DISTRIBUTED SPACE-TIME CODING WITH ULTRA-WIDEBAND SYSTEMS

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ABSTRACT

In this paper, we extend the Amplify-and-Forward (AF) cooperative diversity scheme to the context of impulse radio ultra-wideband (IR-UWB). In particular, we apply a space-time (ST) coding scheme based on totally-real cyclic division algebras in order to achieve full diversity with no data rate losses. At a second time, the pulse repetitions are exploited in order to enhance the performance of the first scheme and to propose a new scheme that achieves comparable performance with lower complexity. These schemes are associated with pulse position modulation (PPM), pulse amplitude modulation (PAM) and hybrid pulse position and amplitude modulation (PPM-PAM).

I. INTRODUCTION

Despite the high frequency selectivity of the ultra-wideband (UWB) channels, profiting from multi-path diversity can necessitate Rake receivers with very high orders. This follows from the very important delay spread of these channels. In this context, the utility of multi-antenna UWB techniques was proven in [1], [2] where it was shown that these systems profit from enhanced diversity gains and multiplexing. On the other hand, in the Amplify-and-Forward (AF) cooperative diversity schemes, the spatial diversity is exploited in a distributed manner among the different terminals [3], [4]. However, these schemes were never investigated with UWB systems.

Explicit algebraic constructions of short ST block codes based on the nonorthogonal-AF scheme [3] were proposed in [4]. A second class of cooperation strategies based on the perfect codes [5] was recently discussed in [6]. However, these complex-valued codes based on phase rotations are not adapted to real carrier-less impulse radio (IR) UWB.

In this work, a new cooperation strategy based on totally-real ST codes constructed from cyclic division algebras [5], [7] is presented. This strategy is associated with multi-dimensional hybrid pulse position and amplitude modulations (PPM-PAM). In time-hopping (TH) IR-UWB systems, each information symbol is conveyed by a train of pulses. We take advantage of these repetitions in order to distribute the transmitted energy in a more balanced way among the symbols of a given codeword resulting in better performance with the same decoding complexity. At a second time, an intra-symbol coding scheme is evaluated and shown to have good performance with lower decoding complexity. Rake receivers are implemented at the destination and the cooperating relays and the performance is evaluated over realistic indoor UWB channels [8].

Notations: I_n is the n × n identity matrix, 1_{m×n} and O_{m×n} are the m × n matrices whose elements are equal to 1 and 0 respectively. * and ⊗ stand for convolution and Kronecker product respectively. The function diag(X_1,...,X_n) stacks the corresponding matrices on the principal diagonal.

II. SYSTEM MODEL

We consider the case of half-duplex terminals each equipped with a single antenna. For M-PPM-M′-PAM modulations, the j-th information symbol is mapped onto the amplitude a_j ∈ {2m′ − 1 − M′ : m′ = 1, ..., M′} and the position d_j ∈ {0, ..., M − 1}. This symbol is represented by the M-dimensional vector A_j = [a_j,0 ⋯ a_j,M−1]T where a_j,m = a_jδ(d_j − m) for m = 0, ..., M − 1 and δ(·) is the Dirac delta function. PPM and PAM follow as special cases by setting a_j = 1 and d_j = 0 respectively. The transmitted signal corresponding to the j-th symbol can be expressed as:

\[ s_j(t) = \sqrt{\frac{\beta_1}{N_f}} \sum_{n=0}^{N_f-1} \sum_{m=0}^{M-1} a_j,m b_j,n w(t - nT_f - m\delta) \]  (1)

where \( w(t) \) is the pulse waveform of duration \( T_w \) normalized to have unit energy and \( \beta_1 \) is a normalization factor. The symbol duration is given by \( T_s = N_f T_f \) where \( T_f \) is the frame duration. \( \delta \) is the position modulation delay chosen to be larger than \( T_w \). For the transmission of a given information symbol, \( N_f \) pulses with different positions and amplitudes are used. The amplitude spreading sequence \{b_j,n\} introduces additional coding between the pulses and can be exploited in the cooperative mode. No reference to the TH sequence was made since multiple access interference is not taken into consideration.

K neighboring terminals (relays) can help the source in delivering its message to a given destination. During the cooperative mode, the K nodes share the same TH sequence as the source. Denote by \( h_k^t(t) \), \( h_k^d(t) \) and \( g_k^t(t) \) the impulse responses of the channels from the source to the destination, the source to the k-th node and the k-th node to the destination respectively for \( k = 1, ..., K \).

The signal received at the k-th relay can be expressed as:

\[ r_{k,j}(t) = \frac{\beta_k \rho_k}{N_f} \sum_{n,m} a_j,m b_j,n h_k(t - jT_s - nT_f - m\delta) + n_k(t) \]  (2)

where \( n_k(t) \) is the noise at the k-th node which is supposed to be real AWGN with double sided spectral density \( N_0/2 \). The time origin of each relay is the arriving time of the first multi-path component. The received signal at the destination is obtained by setting \( k = 0 \), \( \rho_k \) is a gain factor corresponding to the relative quality of the channel between the source and the k-th

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relay with respect to the channel between the source and the destination \((\rho_0 = 1)\), \(h_k(t)\) is \(h_k^\prime(t) + w(t)\) for \(k = 0, \ldots, K\). In the same way, \(g_k(t) = g_k^\prime(t) + w(t)\) for \(k = 1, \ldots, K\).

In order to eliminate the Inter Symbol Interference (ISI), \(T_f\) is chosen to be larger than the channel delay spread \((\Gamma)\). A simple AF strategy at each relay will result in a received signal that is spread over a duration of \(2\Gamma\) at the destination. This implies two major inconveniences. ISI can be only eliminated by choosing \(T_f \geq 2\Gamma\) resulting in larger symbol durations and lower data rates. On the other hand, higher order Rake receivers must be implemented at the destination since the energy collected by each finger is now very small. In order to overcome these limitations, multi-path combining is performed at each relay prior to amplification and retransmission.

This choice is not penalizing. In fact, the channel parameters between the source and the relays must be known at the destination in order to achieve optimal detection with all AF schemes [3], [4]. For example, each relay can estimate its underlying channel and then it forwards the estimated parameters to the destination during the control phase. Therefore using the available channel parameters to perform maximum ratio combining (MRC) does not result in a significant increase in the complexity of the relays.

The output of the \(l\)-th Rake finger corresponding to the \(m\)-th position of the \(n\)-th pulse is given by:

\[
x_{k,j,n,l,m} = \int_{(jN_f+n+1)T_f}^{(jN_f+n+1)T_f} r_{k,j}(t)\tilde{w}_{j,n,l,m}(t)dt
\]

where:

\[
\tilde{w}_{j,n,l,m} = w(t-jN_fT_f-nT_f-\Delta_l-md)
\]

Equation (3) follows from the condition of no ISI. \(\Delta_l = lMT_0\) is the \(l\)-th finger delay for \(l = 0, \ldots, L-1\). For the \(k\)-th relay and the \(l\)-th Rake finger, \(h_{k,l,m,m'}\) corresponds to the effect of the signal transmitted at the \(m^\prime\)th position on the \(m\)-th matched filter corresponding to the \(m\)-th position. \(n_{k,j,n,l,m}\) is a white Gaussian noise which follows from the choice \(|(m-m')\delta - \Delta_l| \geq T_0\) for \(l \neq l'\) and \(m \neq m'\).

The proposed strategy is as follows. We suppose that the channel is shared in a TDMA manner. The source and the relays transmit during the time slot allocated to the source. The cooperation between the source and the \(K\) relays is periodic with a period of \((K+1)N_s\) symbol durations. This period is divided into \((K+1)\) frames of \(N_s\) symbols each. This period is supposed to be smaller than the channel coherence time. The \(k\)-th relay listens to the source during the symbol durations \((k-1)N_s + 1, \ldots, kN_s\). It performs MRC, scales the received symbols and retransmits them during the symbol durations \(K\,N_s + 1, \ldots, (K+1)\,N_s\). In other words, the \(k\)-th relay retransmits the \(k\)-th frame during the \((K+1)\)-th frame duration. The source and the \(K\) relays transmit simultaneously during this frame duration. The \(1 \times N_f\) amplitude spreading vectors corresponding to \(k\)-th frame will be denoted by \(B_k\) for \(k = 1, \ldots, K+1\). Denote by \(A(k)\) the \(M \times N_s\) matrix given by \(A(k) = [A_{(k-1)N_s+1} \cdots A_{kN_s}]\).

Taking these notions into consideration, eq. (3) can now be expressed in matrix form as (for \(k = 1, \ldots, K\)):

\[
X_k = \sqrt{\frac{\beta_1\rho_k}{N_f}} H_k \left( A(k) \otimes B_k \right) + N_k
\]

where \(H_k\) is the \(LM \times M\) channel matrix whose \((LM + m + 1, m'+1)\)-th element is equal to \(h_{k,l,m,m'}\) for \(l = 0, \ldots, L-1, m' = 0, \ldots, M-1\) and \(m = 0, \ldots, M-1\). \(X_k\) is the \(LM \times N_fN_s\) decision matrix whose \((LM + m + 1, jN_f + n+1)\)-th element is equal to \(x_{k,j,n,l,m}\) for \(l = 0, \ldots, L-1, m = 0, \ldots, M-1, j = 0, \ldots, N_s-1\) and \(n = 0, \ldots, N_f-1\). \(N_k\) is the noise matrix and it is constructed in the same way as \(X_k\).

The decision matrix at the destination during the \(k\)-th frame is denoted by \(X_k'\) and it takes the form:

\[
X_k' = \sqrt{\frac{\beta_1}{N_f}} H_0 \left( A(k) \otimes B_k \right) + N_k'
\]

At each relay, MRC is performed before retransmitting. In other words, the signal transmitted by the \(k\)-th relay during the \((K+1)\)-th frame is given by:

\[
y_k = \sqrt{\frac{\beta_2}{\Psi_k}} H_k^T X_k
\]

\[
\Psi_k = (H_k^2 H_k) \frac{1}{2} \left( \frac{\beta_1 \rho_k}{N_f} H_k^T H_k + \frac{N_0}{2} I_M \right) \frac{1}{2}
\]

The transmitted energy is scaled by \(\beta_1\) for the first \(K\) frames. Moreover, when transmitting simultaneously during the \((K+1)\)-th frame duration, the energy of the source and each one of the relays is scaled by \(\beta_2\). Transmitting the same energy as in non-cooperative systems corresponds to fixing \(K\beta_1 + (K+1)/\beta_2 = (K+1)\). Finally, the decision matrix at the destination corresponding to the \((K+1)\)-th frame can be expressed as:

\[
Y = \sqrt{\frac{\beta_1\beta_2}{N_f}} \sum_{k=1}^{K} \sqrt{\rho_k} \delta_k G_k \Psi_k H_k^T H_k (A(k) \otimes B_k) + N_k''
\]

\[
+ \sqrt{\frac{\beta_2}{N_f}} H_0 (A(K+1) \otimes B_{K+1}) + \sqrt{\frac{\beta_2}{N_f}} \sum_{k=1}^{K} \sqrt{\delta_k G_k} \Psi_k H_k^T N_k
\]

where \(\delta_k\) is determined in a similar way as \(\rho_k\) and it stands for the quality of the channel between the \(k\)-th relay and the destination. \(G_k\) is a \(LM \times M\) matrix corresponding to the channel between the \(k\)-th relay and the receiver. It is constructed in a similar way as \(H_k\). The \((LM + m + 1, m'+1)\)-th element of \(G_k\) is equal to \(g_{k,l,m,m'} = \int_{(jN_f+n+1)T_f}^{(jN_f+n+1)T_f} g_k^\prime(t) w(t-(m-m')\delta - \Delta_l) dt\) for \(l = 0, \ldots, L-1\) and \(m, m' \in \{0, \ldots, M-1\}\). \(g_k^\prime(t)\) is related to the channel response \(g_k(t)\) by the following relation:

\[
g_k^\prime(t) = g_k(t - e^{(k)})
\]
where $\varepsilon^{(k)}$ corresponds to the propagation delay between the k-th relay and the destination. It is determined with respect to the delay between the source and the destination. In general, the relay is closer to the destination than the source and $\varepsilon^{(k)}$ is a negative quantity. Based on this formulation, we can differentiate between two cases. In the first case, a feedback between the destination and the relays indicates the quantity by which each relay must advance or delay its transmission. In this case, the different delays are compensated and we can write $\varepsilon^{(k)} = 0$ for $k = 1, \ldots, K$. In other words, the first arriving rays of the different relays are aligned at the receiver before performing matched filtering. In the second case, the first finger delay of the Rake corresponds to the first ray between the source and the destination ($\varepsilon^{(k)} \neq 0$ for $k = 1, \ldots, K$).

Combining eq. (6) and eq. (9), the decision variables collected during the $K + 1$ frames can be expressed as:

$$
\begin{bmatrix}
X' \\
Y
\end{bmatrix} = \begin{bmatrix}
\sqrt{\beta_1} I_{K} \otimes H_0 & O_{K \times M} \\
\overline{H}^T \otimes \sqrt{\beta_2} H_0
\end{bmatrix} A^{(1)} \otimes B_1 + N
$$

where $K' = K + 1$ and the matrix $X'$ is obtained by concatenating the matrices $X_1, \ldots, X_K$ vertically. $N$ is a white noise whose variance is equal to $\overline{\beta_2}$. $\overline{H} = [H_1 \cdots H_K]$ and its constituent matrices are given by:

$$
\overline{H}_k = \sqrt{\beta_1} \delta_k G_k \Psi_k H_k^T H_k
$$

$\Sigma$ is the noise whitening matrix given by:

$$
\Sigma = \left( I_{L,M} + \beta_2 \sum_{k=1}^{K} \delta_k G_k \Psi_k H_k^T H_k \Psi_k^T G_k^T \right)^{-\frac{1}{2}}
$$

In a more simplified form, eq. (11) becomes (the subscripts indicate the matrices’ dimensions):

$$
Z_{(K'LMN,NJ)} = \overline{H}_{(K'LMK'M)} A_{(K'MN,NJ)} + N
$$

### III. Coding schemes

We distinguish between Inter Symbol Coding (ISC) and Inter Pulse Coding (IPC). The first scheme encodes adjacent symbols while the second scheme encodes the pulses used to convey one information symbol. For ISC, $B_k = B_1 \otimes 1_{1 \times N_i}$, for $k = 1, \ldots, K'$. For IPC, the $K'$ spreading sequences are taken to be orthogonal to each other.

We first consider the case of ISC with PAM. Combining the decision variables corresponding to the pulses of the same information symbol is equivalent to calculating:

$$
Z' = Z (I_{N_s} \otimes B^T) = N_f \overline{H} C + N'
$$

where $C$ is the $K' \times N_s$ matrix obtained from concatenating $A^{(1)}, \ldots, A^{(K')}$. It is easy to show that the noise term is still white. In order to achieve a diversity order of $K'$, there is no interest in choosing $N_s > K'$. Therefore, in what follows, we fix $N_s = K'$. $C$ has the structure of a minimal-delay $K' \times K'$ ST code. For carrier-less IR-UWB, the various existing codes [5], [7] cannot be applied since they are complex valued. In what follows, $C$ will be constructed following the steps given in [5], [7]. This construction is based on cyclic division algebras. Let $n = K'$, the code is constructed over the ring of integers of the field extension $K = Q(\theta)$ where $\theta = 2 \cos(\frac{\pi}{15})$ and $N$ verifies $E(N) = 2n$, $E(\lambda)$ being the Euler function. Energy-balanced $n \times n$ codewords constructed from this algebra can be expressed as [7]:

$$
C = \begin{bmatrix}
c_1 & c_2 & \cdots & c_n \\
\sigma(c_1) & \sigma(c_1) & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\sigma^{n-1}(c_2) & \cdots & \sigma^{n-1}(c_1)
\end{bmatrix}
$$

where $c_k = \phi(\gamma) \lambda k^{\frac{\alpha_i}{4}} x_k$ for $i = 1, \ldots, n$. The values $x_1, \ldots, x_n \in K$, they are related to the information symbols by: $x_i = \sum_{j=0}^{n-1} a_{i-j} \gamma^{j+1} \theta^j$ where $a_1, \ldots, a_n$ are the PAM symbols. $\gamma \in Z$ is chosen such that there is no element in $K$ whose norm is equal to $\gamma^1, \ldots, \gamma^{n-1}$. This choice results in fully diverse codes with a non-vanishing determinant [5]. $\phi(\gamma) = (n/\sum_{i=0}^{n-1} \gamma^i)^{\frac{1}{2}}$ is a normalization factor. $\lambda$ is an element whose norm verifies $N_{K/\mathbb{Q}}(\lambda) = \frac{1}{\sqrt{\alpha}}$, where $d_K$ is the discriminant of $K$. As explained in [5], this limits the construction in an appropriate ideal of $K$.

Using KANT software [9], we find that the principal ideal generated by $\gamma = 2$ is prime for $n = 2, \ldots, 6$. This proves that $\gamma = 2$ is a “non-norm” element for $n = 2, \ldots, 6 \ (N = 5, 7, 11, 13)$.

When $n$ is odd, we have $d_K = N^{n-1}$. In this case, $\lambda$ is determined from $\lambda = \beta^{\frac{n-1}{2}} / N$ where $\beta$ is obtained by performing the prime factorization (using KANT) of the ideal $N \mathcal{O}_K$ where $\mathcal{O}_K$ is the ring of integers of $K$. In this case, $N_{K/\mathbb{Q}}(\lambda) = N^{-\frac{n-1}{2}} = \frac{1}{\sqrt{\alpha}}$ since $N_{K/\mathbb{Q}}(\beta) = N$. For $n = 3$ (2 relays), $\beta = 2 + \theta - \theta^2 (\theta = 2 \cos(\frac{2\pi}{15}))$. For $n = 5$ (4 relays), $\beta = 2 + \theta^2 (\theta = 2 \cos(\frac{2\pi}{15}))$.

When $n$ is even, we fix $\lambda = \sqrt{\alpha}$ where $\alpha$ is a totally-positive element of $K$ verifying $N_{K/\mathbb{Q}}(\alpha) = \frac{1}{\sqrt{\alpha}}$. For $n = 2$ (1 relay), using KANT, we find that we can choose $\alpha = 3 - \theta$. In the same way, we choose $15\alpha = 5 + \theta + \theta^2 - \theta^3$ and $13\alpha = [3, -6, -9, 12, 4, -4]$ for $n = 4$ and $n = 6$ respectively. Now the coding scheme for a given number of relays is obtained from eq. (16) by replacing the corresponding values of $\gamma$, $\lambda$ and $\theta$ for $K = 1, \ldots, 5$.

Consider eq. (16). If the scalars $c_i$ are replaced by $x_i$, for $i = 1, \ldots, n$, we obtain $\det(C) \in \mathbb{Z}$ [5]. Following from $N_{K/\mathbb{Q}}(\lambda) = \frac{1}{\sqrt{\alpha}}$, we obtain the following relation:

$$
g_{\min} = \min_{\{C \neq 0, n \times n\}} |\det(C)| = \phi^n(\gamma) d_K^{-\frac{1}{2}}
$$

Since $\gamma > 1$, the transmitted energy is not evenly distributed among the elements of each codeword. For example, the $n$ symbols included in the conjugates of $x_1 (a_1, \ldots, a_n)$ are the most vulnerable to error events. On the other hand, the biggest portion of the energy is used to transmit the symbols included in the conjugates of $a_{(n-1)} \ldots, a_n$ since the amplitude of $x_n$ is multiplied by the biggest power of $\gamma$ (which is equal
to $\gamma^{-\frac{2}{n^2}}$). However, when $N_f > 1$, we can profit from the pulse repetitions in order to distribute the transmitted energy in a more balanced way among the $n^2$ symbols that constitute each codeword. Suppose that $N_f$ is a multiple of $n$ and designate by $\Omega$ the $n \times n$ permutation matrix given by:

$$
\Omega = \begin{bmatrix}
O_{1 \times (n-1)} & 1 \\
I_{n-1} & O_{(n-1) \times 1}
\end{bmatrix}
$$

(18)

Consider the $K'$-th frame duration of a given cooperation period. The coded pulses transmitted by the source and the relays are given by $C \otimes 1 \times N_f$. We propose the following “balanced” version of eq. (16):

$$
C_k = \begin{bmatrix}
C_0 & \Omega C_1 & \ldots & \Omega^{n-1} C_{n-1}
\end{bmatrix} \otimes 1 \times N_f/n
$$

(19)

$C_k$ has the same structure as $C$ and it is obtained from a permuted version of the information symbols. For determining $C_k$, a cyclic permutation of order $k n$ is applied on the symbols $a_1, \ldots, a_n$. In other words, $C_k$ is obtained by applying eq. (16) on the symbols $(\Omega^k \otimes I_n)[a_1 \ldots a_n]^T$. In this way, the $N_f$ pulses of each symbol are transmitted periodically with their amplitudes being multiplied by $1, \gamma^{\frac{1}{2}}, \ldots, \gamma^{-\frac{n^2}{2}}$. In eq. (19) we also took advantage of the pulse repetitions in order to permute the $n$ frames among the different relays (and the source). In this way, the $N_f$ pulses of each coded symbol are relayed periodically by the different nodes.

We now discuss the possibility of directly encoding the pulses of each symbol. When the spreading sequences $B_1, \ldots, B_K$, are orthogonal to each other, despreading the decision variables is equivalent to calculating:

$$
Z' = Z' \begin{bmatrix}
I_{N_s} & \otimes B_1^T & \ldots & \otimes I_{N_s} & \otimes B_K^T
\end{bmatrix}
$$

$$
= N_f \tilde{H} C' + N'
$$

(20)

where $C'$ is a $K' \times K' N_s$ matrix given by:

$$
C' = \text{diag}(A^{(1)} \ldots A^{(K')})
$$

(21)

Following from the dimensions of $C'$, we can see that there is no benefit in choosing $N_s > 1$, therefore we fix $N_s = 1$. In this case, $C'$ becomes a $K' \times K'$ diagonal matrix having the information symbols $a_1, \ldots, a_{K'}$ on its diagonal. Therefore, any full diversity rotation [10] of the $K'$ information symbols is sufficient for achieving full diversity. Since for the considered dimensions, these matrices achieve a minimum product distance of $d_g^{-\frac{1}{2}}$, this implies that:

$$
g_{\text{min}}' = \min_{C' \neq 0_{K \times K}} \text{det}(C') = d_g^{-\frac{1}{2}}
$$

(22)

It is worth noting that IPC is still a non-orthogonal AF scheme [3], [4] even though the source and the relays are using low-dimensional orthogonal vectors. In fact, the multiple access is controlled by the TH sequence which is common to the source and its cooperating relays. If there is another source transmitting at the same moment, its corresponding relays can use the same vectors $B_1, \ldots, B_K$.

With respect to ISC, IPC presents the advantage of lower decoding delays, peak to average power ratios (PAPR) and decoding complexity. The above coding schemes can be used with $M$-dimensional PPM-PAM. In this case, matrix $C$ in eq. (16) becomes a $n M \times n$ matrix obtained by replacing the symbols $a_1, \ldots, a_n$ by their vector representations. For PPM-PAM, by rearranging the rows of $C$, it can be expressed as:

$$
C = [C^{(1)}^T \ldots C^{(M)}^T]^T
$$

$C^{(m)}$ is a $n \times n$ matrix containing rows $(k M + m)$ of $C$ for $k = 0, \ldots, n - 1$. Since $C^{(m)}$ has the same structure as eq. (16), it verifies eq. (17) implying that $C$ achieves full diversity with these constellations. For IPC with PPM-PAM, the rotation matrix $(R)$ is replaced by $R \otimes I_M$.

IV. SIMULATIONS AND RESULTS

The channels between the different terminals are generated independently according to the channel model recommendations CM1 and CM2 [8]. The pulse shaper $w(t)$ is taken to be the second derivative of the Gaussian function with a duration of 0.5 ns. The frame duration $T_f$ is chosen to be $T_f = 100$ ns which is larger than the channel delay spread. We fix $N_f = K + 1$, $\delta = 0.5$ ns and $\beta_1 = \beta_2$. At the destination, the sphere decoder [11] is used for detection.

In order to highlight the effect of the achieved diversity, we first consider systems that do not have any energetic gain. In other words, we fix $\rho_k = \delta_k = 1$ in eq. (12) and eq. (13). In this case, the distances source-relay, relay-destination and source-destination are supposed to be the same. Moreover, we fix $\varepsilon^{(k)} = 0$ for $k = 1, \ldots, K$ in eq. (10).

Fig. 1 and Fig. 2 show the performance on CM2 with 2 PAM. The performance gains resulting from the proposed distributed ST codes are evident. Results show the importance of error balancing. The unbalanced ISC from eq. (16) shows poor performance especially at low signal to noise ratios since the performance is limited by the error events occurring on the less protected symbols $a_1, \ldots, a_n$. Applying eq. (19) overcomes this problem and results in a better performance. In comparison with IPC, we observe that the performance of ISC (and balanced-ISC) gets worse when increasing the number of relays. Balanced-ISC shows the best performance with 1 and 2 relays. It shows practically the same performance as IPC with 3 relays. Finally IPC shows the best performance with 4 and 5 relays. This difference follows from the second derivative of the Gaussian function in eq. (17) being a decreasing function of $\gamma$ for $\gamma = 2$. Similar results are obtained in Fig. 3 with multi-dimensional $M$-PPM-$M'$-PAM.

In the second simulation setup, $\rho_k$ and $\delta_k$ are determined from the relative positions of the terminals assuming free space propagation. The source and destination are separated by a distance of $d = 10$ m. The positions of the $K$ relays are uniformly distributed in the surface determined from the intersection of the two circles whose radii are equal to $d$ and centered at the source and the destination respectively. We compare the case $\varepsilon^{(k)} = 0$ with the case where $\varepsilon^{(k)}$ is determined from the positions of the destination and the relays. IPC is used with 2 PAM, 1 relay and a 4-finger Rake. Fig. 4 shows the performance losses that result when the propagation delays between the relays and the destination are not compensated. CM1 is more vulnerable to this “misalignment”. In fact, the strongest multipath component between the relay and the destination may not be combined if $|\varphi|_1$ has a relatively large value.
Figure 1: Performance on CM2 with 2 PAM and 5 fingers Rake.

Figure 2: Performance of 2 PAM on CM2 with 4 and 5 relays.

Figure 3: Performance on CM2 with PPM-PAM using 1 and 3 relays.

Figure 4: Effect of feedback on the performance of 2 PAM with $K = 1$ and $L = 4$.

V. Conclusion

In this work, we discussed the utility of cooperative schemes with IR-UWB. The proposed system takes into consideration the multi-path propagation and the properties of multi-dimensional constellations. The additional constraint of real-valued carrier-less transmissions was taken into consideration and the pulse repetitions were appropriately encoded. AF cooperation turned out to be beneficial with various constellations and Rake orders. The IPC scheme is appealing especially with large number of relays.

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