm for the estimation of local deformations that global shape modeling primitives are unable to characterize. Section V shows the estimation results obtained by the algorithms developed in Sections III and IV. Visualization techniques for analyzing the estimated left ventricle motion and deformations are presented in Section VI. We describe how we can synthesize the 3-D dynamics of the left ventricle by means of enhancing the spatial dimensionality and color coding the local deformations. Section VII concludes this paper with some discussions on future research directions.

II. LEFT VENTRICLE MOTION AND SHAPE MODELING

The advantage of modeling in image sequence-based cardiac research is twofold. First, a priori knowledge of the heart motion and shape can be easily incorporated into the modeling primitives. Second, the modeling primitives are generally parameterized so that we are able to devise well-defined parameter estimation algorithms. In the following, we first present a hierarchical motion model that combines various simple motion models. Then we introduce the left ventricle shape modeling primitives that correspond to different levels of shape representations. We also identify the relationship between the level of shape representation and the hierarchy of motion description in order to show the correlation between the shape modeling and the motion modeling.

A. Left Ventricle Motion Modeling

The importance of motion modeling has long been recognized by the cardiac researchers. Based on various motion models, the noninvasive evaluation of cardiac dynamics using image sequences such as cineangiograms has been investigated. Most motion and deformation models are simple in the sense that they are based on crude approximation or localized information. Many simple motion models provide fairly good qualitative intuition but not quantitative characterization of heart dynamics. The complexity of the motion model has grown over the years of research. However, no unified model that combines localized models into a generalized one has been proposed. We present here a hierarchical motion model, a superposition of several simple models, in an attempt to characterize the overall dynamics of the heart. In the following, we first review existing motion models that are based on only partial information of heart dynamics. Then we discuss how the hierarchical motion model can be derived from these simple motion models. The advantages of hierarchical motion modeling are also explored so that the philosophy of hierarchical modeling can be applied to the implementation of estimation algorithms.

Existing Motion Models: Existing models of left ventricle motion and deformation can be roughly divided into two major categories: global motion and deformation models, and local deformation models. The global motion and deformation models generally describe the overall changes in the ventricular shape, size, and orientation [13]. Although global rigid motion of the heart is part of the heart dynamics, cardiac researchers have been focusing mainly on the global deformations, such as volume change, shape dilation and twisting. For a crude estimate of global cardiac dynamics, simple models of motion and deformation have been considered good enough. One typical example of such models is the pressure-volume model [14], which indicates the global dynamic changes of the left ventricular wall. Another example is the linear transformation model, which computes the global deformation (ventricular ejection fractions) based on eigenvalues calculated from implanted markers [13].

The local deformation models concentrate on the local fibrous structure and stress distribution of the left ventricle wall [6], [15]. To isolate local deformations from chamber global translation and rotation, these models are usually based on a reconstructed local cardiac coordinate system. Two popular methods have been employed in the analysis of the left ventri-
cile local deformation. Tensor analysis of the local deformation is applied when the correspondences of the markers on the surface of left ventricle are available or can be established [7], [16], [17]. Otherwise, finite element method is a good alternative since it is capable of interpolating the data points into nodes of nearby elements and estimating the deformation based on these nodal values of displacement [11], [15].

These models, global or local, reflect only partially the complex dynamic behavior of the left ventricle. It is necessary to combine many aspects of these simple models into a generalized model in order to more completely characterize the left ventricle dynamics.

The Hierarchical Motion Model: Our hierarchical motion model decomposes a general motion into two major parts: global movement and local movement. The global movement is further decomposed into global rigid motion and global nonrigid motion, while the local movement is decomposed into local rigid motion and local deformations. Such a generalized motion model is built via the rearrangement of individual global and local motion models into a hierarchy of motion and deformations. This motion hierarchy can be applied to other nonrigid object motion analysis problems as well. However, in the case of left ventricle motion analysis, this motion model includes several well-recognized cardiac motion patterns, especially the patterns of global and local deformations. These patterns have been used separately in existing motion models, and each has been confirmed by medical observations or biomedical engineering experiments.

We now identify the components of the hierarchical motion model for the left ventricle. Global rigid motion of the left ventricle is parameterized by the overall chamber translation and rotation. Global nonrigid motion consists of dilation (scaling) deformation in three orthogonal directions and twisting deformation about the long axis of the left ventricle. Local rigid motion and nonrigid deformation can be described together using the well-known Helmholz theorem of kinematics [18], which represents the local deformation by stretching tensors in a localized volume element. Naturally, we will employ the tensor description-based method in our local deformation estimation algorithms.

B. Left Ventricle Shape Modeling

The left ventricle as a whole is a nonrigid object, and it is well known that its shape changes periodically over each cardiac cycle. The shape change of the left ventricle is due to the global and local deformations during the pumping of the heart. Naturally, deformation modeling of the left ventricle, global or local, can be realized by shape modeling of the left ventricle, if one is able to incorporate these deformation parameters into the shape modeling primitives. Therefore, many cardiac researchers have implemented the left ventricle deformation analysis algorithms through modeling the left ventricle surface by simple shape primitives, such as sphere and cylinder [19], [20]. However, the success of existing shape-modeling-based approaches are limited due to the oversimplified surface primitives that are in sharp contrast to the complex nature of the left ventricle dynamics.

We present here two types of shape modeling primitives; each describes a different level of shape representation, or deformation description. For global deformation analysis, global shape is modeled by the superquadric modeling primitives. Superquadric shape modeling primitives presented in this paper are considered better than the existing simple shape models, such as spheres or cylinders, since their parameters are capable of describing the global deformations that simple shape models are unable to characterize. However, due to the intrinsic axial symmetric property of the superquadrics, they are still not flexible enough to characterize local deformations, which are different from one surface patch to another. For local deformation analysis, we propose a local surface estimation scheme based on spherical harmonic fitting of the residual distances computed from the given data and the fitted superquadric surface. These residual distances are the measures of the deviation of the global superquadric surface from the given data. A finite expansion of spherical harmonic basis functions is chosen in such a way that the fitted surface is a good interpolation of these residual distances in terms of both surface smoothness and variation. In other words, the spherical harmonic shape modeling is used to characterize the details of the left ventricle shape that superquadric modeling primitives are unable to characterize. The combination of these two shape modeling primitives forms the left ventricle shape that captures the global as well as local surface properties.

Superquadric Shape Modeling: We propose here a parameterized family of shapes, known as superquadrics, as the global shape modeling primitives. In contrast to existing models, they are flexible enough to capture the globally deformable nature of the left ventricle. They have been used for shape representation in computer graphics [21], [22] as well as computer vision [23], [24], [25], [26]. A superquadric surface is the spherical product of two superquadric curves and can be defined in vector form as follows:

$$S(\theta, \phi) = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_x \cos^\epsilon(\theta) \cos^\epsilon(\phi) \\ a_y \cos^\epsilon(\theta) \sin^\epsilon(\phi) \\ a_z \sin^\epsilon(\theta) \end{bmatrix},$$

(1)

where $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ and $-\pi \leq \phi \leq \pi$. Parameters $\theta$ and $\phi$ correspond to latitude and longitude angles, respectively, expressed in the object-centered spherical coordinate system. Angle $\phi$ lies in the $x - y$ plane, while $\theta$ corresponds to the angle between the vector $\mathbf{S}(\theta, \phi)$ and the $x - y$ plane. Scale parameters $a_x, a_y, a_z$ define the size of the superquadrics in the $x, y$, and $z$ directions, respectively, $\epsilon_1$ is the squareness parameter along the $z$-axis, and $\epsilon_2$ is the squareness parameter in the $x - y$ plane. By varying these parameters, superquadrics can model a large set of standard building blocks such as spheres, cylinders, and parallelepipeds as well as shapes in between. An implicit equation of the superquadrics can be obtained by manipulating the components of the vector in (1),

$$\left( \frac{x}{a_x} \right)^{\frac{2}{\epsilon_1}} + \left( \frac{y}{a_y} \right)^{\frac{2}{\epsilon_2}} + \left( \frac{z}{a_z} \right)^{\frac{2}{\epsilon_1}} = 1.$$

The modeling power of superquadrics is augmented by the application of various deformation operations to the basic models [27]. In particular, certain classes of deformed
superquadrics are able to model nonrigid objects that are nonsymmetric. A typical example of deformation operation that can be applied to the basic superquadrics is the tapering deformation. If a point \((x, y, z)\) is transformed to \((X, Y, Z)\), the tapering deformation can be written as

\[
\begin{align*}
X &= f_x(z)x \\
Y &= f_y(z)y \\
Z &= z,
\end{align*}
\]

where \(f_x(z)\) and \(f_y(z)\), the tapering functions, are usually piecewise linear functions of \(z\). An object’s cross section increases its size when the derivative of the tapering function is positive and decreases its size when the derivative is negative.

Twisting and bending deformations can also be applied to the basic superquadrics to model the complex nature of the left ventricle shape. The basic superquadrics of (1) along with possible deformation transformations define the parametric model that we use for left ventricle motion analysis. The parameters we need to recover in this model include three rotation parameters and three translation parameters for global motion, three scale parameters and two squarness parameters for basic superquadrics, and additional shape deformation parameters for deformed superquadrics.

As previously indicated, the choice of modeling primitive depends heavily on our \textit{a priori} knowledge of the object. The modeling primitive plays an important role in recovering global motion, especially when the 3-D feature points are distributed with bias. In the case of the left ventricle, \textit{a priori} shape knowledge leads us to choose an ellipsoid with tapering and twist deformations to model the left ventricle shape. Although the modeling of the left ventricle as a tapered and twisted ellipsoid will not model all localized deformations, it is a good approximation of the global shape. The tapering deformation allows us to model the varying cross sectional areas perpendicular to the long axis of the left ventricle. For simplicity’s sake, we make the tapering functions the same for two global axes:

\[
f_x(z) = f_y(z) = kz + 1,
\]

where \(k\) is a tapering constant and \(-\frac{1}{n} < k < \frac{1}{n}\). The axial twist deformation models the shape change caused by torsion of the left ventricle during the ejection phase. Its functional form is usually linear with respect to the axis \(z\). The introduction of twist deformation to model the shape of the left ventricle is justified in [28] and [29].

\textit{Spherical Harmonic Shape Modeling:} The superquadric modeling primitives presented earlier capture only the global deformations of the left ventricle. The residual distances computed from the fitted global shape and the given data are the measures of local deformations that the superquadric modeling primitives are unable to capture. To analyze the local deformation of the left ventricle shape in the neighborhood of the data points, these discretely distributed residual distances should be interpolated over their neighborhood surface patches. We introduce here the spherical harmonic shape modeling primitives in order to construct a smooth surface from the residual distances.

According to Ballard and Brown [30], spherical harmonic surfaces are closed surfaces that are functions on the sphere and can be decomposed into a set of orthogonal functions. Spherical harmonics may be parameterized by two numbers, \(m\) and \(n\); thus they are a doubly infinite set of functions that are continuous, orthogonal, single valued, and complete on the sphere. The basis functions \(U_{nm}(\phi, \theta)\) and \(V_{nm}(\phi, \theta)\) are defined in a spherical coordinate system as

\[
\begin{align*}
U_{nm}(\phi, \theta) &= \cos m\phi P_n(\cos \theta) \\
V_{nm}(\phi, \theta) &= \sin m\phi P_n(\cos \theta),
\end{align*}
\]

where \(P_n(\cdot)\) is the general Legendre function and given by

\[
P_n(x) = (1 - x^2)^{\frac{m}{2}} \frac{d^m}{dx^m} P_n(x)
\]

and \(P_n(\cdot)\) is the Legendre polynomial of degree \(n\).

In combination, the spherical harmonics are able to produce various well-behaved functions. To represent an arbitrary shape, the radius \(r(\phi, \theta)\) in spherical coordinate system can be written as a linear sum of spherical harmonic base functions:

\[
r(\phi, \theta) \approx \sum_{n=1}^{N} \sum_{m=0}^{n} \{A_{nm}U_{nm}(\phi, \theta) + B_{nm}V_{nm}(\phi, \theta)\}.
\]

In other words, any continuous surface on the sphere may be represented by a set of real coefficients \(A_{nm}\) and \(B_{nm}\).

We propose spherical harmonics as shape modeling primitives for left ventricle deformation analysis mainly because they can produce a closed surface with well-defined functional properties. One distinct nature of the spherical harmonic modeling primitives different from the superquadric modeling primitives is their ability to capture the details of local surface patch without enforcing any kind of symmetric constraints. Moreover, the size of the coefficient set can be chosen so that the spherical harmonic modeling primitives characterize enough details in a localized region. Therefore, the reconstruction of the surface details that superquadric modeling primitives fail to capture enables us to analyze local deformation of the left ventricle based on spherical harmonic modeling.

To estimate the local deformations from a spherical harmonic fitted surface, we propose to establish the displacement fields of left ventricle surface over the cardiac cycle. The details of how we estimate the displacement fields will be presented in Section IV. The spherical harmonic surfaces will be combined with the displacement field to produce correspondences of surface points so that tensor-description-based analysis can be applied to local surface patches.

III. MODEL-BASED MOTION AND DEFORMATION ANALYSIS

In the previous section, we presented motion and shape modeling primitives for the left ventricle. Our objective of introducing these modeling primitives is to devise a model-based approach for estimating left ventricle motion and deformations. Through modeling, we are able to incorporate into the model \textit{a priori} knowledge of left ventricle motion and shape. Furthermore, the modeling primitives are generally
parameterized so that the model-based approach can be easily implemented.

In this section, we first present the hierarchical decomposition of the proposed motion model. The philosophy of the simple motion-model-based approaches is isolating different components of motion and deformations by adopting a simple model that describes only the component of motion or deformation of particular interest. In contrast, our hierarchical decomposition of motion does not isolate the motion and deformation components. The well-defined hierarchy of the motion model provides the interconnections between each of these components. This model-based analysis of left ventricle motion and deformation is expected to produce better estimation results because of the hierarchical structure of the motion model.

A. Motion Analysis by Hierarchical Decomposition

Various generalized motion models have been proposed in the analysis of nonrigid motion in the field of computer vision. One popular approach to nonrigid motion analysis based on a generalized model analysis is the overall motion parameterization without identifying and utilizing the interrelations of each of the components of motion or deformations in the whole parameter set. It is obvious that such an overall parameterization approach is difficult to implement since it usually involves a nonlinear optimization with a large number of parameters. In the following, we first discuss the need for motion decomposition when a generalized motion model is used for nonrigid motion analysis. We argue that the overall parameterization approach is suitable only for motion analysis of nonrigid objects whose pattern of motion is not clear. To perform the model-based motion analysis of the left ventricle, we present the hierarchical decomposition of the our motion model described in Section II. Through decomposition, our estimation scheme reflects the interrelation of each motion and deformation component in our hierarchical motion model and leads to the implementation of coarse-to-fine estimation algorithms.

The Needs for Motion Decomposition: Terzopoulos et al. [31], [32] are among the first computer vision and graphics researchers [23], [31], [32], [33] to start working on nonrigid object motion analysis based on motion and surface modeling. They developed algorithms [31], [34] that are able to recover the geometric shape of the object at different time instants using intrinsic and extrinsic constraints. Recently, they have extended their previous approach and proposed a family of modeling primitives called deformable superquadrics [35], [36]. This is a unified motion and shape model with the parameters of global motion, global deformation, and local deformations. These classified motion and deformations are combined into an overall set of parameters that is estimated simultaneously through manipulation of dynamic equations derived for the purpose of implementation only. Chen and Huang [37] have independently proposed a unified motion model similar to that of Terzopoulos [35] when developing estimation algorithms for left ventricle motion analysis based on computer vision techniques. However, they have implemented their estimation algorithms by decomposing the global motion from the local deformations. Such a decomposition is based on the interrelations between motion and deformation components of the left ventricle and leads to computationally efficient estimation algorithms.

The deformable superquadrics proposed by Terzopoulos et al. [35], [36] is an elegant formulation of dynamic 3-D surface modeling. However, the motion and deformation parameters so recovered do not reflect the interrelations of each component of nonrigid motion and deformations. The model can be applied to many practical problems that involve shape fitting as well as motion analysis. However, this approach is not practical for general nonrigid motion analysis, mainly due to the complex overall optimization implementation of dynamic equations used in their energy minimizing schemes. For many nonrigid objects, it is possible to implement a decomposition of optimization algorithm if the knowledge of the motion pattern of the object is available. It is also not practical to apply their approach directly to model-based heart motion analysis for following reasons. First of all, since the pattern of left ventricle motion is approximately known, the recovery of such motion patterns instead of overall motion and deformations becomes necessary. Second, the global deformations, including expansion/contraction and twisting, are all in terms of a localized coordinate system with its origin on the center of contraction. This kind of global deformation directly reflects the functioning of a beating heart but is generally different from the deformation parameters recovered from the deformable superquadrics unless the origin of the superquadrics, in fact, coincides with the center of contraction. In our work, we also make use of the modeling primitive superquadrics, but only after we have established the object-centered coordinate system. That means we will separate the global rigid motion estimation from the global nonrigid motion estimation process. Finally, we will show later that, by hierarchical decomposition, the implementation of the estimation scheme can be converted into a series of simple or linear estimation algorithms. The implementation of these simpler algorithms is easier than the approach based on deformable superquadrics that combines all of the parameters into one single set. Therefore, instead of directly applying the hierarchical motion model, we devise a hierarchical decomposition of motion model based on a priori knowledge of the motion patterns of the left ventricle.

Hierarchical Decomposition Approach: Our hierarchical motion model includes four parts: global rigid motion, global deformation, local rigid motion, and local deformation. The goal of hierarchical decomposition is to devise algorithms that can estimate each part individually. We described earlier the necessity and advantages of such decomposition in the case of left ventricle motion analysis based on the hierarchical model. Here we will discuss the interrelation of these four components of left ventricle motion and how the interrelation can be used in our concrete implementation of the proposed hierarchical decomposition.

We first examine carefully the overall motion and deformation patterns of the left ventricle known to cardiac researchers. Although many models of left ventricle motion and deformation have been proposed, until recently there was no attempt
being made to recognize the interrelations among different motion and deformation components of heart dynamics. Potel et al. [38] are among the first few researchers who looked into the overall dynamics of the left ventricle and identified the relationship among different motion and deformation components. According to their findings, a moving coordinate system seems to describe the left ventricle motion better than a fixed coordinate system. This suggests that the left ventricle does have a global rigid motion in addition to the well-known global deformation, such as expansion/contraction, and local deformations. They also found that, at each time instant, about 90% of the entire left ventricle wall motion is directed toward the center of contraction, the origin of the moving spherical coordinate system. This tells us that the expansion and contraction in terms of the center of contraction are far more significant than the circumferential rotation or twisting motion that have also been observed in their research. All of these findings prompted our idea of hierarchical motion decomposition, which allows us to devise a series of efficient estimation algorithms. We shall show later that some of the hierarchical decomposition scheme can be derived analytically based on the Potel’s findings.

Three basic assumptions have been adopted in our hierarchical decomposition of overall motion and deformations. These assumptions are summarized as follows.

1) Among the various global and local deformation—such as expansion or contraction, twisting, bending, and local stretching, and so on—the deformation due to expansion or contraction accounts for a commanding high percentage of the total deformation.

2) The global shape of the left ventricle can be approximated as an ellipsoid with tapering and twisting deformations. The local deformations can be defined as the minimum deviation from the global shape.

3) The local deformations are spatially smooth, or, in other words, the neighboring areas have similar local stretching deformation in terms of both direction and magnitude.

These three assumptions have all been confirmed by medical observations. The first assumption is the crucial one and has been supported by Potel’s findings. The second assumption is based on many medical observations and experiments that the global shape of the left ventricle can be roughly approximated as ellipsoid [13]. However, our modeling primitives are not as restrictive as these ellipsoid models, since we allow the tapering and twisting deformations to be performed on the ellipsoid models to capture the changing sizes of cross section and transverse or shear stresses during contraction. The third assumption has also been widely used in cardiac research and shown to match medical observations [7].

Here is an outline of our hierarchical motion decomposition, which divides the model-based estimation of left ventricle motion into three stages. First, the global rigid motion of the left ventricle is computed by constructing an object-centered moving coordinate system. The origin of the coordinate system is defined as the center of contraction and the orientation of the coordinate system as the principal axes. We will show in Section III-B that the origin and the orientation of the coordinate system calculated directly from the 3-D data are approximately invariant to the global deformations, assuming that the twisting deformations are small compared with the rigid motion and expansion or contraction during a cardiac cycle. Upon compensation of global rigid motion, the left ventricle undergoes global as well as local deformations. Then, the global deformations are estimated by performing superquadric surface fitting on the time-varying object-centered coordinate system. The superquadric surface fittings will capture the global deformation characteristics of the left ventricle shape, including expansion/contraction and twisting. Finally, the spherical harmonic fitting of residual distances is performed to characterize the local deformations of the left ventricle that global surface modeling primitives are unable to capture. In this stage, we will combine the global superquadric surface with the spherical harmonic interpolated residual surface to form an overall surface on which the tensor-based deformation analysis is carried out.

### B. Global Rigid Motion Analysis

In the first stage of hierarchical decomposition, we propose to estimate the global rigid motion of the left ventricle by extracting the global rigid motion from the overall motion. In the following, we present the estimation of the left ventricle centroid and principal axes, which serve as the origin and orientation of the object-centered moving coordinate system. We show that the global rigid motion of the left ventricle between two consecutive time instants can be easily calculated from the estimated moving coordinate system for each time instant.

**Estimation of Left Ventricle Centroid:** According to our first assumption, the contribution of local deformation and global twisting to the overall motion is small compared with the global rigid motion and global expansion or contraction. This means that the motion of the centroid of these data points is mainly due to global rigid motion and global expansion or contraction. It can be shown further that pure expansion or contraction of the left ventricle contributes null to the motion of the centroid. Therefore, the centroid computed from the data points can be a good approximation for the origin of the moving coordinate system.

Suppose there are n points of interests on the surface of the left ventricle with coordinates \((x_i, y_i, z_i), i = 1, \cdots, n\); then, the centroid of these points will be

\[
\begin{align*}
    x_m &= \frac{1}{n} \sum_{i=1}^{n} x_i, \\
    y_m &= \frac{1}{n} \sum_{i=1}^{n} y_i, \\
    z_m &= \frac{1}{n} \sum_{i=1}^{n} z_i.
\end{align*}
\]  

(9)

The centroid computed from data points using (9) is used to represent the center of contraction of the left ventricle. The displacement of the centroid over two time instants represents the translation vector of the global rigid motion. Based on the estimated centroid, the principal axes can be estimated and the object-centered coordinate system can therefore be constructed.
Estimation of Left Ventricle Principal Axes: As discussed above, the translation vector of the global motion over consecutive time instants can be determined by the centroids of the data sets. However, to determine the rotation matrix of global motion, we need more information than the location of the centroids. It is well known in the motion analysis of rigid objects that the correspondences between two noncollinear vectors over two time instants is sufficient to determine the rotation matrix. Thus, for the purpose of determining the rotation matrix of global motion, we need to find the principal axes that are invariant to the expansion or contraction of the left ventricle, so that the relative orientation of these axes over the consecutive time instants represents the global rotation of left ventricle.

For a given set of 3-D points on the surface of an object, a principal axis can be defined as an axis that goes through the centroid of these points with its orientation in such direction that the sum of the squared distances between the axis and the individual points is minimum. Assume that the directional cosines of the axis are $\alpha, \beta, \gamma$. Then the parameters are the solutions of the following constrained optimization problem:

Minimize: $\sum_{i=1}^{n} \left\| \mathbf{D} \times (\mathbf{P}_i - \mathbf{P}_m) \right\|^2$

subject to: $\| \mathbf{D} \| = 1$, \hspace{1cm} (10)

where $\mathbf{D} = (\alpha, \beta, \gamma), \mathbf{P}_i$'s are position vectors representing the $i$th 3-D points, $\mathbf{P}_m$ is the position vector of the centroid, and $\times$ is the vector cross product. In fact, $\| \mathbf{D} \times (\mathbf{P}_i - \mathbf{P}_m) \|$ is the distance between point $\mathbf{P}_i$ and the axis that goes through the centroid $\mathbf{P}_m$ with directional vector $\mathbf{D}$. After some algebraic manipulation, the above optimization problem is converted into the problem of minimizing a quadratic objective function, as follows:

Min: $a_{11}\alpha^2 + a_{22}\beta^2 + a_{33}\gamma^2 + (a_{12} + a_{21})\alpha\beta + (a_{13} + a_{31})\alpha\gamma + (a_{23} + a_{32})\beta\gamma$

subject to: $\alpha^2 + \beta^2 + \gamma^2 = 1$ \hspace{1cm} (11)

where

$a_{11} = \sum_{i=1}^{n} [(y_i - y_m)^2 + (z_i - z_m)^2]$ \hspace{1cm} (12)

$a_{12} = a_{21} = -\sum_{i=1}^{n} (x_i - x_m)(y_i - y_m)$ \hspace{1cm} (13)

$a_{13} = a_{31} = -\sum_{i=1}^{n} (x_i - x_m)(z_i - z_m)$ \hspace{1cm} (14)

$a_{22} = \sum_{i=1}^{n} [(x_i - x_m)^2 + (z_i - z_m)^2]$ \hspace{1cm} (15)

$a_{23} = a_{32} = -\sum_{i=1}^{n} (x_i - x_m)(z_i - z_m)$ \hspace{1cm} (16)

$a_{33} = \sum_{i=1}^{n} [(x_i - x_m)^2 + (y_i - y_m)^2]$, \hspace{1cm} (17)

or in matrix form,

\[
\text{minimize} : \mathbf{D}^T \mathbf{A} \mathbf{D} \\
\text{subject to} : \| \mathbf{D} \| = 1.
\] \hspace{1cm} (18)

Equivalently, the problem now is to find the smallest eigenvalue of $\mathbf{A}$, a $3 \times 3$ symmetric and positive definite matrix. The solution to the optimization problem will be the eigenvector that corresponds to the smallest eigenvalue of the matrix. The $3 \times 3$ symmetric matrix is of the following form:

\[
\mathbf{A} = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}.
\] \hspace{1cm} (19)

Suppose that the expansion or contraction factor between two consecutive time instants is $k$. Then, according to (6) through (14), the relationship between the two matrices $\mathbf{A}$ and $\mathbf{A}'$ will be $\mathbf{A}' = k^2 \mathbf{A}$, where $\mathbf{A}$ and $\mathbf{A}'$ are matrices used to find the principal axes that correspond to two consecutive time instants. Therefore, the eigenvectors of $\mathbf{A}$ and $\mathbf{A}'$ are clearly the same. This shows that the rotation of the estimated principal axis represents global rigid rotation of the left ventricle if the contributions due to global twisting and local deformations can be neglected since the expansion or contraction does not contribute to the rotation of the axis.

We indicated earlier that two noncollinear vectors associated with the surface are needed to determine the rotation matrix of global motion. We have already discussed how to locate an axis that is approximately invariant to certain global and local deformations. Note that the objective function we used in the above optimization problem is the sum of the squared distances from the points of interest to an axis we intended to determine. For the same reason, we can determine an axis such that the objective function is actually maximized. The solution of this optimization problem is the eigenvector of the constructed matrix corresponding to the largest eigenvalue. Furthermore, since $\mathbf{A}$ is a symmetric, positive definite matrix, the two axes are orthogonal because they are the eigenvectors corresponding to smallest and largest eigenvalues. Therefore, the noncollinearity requirement of the vectors is satisfied and global rotation can be determined using these two estimated principal axes.

Global Rigid Motion Computation: Assume that we have already found the centroids and principal axes that are associated with the bifurcation points over the successive time instants. Then, the translation vector and the rotation matrix of the equivalent global motion of the left ventricle surface between successive time instants can easily be identified from the position and the orientation change of these axes.

The translation vector of the global motion is simply the difference between the two centroids. For the rotation matrix, it is the change in orientation of the principal axes between two consecutive time instants. Since these two principal axes estimated from the given set of points are orthogonal, we can construct a mutually orthogonal 3-D coordinates fixed to the left ventricle surface by using the two principal axes and their cross product as the third axis. Now, the rotation matrix of the global motion becomes the transformation matrix of the
coordinate system from one time instant to the next. In matrix form, the global motion parameters can be written as
\[
T = \begin{bmatrix}
  x'_m \\
  y'_m \\
  z'_m
\end{bmatrix} - \begin{bmatrix}
  x_m \\
  y_m \\
  z_m
\end{bmatrix}, \quad (20)
\]
\[
R = [e_1' \times e_2'][e_1'e_2]_T, \quad (21)
\]
where \((x_m, y_m, z_m)\) and \((x'_m, y'_m, z'_m)\) are the centroids, \(e_1, e_2\), and \(e_1', e_2'\) are principal axes of the left ventricle at two consecutive time instants. The main assumption for estimating the global rigid motion has been that, in addition to global rigid motion, the component of global expansion or contraction is much larger in magnitude than the rest of the possible deformations.

C. Global Deformation Analysis

Suppose that we have successfully found the translation vectors and rotation matrices of the global rigid motion over successive time instants. Then, the position and orientation of the left ventricle are all known at these time instants. Upon transformation of original data to the object-centered coordinate system using the rigid motion parameters, we are ready to perform the global deformation analysis through fitting the transformed data to the superquadric modeling primitives. Generally, the parameters to be estimated in the superquadric surface fitting would include six parameters for position and orientation since the given data are usually with respect to a fixed coordinate system due to the fixed spatial position of the imaging devices. It is therefore a great advantage in the shape recovery process based on superquadrics that we know the position and orientation of the object. In recent research on recovering superquadric model from 3-D information [24], [25], [26], [27], various error of fit measures have been investigated, leading to nonlinear optimization algorithms. By reducing the number of parameters involved, we reduce the dimension of search space, reduce the computational complexity, and increase the probability of convergence to the right solution.

Among various optimization schemes in recovering superquadrics investigated by many researchers, the most common one is based on the inside–outside function defined as:
\[
f(x, y, z) = \left(\frac{x}{a_x} \right)^{\frac{2}{n_1}} + \left(\frac{y}{a_y} \right)^{\frac{2}{n_2}} + \left(\frac{z}{a_z} \right)^{\frac{2}{n_3}}, \quad (22)
\]
where if \(f(x_0, y_0, z_0) = 1\), then \((x_0, y_0, z_0)\) is on the surface; if \(f(x_0, y_0, z_0) < 1\), then \((x_0, y_0, z_0)\) lies inside the surface; if \(f(x_0, y_0, z_0) > 1\), then \((x_0, y_0, z_0)\) lies outside the surface. The objective function for the optimization is defined as:
\[
\text{Minimize : } \sum_{i=1}^{n} |f(x_i, y_i, z_i) - 1|^2, \quad (23)
\]
where the summation is over all known 3-D points.

D. Local Motion and Deformation Analysis

The well-known Helmholtz decomposition states that [18], locally, the motion of a sufficiently small volumetric element of a deformable body can be decomposed into the sum of a translation, a rotation, and an expansion (contraction) in three orthogonal directions. Notice that the translation and rotation here are that of the small element and therefore are different from the global rigid motion of the whole deformable body, assuming that the deformable object undergoes both local deformations and global movement. For a given reference coordinate system, the mathematical expression of the local deformation based on tensor transformation can be written as
\[
P_{i+1} = T_i(P_i) + R_i(P_i)F_i + E_i(P_i), \quad (24)
\]
where \(P_i\) and \(P_{i+1}\) are point vectors within the small volumetric element at time instant \(i\) and \(i + 1\), respectively. \(T_i\) is a translation vector, \(R_i\) a rotation tensor, and \(E_i\) an expansion tensor, all varying with space and time. However, within a local small volumetric element, \(T_i\), \(R_i\), and \(E_i\) can be considered approximately the same for all points. Furthermore, the rotation tensor \(R_i\) is orthonormal and the expansion tensor symmetric. It has been shown in [17] that local deformation of the cardiac wall can be approximated as homogeneous changes and described by the tensor model of deformable object, as illustrated above.

There are 12 unknowns in (24). For each 3-D point correspondence, three equations can be established to specify the relationship of motion and deformation between two successive time instants using (24). Hence, in order to determine all twelve unknowns, four point correspondences within the local element are needed. Assuming that the deformable surface is specified by the sample points on the surface and the correspondences of these points over consecutive time instants are given, then the tensor analysis parameters can be estimated over each local surface patch containing at least four points. Equation (24) is nonlinear with respect to the three rotation parameters, and no closed form solution is known. However, as Chen and Huang [9] pointed out, if small angle rotation is assumed, the original nonlinear problem can be transformed into a linear one. The rotation matrix can be approximately expressed as
\[
R = \begin{bmatrix}
  1 & -\gamma & \beta \\
  \gamma & 1 & -\alpha \\
  -\beta & \alpha & 1
\end{bmatrix}, \quad (25)
\]
where \(\alpha, \beta, \gamma\) are rotation angles around \(x-, y-, z-\) axis, respectively. Then, (24) reduces to a set of linear equations. Singular value decomposition method may be introduced to overcome the possible ill condition of the system. According to [18], the eigenvectors of the expansion tensor \(E_i\) give the directions of extreme deformation, and the corresponding eigenvalues specify the magnitudes of deformation.

IV. IMPLEMENTATION OF ESTIMATION ALGORITHMS

We have discussed our methodology for motion and deformation analysis of left ventricle from angiographic data using hierarchical decomposition. The idea of decomposing
a complex and nonlinear problem into several simple or linear stages has been applied throughout the development of these estimation algorithms. We describe here the implementation of some of these algorithms and discuss their simplicity and robustness. One special feature of this approach is its ability to provide a good estimate of left ventricle global motion and shape even when the given bifurcation points derived from angiographic data are distributed with bias.

A. A Recursive Algorithm for Global Motion and Deformation Analysis

The global rigid motion analysis algorithm presented in Section III–B is based on 3-D coordinates of the data points at successive time instants. We have implicitly assumed that these data points are uniformly distributed on the surface of the left ventricle. In reality, however, the data points used are bifurcation points extracted from the biplane angiogram sequences. The estimate of the centroid of the left ventricle using the proposed scheme based on only bifurcation points is inaccurate since these bifurcation points do not cover the whole surface of left ventricle. The inaccurate estimate of the centroid then causes faulty computation of the object orientation and global deformations. Therefore, the estimation of the object centroid is critical. In this section, we present a recursive algorithm that can adjust the centroid estimate based on the knowledge that has been incorporated in the global shape modeling primitives of the left ventricle.

Combat the Bias via Modeling Primitives: In the bifurcation point-based heart motion analysis, we notice that complete and unbiased 3-D data points are not available due to the structural configuration of left ventricle coronary arteries. Fortunately, along with the biased data, a priori shape information of the left ventricle can also be used in the development of estimation algorithms. In Section II the a priori shape knowledge of the left ventricle has been incorporated into global shape modeling primitives, which are used to estimate the global deformation of the left ventricle. Although the global deformation estimation is based on the results of global rigid motion estimation, the shape modeling primitives can also be used to develop a recursive algorithm for improved estimation of global rigid motion and global deformations. Our recursive algorithm intends to use the shape information of the left ventricle incorporated in the global shape modeling primitives to guide the adjustment of the estimated centroid until the fitted surface has the smallest error of fit, which implies that the fitted surface is considered consistent with a priori knowledge of the left ventricle shape.

Let us examine carefully the given 3-D data, which consist of coronary artery bifurcation points of the left ventricle. It is known that the coronary arteries of the left ventricle encircle only about three-quarters of the left ventricle surface. Hence, the 3-D bifurcation points are distributed with bias on the surface of left ventricle. The algorithm for estimating global rigid motion presented in Section III–B provides unacceptable results if we use only the bifurcation points that are distributed with bias. Here, a recursive algorithm is proposed that utilizes a priori knowledge of the left ventricle shape in the estimation process and produces potentially unbiased estimation results.

The recursive algorithm is implemented by incrementally adjusting the estimated centroid. The adjustment of estimated centroid is accomplished through the application of a priori knowledge of left ventricle shape and the data acquisition limitations. After analyzing the geometric position of the coronary arteries, we know that the actual centroid of the left ventricle is different from the geometric center of the given bifurcation points. Compared with the estimated centroid directly using (9), the actual centroid is located further away from the side of the left ventricle surface encircled by the coronary arteries. This side of the surface is identified as the side in which the size parameter of the surface primitives, estimated from fitting the given data, is the smallest. Our argument is based on the observation that a left ventricle modeled by the surface primitives may be elongated along only the long axis of the left ventricle. This a priori knowledge is utilized to guide the adjustment of the estimated centroid towards the direction in which we believe the actual centroid should be located. This adjusted centroid, together with the newly estimated principal axes, is used to fine tune the left ventricle shape again through surface fitting. The updated error-of-fit measure, computed using (23), is used to control the adjustment to prevent over adjusting. If the updated error of fit is greater or equal than the previous one, then the centroid may have been over adjusted. One cycle of such adjustment is considered as one step in a recursive algorithm to be described in the following. We show in Section V that the results obtained by this recursive algorithm are much better in terms of their compatibility with the apparent shape knowledge of the left ventricle.

A Recursive Algorithm: In the above, we presented a scheme to adjust the biased centroid estimate according to the available left ventricle shape information. Since the optimal adjustment often cannot be accomplished in just one step, we propose a recursive algorithm that adjusts the estimated parameters incrementally until they are in accordance with a priori knowledge of the left ventricle. A single step of the adjustment includes centroid adjustment, principal axes modification, and error-of-fit computation. Based on the current error of fit, we decide whether the adjustment is in the correct direction and the amount of adjustment is right. The adjustment will continue until the error-of-fit measure stops decreasing. Intuitively, this recursive algorithm for estimating global motion and object shape will converge to the right solution. A solid theoretical study of the convergence properties of the recursive algorithm is yet to be made, but the estimation results presented in Section V show the desired convergence in our application of this recursive algorithm to left ventricle motion analysis.

This recursive algorithm consists of the following steps:

Step 1: Estimate the centroid and the principal axes using the algorithms proposed for the ideal case, in which only the given 3-D coordinates of bifurcation points are used.

Step 2: Recover the global shape of the left ventricle using the position (centroid) and orientation (principal axes) parameters via the superquadric-modeling-based approach.
and calculate the error of fit.

Step 3: Adjust the position parameters according to the \textit{a priori} knowledge of the distribution of bifurcation points and estimate the new orientation parameters.

Step 4: Recover the global shape parameters using the adjusted position and orientation parameters and calculate the error of fit again.

Step 5: Stop the recursive algorithm if the current error of fit is greater than or equal to the previous one; otherwise go to Step 3 and continue the recursive algorithm.

The recursive estimation algorithm provides us the adjusted centroid, adjusted principal axes, and adjusted parameters for superquadric modeling primitives. For a given pair of centroid and principal axes set, the global rigid motion of the left ventricle can be obtained using (20) and (21). The global deformation parameters can be obtained by comparing the parameters of the superquadric surfaces at two consecutive time instants.

B. Spherical Harmonic Interpolation of Deformable Surface

The recursive algorithm presented in the previous subsection produces the global rigid motion parameters and the global deformation parameters for the left ventricle. However, the left ventricle shape obtained from the recursive algorithm represents only the global shape at a given time instant. The local deformations, which are different from one localized surface patch to another, have been smoothed out in this globally oriented modeling approach because of the axial symmetric property of the modeling primitives. A fine tuning of the surface estimation is thus needed in order to accurately describe the local deformable left ventricle surface. In particular, the local deformable surfaces are very much needed for the tensor analysis based on the correspondences of local feature points.

We notice that the given bifurcation points are not necessarily on the fitted surface of the left ventricle represented by the shape modeling primitives. The distances between these points and the estimated surface provide information on the estimation residues for the localized region around these points. To obtain a better representation of the left ventricle surface, the distances between these points and the estimated surface are interpolated over the spherical coordinate system as a function of spherical coordinates \((\phi, \theta)\) to produce a residual surface for each time instant. These residual surfaces are added to their corresponding global shapes estimated in previous subsections to compose a more accurate description of the left ventricle surface.

Several things must be taken care of when we produce the residual surface from the estimated global shape and the given data points. First of all, the distances between the estimated superquadric surface and the given points are calculated along the radial direction since we intend to interpolate these distances over the spherical coordinates \((\phi, \theta)\). Second, these bifurcation points are distributed over only three-quarters of the left ventricle surface. This means that we have to pad some zero residues for that side of the residual surface where there are no bifurcation points. Finally, these points can be inside or outside the estimated surface and thus a base value has to be set for these distances in order to produce a surface that has a one-to-one mapping onto the unit sphere.

Suppose the 3-D coordinates of the given points are \((x_i, y_i, z_i), \; i = 1, \ldots, n\), the estimated global shape is specified by the parameters \((a_x, a_y, a_z, k)\), and the base value is set as \(b_0\); then the following is a mathematical procedure of producing sample points that are used to interpolate the residual surface. The spherical coordinates of these bifurcation points are calculated as

\[
\begin{bmatrix}
\phi_i \\
\theta_i \\
r_i(\phi_i, \theta_i)
\end{bmatrix} = \begin{bmatrix}
\arctan\left(\frac{2x_i}{z_i}\right) \\
\arctan\left(\frac{x_i^2 + y_i^2}{z_i}\right) \\
\sqrt{x_i^2 + y_i^2 + z_i^2}
\end{bmatrix}.
\]

(26)

The radial component of their corresponding points on the estimated superquadric surface with the same longitudinal and latitudinal coordinates as the given bifurcation points can be calculated as shown in (27) at the bottom of this page, where \(f_x(\theta_i) = f_y(\theta_i) = (ka_x \sin(\theta_i) + 1)\) is the tapering function.

On that side of the estimated surface where there are no bifurcation points, we have no information indicating the difference between the estimated global surface and local deformed surface. To get a closed residual surface that represents the estimation residue distribution over the whole surface of the estimated global shape, we need to zero-pad some residual distances. Thus, the residual sample points are defined as follows:

\[
\begin{align*}
\mathbf{r}_i &= r_i(\phi_i, \theta_i) - r(\phi_i, \theta_i), & i = 1, \ldots, n; \\
r_i &= 0, & i = n + 1, \ldots, p.
\end{align*}
\]

(28)

As we have already pointed out, the given bifurcation points may be inside or outside the estimated surface, and therefore the above defined \(r_i\)'s may not necessarily be positive. Since the interpolation of residual surface is over the whole spherical coordinate system, a one-to-one mapping between the residual surface and the unit sphere is generally required. Under this assumption, the interpolation algorithm will require that the sample points be all positive. To meet this condition, we need to set a base value for all the sample points such that the radial coordinate of all sample points become positive by adding such a base value. A natural choice of the base value is the average of estimated global shape size parameters since it represents the approximate size of left ventricle. Therefore, the radial coordinate of these sample points is represented as

\[
r_i = r_i + b_0 > 0, \; i = 1, \ldots, p.
\]

(29)

where \(b_0\) can be set as \(\frac{1}{3}(a_x + a_y + a_z)\).

\[
r_i(\phi_i, \theta_i) = \sqrt{f_x^2(\theta_i)a_x^2 \cos^2 \theta_i \cos^2 \phi_i + f_y^2(\theta_i)a_y^2 \cos^2 \theta_i \sin^2 \phi_i + a_z^2 \sin^2 \theta_i}
\]
Now we are ready to interpolate these sample points to form the residual surface. Among these \( p \) points, \( n \) of them are calculated as the distances between the given bifurcation points and estimated superquadric surface, and \( p - n \) of them are exactly on the surface of a sphere with radius \( b_0 \).

In Section II-B, we indicated that the radius \( r(\phi, \theta) \) of an arbitrary surface in spherical coordinate system can be written as a linear sum of spherical harmonic base functions:

\[
r_a(\phi, \theta) = \sum_{n=1}^{N} \sum_{m=-n}^{n} \left[ A_{nm} U_{nm}(\phi, \theta) + B_{nm} V_{nm}(\phi, \theta) \right].
\]  

(30)

If we denote the coefficients of the above approximation by \( \alpha_i \) and the basis functions as \( B_i(\phi, \theta) \), then the approximation will be:

\[
r_a(\phi, \theta) \approx \sum_{i=1}^{M} \alpha_i B_i(\phi, \theta).
\]  

(31)

The number of coefficients \( M \) may be large depending upon the order of approximation \( N \). In the case of left ventricle, according to Coppini et al. [7], we can take the order of approximation \( N = 3 \) (\( M = 16 \)) to get a satisfactory interpolation over the given sample points. The objective function for the interpolation is therefore of the following form:

\[
\epsilon = \sum_{j=1}^{p} \left[ r_a(\phi_j, \theta_j) - \sum_{i=1}^{16} \alpha_i B_i(\phi_j, \theta_j) \right]^2.
\]  

(32)

Least squares methods are applied to get the coefficients of the interpolation. In this case, the sample points are set as \( r_a(\phi_i, \theta_i) = d_i, i = 1, \ldots, p \) by the definition given in (29). This algorithm is easy to implement since the coefficients to be estimated can be obtained by solving a system of linear equations.

The surface obtained using (32) is a superposition of a sphere with radius \( b_0 \) and the interpolated error distances. For any given longitudinal and latitudinal coordinates, the corresponding residual distances can be calculated from the interpolated surface. For example, if the given longitudinal and latitudinal coordinates are \( (\phi, \theta) \), then the residual distances will be:

\[
r_e(\phi, \theta) = r_e(\phi, \theta) - b_0.
\]  

(33)

If we represent the estimated global shape by its radial coordinates as a function of longitudinal and latitudinal coordinates, we can get a deformable surface by combining (27) and (33). The deformable surface can be written as follows:

\[
r_f(\phi, \theta) = r_e(\phi, \theta) + r_r(\phi, \theta),
\]  

(34)

where \(-\pi < \phi \leq \pi\) and \(0 \leq \theta \leq \pi\).

C. Displacement Field Estimation and Tensor Analysis

We indicated in Section III-D that four-point correspondences within a localized surface are needed to perform the tensor-based deformation analysis. In the case of estimating left ventricle motion and deformation from coronary artery bifurcation points, we are given the correspondences of these bifurcation points over consecutive time instants within a cardiac cycle. As we have already pointed out, these bifurcation points are sparsely and biasedly distributed over the surface of left ventricle. To analyze the local deformation using the tensor-based approach, these correspondences are again interpolated over their neighborhood to obtain a displacement field that specifies how each point on the estimated left ventricle surface moves from one time instant to another.

The algorithm developed in Section IV-B produces surfaces that are represented by their radial components and are functions of longitudinal and latitudinal coordinates \( (\phi, \theta) \). It is natural that the displacement fields to be established for each consecutive time instant are also expressed as the longitudinal and latitudinal coordinates variations for each point. These variations are in turn functions of the spatial position, or functions of longitudinal and latitudinal coordinates since the radial coordinate of given point is fixed by the constraint that the point is on the estimated surface. Without loss of generality, we can write:

\[
\begin{aligned}
\phi_{t+1} &= \phi_t + \Delta\phi(\phi_t, \theta_t) \\
\theta_{t+1} &= \theta_t + \Delta\theta(\phi_t, \theta_t),
\end{aligned}
\]  

(35)

where \( t \) and \( t + 1 \) denote two consecutive time instants associated with the displacement field. The functions \( \Delta\phi(\phi_t, \theta_t) \) and \( \Delta\theta(\phi_t, \theta_t) \) can also be approximated by spherical harmonics as we did for the interpolation of local deformations. The sample points we use for the interpolation are the displacements of the given bifurcation points. This algorithm again is implemented by solving a system of linear equations.

Once we have established the displacement field, we can calculate its corresponding point in the next time instant for any given point using (35). A tensor-based approach for local deformation estimation can be performed based on the left ventricle surface estimated in Section IV-B and the displacement field specified above. Given four point correspondences on a localized surface, (24) and (25) are combined to derive a system of 12 linear equations. The local translation, local rotation, and local expansion tensor can be obtained simultaneously by solving this system of linear equations. The magnitudes and directions of extreme deformation within the localized surface patch can be represented by the eigenvalues and corresponding eigenvectors of the expansion tensor.

V. Estimation Results

We present here the estimation results of the angiographic data-based left ventricle motion and deformation analysis. The estimated trajectory of the left ventricle centroid is shown in Fig. 1. A complete cardiac cycle consists of 16 consecutive image frames, but we have obtained only 10 of these 16 image frames. The estimated trajectory shows that the left ventricle centroid has a tendency to move back to its starting position at the eighth time instant and therefore corresponds well to the supposed periodic motion of the heart. Figs. 2 and 3 show the 3-D view and top view of the estimated left ventricle global shape without and with the recursive algorithm. The results indicate that the estimated global shape using the recursive
algorithm is much closer to the intuitive shape of a left ventricle.

The numerical results of estimated local deformation is difficult to present since the left ventricle surface is composed of many pieces of small quadrilateral patches and each would be quantified by a vector system of extreme deformations. One way of analyzing and understanding such estimated results is to animate the moving surface and its quadrilateral meshes using scientific visualization techniques. The details of the animation are discussed in Section VI. We have produced a videotape based on the animation that shows the motion and deformation of left ventricle estimated by the proposed algorithms developed in this paper.

VI. VISUALIZATION OF DYNAMIC LEFT VENTRICLE GEOMETRY

Scientific visualization is a very effective tool both for interpreting image data and for generating images from complex multidimensional data sets. It embraces both image understanding and image synthesis. The visual representation of the data often reveals more insight than the numerical data itself. However, visualization of data poses not only a question of how we should represent numerical data but also, more importantly, what we should represent, since the visual domain is limited in how much can be shown at any moment in time. Once we have decided what we want to look at, the "how to" is a matter of communication and technology. Communication deals with data representation and the format of the visual presentation. Technology deals with the mechanics that we have at our disposal to carry the communication forward.

In this research, once we have obtained the estimation results for the left ventricle shape, motion and deformation, we need to present the 3-D geometry in such a way that evolution of spatially varying left ventricle surface or volume is clearly reflected by the estimated results. Since the motion and deformation of a beating heart is a time-varying 3-D process, these estimation results are difficult to present numerically. It is appropriate to inspect and analyze the numerical results visually by generating an animation of a beating heart using the estimated left ventricle model driven by the estimated motion and deformation parameters. With the help of visualization tools, we can observe the estimation results at various stages of the analysis; this allows the computation to be interactive. Also, the display of the estimated left ventricle surface reveals the performance of the estimation algorithm, which is not apparent from numerical representation. Furthermore, the visualization of spatially varying left ventricle surface or volume with encoded deformation quantities provides the physicians convenient means of clinical evaluation and understanding.

In the following, we first discuss the representation of a beating left ventricle as 3-D geometry. Several techniques are applied in the generation of such 3-D geometry so that the visual effects of the dimensionality is enhanced. We then present schemes we use to code the local deformations...
represented by tensor quantities. One easy way of coding these
tensor quantities is to extract scalar quantities from the tensor
and represent them by visually sensitive quantities, such as
color.

A. Beating Left Ventricle as a 3-D Geometry

When visualizing 3-D forms, it is useful to give each
form a three dimensional context, as opposed to the use
of black background. The context, or stage, reinforces the
dimensionality of the form against the inherent ambiguity and
loss of information caused by the projection of the 3-D
geometric shape onto the 2-D image plane. In our approach to
the left ventricle motion analysis, the 3-D surface or volume of
the left ventricle at each time instant is estimated. We inspect
these 3-D geometric forms using visualization techniques to
improve the depth and fidelity of the constructed image.

In the visualization of left ventricle geometry, we sur-
rounded the estimated form of the heart with three orthogonal
planes. This was done in the same fashion and for the same
reasons that a 2-D graph is surrounded by its axes. However,
for a 3-D geometric form that is time varying, the context or
stage will not be enough to reflect the dimensionality of the
geometric motion and deformation. To reinforce the dimen-
sionality of time-varying property, we then cast orthogonal
projections of the form onto each of the planes as shadow
objects. These shadow objects serve two purposes. The first
is to visually establish the relationship of the form with the
surrounding planes. The second purpose is that the shadows
and the planes are themselves 2-D plots of the position or
extent of deformation of the form on the given projection
planes. The projection planes are gridded so as to give a
reference for motion and deformation of the geometric form.
A typical frame of this animation sequence is shown in Fig. 4.

B. Visualization of Tensor-Represented Deformation

When we analyze the heart motion and deformation based
on the angiographic data, the estimated heart form is visualized
as a shaded polygonal mesh on the surface in order to identify
each localized surface area. The polygons are outlined to
denote the extent of the underlining quadrilateral mesh. The
outlines also serve to give depth to the geometric form. The
estimated local deformations associated with each quadrilateral
patch are represented by stretching tensors. Although a stretch-
ing tensor can be depicted as a vector system at the center of
the quadrilateral, such a representation would be unclear or
too minute on a global scale. These tensor quantities must
be converted into some scalar quantity and linked to visually
sensitive quantities, such as color or intensity.

Following are the details of how we have investigated the
visualization of local deformations. First, we applied color
to the polygon mesh as a function of area change of the
quadrilaterals. The color ranged from blue for negative area
change to yellow for positive area change. The color swing
was through red, which represented no area change. The color
map was based on the LAB color space, where red and green
are the extents of one axis, yellow and blue are the extents of
the perpendicular axis, and luminance is the vertical axis. We
used this visualization of area change to guide us into regions of
interest to look at the quadrilateral mesh in close up. This
was to analyze the stretch tensors in these regions of interest,
in which we could show the directions and magnitudes of
extreme deformations using a vector system at the center of
that quadrilateral patch. A typical frame of this animation is
shown in Fig. 5.

The visualization part of research is performed at the
National Center for Supercomputing, NCSA, the University of
Illinois Urbana-Champaign. The data representation soft-
ware is an internal product of the Visualization Group at
NCSA. The choreography and render software used is part
of the Wavefront Technology "Advanced Visualizer" software
package. The image sequences were assembled on an Abekas
A64 digital video disk system, and the video was mastered
on a Bosh D1 digital video record deck. Post production of
the video was done in conjunction with the Media Services Group at NCSA.

VII. CONCLUSION

We have presented a hierarchical-decomposition-based approach to the modeling, analysis, and visualization of left ventricle motion and deformation using angiographic data. In modeling left ventricle motion and shape, we have combined several existing simple motion models into a hierarchy of motion and deformations. Such a hierarchical approach can also be applied to motion analysis of other nonrigid objects. Our left ventricle motion model characterizes all major motion and deformation components that are confirmed by medical observations and thus is more realistic than any of the existing simple models. More importantly, the hierarchical decomposition enables us to convert the seemingly complex estimation procedure of the coupled parameterization into coarse-to-fine estimation subsequences. Each of these subprocedures is well defined and relatively simple so that computationally efficient algorithms can be implemented. We have also developed an animation procedure for left ventricle dynamic geometry by means of scientific visualization techniques to inspect and analyze the estimation results of such time-varying 3-D processes. Our animation of left ventricle dynamic shape is generated through enhancing the spatial dimensionality and color coding the vector-represented local deformations.

In summary, we have shown that the bifurcation points obtained from biplane cineangiography can be used for left ventricle motion and deformation analysis, even though they are sparse and distributed with bias. However, denser and more uniformly distributed data are still desired. Currently, we are investigating the analysis of left ventricle motion and deformation using dynamic computed tomographic data. The volumetric nature of the computed tomographic data provides us accurate 3-D shape description of the left ventricle. Furthermore, the hierarchical decomposition of the left ventricle motion and deformation can be easily adapted to the computed tomographic data case. However, the difficulty lies in the estimation of 3-D displacement field from the time-varying volumetric data in order to establish the correspondences over consecutive image frames.

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