Active Noise Control in a Duct With Flow

Active noise control in a one dimensional acoustic duct, in which fluid medium inside the duct has a mean flow velocity, is studied. The acoustic duct model with general boundary conditions is solved in Laplace domain and infinite dimensional system transfer functions are obtained. For controller designs, appropriate microphone, and noise canceling source locations are determined. Low order finite dimensional transfer function approximations of actual system transfer functions are obtained. It is found that, in a selected frequency range, approximations represent actual system in a satisfactory way. By using approximated system transfer functions, finite dimensional, low order, optimal $H_2$ and $H_{\infty}$ controllers are synthesized via linear matrix inequalities method. Closed loop frequency response and time domain simulations show that the controllers successfully suppress unwanted sound, which propagates along the duct. [DOI: 10.1115/1.4026410]

1 Introduction

Acoustic noise arousing from heating ventilating air conditioning or exhaust ducts is an important problem that can influence human health and comfort negatively. Traditionally, passive noise control techniques, which include reactive techniques (e.g., using an expansion chamber) or dissipative techniques (e.g., using a porous lining material) are employed in ducts [1]. However, in order to cope with low frequency noise problems in ducts, large passive silencers or thick and heavy lining (absorbing) materials are needed. Large silencers can cause pressure drop inside the duct. Thick absorbing materials inhibit ventilation and increase the weight of the system. As an alternative to passive techniques, low frequency noise problem can be handled with active noise control (ANC). As a result, weight and size reduction can be achieved and the pressure drop problem can be eliminated.

ANC studies in ducts can be grouped into two. In the first group, empirical models with adaptive control are utilized [2,3]. In this group, some studies [4–6] concentrate on hybrid active and passive noise reduction techniques. Physical models with fixed-gain feedback control are used in the second group. In the literature, there are numerous physical model based duct ANC studies which do not consider the flow of fluid medium inside the duct. Some of early works [7–10] utilized classical control theory. In Ref. [11] pole placement feedback algorithm, in Refs. [12–14] Linear Quadratic Gaussian (LQG) optimal controllers, in Refs. [15,16] infinite dimensional $H_{\infty}$ optimal controllers and in Ref. [17] proportional integral derivative controller are utilized. In Ref. [18], controller synthesis was performed after obtaining finite dimensional transfer function approximations whereas in Ref. [19], sound dynamics of finite length ducts were studied with infinite dimensional system transfer functions. In order to obtain a realistic duct model, gas flow inside the duct should be taken into consideration. In the literature, there exists relatively few physical model based studies, which also consider mean flow of fluid medium inside the duct. Some early works [20,21] utilized classical control theory. In Refs. [22,23] infinitely long ducts, many sources and microphones, and unidirectional plane wave were considered. In Ref. [24] the effects of mean flow and frequency dependent radiation impedance at duct end are studied separately. Modal analysis is employed and transfer function approximations of system were obtained for ideal open boundary condition (BC) case in Refs. [24,25].

From an engineering point of view, low cost and efficient ANC systems are needed. By using finite dimensional, low order, optimal controllers, and less amount of microphones and sources (e.g., a single microphone and a single canceling source loudspeaker), these requirements can be met. Solving the system model in Laplace domain provides exact system frequency response. Thus, the only remaining item for synthesizing a low order, finite dimensional controller is, to find a suitable low order approximating scheme that fits the original system data. To have a more realistic duct system, both mean flow and frequency dependent impedance BC should be considered. For synthesizing optimal controllers, $H_2$ and $H_{\infty}$ optimal control theory and linear matrix inequalities (LMIs) can be employed. If a control problem can be written in terms of LMIs, a solvable convex optimization problem is obtained. However, LMIs were not utilized in the literature often [26].

For the physical model based studies in the literature, to our knowledge, the effects of mean flow of fluid medium inside the duct and frequency dependent specific acoustic impedance of the duct end are not studied together. Taking into consideration these effects provide a more realistic duct system. Moreover, synthesizing low order, optimal, finite dimensional controllers provide easily implementable ANC systems. Thus, our main motivation for this paper is to study a realistic ANC system in which the effects of mean flow of fluid medium inside the duct and frequency dependent specific acoustic impedance of the duct end are considered, simultaneously.

In the light of previous discussions, in this paper, a physically applicable single input single output ANC system in ducts with flow, which aims to reduce low frequency acoustic noise is considered. Realistic (frequency dependent impedance BC) duct systems with flow are analyzed in Laplace domain. Low order, real, rational, finite dimensional approximations for system transfer functions are obtained in frequency domain. Then, low order, finite dimensional optimal $H_2$ and $H_{\infty}$ controllers are synthesized with a powerful but relatively less utilized optimization algorithm (LMIs).

2 Model of the Acoustic Duct System

The following assumptions are made for the duct model: fluid inside the duct is inviscid; the process is adiabatic; duct is rigid walled; transverse dimensions of the duct are small compared to acoustic wavelengths considered, so only plane waves inside the duct can progress; there is a known and constant uniform flow of fluid along the duct axis and high order nonlinear effects are
neglected. Acoustic wave propagation equation along this duct (see Fig. 1) is given below

\[\frac{\partial^2 p}{\partial t^2} + 2\nu_0 \frac{\partial p}{\partial x} + (\sqrt{c} - c)^2 \frac{\partial^2 p}{\partial x^2} = \left[\frac{\partial q_s(t)\delta(x-x_s)}{\partial t} + u_0 \frac{\partial[q_s(t)\delta(x-x_s)]}{\partial x}\right] \]

(1)

where \(c\) is the speed of sound (m/s), \(u_0\) is the mean flow velocity of fluid inside the duct (m/s), \(t\) is the time (s), \(x\) is any spatial coordinate in duct (m), \(p(x, t)\) is the acoustic pressure (Pa), \(q_s(t)\) is the mass injection rate per unit area by point source (kg/m²s), \(\delta(x)\) is the spatial Dirac delta function indicating the point source location (1/m), \(x_s\) source location in duct (m). Our nonhomogeneous one-dimensional wave propagation model can be found in Refs. [22], [27], and [28].

For the purpose of obtaining transfer functions of the system, “zero” initial conditions \((p(x, 0) = 0 \text{ and } \partial p(x, 0)/\partial t = 0)\) are chosen as encountered widely in control theory. Boundary conditions of the system are

\[p(0, t) = d(t)\]

(2)

\[
\frac{\partial p(L, t)}{\partial x} = \rho_0 \left(\frac{\partial u(L, t)}{\partial t} + u_0 \frac{\partial u(L, t)}{\partial x}\right)
\]

(3)

where \(d(t)\) is time dependent disturbance sound pressure (Pa), \(u(x, t)\) is the velocity change of fluid particle due to acoustic excitation (m/s), \(\rho_0\) is equilibrium density of fluid (kg/m³), \(L\) is the length of the duct (m). Equation (2) forms disturbance BC at \(x = 0\). Equation (3) indicates frequency dependent impedance BC at \(x = L\). It represents the fact that momentum equation should be satisfied at \(x = L\).

3 System Transfer Functions

In order to obtain the transfer functions of the system, Laplace transform method is used [15,18,19,29]. Laplace transform of Eq. (1) for the case of “zero” initial conditions can be obtained as follows:

\[s^2P(x, s) + 2\nu_0 sP'(x, s) + (\sqrt{c} - c)^2P''(x, s) = c^2[sq(s)\delta(x-x_s) + u_0q(s)\delta'(x-x_s)]\]

(4)

where \(s\) is the Laplace variable, \(P(x, s)\) is acoustic pressure in Laplace domain, \(P'(x, s)\) and \(P''(x, s)\) are first and second derivatives of \(P(x, s)\) with respect to \(x\), respectively. Similarly, Laplace transform of BCs in Eqs. (2) and (3) are obtained as follows:

\[P(0, s) = d(s)\]

(5)

\[
\frac{\partial P(L, s)}{\partial x} = \rho_0 \left(\frac{\partial u(L, s)}{\partial t} + u_0 \frac{\partial u(L, s)}{\partial x}\right)
\]

(6)

Defining the acoustic impedance at \(x = L\) as \(Z_L(s) = P(L, s)/u(L, s)\) and then, using it in Eq. (6) and rearranging the terms gives

\[P(L, s) = -\left(\frac{Z_L(s)}{\rho_0 s}\right) + \frac{u_0}{s} \frac{\partial P(L, s)}{\partial x}\]

(7)

\[Z_L(s)\] is the frequency dependent specific acoustic impedance of the duct end at \(x = L\). \(Z_L(s)\) is approximated with a rational transfer function in Ref. [30] as

\[Z_L(s) = \frac{\pi r^2 CR_1R_2Ms^2 + M(R_1 + R_2)s}{MCR_1s^2 + (M + CR_1R_2)s + (R_1 + R_2)}\]

(8)

This approximation is based on an electrical circuit analogy in which radiation impedance of the plane circular piston vibrating sinusoidally in the end of a long tube is represented [31]. Parameters used in this approximation are given in Ref. [31] as \(R_1 = \rho_0 c/\pi r^2, R_2 = 0.504R_2, C = 5.44\rho c^2/\rho_0 c^2, M = 0.1952\rho_0 /r\), where \(r\) is the radius of the duct (m).

Equation (4) together with Eqs. (5) and (7) form the boundary value problem, where frequency dependent specific acoustic impedance of the duct end is given by Eq. (8). This boundary value problem is solved by using a commercial software and infinite dimensional system transfer functions are obtained.

3.1 Disturbance to Output Pressure Transfer Function.

Consider the duct shown in Fig. 1. Here, disturbance to output pressure transfer function for frequency dependent impedance BC for mean flow case is obtained as

\[\frac{P(x, s)}{d(s)} = \frac{g_2}{g_1} \left(\frac{g_2g_1h_c(c + u_0) + h_2(-c + u_0)}{g_2h_1(c + u_0) + h_2(-c + u_0)}\right)\]

(9)

where all parameters in Eq. (9) are given in the Appendix.

3.2 Input to Output Pressure Transfer Function.

Input to output pressure transfer function for frequency dependent impedance BC for mean flow case is obtained as

\[\frac{P(x, s)}{q(s)} = G_{q1} + \frac{c^2}{2(c^2 - u_0^2)} \left(\frac{h_1k_2(c + u_0)^2 + h_2(k_3 - k_4)(c - u_0)^2}{g_2h_1(c + u_0) + h_2(-c + u_0)}\right)\]

(10)

for \(x > x_s\)

where all parameters in Eqs. (10) and (11) are given in the Appendix.

4 Duct Natural Frequencies and Nodes

When Eq. (4) is solved for open BC \((P(L, s) = 0)\), \(n\)th natural frequency of the duct is obtained as

\[f_n = \frac{cn}{2L} \frac{c^2 - u_0^2}{c^2}\]

(12)

Existence of mean flow of fluid medium in the open end duct shifts the poles and zeros of the system by the factor \((c^2 - u_0^2)/c^2\). Similar results hold for the nodes, as well. For the open end duct
with flow the frequency $f_m$ at which microphone located at $x_m$ measures zero pressure is found as

$$f_m = \frac{c}{2|x_e - x_m|} \left( \frac{c^2 - u_0^2}{c^2} \right)$$

(13)

where $x_m$ is the microphone location in duct (m), $x_e$ is the duct end closest to the $x_m$. Value of the $x_e$ could be 0 or $L$. Similarly, the frequency $f_s$ at which source located at $x_e$ creates zero pressure is found by replacing $f_m$ with $f_s$ and $x_m$ with $x_e$ in Eq. (13).

5 Optimal Controller Designs

System transfer functions obtained in Sec. 3 are infinite dimensional. In order to apply linear time invariant (LTI) finite dimensional control theory, finite dimensional transfer functions, which represent the system are needed. Control problem formulation below is performed for infinite dimensional transfer function approximations.

The transfer function that relates disturbance ($d(s)$) to pressure at a point $x$ inside the duct ($P(x, s)$) is given by $G_d(x, s) = P(x, s)/d(s)$ and the transfer function that relates control input ($q(s)$) to pressure at a point $x$ inside the duct ($P(x, s)$) is given by $G_q(x, s) = P(x, s)/q(s)$. Then, the total pressure at a point $x$ inside the duct resulting from both disturbance and control input is found by

$$P(x, s) = G_d(x, s)d(s) + G_q(x, s)q(s)$$

(14)

As can be seen from Fig. 1, our control input is

$$q(s) = C(s)P(x_m, s)$$

(15)

where $C(s)$ is the controller transfer function, $P(x_m, s)$ is the measured pressure value by microphone at sensing point. Realizing that $P(x_m, s)$ can be obtained by inserting $x_m$ instead of $x$ in Eq. (14) and then, combining Eqs. (14) and (15) results in

$$\frac{P(x, s)}{d(s)} = G_d(x, s) + G_q(x, s)C(s) \frac{G_d(x_m, s)}{1 - G_q(x_m, s)C(s)}$$

(16)

Equation (16) describes the closed loop system between $P(x, s)$ and $d(s)$. In order to accomplish the control objective; $H_2$ and $H_\infty$ norms of closed loop system are minimized, respectively, to get optimal $H_2$ and $H_\infty$ controllers while maintaining closed loop stability. State space description of closed loop system in Eq. (16) is used for optimal controller designs. In controller synthesis LMI formulations are utilized [32].

6 Numerical Results

In this section, numerical simulation results for the considered ANC system is given. The aim is to suppress low frequency noise in target frequency range. For this purpose, a feedback configuration shown in Fig. 1 is proposed. To have a feedback configuration that functions throughout the whole frequency range of interest, antiresonance frequencies should be avoided. To that end, the microphone and the source should be placed sufficiently close to the two duct ends.

The parameters used in numerical studies are determined as:

- Speed of sound: $c = 340$ m/s; mean flow velocity of fluid: $u_0 = 0$ m/s, 34 m/s, 102 m/s; length of the duct: $L = 3.4$ m; microphone location: $x_m = 2.8$ m; source location: $x_e = 0.6$ m; equilibrium density of fluid: $\rho_0 = 1.2$ kg/m$^3$; radius of the duct: $r = 0.1$ m; and target frequency range: 0–200 Hz.

For a circular duct of radius $r$, transverse waves attenuate rapidly below the cutoff frequency of 0.293 $c/r$ (see Ref. [33] (Sec. 9.2)). In our case, $r = 0.1$ m and $c = 340$ m/s, resulting in a cutoff frequency of 996 Hz. Thus, in the frequency range of interest (0–200 Hz), the one-dimensional model is valid.

Before obtaining finite dimensional system transfer function approximations and controllers, it is beneficial to see the findings of Sec. 4, numerically. In Fig. 2(a) frequency response of $d(s)$ to $P(x, s)$ transfer function at $x = 2.8$ m for three different Mach numbers ($Ma = u_0/c$) for open BC is shown. This figure is in agreement with Eq. (12). For open BC, system resonance and antiresonance frequencies, as well as shifts in those frequencies due to variations in Mach number, can be seen clearly. Although the analysis mentioned in Sec. 4 does not provide exact results for the frequency dependent impedance BC case, it still gives some insight about approximate shifts for the ducts with general BCs as can be seen from Fig. 2(b). This information is used to place the microphone and the source to proper places in the duct for ANC system design. Notice that, for the selected $x_e$ and $x_m$ values, there is no antiresonance in the target frequency range (see Figs. 2(a) and 2(b)).

6.1 Transfer Function Approximations. Here, a finite dimensional approximation scheme is proposed to approximate system transfer functions in the target frequency range. Writing down the transfer function form used in approximations as
where $C_0$, $A_i$, and $B_i$ are the real coefficients to be determined, $\omega_o$ is the corner frequency of the first order system near the “zero” frequency region, $\omega_i$ is the ith natural frequency of the duct, $\zeta_i$ is the damping ratio corresponding to the ith mode, $G_a$ is the transfer function approximation. Summation index goes from 1 to 5 to consider only the first five modes. Even though we are interested with the modes up to 200 Hz, we have taken first five modes in order to obtain a more accurate transfer function approximation in our target frequency range. Note that, in Eq. (17), for the data point $j\omega_o$, only unknowns are $C_0$, $A_i$‘s, and $B_i$‘s. Therefore, these equations form a linear least squares problem as mentioned below:

\[
\text{Re}(G_a(j\omega_o)) \approx C_0 x_0 + A_1 x_1 + \ldots + A_5 x_5 + B_6 x_6 + \ldots + B_5 z_5 + B_1 z_6 + \ldots + B_5 z_{10} \tag{20}
\]

\[
\text{Im}(G_a(j\omega_o)) \approx C_0 z_0 + A_1 z_1 + \ldots + A_5 z_5 + B_6 z_6 + \ldots + B_5 z_{10} \tag{21}
\]

where $G_a(j\omega_o)$ is the exact system transfer function value at $j\omega_o$, $\text{Re}(G_a(j\omega_o))$ is the real part of the exact system transfer function to be approximated at $j\omega_o$, $\text{Im}(G_a(j\omega_o))$ is the imaginary part of the exact system transfer function to be approximated at $j\omega_o$. For $N$ number of data points, this linear regression problem can be expressed in matrix form as

\[
\hat{\beta} = (X^T X)^{-1} X^T Y \tag{22}
\]

where

\[
X = \begin{bmatrix}
\vdots & \vdots & \vdots & \vdots \\
x_0 & x_1 & \ldots & x_{10} \\
0 & 0 & \ldots & 0 \\
z_0 & z_1 & \ldots & z_{10} \\
\vdots & \vdots & \ddots & \vdots 
\end{bmatrix}_{N \times 11}
\]

\[
Y = \begin{bmatrix}
\text{Re}(G_a(j\omega_o)) \\
\text{Im}(G_a(j\omega_o)) \\
\vdots \\
\text{Re}(G_a(j\omega_o)) \\
\text{Im}(G_a(j\omega_o)) \\
\vdots 
\end{bmatrix}_{N \times 1}
\]

\[
\hat{\beta} = [C_0 A_1 \ldots A_5 B_1 \ldots B_5]
\]

Fig. 3 (a) Bode magnitude plot and (b) Bode phase plot for $d(s)$ to $P(x, s)$ transfer function at the duct end $x = 3.4$ m. (c) Bode magnitude plot and (d) Bode phase plot for $q(s)$ to $P(x, s)$ transfer function at the duct end $x = 3.4$ m (dotted line—exact system transfer function, solid line—approximated transfer function). Here, $M_w = 0.1$. 

\[
G_a(s) = \frac{C_0}{s + \omega_o} + \sum_{i=1}^{5} \frac{A_i + B_i s}{s^2 + 2 \zeta_i \omega_i s + \omega_i^2} \tag{17}
\]
The results for proposed approximation scheme are shown in Figs. 3 and 4. In these figures, $N = 25,000$ data points, which are equally spaced between 0 and 250 Hz frequency range, are used. As it can be seen in these figures, proposed low order approximation scheme is quite satisfactory in the frequency range of interest.

6.2 Performance of Controllers. In this part, $H_2$ and $H_\infty$ controller performances, which are synthesized according to Sec. 5, are presented. For the spatial region, $x = 2.8$ m to $x = 3.4$ m in the duct, maximum Bode magnitude value for $d(s)$ to $P(x, s)$ transfer function occurs at $x = 2.8$ m with 18.73 dB. So, it would be reasonable to suppress the noise levels at that point in the duct. Thus, performance point is selected as $x_p = 2.8$ m. Aim is to suppress the noise levels at the selected performance point. Furthermore, we claim that by this design, noise levels at duct section from $x = 2.8$ m to $x = 3.4$ m will be suppressed, as well.

Controlled output ($z$) and measured output ($y$) are the same since, measurements are taken from $x_m = 2.8$ m and performance point is at $x_p = 2.8$ m in the duct. This condition automatically reduces plant order by half. To further reduce the order, we use the equivalence of damping ratios and damped natural frequencies of approximated system transfer functions. However, due to numerical errors, there exists a maximum of 0.1% difference between damped natural frequencies of the same mode that are calculated using these two different transfer functions. Thus, the low order plant description is obtained by averaging the $f_{os}$ values of disturbance to output pressure and input to output pressure transfer functions at $x = 2.8$ m. Moreover, the damping ratio values of all second order modes ($\zeta$) and the corner frequency of the first order mode ($\omega_o$) are averaged, as well.

Applying the procedure above, the order of the plant is reduced to 11 states. It implies that the controllers have 11 states, as well. Using a low-order controller generally increases numerical robustness.

Figure domain uncontrolled and controlled system performances are given in Fig. 5(a) for performance point $x = 2.8$ m, and in Fig. 5(b) for the duct end $x = 3.4$ m, respectively. These plots are obtained by using the approximated system transfer functions. As it can be seen from these graphs, controllers successfully suppress the first four modes of the duct. However, around zero frequency, the controller’s attenuation performance is slightly inferior than that of the uncontrolled case. In Fig. 5(a) closed loop performance for $x = 2.8$ m is very smooth since controllers are designed to attenuate the noise at that particular point. Frequency response in Fig. 5(b) shows similar characteristics as in Fig. 5(a). Whereas around the resonance frequencies in Fig. 5(b), there are tiny peaks and valleys. Those small variations in performance are encountered because controllers are not designed to reduce noise levels at $x = 3.4$ m. But, since variation in damping ratios and damped natural frequencies along the duct due to numerical errors are low, low order controller works reasonably well in most of the target bandwidth. Thus, our earlier claim for suppressing noise from $x = 2.8$ m to $x = 3.4$ m along the duct, is verified.

When Figs. 5(a) and 5(b) are investigated, one can see that $H_2$ and $H_\infty$ controller performances are quite similar. However, at low frequencies $H_\infty$ controller has slightly better performance whereas at high frequencies $H_2$ controller is slightly superior. At midfrequencies no significant distinction is observed. But for the performance point($x = 2.8$ m), $H_\infty$ shows better results since it does not exceed 0 dB value within all the target bandwidth as can be seen from Fig. 5(a). Therefore, the output pressure is always lower than the input pressure throughout the whole frequency range of interest.

Time domain simulations shown in Fig. 6 validate the results obtained in the frequency domain. A periodic signal, which is a summation of eight sine waves, is significantly suppressed. All of these sine waves have amplitude 1 Pa. Eight different frequencies for these sine waves are: 25, 48.67, 75, 97.28, 125, 145.91, 175, and 194.54 Hz. Note that: 48.67, 97.28, 145.91, 194.54 Hz

![Fig. 4](image-url) (a) Bode magnitude plot and (b) Bode phase plot for $d(s)$ to $P(x_m, s)$ transfer function at $x_m = 2.8$ m. (c) Bode magnitude plot and (d) Bode phase plot for $q(s)$ to $P(x_m, s)$ transfer function at $x_m = 2.8$ m (dotted line—exact system transfer function, solid line—approximated transfer function). Here, $Ma = 0.1$. 

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Fig. 5  (a) Frequency response of uncontrolled and controlled systems at $x = 2.8$ m. (b) Frequency response of uncontrolled and controlled systems at the duct end at $x = 3.4$ m (dotted line—uncontrolled, dashed line—$H_2$ controlled, solid line—$H_\infty$ controlled). Here, $Ma = 0.1$.

Fig. 6 (a) Time domain uncontrolled and controlled system responses at $x = 2.8$ m. (b) Time domain uncontrolled and controlled system responses at the duct end $x = 3.4$ m (dotted line—uncontrolled, dashed line—$H_2$ controlled, solid line—$H_\infty$ controlled). Here, $Ma = 0.1$.

Fig. 7  (a) Time domain uncontrolled and controlled system responses for white noise at $x = 2.8$ m. (b) Time domain uncontrolled and controlled system responses for white noise at the duct end $x = 3.4$ m (dotted line—uncontrolled, dashed line—$H_2$ controlled, solid line—$H_\infty$ controlled). Here, $Ma = 0.1$. 

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frequencies are the first four damped natural frequencies of the disturbance to output pressure transfer function at \( x = 2.8 \) m. In Fig. 5 Bode plots show the steady state frequency response of the system. Therefore, once transients die out, there is considerable attenuation in both Figs. 6(a) and 6(b), which happens after approximately 0.5 s. Furthermore, in Fig. 7, response of the system to white noise is shown. For this input, steady state response cannot be achieved. As a result, the attenuations are not as good as in Fig. 6. Nevertheless, at the performance point (\( x = 2.8 \) m), both controllers can suppress the broadband input. Finally, at the end of the duct, \( H_2 \) controller can achieve better suppression.

7 Conclusions

In this study, including frequency dependent impedance BC together with the mean flow of fluid resulted in a quite realistic duct model. System resonance and antiresonance frequencies are found to be shifted by the existence of mean flow of fluid inside the duct. Proposed feedback control configuration using one microphone and one canceling source is observed to be adequate for achieving satisfactory noise attenuation levels.

Designed low order optimal feedback controllers provide global noise reduction at the duct end, which is supported by frequency domain and time domain simulations. Frequency domain simulations show that all the resonance peaks are well suppressed. But, at very low frequencies performance is slightly worsened. \( H_2 \) and \( H_\infty \) controllers show similar characteristics, however because of the performance superiority at very low frequencies, \( H_\infty \) controller is considered to be slightly better than \( H_2 \). Though, when time domain simulations for white noise input are considered, \( H_\infty \) controller achieves slightly better noise reduction. It is shown that the proposed approximation scheme for obtaining low order finite dimensional transfer functions from the actual system transfer functions works quite well throughout the whole target frequency range.

Robustness is not considered in this paper. However, closed loop performances of optimal controllers can be improved by considering the uncertainties of the duct parameters. If one aims to apply the existing controllers to a system with higher number of approximated modes, then modal spillover would occur. As a future study, this phenomenon can be investigated.

Appendix: System Transfer Function Parameters

\[ g_1 = e^{-\frac{s}{k_1}}, \]
\[ g_2 = e^{-\frac{s}{k_2}}, \]
\[ g_3 = e^{-\frac{s}{k_3}}, \]
\[ g_4 = e^{-\frac{s}{k_4}}, \]
\[ g_5 = e^{-\frac{s}{k_5}}, \]
\[ h_1 = c_0 + Z_e(s), \]
\[ h_2 = c_0 - Z_e(s), \]
\[ k_1 = g_1 g_2 g_3 g_4, \]
\[ k_2 = g_2 g_3 2 g_5 g_6, \]
\[ k_3 = g_3 2 g_4 g_5, \]
\[ k_4 = g_2 2 g_4 g_5, \]
\[ k_5 = g_3 2 g_4 g_5, \]
\[ k_6 = g_1 2 g_4 g_5. \]

References


