Self-routing switching of solitonlike pulses in multiple-core nonlinear fiber arrays

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A unified framework to optimize power discrimination for self-routing switching of solitonlike pulses in multiple-core fiber arrays is presented. This framework, based on the variational approach, is applicable to all-optical switching in \(N\)-core nonlinear fiber arrays for any configuration of the cores. By defining a quality factor as the product of the maxima of all the transmission curves characterizing the switching process in the fiber array, we show that when linearly varying coupling coefficients are considered improved performance can be achieved. By use of the split-step Fourier method the accuracy of the reduced formulation is ensured.

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1. INTRODUCTION

Fiber solitons are probably the best solution for overcoming the dispersion-limited performance of long-haul light-wave systems. In fact, the combined use of dispersion management and transmission control makes wavelength-division-multiplexed solitons one of the best solutions for fiber-optic communications systems, especially in long-haul links. The deployment of soliton transmission systems will raise, however, another issue: the need for soliton switching to guarantee total transparency in optical networks.

Optical power can be exchanged in waveguide couplers as a result of the weak overlap stemming from evanescent-field coupling. Nonlinear couplers have been proposed as building blocks for all-optical processing.\(^1,2\) A major goal is to be able to switch from one input channel to one of \(N\) output channels. Hence multiport waveguide couplers are required. However, power discrimination decreases rapidly with the number of coupled waveguides: Only dual-core couplers allow almost total power transfer.\(^3\)

It has been shown that fiber solitons are suitable for all-optical signal processing: Owing to their particlelike behavior, they do not break up in the switching process.\(^4-6\) Propagation and switching of solitons (or, more rigorously, of solitonlike pulses) in multicore nonlinear fiber arrays have been considered in the literature.\(^7,8\) However, an optimized design of the fiber array, to increase power discrimination, is still lacking.

In multiple-core nonlinear fiber arrays the optimization procedure is computationally time consuming when full numerical simulations are used. In this paper, by using a reduced analytical formulation based on the variational approach (VA),\(^9\) we present a new design technique that is able, through a full range analysis of the parameter space, to improve the performance of \(N\)-core nonlinear fiber arrays. The analysis presented herein is a generalization of our previous work on twin-core fiber couplers.\(^10,11\)

The appropriate definition of trial functions in our Lagrangian formulation, to match as accurately as possible the dynamic evolution of solitonlike pulses, reduces the system of \(N\) coupled nonlinear Schro"dinger equations to a set of \(2N + 2\) ordinary differential equations (ODE’s). Although our Lagrangian formulation is applicable to any configuration of the cores, we will consider only linearly or circularly distributed multiple cores. Moreover, we will assume that the coupling coefficients may have a linear variation along the longitudinal direction.

By introduction of the quality factor, a figure of merit for pulse switching in multicore fiber arrays, the optimization procedure of self-routing switching can readily be rated. The quality factor is defined as the product of the maxima of all \(N\) transmission curves. It is also shown that, with coupling coefficients that linearly vary along the propagation distance, the quality factor can be significantly improved.

To ensure the accuracy of our reduced semianalytic framework, the results of our optimization procedure are compared with full numerical simulations based on the split-step Fourier method (SSFM).

2. GOVERNING EQUATIONS

In this section we present a unified formalism to describe solitonlike pulse switching in any multiple-core fiber array. Defining \(u_j\) as the normalized amplitude in the \(j\) core (\(j = 1,\ldots,N\)), we can write the system of equations that governs the propagation in the \(N\) cores as

\[
\frac{\partial u_j}{\partial \zeta} + \frac{1}{2} \frac{\partial^2 u_j}{\partial \tau^2} + |u_j|^2 u_j + \sum_{n=1}^{N} \kappa_{jn} u_n = 0, \tag{1}
\]

where\(^12\)

\[
\tau = \frac{t - z/\nu_g}{\tau_0} \tag{2}
\]
is the normalized time moving at the group velocity \( v_g \) and

\[
\zeta = \frac{z}{L_D} = \frac{|\beta_2|}{\tau_0^2}
\]  

(3)

is the normalized longitudinal distance. \( L_D \) is the dispersion length, \( \beta_2 \) is the group-velocity dispersion coefficient, and \( \tau_0 \) is related to the pulse width (i.e., \( \tau_0 = \tau_s/1.763 \), where \( \tau_s \) is the pulse width at half-maximum).

In Eq. (1) the second term is responsible for the group-velocity dispersion effect, while the third term represents the nonlinearity caused by the Kerr effect. The final term represents the linear coupling between the cores. Although all the \( k_{jn} \) terms are present in Eq. (1), some of these elements may be neglected. One should also note that \( k_{jj} = 0 \) and \( k_{jn} = k_{nj} \) (i.e., all the cores are identical). The configuration of the array can be described by a matrix \( (K) \) with \( N \) columns and \( N \) rows. This matrix is symmetric, and all the principal diagonal elements are null. For example, in a twin-core fiber coupler this matrix is

\[
K = \begin{bmatrix}
0 & k_{12} \\
k_{12} & 0
\end{bmatrix}
\]  

(4)

In the continuous-wave (cw) regime the group-velocity dispersion term can be neglected, since the pulses change slowly along \( \tau \). This regime is realistic only for pulses with large widths or for couplers considerably shorter than \( L_D \)—i.e., \( \xi_C \ll 1 \), where \( \xi_C \) is the normalized length of the coupler.

It is customary to define two reference lengths in the linear cw regime to study these couplers. The full-beat length, \( \xi_B \), is the distance at which the initial power applied to core 1 has fully returned to its original core. The half-beat length, \( \xi_H \), is half the full-beat length. Although all the previous studies on the present topic published have been confined to these two reference lengths, one should note that some gains in the transmission coefficients can be obtained by a simple optimization of the coupler’s length.

The published results for nonlinear fiber couplers may be divided into two categories: (i) numerical simulations based on algorithms such as the SSFM, (ii) a reduced system of ODE’s based on a VA and a specific set of trial functions. The numerical solution of these ODE’s can largely reduce the computational time necessary for large optimization tasks such as core separation adjustment, length optimization, and phase-controlled optimization.

The transmission coefficient for core \( j \) is defined as the percentage of the total power present in this core:

\[
T_j(\zeta) = \int_{-\infty}^{\infty} |u_j(\zeta)|^2 d\tau / \sum_{n=1}^{N} \int_{-\infty}^{\infty} |u_n(\zeta)|^2 d\tau.
\]  

(5)

The denominator of Eq. (5) is the total normalized energy. Since losses were disregarded, the total energy remains constant.

### 3. VARIATIONAL APPROACH

The VA based on the Lagrangian formalism was first applied to nonlinear fibers by Anderson.\textsuperscript{9} Since then, several applications of nonlinear fibers have taken advantage of this formalism for a better understanding of the underlying physics of nonlinear fiber optics.

This method is based on the reduction of the original infinite-dimensional system into a finite-dimensional one. The initial step consists in finding the Lagrangian density of the system. This is done with the knowledge that, if we apply the Lagrangian density to the Euler–Lagrange equations, we must obtain the original system. For the system under study, the Lagrangian density is

\[
L = \sum_{j=1}^{N} \left[ \frac{i}{2} \left( u_{j,\tau}^* \frac{\partial u_j}{\partial \tau} - u_j \frac{\partial u_j^*}{\partial \tau} \right) \right] - \frac{1}{2} \left| \frac{\partial u_j}{\partial \tau} \right|^2
\]

\[
+ \frac{1}{2} |u_j|^4 + \sum_{j=1}^{N} \sum_{n=1}^{N} k_{jn} u_j u_n^*.
\]  

(6)

The last term in Eq. (6) is responsible for the linear coupling between the cores. For \( N = 2 \), Eq. (6) reduces to the Lagrangian densities presented in Refs. 11, 13, and 14. The most important step in the VA is the choice of the trial functions (or ansatz). These functions should describe, with a minimum number of free parameters and as closely as possible, the evolution of the amplitudes in the cores. The functions are usually a generalization of known solutions for a given set of initial conditions. For the coupling of nonlinear fibers, the solutions are based on the trivial solutions of Eq. (1) for \( N = 2 \).\textsuperscript{15}

Owing to the coupling between the cores, the power in each core must change along \( \xi \). Therefore we must introduce a free amplitude parameter \( A_j \) that controls the energy in core \( j \). This variation in the core energy must be accompanied by a varying phase \( \phi_j \). In Ref. 14 the authors showed that a varying width (and chirp) can improve the final results. Although we could consider a width (and chirp) for each core, this would greatly increase the complexity of the problem. We will, therefore, consider a single, common width and chirp (\( \eta \) and \( \zeta \), respectively). Hence the chosen ansatz is

\[
u_j = A_j \sqrt{\eta} \text{ sech}(\eta \tau) \exp[i(\phi_j + C \tau^2)],
\]  

(7)

where \( A_j, \phi_j, \eta, \zeta \), and \( C \) are all free parameters (i.e., we have \( 2N + 2 \) free parameters). With this ansatz the normalized energy in each core is

\[
Q_j = 2A_j^2,
\]  

(8)

and the transmission coefficients are given by

\[
T_j(\zeta) = \frac{A_j^2(\zeta)}{\sum_{n=1}^{N} A_n^2(\zeta)}.
\]  

(9)

We obtain a reduced Lagrangian by substituting Eq. (7) into Eq. (6) and integrating over \( \tau \):
where
\[ \mathcal{L}_j = -2A_j^2 \left( \frac{d\psi_j}{d\zeta} + \frac{\pi^2}{12\eta^2} \frac{dC}{d\zeta} \right) - \frac{A_j^2}{3} \left( \frac{\eta^2}{\eta^2} + \frac{C^2\pi^2}{\eta^2} \right) + \frac{2\eta A_j^4}{3} \] (11)
is the reduced Lagrangian of each core in isolation and
\[ \mathcal{L}_N = \sum_{j=1}^{N} \sum_{n=1}^{N} 2\kappa_{jn} A_j A_n \cos(\psi_j - \psi_n) \] (12)
is the term responsible for the linear coupling. The last term in Eq. (11) corresponds to the nonlinear effect.

After determining the reduced Hamiltonian and using Hamilton’s equations of motion, we obtain the desired system of ODE’s:
\[
\frac{d\psi_j}{d\zeta} = \frac{2\eta}{3} A_j^2 - \frac{\pi^2}{6} - \frac{C^2\pi^2}{6\eta^2}
\]
(13a)
\[
\frac{dC}{d\zeta} = \frac{2\eta}{\pi^2} \left( \eta - \frac{C^2\pi^2}{\eta^2} - \sum_{j=1}^{N} A_j^4 / \sum_{j=1}^{N} A_j^2 \right),
\] (13c)
\[
\frac{d\eta}{d\zeta} = -2C \eta.
\] (13d)

Equation (13d) shows the relation between the chirp and the pulse width. From Eq. (13c) we can see that the chirp depends basically on the pulse width and on the total energy. The energy in each core depends on the coupling with the other cores and on the relative phase between them [Eq. (13b)]. The first three terms in Eq. (13a) correspond to the normal pulse propagation, while the final term is responsible for the coupling with the other cores.

The numerical solution of this system of equations can be obtained (for a particular set of initial conditions) by for example, a fourth-order Runge-Kutta method.16

4. NUMERICAL RESULTS AND DISCUSSION

The results presented in this section will be divided into subsections according to the number of cores. Although extensive results for the two-core device may be found in the literature, we will, nevertheless, present some results for comparison and some new results obtained through an optimization procedure. Namely, we will optimize the length of the coupler and will include longitudinal variations for the coupling coefficient.

For the three-core case, linear and circular configurations will be analyzed with special emphasis on the linear case with different coupling coefficients between cores 1 and 2 and between cores 2 and 3. The circular configuration, for self-routing switching, is a symmetric device that presents no relevant application to switching (however, it can be useful as a power splitter).

We should note that numerical results for multiple cores in the cw regime were presented in Ref. 3.

Although higher-numbered core devices can be fitted into our formulation, their application to switching is reduced, since high-transmission coefficients cannot be obtained for all the cores.

For self-routing switching, we will launch pulses only into core 1:
\[
u_1(\zeta = 0, \tau) = \sqrt{p} \text{sech}(\sqrt{p} \tau),
\] (14a)
\[
u_j(\zeta = 0, \tau) = 0 \quad \text{for } j = 2, \ldots, N,
\] (14b)
where \( p \) is the normalized input peak such that
\[ P = pP_0 = p/(\gamma L_D). \] (15)

\( P_0 \) is the peak power necessary to support the fundamental soliton in a single core, and \( \gamma \) is the nonlinear coefficient of the fiber.

Matching Eqs. (14) with the ansatz, we obtain the initial conditions for the free parameters of the trial functions: \( \psi(\zeta = 0) = 0 \) for all the cores; \( A_j(\zeta = 0) = p^{1/4}, A_j(\zeta = 0) = 0 \) for \( j = 2, \ldots, N; \eta(\zeta = 0) = \sqrt{p}; \) and \( C(\zeta = 0) = 0 \).

For the assessment of the transmission curves, it is useful to define a unique parameter that can describe the merit of each transmission curve. To achieve a good power discrimination for all the cores, there must exist \( N \) power levels \( p_j \), where \( T_j(p_j) \) is as close to 1 as possible. With this in mind, we define a quality factor for the transmission characteristics as
\[ Q_T = \prod_{j=1}^{N} \max[T_j(p)]. \] (16)

Therefore the best transmission situation would have \( Q_T = 1 \). The power levels \( p_j \) correspond to the powers at which the maxima of \( T_j \) occur.

The cross-talk parameter is defined as
\[ CT_{j}[\text{dB}] = 10 \log_{10} \left[ \frac{Q_j(\zeta)}{\sum_{j=1}^{N} Q_j(\zeta j)} \right]. \] (17)

A. Two-Core Nonlinear Device

In Fig. 1 we present the transmission curve for a two-core nonlinear device with fixed coupling coefficient (in this case we will consider \( \kappa_{12} = \kappa_{21} = 1 \) without loss of generality) and with half-beat length—i.e., \( \xi_{12} = \pi/2\kappa_{12} \). The results obtained by the VA are compared with the SSFM results. It can be seen that the results obtained by our ansatz are the same as the ones obtained with more-complex trial functions (see, e.g., Refs. 13 and 14).

Since the device does not present the ideal step transmission curve, we must choose two reference power levels for a nonswitching \( (p_1) \) and a switching pulse \( (p_2) \) (see Fig. 2). The power levels were chosen to ensure transmission coefficients above 90%. Figure 3 depicts the longitudinal evolution of the transmission coefficients for the two power levels.

In Table 1 the transmission coefficients for the SSFM and the VA are presented. As we have already men-
tioned, the half-beat length is defined for the linear and cw regimes. From Fig. 3 we can see that, for a slightly longer coupler, better transmission coefficients can be obtained, especially for $p_2$.

To optimize the length of the coupler we must select a distance $\zeta_{OPT}$ at which we obtain a high $CT_1$ value for $p_1$ and a high $CT_2$ value for $p_2$. In Fig. 4 we present the longitudinal variation of the cross-talk parameters for the previously selected switching powers. For a high input peak power ($p_1$), the energy remains mainly in the first core (i.e., $CT_1$ remains well above zero). For the lower peak power ($p_2$), the coupler is nearly in the linear regime and presents a maximum near $\zeta_H$ (note that, in the linear regime, we have $CT_1 = -\infty$ and $CT_2 = \infty$ at $\zeta = \zeta_H$).

Improved transmission characteristics can be obtained for $\zeta_{OPT} = 1.15\zeta_H$. In fact, a 2.6-dB gain was achieved for $p_2$ (Table 2). This 2.6-dB difference represents an output energy decrease of 42.5% in the first core.

Although most of the studies published so far have focused on two identical and parallel cores (resulting in equal and constant coupling coefficients), it is useful to analyze the influence of a linear variation of the coupling coefficients. We will consider the following generic linear variation:

$$k_{ij}(\zeta) = k_{ji}(\zeta) = k_{0ij} + m_{ij} \frac{\zeta}{\zeta_C},$$  (18)

Table 1. Transmission Coefficients for the Two Input Peak Powers $p_1$ and $p_2$ at $\zeta_C = \zeta_H$

<table>
<thead>
<tr>
<th>Numerical Method</th>
<th>$T_1(p_1 = 10)$</th>
<th>$T_2(p_2 = 4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSFM</td>
<td>0.9776</td>
<td>0.9012</td>
</tr>
<tr>
<td>VA</td>
<td>0.9788 (+0.1%)</td>
<td>0.9347 (+3.7%)</td>
</tr>
</tbody>
</table>

Table 2. Transmission Coefficients $T_j$ and Cross-Talk Parameters $CT_j$

<table>
<thead>
<tr>
<th>$\zeta_C$</th>
<th>$T_2(p_2)$</th>
<th>$T_1(p_1)$ (dB)</th>
<th>$CT_1(p_1)$ (dB)</th>
<th>$CT_2(p_2)$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta_H$</td>
<td>0.9012</td>
<td>0.9776</td>
<td>16.40</td>
<td>9.60</td>
</tr>
<tr>
<td>$1.15\zeta_H$</td>
<td>0.9432</td>
<td>0.9675</td>
<td>14.74</td>
<td>12.20</td>
</tr>
</tbody>
</table>
where $k_{ij}$ is the initial value and $m_{ij}$ controls the inclination ($m_{ij} = m_{ji}$). For $m_{ij} > 0$ ($m_{ij} < 0$), the coupling coefficient increases (decreases); i.e., the distance between cores decreases (increases) along $z$.

The linear decreasing coupling coefficient corresponds to an exponential increasing separation between the cores. From a practical point of view, it is easier to construct a coupler with a linear separation than to construct one with an exponential separation. However, if we replace the exponential separation with a linear approximation, the resulting coupling coefficient remains nearly unaltered and has the same switching characteristics.

In Fig. 5 the contour plot of the transmission coefficient $T_1$ as a function of the input peak power $p$ and coefficient $m_{12}$ is depicted. The level lines correspond to $0.1$ increases from $0.1$ to $0.9$. There is a region, at $|m_{12}| < 0.4$, in which acceptable transmission curves can be obtained. To choose the appropriate value for $m_{12}$ one must define a rule. This rule can be the critical power, the power difference between two specific values of the transmission coefficient (e.g., between $T = 0.1$ and $T = 0.9$), or one can predefine the two peak power levels ($p_1$ and $p_2$) and pick the best curve for those two cases. Since each criterion has its own merits and deficiencies, we will simply opt for a decrease of in $10\%$ in the critical power. This objective is accomplished with $m_{12} = -0.227$. In this case the critical power is $5.92$: For $m_{12} = 0$, the critical power is $6.60$.

The length of the device and the required input peak power are the two major problems of twin-core fiber couplers. For silica fibers with $\gamma = 2 \text{ W}^{-1} \text{ km}^{-1}$ and pulses with $\tau_0 = 10 \text{ ps}$, the length of the coupler is $78.5 \text{ km}$ ($\kappa = 0.1$), and the input peak power for nonswitching pulses is $P_1 = 0.1 \text{ W}$. The switching pulses have peak powers of $P_2 = 40 \text{ mW}$. In this situation the construction of the device is not practical, and the fiber losses must also be taken into account. However, for erbium-doped fiber couplers (see Ref. 17), the value of $\gamma$ can increase by several orders of magnitude. Assuming that $\gamma = 2000 \text{ W}^{-1} \text{ km}^{-1}$, the length of the device is reduced to $78.5 \text{ m}$ ($\kappa = 10$) for the same input peak powers. In fact, owing to the amplification of the erbium-doped cores, the input peak power is even smaller, although with some side effects.$^{18}$

B. Three-Core Nonlinear Device

The geometric position and the separation of the three cores determined the values of the coupling coefficients. Generically, there are three independent coupling coefficients: $\kappa_{12}$, $\kappa_{23}$, and $\kappa_{13}$. The two most studied devices with three cores are (i) the linear configuration, in which the centers of the three cores are aligned along a single line [Fig. 6(a)]; (ii) the circular (or triangular) configuration, which corresponds to equal distances among all the cores [Fig. 6(b)].

In the first case the coupling between the first and the third cores is usually disregarded; i.e., $\kappa_{13} = 0$. For the circular configuration, one should note that there is a geometric symmetry around the first core. This fact makes it impossible, from the point of view of the first core, to distinguish the second from the third core (i.e., when $\kappa_{12} = \kappa_{13}$, $T_2 = T_3$). This feature of the circular configuration makes it unsuitable for a three-core switching device (at least for self-routing).

In Fig. 7 the transmission curves for the linear configuration are shown (when $\kappa_{12} = \kappa_{23}$ the half-beat length is given by $\xi_H = \pi / \sqrt{2} \kappa_{12}$). As for the twin-core coupler, the differences between the results obtained by the SSFM and those obtained by the VA are minimal. Overall, this device does not present good transmission characteristics for the second core. In fact, there is no input peak power at which $T_2 > 65\%$. This fact limits the quality factor to $0.629$. One of the possible solutions to improve the device consists in using different coupling coefficients between the consecutive cores (i.e., $\kappa_{12} \neq \kappa_{23}$, which can be obtained with different core separations, and consequently $\xi_H = \pi / \sqrt{\kappa_{12}^2 + \kappa_{23}^2}$). In Fig. 8 we present the contour plot of the quality factor $Q_T$ as a function of $\kappa_{12}$ and $\kappa_{23}$. This solution, however, does not show significant improvements in the maximum value for $Q_T$.

Another possible way to improve the transmission curves is to use the longitudinal variation of the coupling coefficients. For the linear core configuration, we will consider two independent varying coefficients, $m_{12}$ and $m_{23}$. The initial values are $\kappa_{012} = 1$ and $\kappa_{023} = 1$. The contour plot of the quality factor in the plane ($m_{23}, m_{12}$; Fig. 9) shows improvements for $m_{12} = 0.310$ and $m_{23} = -0.091$, which result in $Q_T = 0.703$.

![Image](image-url)

Fig. 6. The two most used configurations of the three-core fiber coupler: (a) linear configuration, (b) circular configuration.

![Image](image-url)

Fig. 5. Level lines of $T_1$ for different longitudinal inclinations of the coupling coefficient. Each level line corresponds to a $0.1$ increase from $0.1$ to $0.9$. The length of the coupler is $\xi_C = \xi_H$.
In Fig. 10 we present the improved transmission curves. Comparing these results with the original transmission curves (Fig. 7), we can see that the transmission curve for the second core is higher, which thus allows a better power transfer for the second core.

In Fig. 11 the switching of three pulses (with different input peak powers) is shown as an example of the self-routing capabilities of this device. Although each pulse is supposed to switch to a different core, it can be seen that the middle pulse (corresponding to a peak power associated with a high $T_2$ value) is the most fragmented at the end. The poor value achieved for the transmission coefficient of the second core causes this result.

In Fig. 12 the transmission curves for the circular configuration are presented. As mentioned above, the symmetry causes $T_2 = T_3$. One should also note that, for half-beat couplers (i.e., $\xi_H = \pi/3\kappa_{12}$) and low input peak powers, we do not achieve total power transmission.

Geometrically, there are several possibilities for the linear varying coupling coefficient method applied in the previous devices. However, none of these cases can produce relevant improvements in the transmission curves.

C. Four-Core Nonlinear Device

For the four-core fiber coupler, we will consider the linear configuration with coupling between only neighboring cores, i.e., $\kappa_{13} = \kappa_{24} = \kappa_{14} = 0$. In all the devices with a linear configuration the power will start off in the first core and will switch sequentially to the higher-numbered cores. Once the energy reaches the last core, it will start to switch back. This can prevent the concentration of all the energy in the final core. However, in Ref. 19 the authors determined the coupling coefficient values that

![Fig. 7. Three-core fiber coupler with a linear configuration. The transmission curves are shown as functions of the normalized input peak power $p$. Results were obtained by the SSFM and the VA.](image)

![Fig. 8. Contour plot of the quality factor $Q_T$, for the three-core fiber coupler with $k_{13} = 0$, with $Q_T$ shown as a function of the coupling coefficients $k_{12}$ and $k_{23}$.](image)

![Fig. 9. Contour plot of the quality factor $Q_T$, for the three-core fiber coupler with $k_{13} = 0$. The quality factor is presented as a function of inclination factors $m_{12}$ and $m_{23}$.](image)

![Fig. 10. Transmission curves for the three-core fiber coupler with $m_{12} = 0.310$ and $m_{23} = -0.091$.](image)
cause the total energy transfer to the Nth core in the linear cw regime. For the four-core device, these values correspond to

$$\kappa_{12} = \kappa_{34} = \kappa\sqrt{3}, \quad \kappa_{23} = 2\kappa. \quad \text{(19)}$$

Under these circumstances the half-beat coupling length is given by

$$\zeta_H = \pi/(2\kappa), \quad \text{(20)}$$

and the coupling matrix is

$$K = \begin{bmatrix} 0 & \kappa\sqrt{3} & 0 & 0 \\ \kappa\sqrt{3} & 0 & 2\kappa & 0 \\ 0 & 2\kappa & 0 & \kappa\sqrt{3} \\ 0 & 0 & \kappa\sqrt{3} & 0 \end{bmatrix}. \quad \text{(21)}$$

If the input power is high enough, it will remain in the initial core because of the nonlinear effect. In Fig. 13 the transmission curves for this device (with $\kappa = 1$) are presented. At $p = 0$, the energy is all in the fourth core ($T_4 = 1$), as desired.

In this coupler we have $Q_T = 0.280$. If we optimize the device with the linear varying coupling coefficients, we improve to $Q_T = 0.406$. The optimization point corresponds to $m_{12} = 0.705$, $m_{23} = 0.355$, and $m_{34} = -0.193$. In Fig. 14 we present the transmission curves for the optimized four-core device. Comparing Fig. 14 with Fig. 13, we can clearly see the increase in the maximum of the transmission coefficient for the third core.

However, it is also clear that this optimization procedure has its limitations. For devices with a higher number of cores, the power discrimination is greatly reduced, and the optimization cannot significantly improve the overall switching characteristics.

The transmission curve for the first core is steeper than for all the previously analyzed configurations. This characteristic permits its use as an ON/OFF switch. In fact, two slightly different input peak powers can result in drastically different outputs at the first core. In Fig. 15 we present the input and the output of core 1 for two pulses with $p_{ON} = 18.3$ and $p_{OFF} = 12.3$. It can be seen that, in the ON situation, the output pulse is identical to the input pulse. In the OFF case, the output pulse is greatly reduced.
pared with the SSFM), the variational approach permits the realization of extensive optimization procedures. For self-routing switching, these procedures include length optimization, different core separations, and longitudinally varying coupling coefficients.

In the twin-core fiber coupler these optimizations were able to improve the cross-talk parameter $CT_2$ in 2.6 dB or to reduce the critical power by 10%. In the three-core coupler an increase of 11% (from 0.629 to 0.703) in the quality factor was obtained.

Although, for the four-core device, a staggering 56.2% increase (from 0.26 to 0.406) in the quality factor was possible, the final value is too small for efficient $1 \times N$ switching. However, this device can function as an ON/OFF switch. In the ON mode the cross talk is $CT'_1 = 11.17$ dB, while for the OFF mode it is $CT'_1 = -19.3$ dB. The relation between the output energy in the ON mode and that in the OFF mode is 19.89 dB.

Further work in this area includes local-controlled switching and other core configurations.

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