Time Delay and Permittivity Estimation by Ground-Penetrating Radar With Support Vector Regression

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Abstract—In the field of civil engineering, sounding the pavement layers is classically performed using standard ground-penetrating radar, whose vertical resolution is bandwidth dependent. The layer thicknesses are deduced from both the time delays of backscattered echoes and the permittivity of layers. In contrast with conventional spectral analysis approaches, this letter focuses on one of the machine learning algorithms, namely, the support vector machine, to perform time delay estimation and dielectric constant estimation of the medium from backscattered radar signals. This letter shows the super time resolution capability of such technique to resolve overlapping and fully correlated echoes within the context of thin pavement layer testing.

Index Terms—Ground-penetrating radar (GPR), nondestructive testing and evaluation (NDTE), resolution, support vector (SV) machine (SVM), time delay estimation (TDE).

I. INTRODUCTION

GROUND-penetrating radar (GPR) is a common tool for nondestructive testing of civil engineering materials (hydraulic and bituminous concretes and soils). For centimeter-scale wavelengths, GPR, whose vertical resolution is bandwidth dependent, is often used for the specific application of pavement survey [1]–[6]. For this purpose, the roadway is assumed to be horizontally stratified. The vertical structure of roadway can then be deduced from radar profiles by means of echo detection and amplitude estimation. Echo detection provides time delay estimation (TDE) associated with each interface, whereas amplitude estimation is used to retrieve the wave speed (or the dielectric constant) within each layer. In the literature, electromagnetic inversion and layer stripping constitute two methods for estimating the latter parameters [4], [5].

TDE is usually performed using spectral analysis methods, including conventional fast Fourier transform (FFT)-based methods (inverse FFT, matched filter, or cross-correlation methods) and model-based methods (Multiple Signal Classification (MUSIC), Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT), etc.). When enhanced time resolution is required, e.g., thin pavement thickness measurement [7]. Machine learning algorithms (e.g., neural networks) have been introduced more recently in GPR community [8]–[10]. Among this family of algorithms, the support vector (SV) machines (SVMs) have shown promising results for different applications. Originally proposed for document classification and pattern recognition [11], the SVMs have been used to identify and classify buried objects from GPR imagery [12] or to assess railway-ballast conditions [13]. Moreover, the SVMs have been extended for regression problems by the SV regression (SVR) method [14]. In this case, it has been applied to predict the vehicle travel times [15] and to estimate the directions of arrival of multiple waves by smart antenna [16]. The expected advantage of SVMs is its capability to compensate the model uncertainty by a large experimental training data set, which aims at covering the whole possible experimental radar configurations with a specified material. In contrast, spectral analysis methods are model dependent, and their performance is related to the simplified electromagnetic hypothesis sustained by the data model. The SVMs provide a great generalization ability and usually have a low calculation time [14]. In this letter, we propose to use this method to estimate, on the one hand, the time delays of backscattered radar signals and, on the other hand, the dielectric constants of medium in the context where the echoes are totally correlated. Contrary to [17], in this letter, the dielectric constants of layers are unknown and are estimated by the SVMs. The proposed method includes two main steps: feature extraction and regression model. This letter focuses on the context of pavement thickness estimation corresponding to overlapped echoes or nonoverlapped echoes. The layer thickness is deduced from both the TDE of backscattered echoes and the dielectric constant estimation of layers.

Section II provides a simplified model of the backscattered radar data and the features. In Section III, we introduce SVR briefly. In Section IV, this method will be applied to simulated data that represent the backscattered radar data from a horizontal stratified medium in far-field condition in order to determine the performance of the proposed algorithm. A conclusion is drawn in Section V.

II. RADAR DATA MODELING AND FEATURE EXTRACTION

A monostatic GPR is positioned at nadir in far-field condition to control the roadway layers. We consider the backscattered signal from a horizontal stratified medium with \( K - 1 \) layers. Each layer of the medium is characterized by its thickness and its dielectric permittivity. In far-field condition, the received
signal is composed of the $K$ backscattered echoes from each interface. The shape variation of the pulse wavelet within the medium is likely to be very limited not only because of the low conductivity of the pavement material, as indicated in [18], but also because of the small layer thickness to be surveyed. Thus, the backscattered echoes are approximated in the data model by a time-shifted and attenuated replica of the emitted radar pulse $e(t)$. An additive noise $n(t)$ is assumed to represent the measurement uncertainties. Then, the received signal can be written as

$$r(t) = \sum_{k=1}^{K} s_k e(t - T_k) + n(t).$$

(1)

At vertical incidence, the amplitudes $s_k$ depend on the dielectric contrast between layers through the Fresnel coefficients at nadir [7]. Advanced TDE is usually performed in frequency domain, for which the data model is written as a linear combination of cissoids modulated by the radar pulse as follows:

$$\tilde{r}(f) = \sum_{k=1}^{K} s_k \tilde{e}(f)e^{-2j\pi f T_k} + \tilde{n}(f)$$

(2)

where the $\tilde{\cdot}$ symbol represents the Fourier transform of the radar pulse of the associated temporal signal. For $N$ discrete frequencies $(f_n)$ within bandwidth $B$, the received signal, called observation vector $r$, can be written in the following matrix form:

$$r = \Lambda A s + n$$

(3)

with the following notations.

1. $r = [\tilde{r}(f_1) \tilde{r}(f_2) \cdots \tilde{r}(f_N)]^T$ is the data vector which may represent either the Fourier transform of the GPR signal or the measurements from a step frequency radar.
2. $A = \text{diag}(\tilde{e}(f_1), \tilde{e}(f_2), \ldots, \tilde{e}(f_N))$ is a diagonal matrix whose diagonal elements are the amplitudes of the Fourier transform of the radar pulse.
3. $A = [a(T_1) \ a(T_2) \cdots \ a(T_K)]$ is called mode matrix whose columns are defined hereinafter.
4. $a(T_k) = [e^{-2\pi f_1 T_k} e^{-2\pi f_2 T_k} \cdots e^{-2\pi f_N T_k}]^T$ is called either mode or steering vector; $\Lambda a(T_k)$ represents the parameterized model of the steering vector.
5. $s = [s_1 \ s_2 \cdots \ s_K]^T$ is the source vector composed of echo amplitudes $s_k$.
6. $n = [\tilde{n}(f_1) \tilde{n}(f_2) \cdots \tilde{n}(f_N)]^T$ is the complex noise vector in which each element is a complex white Gaussian noise with zero mean and variance $\sigma^2$.
7. $f_n = f_1 + (n - 1)\Delta f$ represents the equispaced frequency samples, where $f_1$ is the beginning of the bandwidth and $\Delta f$ is the frequency difference between two adjacent samples.

The correlation matrix is defined as $\Gamma = \langle rr^H \rangle$, where $\langle \cdot \rangle$ denotes the ensemble average and the superscript $^H$ indicates the complex conjugate transpose. In practice, the correlation matrix $\Gamma$ is estimated as follows from $M$ independent snapshots of the data vector $r$: $\hat{\Gamma} = (1/M) \sum_{m=1}^{M} r_m r_m^H$.

Let $\Gamma_s$ and $\sigma^2 \Sigma$ be the correlation matrices of the source vector and the noise vector, respectively; the correlation matrix of the data vector (3) can be written as

$$\Gamma = \Lambda \Lambda^T \Sigma_s A^H A H^T + \sigma^2 \Sigma.$$

(4)

The cross correlation between echoes is included in the correlation matrix $\Gamma_s$ and needs special attention in practical applications. Its rank is expected to be close to one (because the source vector $s$ in (3) represents the backscattered echoes from different paths of the same original source) and limits the performance of subspace-based TDE algorithms such as MUSIC or ESPRIT. For the sake of simplicity, consider the correlation matrix of the source vector for two echoes only

$$\Gamma_s = \left( \begin{array}{ccc} E_1 & \rho \sqrt{E_1 E_2} & \rho \sqrt{E_1 E_2} \\ \rho \sqrt{E_1 E_2} & E_2 & E_2 \\ \rho \sqrt{E_1 E_2} & E_2 & E_2 \end{array} \right)$$

(5)

where $\rho = 1$ for the totally correlated case, the superscript $^*$ denotes complex conjugate, $E_k = \langle |s_k|^2 \rangle$, and $s_k$ has been defined in (1). In this letter, we consider the smooth surfaces, so the echoes are totally correlated.

The feature extraction is carried out as in [16] from the correlation matrix. The correlation matrix is Hermitian, so only the upper triangular part is used. Afterward, the relevant matrix elements are organized in vector $v$ as follows:

$$v = (r_{11}, \ldots, r_{NN}, \Re(r_{12}), \Re(r_{23}), \ldots, \Re(r_{(N-1)N}), \Re(r_{13}), \ldots, \Re(r_{1N}), \Im(r_{12}), \Im(r_{13}), \ldots, \Im(r_{1N}), \Im(r_{23}), \ldots, \Im(r_{(N-1)N}))^T$$

with $\Re(r_{bc})$ and $\Im(r_{bc})$ being the real and imaginary parts of $r_{bc} = [\Gamma]_{bc}, c \geq b = 1, \ldots, N$, respectively. In order to use SVR, the vector $v$ is normalized to $[0, 1]$ range to obtain the vector $z \subset \mathbb{R}^{N^2}$ which represents the used feature vector.

III. SVR FOR TIME DELAY AND DIELECTRIC CONSTANT ESTIMATION

The SVM algorithm introduced by Vapnik [19] is considered as one of the most powerful supervised machine learning algorithms. This algorithm often achieves superior classification performance compared to other learning algorithms in many domains (e.g., neural networks). The SVM algorithm is based on the structural risk minimization principle [11]. In this section, we propose to use this method to estimate, on the one hand, the time delays of backscattered radar signals and, on the other hand, the dielectric constants (relative permittivity) of the medium in the context of regression. We propose to use $\nu$-SVR [20] which is a modification of $c$-SVR [19]. In $\nu$-SVR, the parameter $\nu$ introduced in [20] allows to control the number of SVs and training errors [21]. The interested reader will find more details of this method in [20]–[22].

For the formalism of SVR, let us define the feature vector $z_i = (z_{1,i}, \ldots, z_{N^2,i})$ of size $(N^2, 1)$, with $i$ being the index of training data and $N^2$ being the number of features. Let $B_x = \{(z_0, x_0), \ldots, (z_{m-1}, x_{m-1})\}$ composed of $m$ pairs of training data. For the sake of simplicity, the basis of the algorithm is explained for only “one” interface ($K = 1$), i.e., two infinite layers where the first one is the air. For $K = 1$, two training data are used: $B_{z_1}$ and $B_{z_2}$. The $x_i$’s of $B_{z_1}$ and $B_{z_2}$ correspond to the first time delay ($t_1$) (only one interface) and the dielectric constant of the second layer ($\varepsilon_2$), respectively. When $K > 1$, $K$ SVRs for TDE and $K$ SVRs for dielectric constant estimation (DCE) are constructed as described for the case of $K = 1$. 

For example, for \( K = 2 \) (two interfaces), four training data are used: \( B_{t_1}, B_{t_2}, B_{\epsilon_{r_2}}, \) and \( B_{\epsilon_{r_3}} \). In this case, the \( x_i \)'s of \( B_{t_1}, B_{t_2}, B_{\epsilon_{r_2}}, \) and \( B_{\epsilon_{r_3}} \) correspond to the first time delay \( (t_1) \), the second time delay \( (t_2) \), the dielectric constant of the second layer \( (\epsilon_{r_2}) \), and the dielectric constant of the third layer \( (\epsilon_{r_3}) \), respectively. The number of echoes \( K \) is either assumed to be known or estimated with some detection criteria, e.g., [23] and [24]. In the following, the SVR principle is explained for only one database \( B_x \).

The basic idea of the regression problem is to determine a function \( f(z) = \langle w, z \rangle + b \) that can accurately approximate future values [15], with \( w \) and \( b \) being parameters to be determined and \( \langle \cdot, \cdot \rangle \) denoting the inner product. To estimate \( w \) and \( b \) from data \( B_x = \{ (z_0, x_0), \ldots, (z_{m-1}, x_{m-1}) \} \), the authors of [20] propose the following optimization problem:

\[
\min_{\phi} \phi(w, \xi(t), \epsilon) = \frac{1}{2} \| w \|^2 + C \left( \nu + \frac{1}{m} \sum_{i=0}^{m-1} (\xi_i + \xi'_i) \right)
\]

subject to the following constraints:

\[
\begin{align*}
(\langle w, z_i \rangle + b) - x_i & \leq \epsilon + \xi_i \\
x_i - (\langle w, z_i \rangle + b) & \leq \epsilon + \xi'_i \\
\xi_i & \geq 0 \quad \epsilon \geq 0
\end{align*}
\]

where \( C > 0 \) is a regularization constant, \( \nu \in (0, 1) \) is a constant allowing to control the number of SVs and training errors, \( \epsilon \) is the tube radius [19], and \( \xi_i \) and \( \xi'_i \) are the slack variables.

The vector \( w \) can also be described (by the solution of the dual problem) as a linear combination of the training features \( z_i \) as follows:

\[
w = \sum_{i=0}^{m-1} (\alpha'_i - \alpha_i) z_i
\]

where \( \alpha_i \) and \( \alpha'_i \) are Lagrange multipliers. Thus, \( f(z) \) can take the following form:

\[
f(z) = \sum_{i=0}^{m-1} (\alpha'_i - \alpha_i) \langle z_i, z \rangle + b.
\]

In the context of nonlinear functions, the function \( f(z) = \langle w, z \rangle + b \) can be written as \( f(z) = \langle w, \Phi(z) \rangle + b \), with \( \Phi \) being a nonlinear transform from \( \mathbb{R}^{N^2} \) to a higher dimensional space. Thus, (10) becomes

\[
f(z) = \sum_{i=0}^{m-1} (\alpha'_i - \alpha_i) \langle \Phi(z_i), \Phi(z) \rangle + b
\]

\[
= \sum_{i=0}^{m-1} (\alpha'_i - \alpha_i) \psi(z_i, z) + b
\]

where \( \psi(z_i, z) \) is the kernel function. The sample points that appear with nonzero coefficients are called SVs [14]. The dual variable \( b \) is computed by using the Karush–Kuhn–Tucker conditions [15]. More details can be found in [20]. The kernel function may transform the data into a higher dimensional space without explicit calculation of the nonlinear transformation.

### IV. Numerical Results

This section aims at illustrating the feasibility of thickness estimation on simulated data by SVR method. The performance of SVR algorithm is assessed on a GPR profile which corresponds to both overlapped echoes and nonoverlapped echoes in time. The simulated test data have been generated with a varying \( B \Delta t \), as shown in Fig. 1. In this section, the simulated data obey the model in (1) with \( \psi(t) \) as a Dirac pulse. They represent two backscattered signals \( (K = 2) \) from two interfaces. The relative permittivities of the two media \( \epsilon_{r_2} \) and \( \epsilon_{r_3} \) typically range between four and eight [25], [26], and their conductivities typically range between 10⁻³ and 10⁻² S/m. For the simulations, we have taken \( \epsilon_{r_2} \) from four to six and \( \epsilon_{r_3} = 8 \), respectively, and their conductivities are assumed to be negligible.

In this section, we will consider the full-correlation case between echoes. It means that the simulated data represent data from roughless surfaces or slightly rough surfaces. This assumption is often confirmed on real pavements in GPR frequency [29]. The simulations are performed with an uncorrelated white Gaussian noise. The frequency bandwidth \( B \) is equal to 2 GHz, and the frequency band is \([0.5; 2.5]\) GHz. The data vector \( r \) in (3) is made of \( N = 11 \) samples. The signal-to-noise ratio (SNR) is defined as the ratio between the power of the strongest echo and the noise variance. The powers of the two echoes are related to the dielectric constants of the media. In the simulations, the performance of the algorithms is evaluated at medium SNR, i.e., 20 dB. The covariance matrix is estimated from \( M = 100 \) independent snapshots. We have used the \( \nu \)-SVR of the Library for Support Vector Machines [27], and only the kernel “radial basis function” (RBF) is considered. The generic form of the “RBF” kernel is \( \psi(x, y) = \exp(-\gamma \| x - y \|^2) \). Several parameters \( (C, \nu, \text{ and } \gamma) \) must be set with the \( \nu \)-SVR. Several solutions can be used to calculate these parameters [22], [28]. In this letter, \( \nu \) is set at \( \nu = 0.54 \) [20], and the other parameters are estimated by the well-known \( k \)-fold cross-validation on training data [20], with \( k = 5 \). In the following, the symbols \( \hat{x} \) and \( \bar{x} \) represent the quantity \( x \) in the training and test phases, respectively. The value \( \hat{x} \) means the estimated value of \( x \).
A. Nonoverlapped Echoes

The training sets have been generated for two echoes ($K = 2$). For the training sets, the first time delay $T_1$ is composed of ten sets of five equispaced time samples within [2–2.6]-ns time interval. The chosen unit of the SVR outputs is nanosecond. Thus, the first time delay $T_1$ will be between 2 and 2.6. The second time delay $T_2$ is generated, on the other hand, from the set of first time delay $T_1$ and, on the other hand, from five differential time shifts ($\Delta \hat{\tau} = \hat{T}_2 - \hat{T}_1$) by varying the thickness of the layer $\hat{h}$ within the interval of [60 65 70 75 80] mm, according to (11). Thus, the second time delay $T_2$ is also composed of ten sets of five equispaced time samples within [2.97–3.90]-ns time interval. The dielectric constant of the first layer $\epsilon_{r1}$ is composed of 11 equispaced samples within [4–6] interval. The dielectric constant of the second layer $\epsilon_{r2}$ is fixed to eight. Thus, for the SVR training phase, $L = 550$ values of $z$ are calculated. The hyperparameters are estimated by cross-validation, and it results in $(C = 239, \gamma = 1.79 \times 10^{-2})$, $(C = 1176, \gamma = 2.37 \times 10^{-2})$, and $(C = 676, \gamma = 1.92 \times 10^{-2})$ for the first “time delay” SVR, the second “time delay” SVR, and the “dielectric constant” SVR, respectively.

After the training phase, the test phase is carried out with a $B\Delta T$ product superior to one, as shown in Fig. 1(a). Fig. 2 shows the first two time delays and the dielectric constant. The first time delay $T_1$ and the first dielectric constant $\epsilon_{r1}$ are constants and equal to 2.1 ns and 4.7, respectively. In this example, the layer thickness varies between 60 and 80 mm, i.e., $T_2 \in [2.96–3.25]$ ns. The sample number is 1348. Fig. 2 shows that the SVRs estimate correctly the first two time delays and the first dielectric constant. For each run (at each sample), the method performs an estimation of the first time delay, the second time delay, and the first dielectric constant, i.e., $\hat{T}_1$, $\hat{T}_2$, and $\hat{\epsilon}_{r1}$, respectively. Then, the layer thickness estimate $\hat{h}$ is deduced from both the estimated time delays of backscattered echoes and the estimated dielectric constant $\hat{\epsilon}_{r2}$, as follows, assuming a lossless medium:

$$\hat{h} \simeq \frac{c(T_2 - T_1)}{2 \sqrt{\hat{\epsilon}_{r2}}}$$  (11)

with $c = 3 \times 10^8$ m/s being the celerity of light inside the vacuum. The estimated thickness $\hat{h}$ is also shown in Fig. 3.

The solid line represents the real thickness, whereas the marker represents the estimated thickness. The SVRs allow to estimate correctly the thickness; this figure shows that the error seems low. Fig. 3 also shows that the SVR method has a great generalization ability. Indeed, the test sets ($\hat{T}_1$, $\hat{T}_2$, and $\hat{\epsilon}_{r2}$) which are not included in the training set are estimated correctly. The results have been carried out with a high speed of computation. In fact, to estimate the $L = 1348$ first time delays, second time delays, and dielectric constants ($\hat{T}_1$, $\hat{T}_2$, and $\hat{\epsilon}_{r2}$), only about 0.1, 0.1, and 0.09 s are necessary, respectively, with a computer equipped by a processor unit (CPU) at 2.3 GHz and 8 Go of random access memory.

B. Overlapped Echoes

For the training set, the first time delay $\hat{T}_1$ and the dielectric constants of the first layer $\hat{\epsilon}_{r1}$ and the second layer $\hat{\epsilon}_{r2}$ are defined like in Section IV-A. Only the second time delay $\hat{T}_2$ is modified. It is generated from five differential time shifts ($\Delta \hat{\tau}$) by varying the thickness of the layer $\hat{h}$ within the interval [10 15 20 25 30] mm, i.e., within [2.16–3.08]-ns time interval. $L = 550$ values of $z$ are calculated. The hyperparameters are estimated by $k$-fold cross-validation ($k = 5$), and it results in $(C = 169, \gamma = 13.4 \times 10^{-2})$, $(C = 388, \gamma = 16.5 \times 10^{-2})$, and $(C = 4096, \gamma = 35.36 \times 10^{-2})$ for the first “time delay” SVR, the second “time delay” SVR, and the “dielectric constant” SVR, respectively. After the training phase, the test phase is carried out with a $B\Delta T$ product inferior to one, as shown in Fig. 1(b). The first time delay $\hat{T}_1$ and the dielectric constant $\hat{\epsilon}_{r1}$ are defined like in Section IV-A (2.1 ns and 4.7, respectively). Only the second time delay $\hat{T}_2$ is modified and defined within [2.24–2.53]-ns time interval which corresponds to the $B\Delta \hat{\tau}$ product within [0.29–0.87]. The sample number is 894. For each run (at each sample), the method performs an estimation of $\hat{T}_1$, $\hat{T}_2$, and $\hat{\epsilon}_{r1}$, respectively. Fig. 4 shows that the three parameters ($\hat{T}_1$, $\hat{T}_2$, and $\hat{\epsilon}_{r2}$) are correctly estimated. Like in Section IV-A, the estimated thickness $\hat{h}$ is deduced from $\hat{T}_1$, $\hat{T}_2$, and $\hat{\epsilon}_{r2}$, and the result is shown in Fig. 5. The SVRs allow to estimate correctly the thicknesses even if the echoes are overlapped. The maximum relative errors of $\hat{h}$, $\hat{T}_1$, $\hat{T}_2$, and $\hat{\epsilon}_{r2}$ are low, i.e., 5.4%, 0.25%, 0.5%, and 2.8%, respectively. The SVR method shows always a great generalization ability and a high speed of computation (on the order of 0.1 s for each SVR).
In this letter, the SVR method has been applied to time delay and relative permittivity estimation in order to find the pavement thicknesses. The results show that the SVR predictor gives a good performance in all contexts, i.e., overlapped echoes and nonoverlapped echoes, with low calculation time. They have also shown that the SVR method can be a sufficiently accurate solution to estimate the pavement thicknesses. The SVR algorithm has shown its capability to distinguish overlapping echoes when the echoes are correlated. Moreover, this method has shown excellent generalization capabilities. Future work would compare performance and computer efficiency between SVR and model-based subspace algorithms (MUSIC and ESPRIT) for time delay and permittivity estimation.

**REFERENCES**


