Note

Edge-colouring of join graphs

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Abstract

A join graph is the complete union of two arbitrary graphs. We give sufficient conditions for a join graph to be 1-factorizable. As a consequence of our results, the Hilton’s Overfull Subgraph Conjecture holds true for several subclasses of join graphs.

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1. Introduction

The graphs in this paper are simple, that is they have no loops or multiple edges. Let $G = (V, E)$ be a graph; the degree of a vertex $v$, denoted by $d_G(v)$, is the number of edges incident to $v$; the maximum degree of $G$, denoted by $\Delta(G)$, is the maximum vertex degree in $G$; $G$ is regular if the degree of every vertex is the same.

An edge-colouring of a graph $G = (V, E)$ is an assignment of colours to its edges so that no two edges incident to the same vertex receive the same colour. An edge-colouring of $G$ using $k$ colours ($k$ edge-colouring) is then a partition of the edge set $E$ into $k$ disjoint matchings.

The chromatic index of $G$, denoted by $\chi’(G)$, is the least $k$ for which $G$ has a $k$ edge-colouring. In [10] it was shown that every graph $G$ with $m$ edges and $\chi’(G) \leq k$ has an equalized $k$ edge-colouring $C$: each colour $f_i$ in $C$ appears on exactly either $\lfloor m/k \rfloor$ edges or $\lceil m/k \rceil$ edges.

A celebrated theorem of Vizing [17] states that

$$\chi’(G) = \Delta(G) \quad \text{or} \quad \chi’(G) = \Delta(G) + 1.$$ 

Graphs with $\chi’(G) = \Delta(G)$ are said to be Class 1; graphs with $\chi’(G) = \Delta(G) + 1$ are said to be Class 2. The graphs that are Class 1 are also known as 1-factorizable graphs. Fournier [6] gave a polynomial time algorithm that finds a $\Delta(G) + 1$ edge-colouring of a graph $G$.

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Since it is NP-complete to determine if a cubic graph has chromatic index three [9], it follows that deciding whether a graph is Class 1 or Class 2 is NP-hard. The problem remains open for several classes of graphs, including the class of graphs that are $P_4$-free (cographs) [1].

The goal of this paper is to find sufficient conditions for a join graph to be Class 1.

2. The join graphs

Let $G = (V, E)$ be a graph with $n$ vertices. We say that $G$ is a join graph if $G$ is the complete union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$. In other words, $V = V_1 \cup V_2$ and $E = E_1 \cup E_2 \cup \{uv : u \in V_1, v \in V_2\}$. If $G$ is the join graph of $G_1$ and $G_2$, we shall write $G = G_1 + G_2$. Note that the class of join graphs strictly contains the class of connected $P_4$-free graphs.

Write $n_1 = |V_1|$, $n_2 = |V_2|$, $\Delta_1 = \Delta(G_1)$, and $\Delta_2 = \Delta(G_2)$. Clearly, $n = n_1 + n_2$ and $\Delta(G) = \max\{\Delta_1, \Delta_2, n_2 + \Delta_1\}$. Fig. 1 shows a join graph $G = G_1 + G_2$ with $n = 5$ and $\Delta(G) = 3$.

Without loss of generality we shall assume that $n_1 \leq n_2$.

To every join graph $G = G_1 + G_2$ we shall associate the complete bipartite graph $BG$ obtained from $G$ by removing all edges of $G_1$ and $G_2$. For every maximum matching $M$ in $BG$, let $G_M$ denote the subgraph of $G$ obtained by removing all edges of $BG$ but the edges in $M$. Fig. 2 shows two $G_M$ for the graph $G$ in Fig. 1.

Our results are based on the following key observation:

**Observation 1.** Let $G = G_1 + G_2$ be a join graph with $n_1 \leq n_2$ such that $\Delta_1 \geq \Delta_2$, or such that $\Delta_1 < \Delta_2$ and $n_1 = n_2$.

If there exists a maximum matching $M$ in $BG$ such that the corresponding graph $G_M$ is Class 1, then $G$ is Class 1.

**Proof.** Let $M$ be a maximum matching of $BG$ such that $\chi'(G_M) = \Delta(G_M)$; and let $B'$ be the bipartite graph obtained from $BG$ by removing all edges in $M$. Note that $\chi'(G) \leq \chi'(G_M) + \chi'(B')$, and that $\chi'(B') = \Delta(B') = n_2 - 1$ (because $n_1 \leq n_2$). If $\Delta_1 \geq \Delta_2$, then $\Delta(G) = \Delta_1 + n_2$ and $\Delta(G_M) = \Delta_1 + 1$, and so $\chi'(G) \leq \Delta(G)$. If $\Delta_1 < \Delta_2$ and $n_1 = n_2$, then $\Delta(G) = \Delta_2 + n_2$ and $\Delta(G_M) = \Delta_2 + 1$, and so $\chi'(G) \leq \Delta(G)$.

If some assumption in Observation 1 does not hold, then $G$ could be Class 2 even though $G_M$ is Class 1 for every maximum matching $M$. This is, for instance, the case of the graph in Fig. 3 (here $n_1 < n_2$ and $\Delta_1 < \Delta_2$).

![Fig. 1](image1.png)

![Fig. 2](image2.png)
In Section 3 we shall study join graphs \( G = G_1 + G_2 \) with \( n_1 \leq n_2 \) and \( \Delta_1 > \Delta_2 \); in Section 4 we shall study join graphs \( G = G_1 + G_2 \) with \( n_1 \leq n_2 \) and \( \Delta_1 = \Delta_2 \); in Section 5 we shall see the relationship of our results with some old conjecture.

3. Join graphs with \( \Delta_1 > \Delta_2 \)

Let \( G = G_1 + G_2 \) be a join graph with \( G_1 = (V_1, E_1) \) and \( G_2 = (V_2, E_2) \) such that \( n_1 \leq n_2 \). In view of Observation 1, it is natural to ask when there exists a maximum matching \( M \) of \( B_G \) such that the corresponding graph \( G_M \) is Class 1. The following result shows that such a matching always exists and that, in fact, every maximum matching of \( B_G \) has the desired property.

**Theorem 1.** Let \( G = G_1 + G_2 \) be a join graph with \( n_1 \leq n_2 \). If \( \Delta_1 > \Delta_2 \) then for every maximum matching \( M \) of \( B_G \), the corresponding graph \( G_M \) is Class 1.

**Proof.** Assume that the theorem is not true. Then there exists a maximum matching \( M \) of \( B_G \) such that the corresponding graph \( G_M \) is Class 2, and so \( \chi(G_M) > \Delta(G_M) = \Delta_1 + 1 \).

We shall find a contradiction. For this purpose, colour \( G_1 \) and \( G_2 \) with \( \Delta_1 + 1 \) colours \( a_0, a_1, \ldots, a_{\Delta_1} \) (this can be done because \( \Delta_1 > \Delta_2 \)). Extend this colouring to as many edges in \( M \) as possible. Until the end of the proof, we shall consider only the graph \( G_M \).

By assumption, not every edge in \( M \) has been coloured; in particular, some edge \( uv \) in \( M \) with \( u \in V_2 \) and \( v \in V_1 \) is not coloured. Now, every neighbor of \( u \), but vertex \( v \), has degree less than or equal to \( \Delta_2 + 1 \) (every such a neighbor of \( u \) is a vertex of \( G_2 \)). Since \( \Delta_2 < \Delta_1 \), and since we used \( \Delta_1 + 1 \) colours, it follows that every neighbor of \( u \), but vertex \( v \), misses at least one colour \( a_i \). Moreover, since \( uv \) is not coloured, both \( u \) and \( v \) miss at least one colour. But then, we are in the same conditions as in the proof of Vizing’s Theorem (see [18, pp. 210–211]). It follows that we can extend our \( (\Delta_1 + 1) \) edge-colouring in the graph \( G_M \) so to colour also edge \( uv \), getting then a contradiction. \( \square \)

Note that the proof of Theorem 1 yields a polynomial time algorithm to colour the edges of a join graph \( G = G_1 + G_2 \) with \( \Delta(G) \) colours, whenever \( n_1 \leq n_2 \) and \( \Delta_1 > \Delta_2 \).

An instant corollary of Theorem 1 and Observation 1 is:
Corollary 1. Let $G = G_1 + G_2$ be a join graph with $n_1 \leq n_2$. If $\Delta_1 > \Delta_2$, then $\chi'(G) = \Delta(G)$.

If we interchange the roles of $G_1$ and $G_2$ and apply Theorem 1, we get the following:

Corollary 2. Let $G = G_1 + G_2$ be a join graph with $n_1 = n_2$. If $\Delta_1 < \Delta_2$, then $\chi'(G) = \Delta(G)$.

4. Join graphs with $\Delta_1 = \Delta_2$

Let $G = G_1 + G_2$ be a join graph with $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ such that $n_1 \leq n_2$. Assume that $\Delta_1 = \Delta_2$. In view of Observation 1, it is natural to ask whether a result similar to Theorem 1 is still valid. Unfortunately, this is not the case. For instance, consider the join graph $G = G_1 + G_2$ in Fig. 4: it is easy to see that, for every maximum matching $M$ of $B_G$, the corresponding graph $G_M$ is Class 2 ($G_M$ is “overfull”). On the other hand, consider the case of the join graph $G = C_5 + C_5$ (where $C_5$ denotes the chordless cycle with five vertices): it is easy to see that there exist maximum matchings $M$ such that the corresponding $G_M$ are Class 1, even though if $M$ is chosen so that $G_M$ is the Petersen graph then $G_M$ becomes Class 2. It follows that we can still make use of Observation 1 by finding sufficient conditions for the existence of “good” matchings $M$.

Theorem 2. Let $G = G_1 + G_2$ be a join graph with $\Delta_1 = \Delta_2$. If one of the following three conditions holds:

(i) both $G_1$ and $G_2$ are Class 1;
(ii) $G_1$ is a subgraph of $G_2$;
(iii) both $G_1$ and $G_2$ are disjoint unions of cliques;

then there exists a maximum matching $M$ of $B_G$ such that the corresponding graph $G_M$ is Class 1.

Proof. Without loss of generality, we can assume that $n_1 \leq n_2$.

First, assume that (i) holds. Let $M$ be an arbitrary maximum matching of $B_G$. Since $G_1$ and $G_2$ are Class 1, and since $\Delta_2 = \Delta_1$, it follows that we can colour the edges of $G_1$ and $G_2$ with $\Delta_1$ colours. If we use an extra colour to colour all edges in $M$, then $\chi'(G_M) \leq \Delta_1 + 1 = \Delta(G_M)$, and so $G_M$ is Class 1.
Secondly, assume that (ii) holds. Let $V_1 = \{u_1, \ldots, u_{n_1}\}$ and $V_2 = \{v_1, \ldots, v_{n_2}\}$. Colour the edges of $G_2$ with $A_2 + 1$ colours $f_1, \ldots, f_{A_2+1}$. Since $G_1$ is a subgraph of $G_2$, it follows that for every edge $u_i u_j$ of $G_1$ $v_i v_j$ is an edge of $G_2$; let $f_k$ be the colour of $v_i v_j$. Colour $u_i u_j$ with colour $f_k$. Hence, we can extend the $A_2 + 1$ edge-colouring of $G_2$ to all the edges of $G_1$. Now, let $u_i$ be an arbitrary vertex of $G_1$ and let $v_i$ be the corresponding vertex of $G_2$ (such a vertex exists because $G_1$ is a subgraph of $G_2$). Let $f_k$ be a colour missing at $v_i$ (such a colour exists because $d_{G_1}(v_i) \leq A_2$). By construction, colour $f_k$ is missing also at $u_i$ and so we can colour $u_i u_j$ with colour $f_k$. Since we can repeat this operation for every vertex of $G_1$, it follows that for the matching $M = \{u_i v_i, i = 1, \ldots, n_1\}$ the graph $G_M$ is $A_2 + 1$ edge-colourable, and so $G_M$ is Class 1.

Finally, assume that (iii) holds. Order the vertices of $G_1$, $u_1, \ldots, u_{n_1}$, so that all the vertices in a same connected component of $G_1$ are consecutive, and such that if $u_i$ belongs to a clique $K_t$ and $u_j$ belongs to a clique $K_s$ with $t > s$ then $i > j$. Similarly, we can order the vertices of $G_2$, $v_1, \ldots, v_{n_2}$, so that all the vertices in a same connected component of $G_2$ are consecutive, and such that if $v_i$ belongs to a clique $K_t$ and $v_j$ belongs to a clique $K_s$ with $t > s$ then $i > j$.

Let $C = \{f_0, f_1, \ldots, f_{A_2}\}$ be the $A_1 + 1$ edge-colouring of $G_1$ obtained in the following way: to every edge $u_i u_j$ assign colour $f_h$ with $h = (i + j) \mod (A_1 + 1)$. To show that this colouring is admissible, we only need verify that any two arbitrary adjacent edges of $G_1$ have different colours. For this purpose, assume that the edges $u_i u_j$ and $u_i u_k$ (with $i \neq k$) have been assigned the same colour $f_h$. Then, by construction, $h = (i + j) \mod (A_1 + 1)$ and $h = (j + k) \mod (A_1 + 1)$. It follows that $h = i + j - t_1(A_1 + 1)$ (for some nonnegative integer $t_1$) and $h = j + k - t_2(A_1 + 1)$ (for some nonnegative integer $t_2$), with $t_2 \neq t_1$ (because $k \neq i$). But then we can write $(A_1 + 1)(t_2 - t_1) = (k - i) = 0$, and so $|k - i| = t_2 - t_1(A_1 + 1)$, which implies that $|k - i| \geq 1$. On the other hand, the chosen ordering of the vertices of $G_1$ implies that $|k - i| \leq A_1$ (because $u_i$ and $u_j$ belong to a same clique whose size is at most $A_1 + 1$), a contradiction.

Note that, by construction, for every $i = 1, \ldots, n_1$, vertex $u_i$ misses colour $f_{(2i) \mod (A_1+1)}$. Since $A_2 = A_1$, we can colour the edges of $G_2$ in a similar way using the same colours in $C$: to every edge $v_i v_j$ of $G_2$, assign colour $f_h$ with $h = (i + j) \mod (A_1 + 1)$. By construction, for every $i = 1, \ldots, n_2$, vertex $v_i$ misses colour $f_{(2i) \mod (A_1+1)}$.

Now we are ready to choose the desired maximum matching $M$ of $B_G$: $M = \{u_i v_i, i = 1, \ldots, n_1\}$. Indeed, for every $i = 1, \ldots, n_1$, we can assign to edge $u_i v_i$ the colour $f_{(2i) \mod (A_1+1)}$. Thus, $G_M$ is Class 1 and the theorem follows. □

An instant corollary of Theorem 2 and Observation 1 is:

**Corollary 3.** Let $G = G_1 + G_2$ be a join graph with $A_1 = A_2$. If both $G_1$ and $G_2$ are Class 1, or if $G_1$ is a subgraph of $G_2$, or if both $G_1$ and $G_2$ are disjoint unions of cliques, then $\chi'(G) = \Delta(G)$.

Note that when $A_1 < A_2$ and $n_1 < n_2$ there are graphs $G = G_1 + G_2$ that satisfy some of the three conditions in Theorem 2 but are Class 2. For instance, every complete graph $G$ with an odd number of vertices satisfies conditions (ii) and (iii); moreover the Class 2 graph in Fig. 3 satisfies conditions (i) and (ii).

The proof of Theorem 2 gives a polynomial time algorithm to colour the edges of a join graph $G = G_1 + G_2$ with $\Delta(G)$ colours, whenever $A_1 = A_2$, and $G_1$ is a subgraph of $G_2$ or both $G_1$ and $G_2$ are disjoint unions of cliques.

We close this section by showing that, if $G$ is a regular join graph with $A_1 = A_2$, then for every maximum matching $M$ of $B_G$, the corresponding graph $G_M$ is Class 1, and so $G$ is Class 1.

**Theorem 3.** Every regular join graph $G = G_1 + G_2$ with $A_1 = A_2$ is Class 1.

**Proof.** Let $m_i$ denote the number of edges of $G_i$, $i = 1, 2$. Since $G$ is regular and that $A_1 = A_2$, it follows that $n_1 = n_2$ and $m_1 = m_2$. Let $C_1 = \{f_1, \ldots, f_{A_1+1}\}$ be an equalized edge-colouring of $G_1$; and let $C_2 = \{g_1, \ldots, g_{A_2+1}\}$ be an equalized edge-colouring of $G_2$.

Since $C_1$ is equalized, each colour $f_i$ ($i = 1, \ldots, A_1+1$) is missed by exactly $n_1 - 2[m_1/(A_1 + 1)]$ or $n_1 - 2[m_1/(A_1 + 1)]$ vertices of $G_1$; similarly, each colour $g_i$ ($i = 1, \ldots, A_2+1$) is missed by exactly $n_2 - 2[m_2/(A_2 + 1)]$ or $n_2 - 2[m_2/(A_2 + 1)]$ vertices of $G_2$. Without loss of generality, we can assume that colours $f_1, \ldots, f_p$ are missed by exactly $n_1 - 2[m_1/(A_1 + 1)]$ vertices of $G_1$, that colours $f_{p+1}, \ldots, f_{A_1+1}$ are missed by exactly $n_1 - 2[m_1/(A_1 + 1)]$ vertices of $G_1$, that colours $g_1, \ldots, g_q$ are missed by exactly $n_2 - 2[m_2/(A_2 + 1)]$ vertices of $G_2$, that colours $g_{q+1}, \ldots, g_{A_2+1}$ are missed by exactly $n_2 - 2[m_2/(A_2 + 1)]$ vertices of $G_2$.\]
Since \( G \) is regular, it follows that \( G_1 \) is \( A_1 \)-regular and that \( G_2 \) is \( A_2 \)-regular, and so each vertex \( u_i \) of \( G_1 \) misses exactly one colour \( f_j \) and each vertex \( v_i \) of \( G_2 \) misses exactly one colour \( g_h \). Thus, we can write
\[
\begin{align*}
n_1 &= p \left( n_1 - 2 \left\lfloor \frac{m_1}{A_1+1} \right\rfloor \right) + (A_1+1-p) \left( n_1 - 2 \left\lfloor \frac{m_1}{A_1+1} \right\rfloor \right), \\
n_2 &= q \left( n_2 - 2 \left\lfloor \frac{m_2}{A_2+1} \right\rfloor \right) + (A_2+1-q) \left( n_2 - 2 \left\lfloor \frac{m_2}{A_2+1} \right\rfloor \right).
\end{align*}
\]
Since \( n_1 = n_2 \) and \( m_1 = m_2 \), we can write
\[
(p-q) \left( n_1 - 2 \left\lfloor \frac{m_1}{A_1+1} \right\rfloor \right) = (p-q) \left( n_1 - 2 \left\lfloor \frac{m_1}{A_1+1} \right\rfloor \right).
\]
But then,
\[
p = q \quad \text{or} \quad \left\lfloor \frac{m_1}{A_1+1} \right\rfloor = \left\lfloor \frac{m_1}{A_1+1} \right\rfloor.
\]
Note that in the latter case, we must have \( p = A_1 + 1 \) and \( q = A_1 + 1 \). Hence, \( p = q \), and so we can assume that \( g_i = f_i \) for every \( i = 1, \ldots, A_1 + 1 \).

Now, let \( M = \{u_i v_i : i = 1, \ldots, n_1\} \). For every \( i = 1, \ldots, A_1 + 1 \), since both \( u_i \) and \( v_i \) miss the same colour, say \( f_k \), we can assign to edge \( u_i v_i \) the colour \( f_k \). But then we get a \( A_1 + 1 \)-edge-colouring of \( G_M \), and so \( G_M \) is Class 1. \( \square \)

Note that the proof of Theorem 3 gives a polynomial time algorithm to colour the edges of a regular join graph \( G = G_1 + G_2 \) with \( \Delta(G) \) colours, whenever \( \Delta_1 = \Delta_2 \).

5. Some final remarks

A graph \( G \) is overfull if
\[
|E(G)| > \Delta(G) \left\lfloor \frac{|V(G)|}{2} \right\rfloor - 1.
\]
An easy counting argument shows that if \( G \) is overfull then \(|V(G)| \) must be odd and \( G \) is Class 2 (in every edge-colouring at most \( 1/2(|V(G)| - 1) \) edges of \( G \) can have the same colour). If \( G \) is not overfull but it contains an overfull subgraph \( H \) with \( \Delta(H) = \Delta(G) \), then \( G \) is Class 2.

Not every Class 2 graph necessarily contains an overfull subgraph with the same maximum degree. Examples of such graphs are very rare. The smallest one is \( P^* \), the graph obtained from the Petersen graph by removing an arbitrary vertex. For all known of these graphs, the maximum degree is relatively small compared with the number of vertices \( \Delta(P^*) = |V(P^*)|/3 \).

In 1985, Hilton proposed the following conjecture, known as Hilton’s Overfull Subgraph Conjecture [3]:

**Conjecture 1** (Hilton). If \( G \) is a graph with \( \Delta(G) > |V(G)|/3 \) and \( G \) contains no overfull subgraph \( H \) with \( \Delta(H) = \Delta(G) \), then \( G \) is Class 1.

Conjecture 1 was proved to be true for many special cases: when \( G \) is a multipartite graph [8]; when \( \Delta(G) \geq |V(G)|-3 \) [4,15,16]; and when the number of the vertices of maximum degree is “relatively small” and some other conditions on the maximum degree or the minimum degree hold [3,5,12].

If Conjecture 1 were true, then the problem of deciding whether a graph \( G \) with \( \Delta(G) > |V(G)|/3 \) is Class 1 would be polynomially solvable [11,13]. One more consequence of the validity of Conjecture 1 is that an old conjecture on regular graphs would be true.

**Conjecture 2.** Let \( G \) be a \( k \)-regular graph with an even number of vertices. If \( k \geq |V(G)|/2 \), then \( G \) is Class 1.
(Here, $k$-regular means that the degree of every vertex is equal to $k$.) Conjecture 2 appeared in [2] but may go back to G.A. Dirac in the early 1950s. To see that Conjecture 1 implies Conjecture 2, it is sufficient to observe that no $k$-regular graph $G$ with an even number of vertices, such that $k \geq \frac{|V(G)|}{2}$, contains an overfull subgraph $H$ with $\Delta(H) = k$ [3,7]. Conjecture 2 was proved to be true when $k \geq \frac{1}{2}(\sqrt{7} - 1)|V(G)|$ [4], and for large graphs when $|V(G)| < (2 - \varepsilon)\Delta(G)$ [14].

Now, a join graph $G$ with $n$ vertices satisfies $\Delta(G) \geq n/2$. Hence, if Conjecture 1 were true, then every join graph $G$ that contains no overfull subgraph $H$ with $\Delta(H) = \Delta(G)$, would be Class 1; moreover if Conjecture 2 were true, then every regular join graph would be Class 1.

As a consequence of our results we have the following:

**Corollary 4.** Let $G = G_1 + G_2$ be a join graph with $n_1 \leq n_2$:

(a) If $\Delta_1 > \Delta_2$, or if $\Delta_1 < \Delta_2$ and $n_1 = n_2$, then Conjecture 1 holds true;
(b) if $\Delta_1 = \Delta_2$, then Conjecture 2 holds true;
(c) if $\Delta_1 = \Delta_2$, and $G$ satisfies one of the three conditions (i), (ii) and (iii) in Theorem 2, then Conjecture 1 holds true.

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